Fundamentals of radio astronomy

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ITN 215212: Black Hole Universe

Many slides taken from NRAO Synthesis Imaging Workshop
(http://www.aoc.nrao.edu/events/synthesis/2010/)

and the radio astronomy course

of J. Condon and S. Ransom

(http://www.cv.nrao.edu/course/astr534/ERA.shtml)

Atacama Large Millimeter/submillimeter Array

Expanded Very Large Array

Robert C. Byrd Green Bank Telescope

Very Long Baseline Array

NRAO

What is radio astronomy?

- The study of radio waves originating from outside the Earth
- Wavelength range 10 MHz – 1 THz
- Long wavelengths, low frequencies, low photon energies
  - Low-energy transitions (21cm line)
  - Cold astronomical sources
  - Stimulated emission (masers)
  - Long-lived synchrotron emission
- Sources typically powered by gravity rather than nuclear fusion
  - Radio galaxies
  - Supernovae
  - Pulsars
  - X-ray binaries

What is radio astronomy?

- Telescope resolution $\theta = 1.02\ \lambda/D$
  - Requires huge dishes for angular resolution
- Coherent (phase preserving) amplifiers practical
  - Quantum noise proportional to frequency $T = h\nu/k_B$
- Precision telescopes can be built $\sigma<\lambda/16$
- Interferometers are therefore practical: $D \leq 10^4$ km



A short historical di... ession

- Extraterrestrial radio ... ed in 1932 by Karl Jansky
  - Electric ... s
  - Att ... atic at 20.5 ...
  - Stead ... min
  - Stronge ...
- Discovery ign ... by Grote Rebe ...
  - 1937-1940: M ...
  - Demonstrated ...
- World War II stimula ... o and radar technology

NEW RADIO WAVES TRACED TO CENTRE OF THE MILKY WAY

Mysterious Static, Reported by K. G. Jansky, Held to Differ From Cosmic Ray.

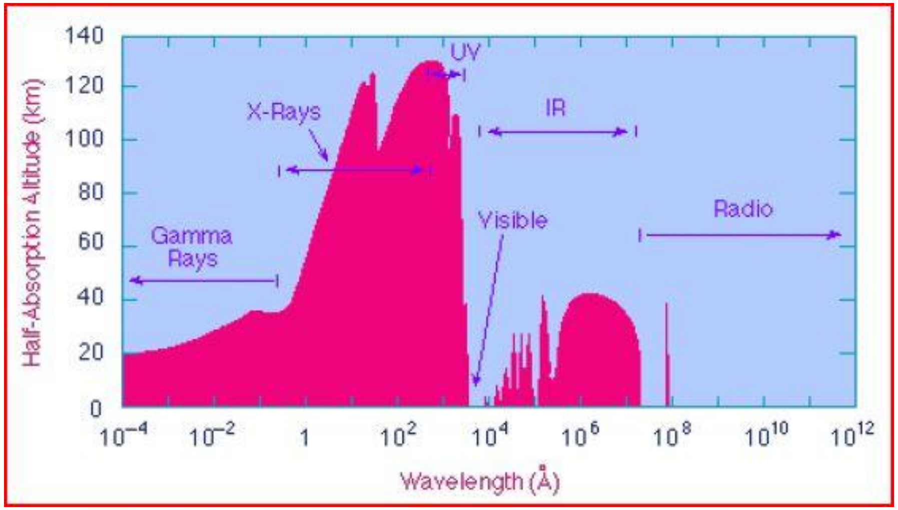
DIRECTION IS UNCHANGING

Figure 18. An obviously contented Grote Reber at the site of the University of Tasmania's 23 metre Mt. Pleasant radio telescope (reproduced with the permission of *The Mercury*).

Why radio astronomy?

- Atmosphere absorbs most of the EM spectrum
  - Only optical and radio bands accessible to ground-based instruments
- Rayleigh scattering by atmospheric dust prevents daytime optical
- Radio opacity from water vapour line, hydrosols, molecular oxygen
  - High, dry sites best at high frequencies
- Stars undetectably faint as thermal radio sources
  - Non-thermal radiation
  - Cold gas emission
  - CMB

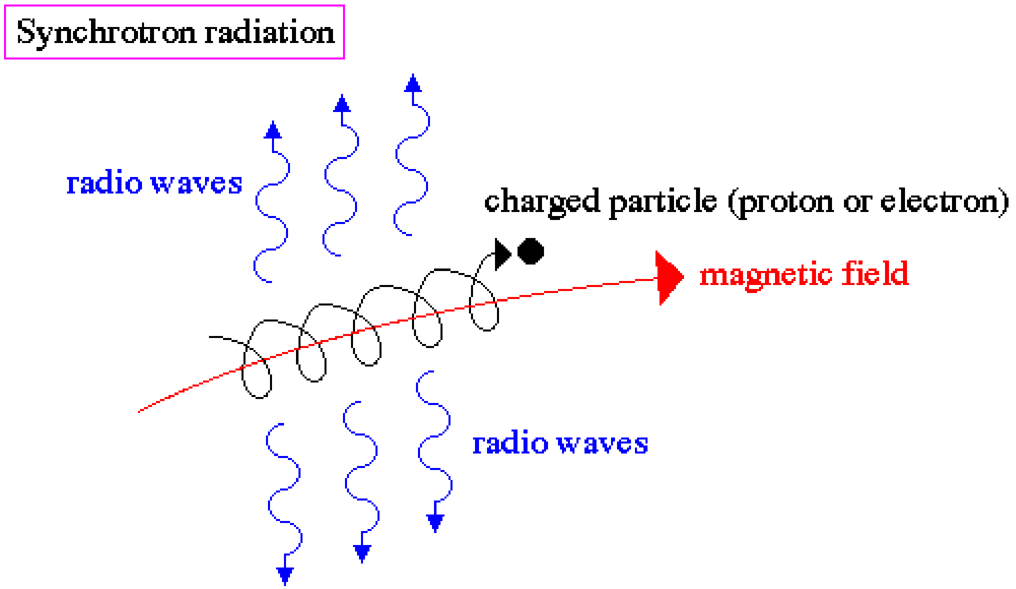
$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{k_BT}}-1} = \frac{2k_BT\nu^2}{c^2} = 0.3\mu\mathrm{Jy}$$



A tour of the radio Universe

What are we looking at?

- Synchrotron radiation
  - Energetic charged particles accelerating along magnetic field lines (non-thermal)

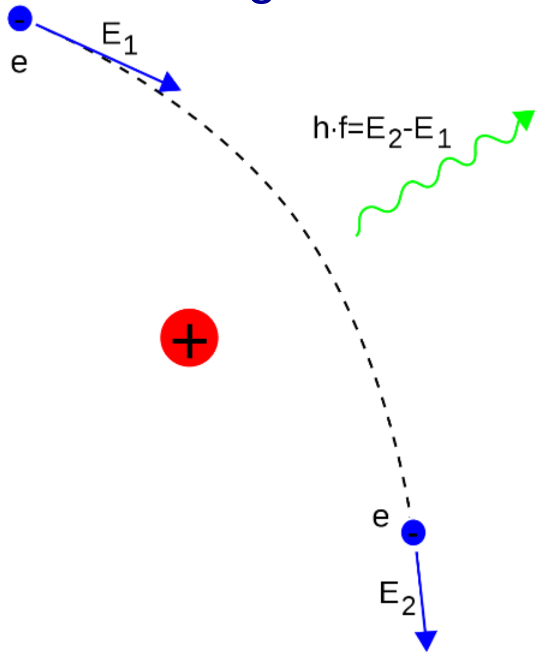


- What can we learn?

- particle energy
- strength of magnetic field
- polarization
- orientation of magnetic field

What are we looking at?

- Thermal emission
  - Blackbody radiation from cool (T~3-30K) objects
  - Bremsstrahlung "free-free" radiation: charged particles interacting in a plasma at T; e^- accelerated by ion
    - H II regions

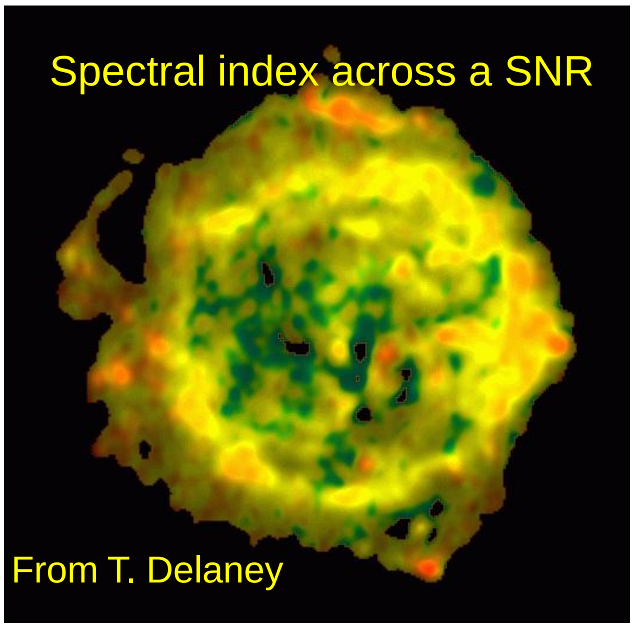


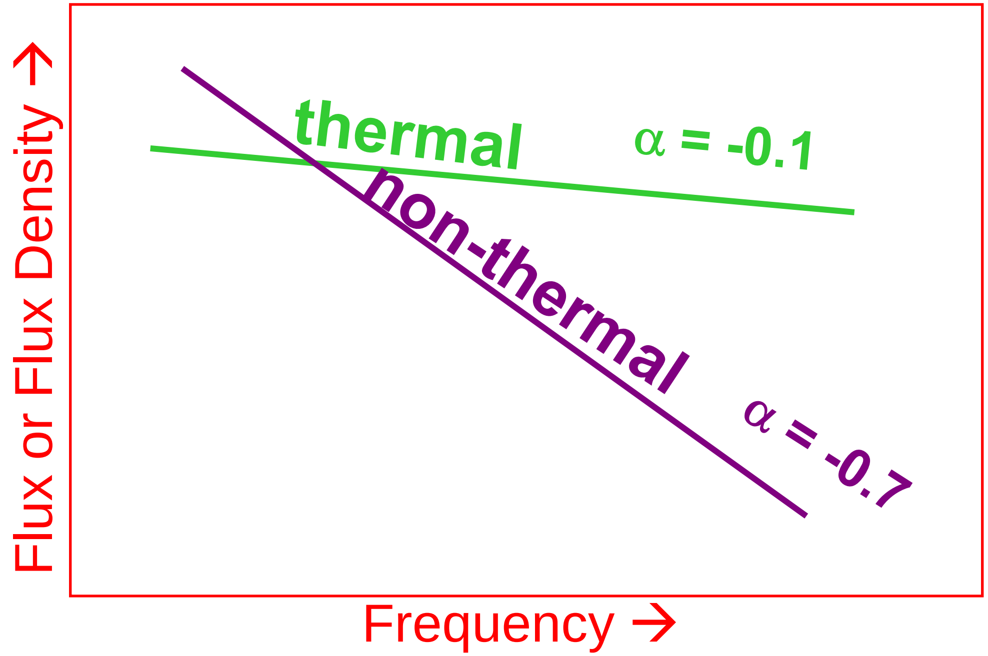
- What can we learn?

- mass of ionized gas
- optical depth
- density of electrons in plasma
- rate of ionizing photons

What are we looking at?

- How to distinguish continuum mechanisms?
- Look at the *spectrum*

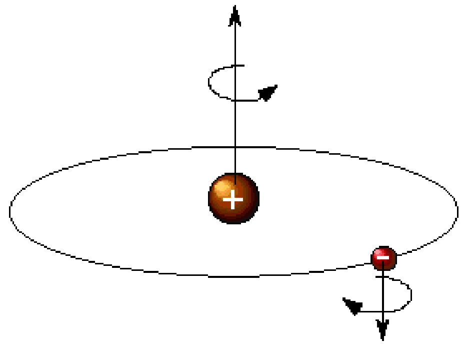




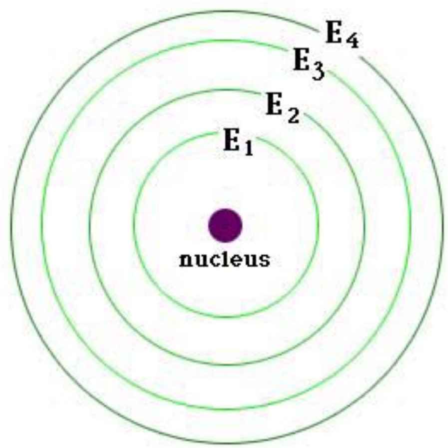
- Spectral index $S_\nu \propto \nu^\alpha$

What are we looking at?

- Spectral line emission (discrete transitions of atoms and molecules)
  - Spin-flip 21-cm line of HI
  - Recombination lines
  - Molecular rotational/vibrational modes

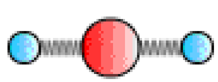


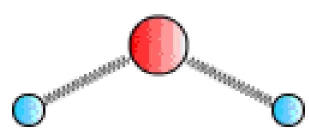
Lower energy state: Proton and electron have opposite spins.



- What can we learn?

- gas physical conditions
- kinematics (Doppler effect)





The Sun

- Brightest discrete radio source
- Atmospheric dust doesn't scatter radio waves

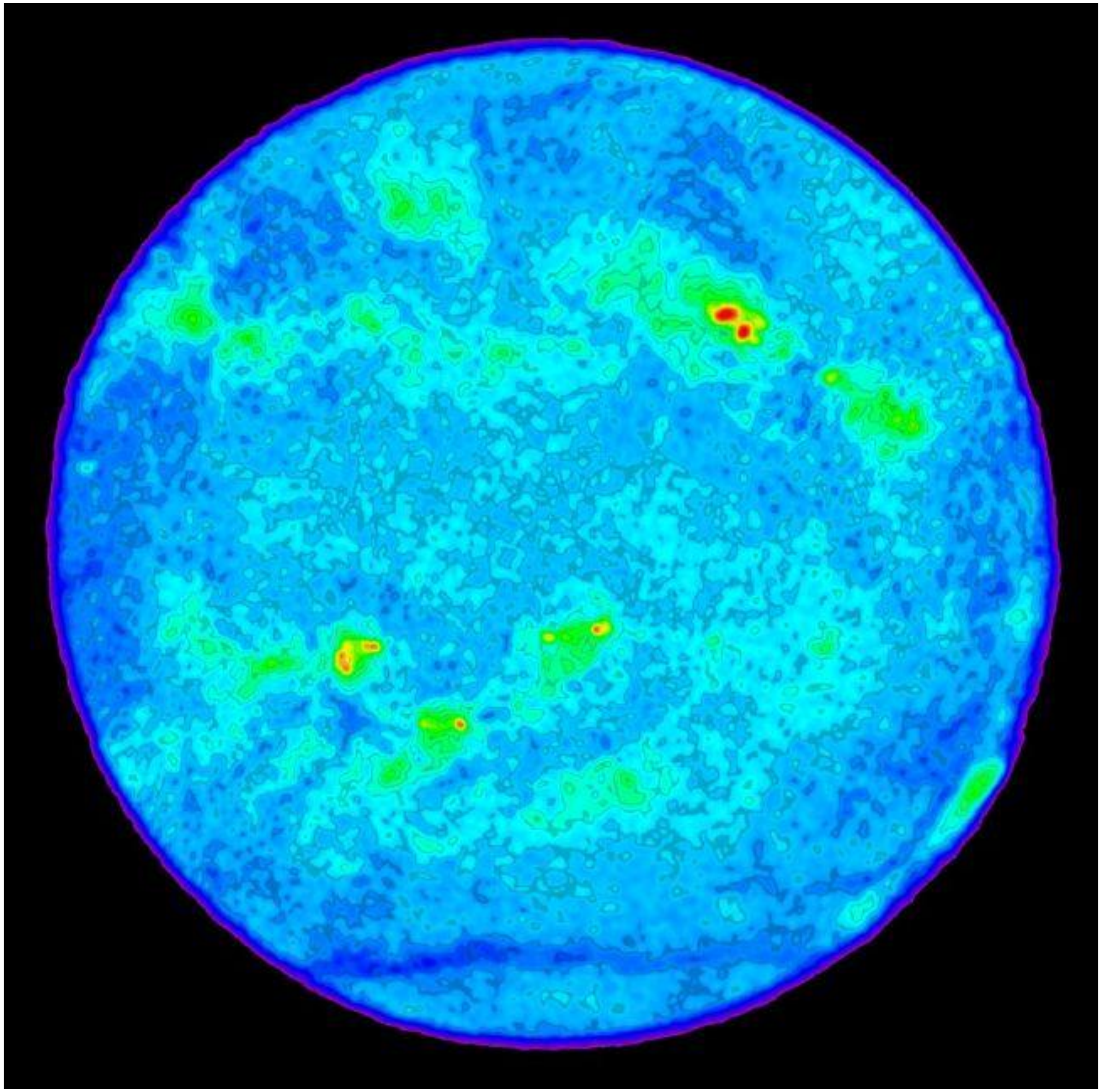
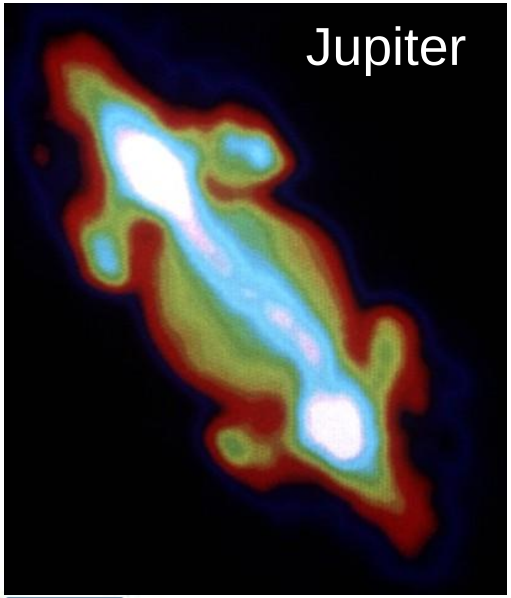
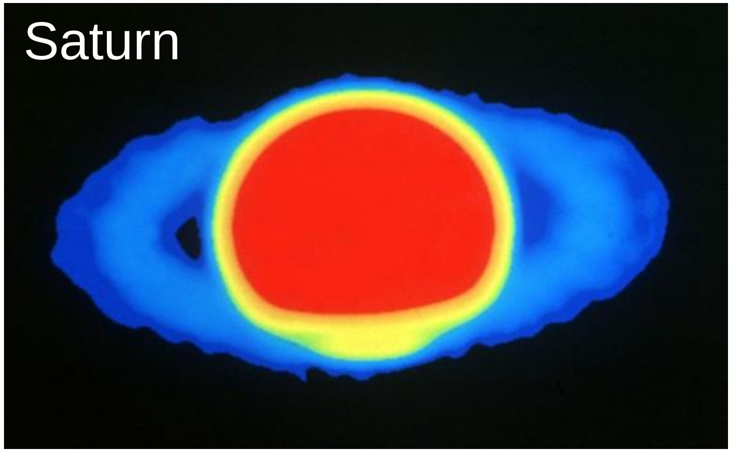


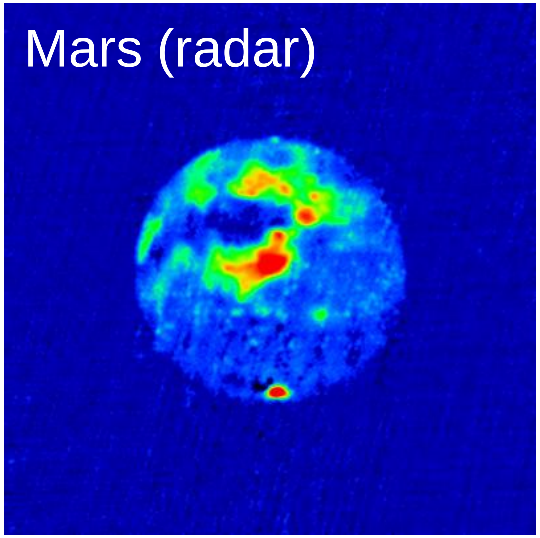
Image courtesy of NRAO/AUI

The planets

- Thermal emitters
- No reflected solar radiation detected
- Jupiter is a non-thermal emitter (B field traps electrons)
- Also active experiments: radar



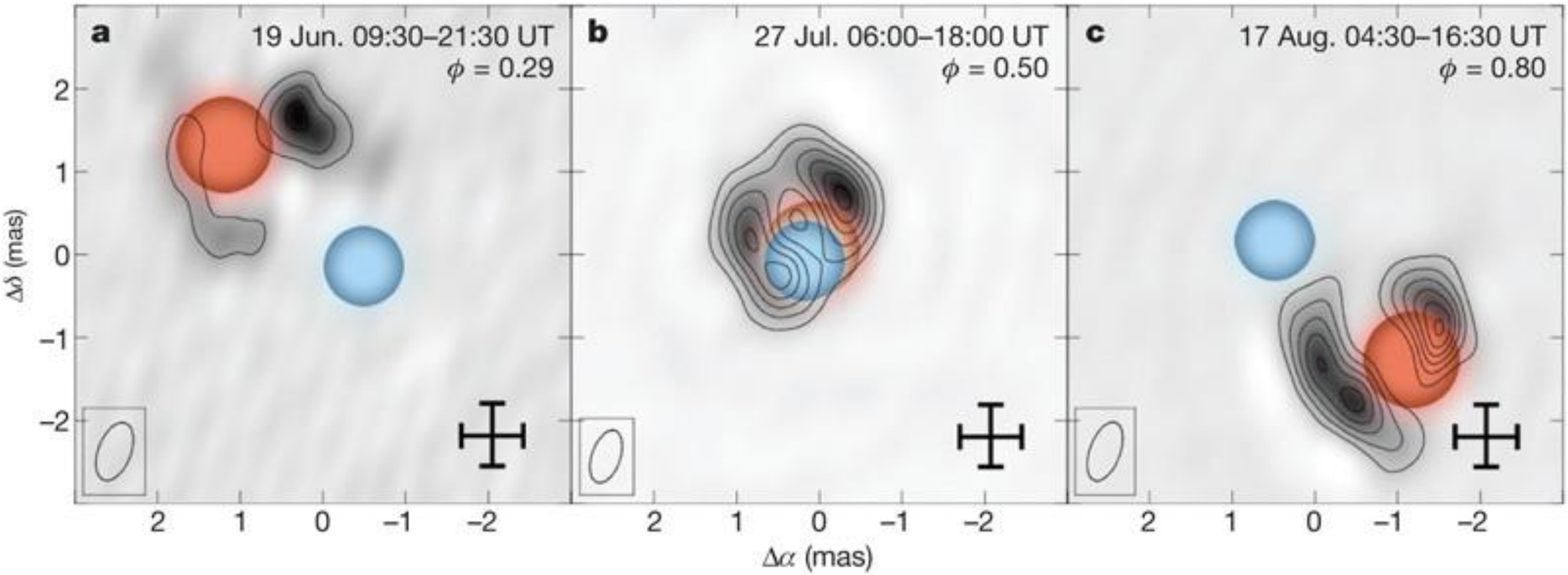




Images courtesy of NRAO/AUI

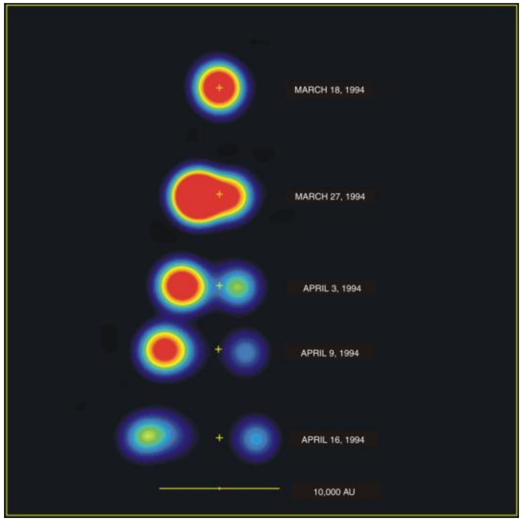
Stars

- Coronal loop resolved in the Algol system
  - Tidally-locked system, rapid rotation drives magnetic dynamo

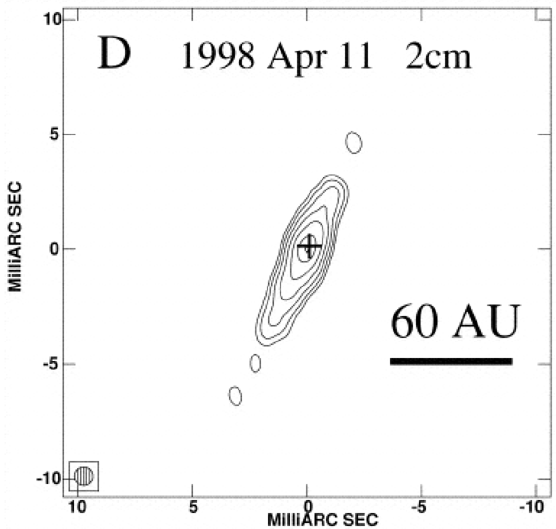


Credit: Peterson, Mutel, Guedel, Goss (2010, Nature)

Black holes and neutron stars

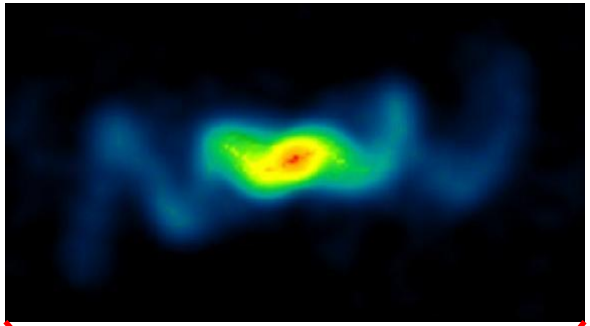


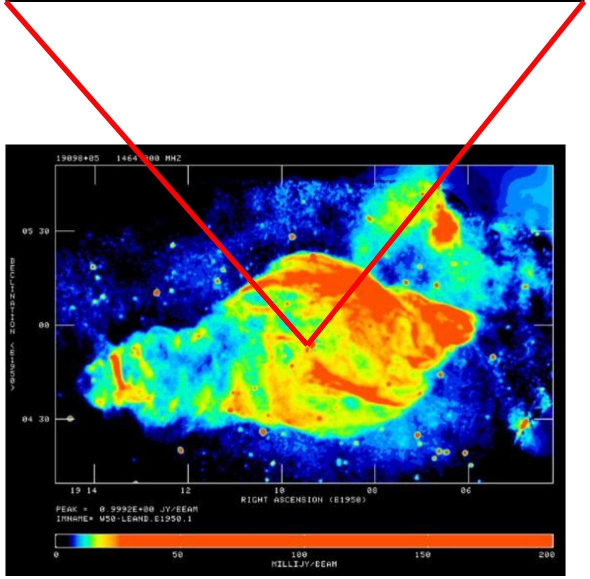
Mirabel & Rodriguez (1994)



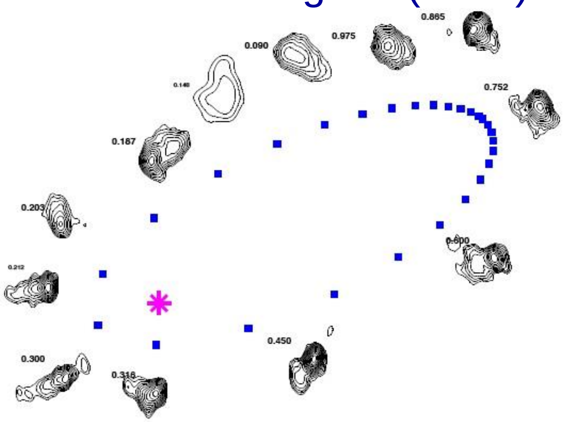
Dhawan et al. (2000)

Blundell & Bowler (2004)

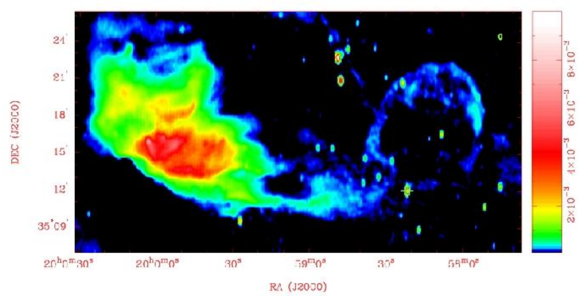




Dubner et al. (1998)

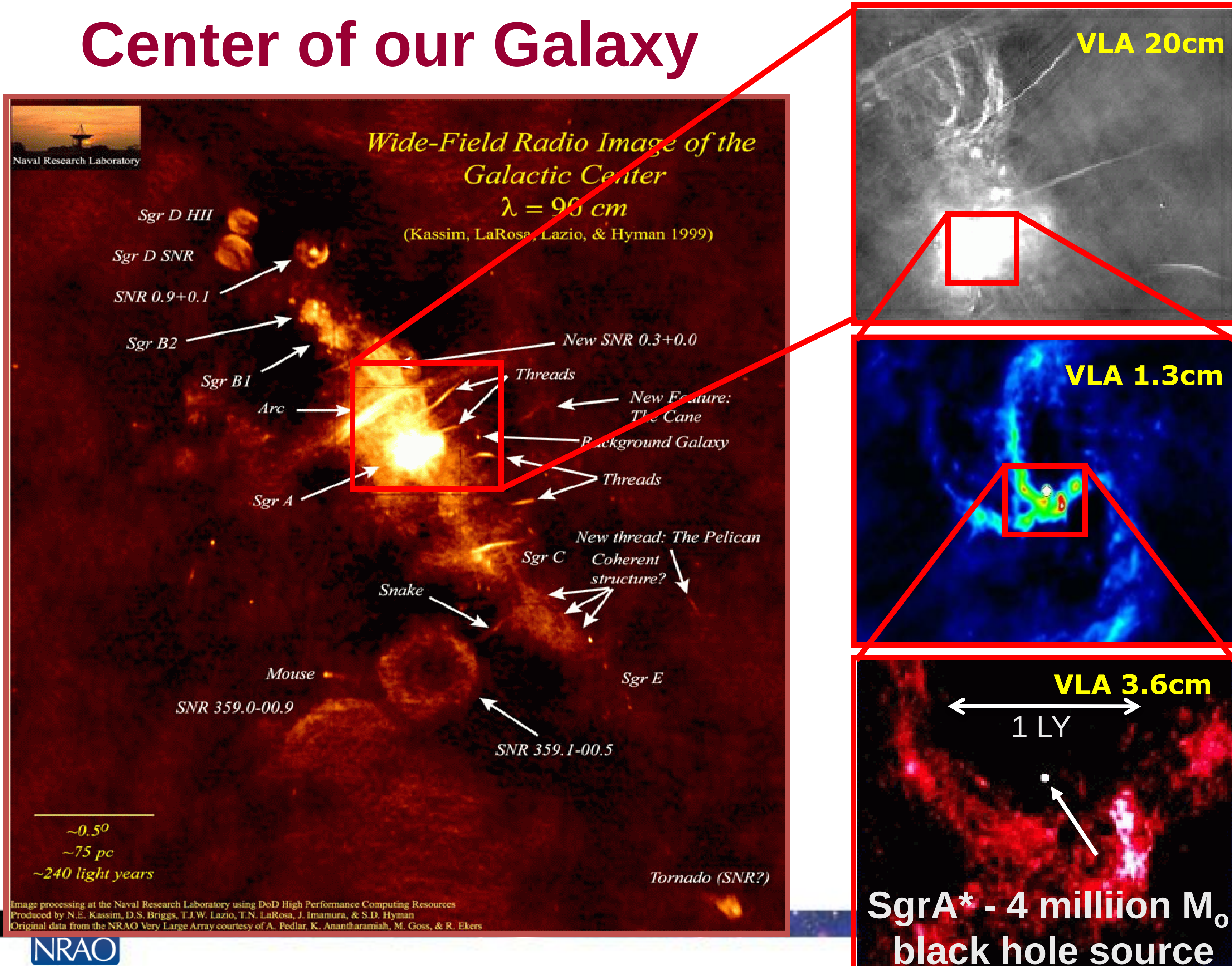


Dhawan et al. (2006)



Gallo et al. (2005)

Center of our Galaxy



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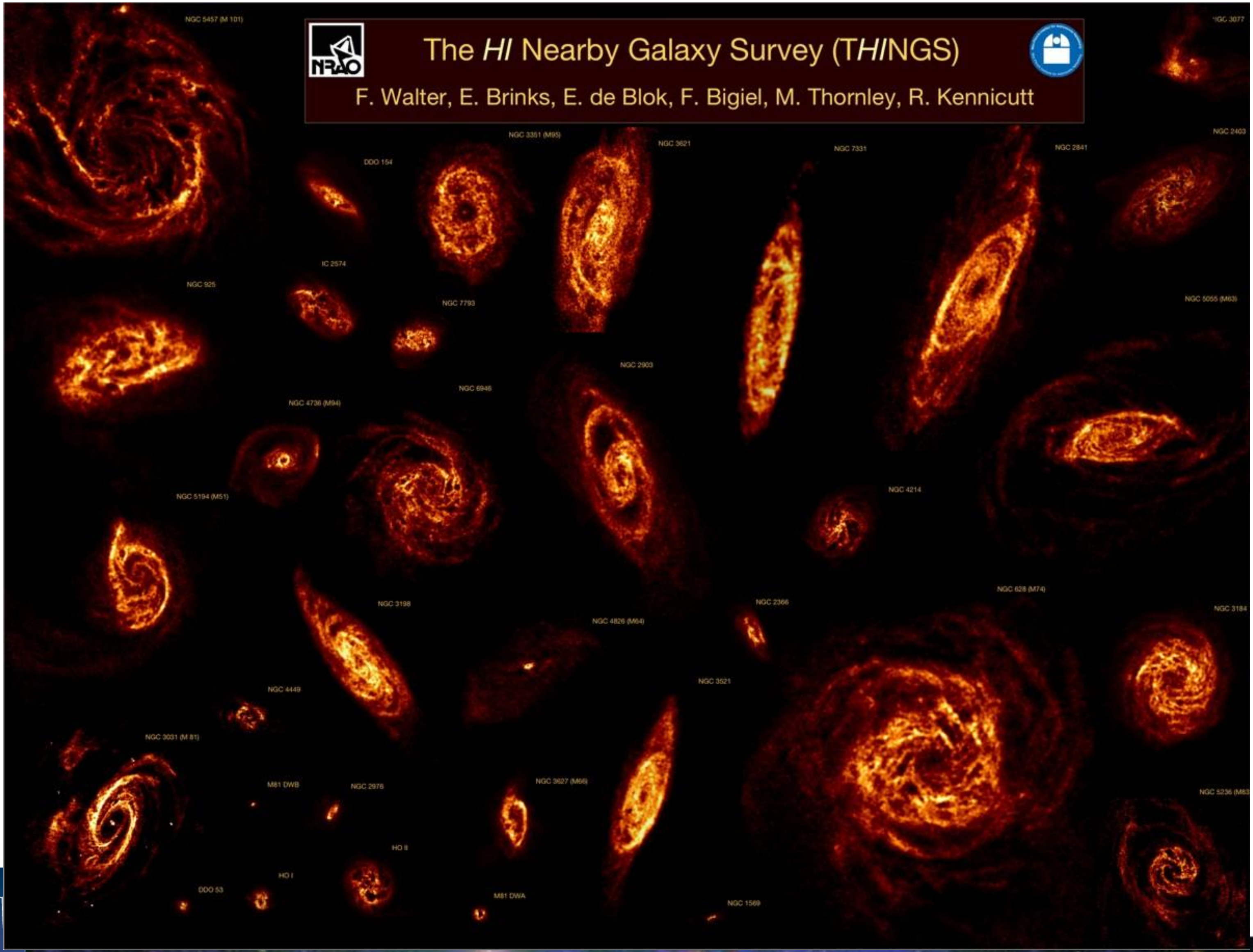
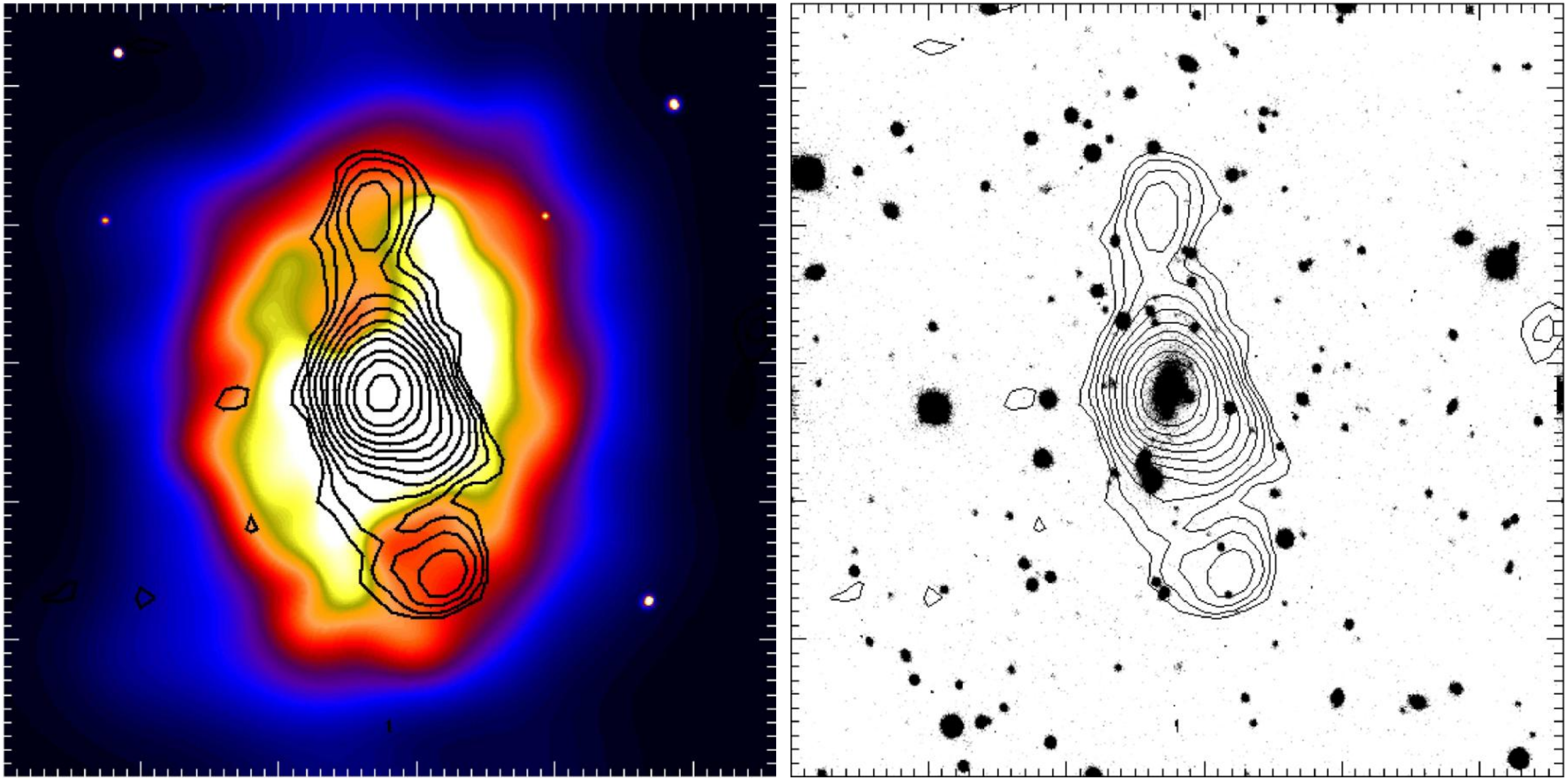
Credits: Lang, Morris, Roberts, Yusef-Zadeh, Goss, Zhao

Radio Spectral Lines: Gas in galaxies

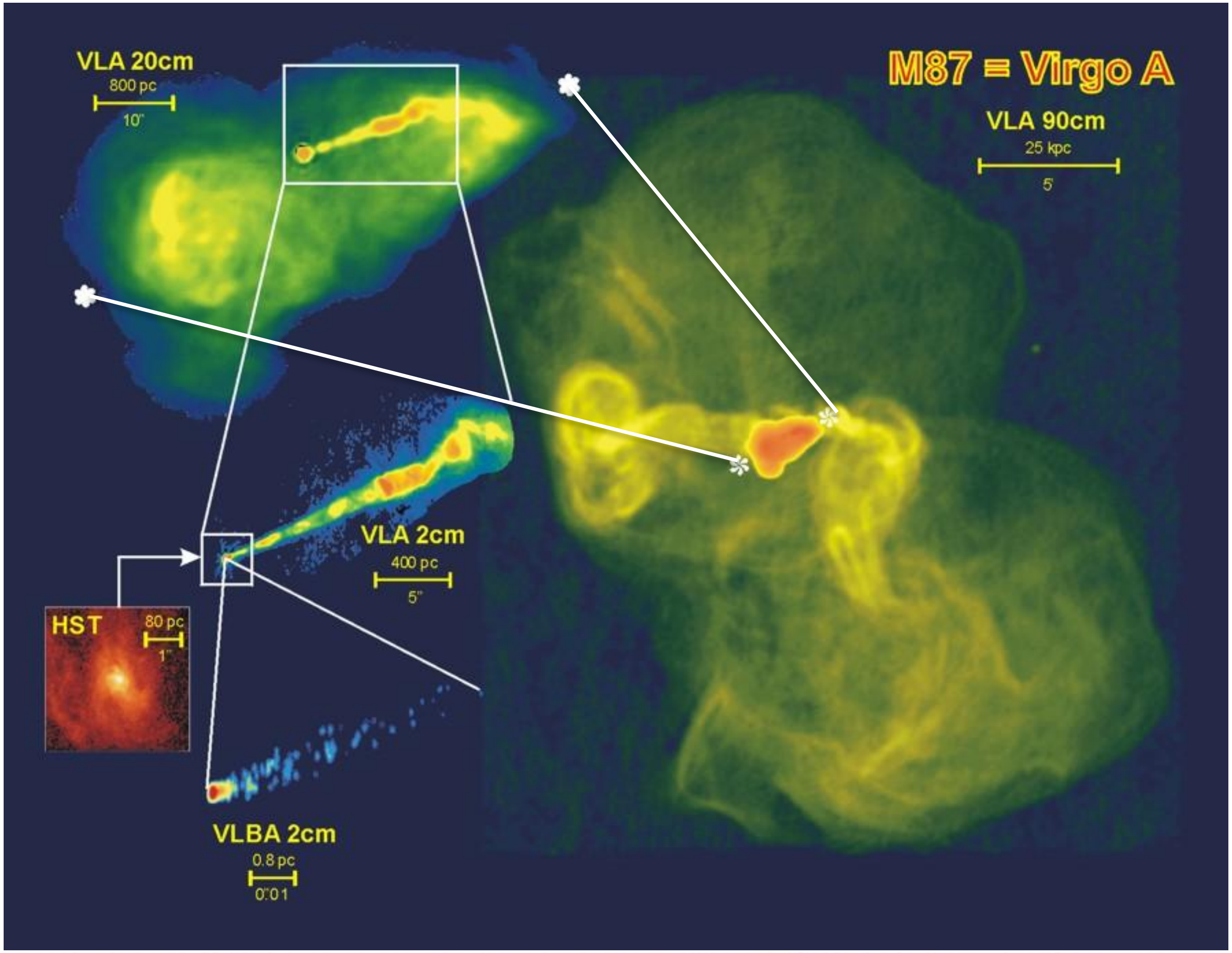


Jet Energy via Radio Bubbles in Hot Cluster Gas



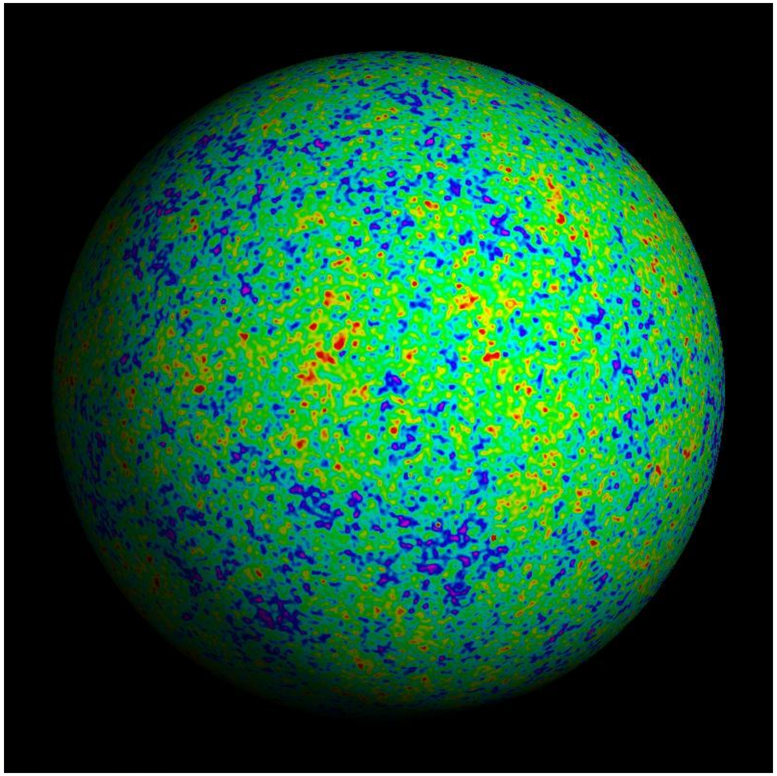
1.4 GHz VLA contours over Chandra X-ray image (left) and optical (right)
6 X 10^{61} ergs ~ 3 X 10^{7} solar masses X c^2 (McNamara et al. 2005, Nature, 433, 45)

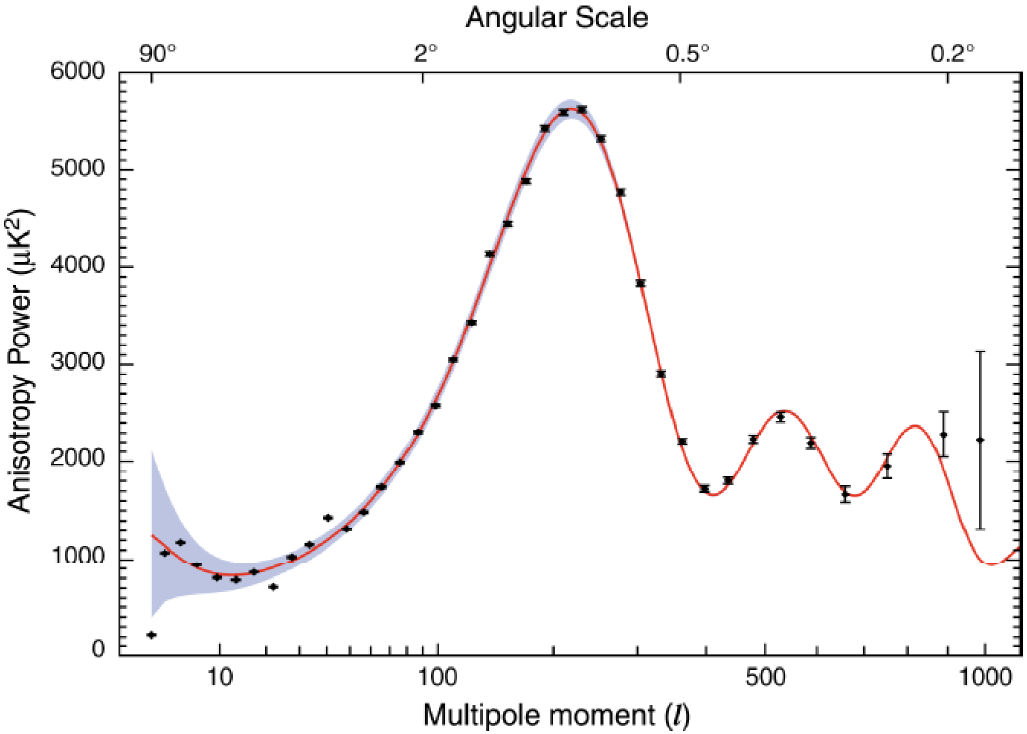
Resolution and Surface-brightness Sensitivity



The Cosmic Microwave Background

- 2.73 K radiation from z~1100
- Angular power spectrum of brightness fluctuations constrains cosmological parameters





Definitions

I_ν (or B_ν) = Surface Brightness : erg/s/cm^2/Hz/sr

(= intensity)

S_ν = Flux density : erg/s/cm^2/Hz $\int I_\nu \Delta\Omega$

S = Flux : erg/s/cm^2 $\int I_\nu \Delta\Omega \Delta\nu$

P = Power received : erg/s $\int I_\nu \Delta\Omega \Delta\nu \Delta \mathrm{A}_{\mathrm{tel}}$

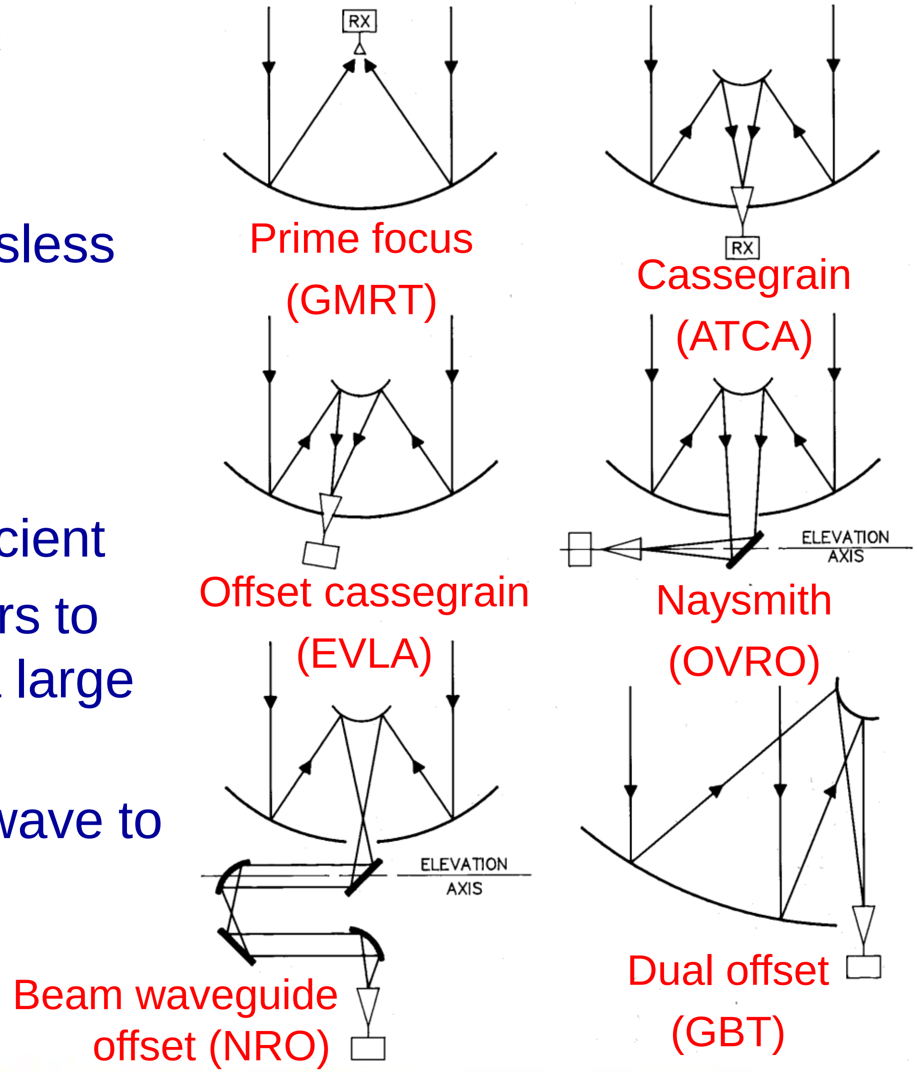
E = Energy : erg $\int I_\nu \Delta\Omega \Delta\nu \Delta \mathrm{A}_{\mathrm{tel}} \Delta t$

Radio astronomy: instrumentation

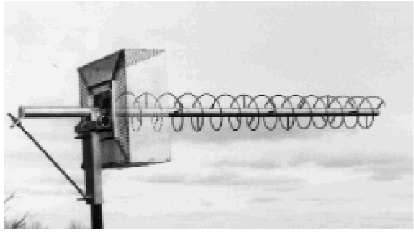
- Antennas convert electromagnetic radiation into electrical currents in conductors
- Average collecting area of any lossless antenna

$$\langle A_e \rangle = \frac{\lambda^2}{4\pi}$$

  - Long wavelength: dipoles sufficient
  - Short wavelength: use reflectors to collect and focus power from a large area onto a single feed
    - Parabola focuses a plane wave to a single focal point



Types of antennas

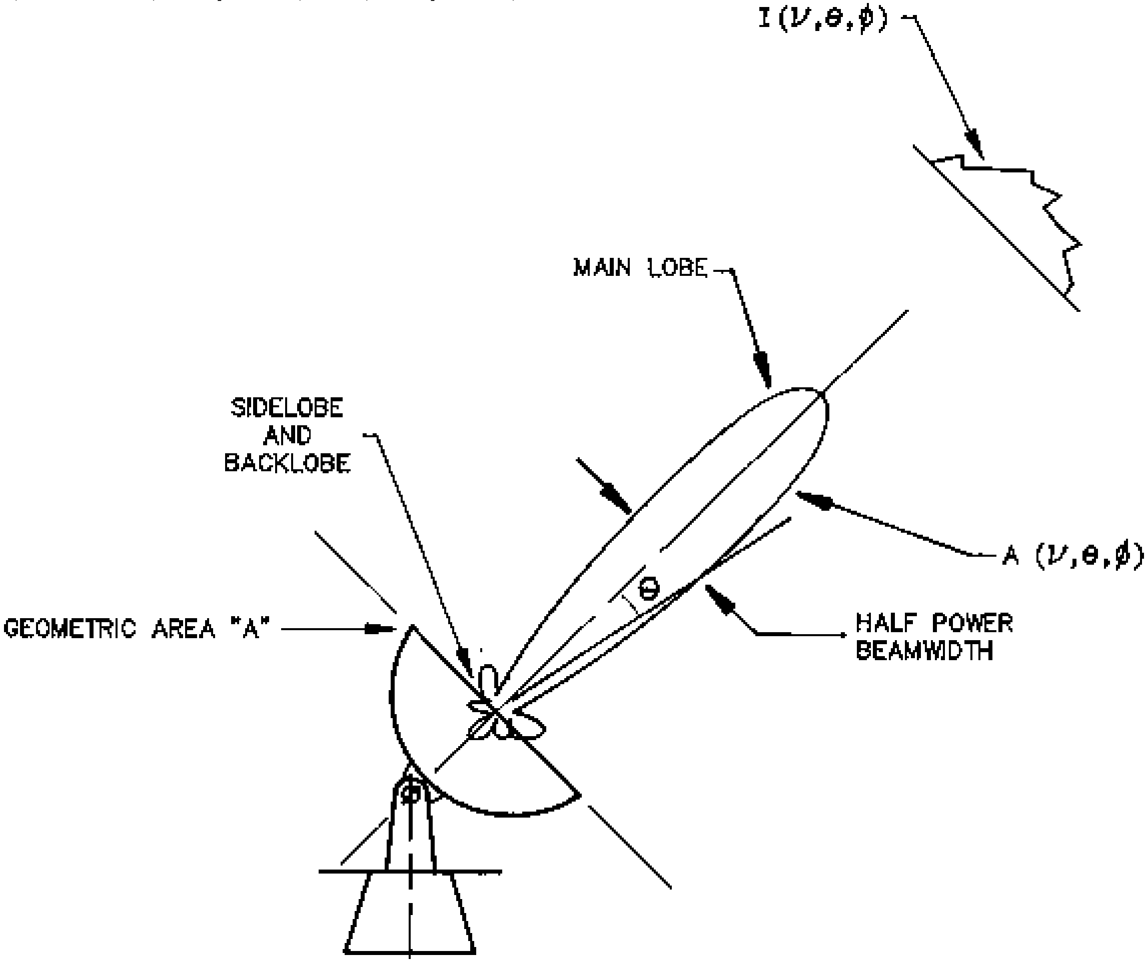




- Wire antennas $(\lambda > 1\text{m})$
  - Dipole
  - Yagi
  - Helix
  - Small arrays of the above
- Reflector antennas $(\lambda < 1\text{m})$
- Hybrid antennas $(\lambda \approx 1\text{m})$
  - Wire reflectors
  - Reflectors with dipole feeds

Fundamental antenna equations

$$P(\theta,\phi,\nu) = A(\theta,\phi,\nu)I(\theta,\phi,\nu)\Delta\nu\Delta\Omega$$

- Effective collecting area $A(\theta,\phi,\nu)$ m^2
- On-axis response $A_o = \eta A$
- Normalized pattern (primary beam) $\boldsymbol{A}(\theta,\phi,\nu) = A(\theta,\phi,\nu)/A_o$
- Beam solid angle $\Omega_A = \iint_{\text{all sky}} \boldsymbol{A}(\nu,\theta,\phi)\, d\Omega$
- $A_o\Omega_A = \lambda^2$



Aperture-Beam Fourier Transform Relationship

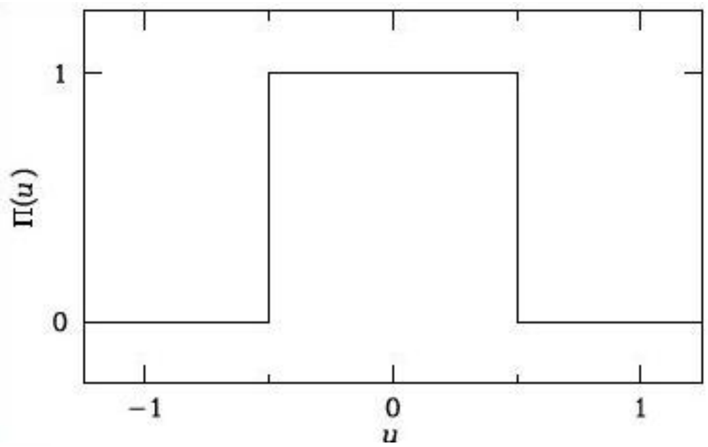
- Aperture field distribution and far-field voltage pattern form FT relation

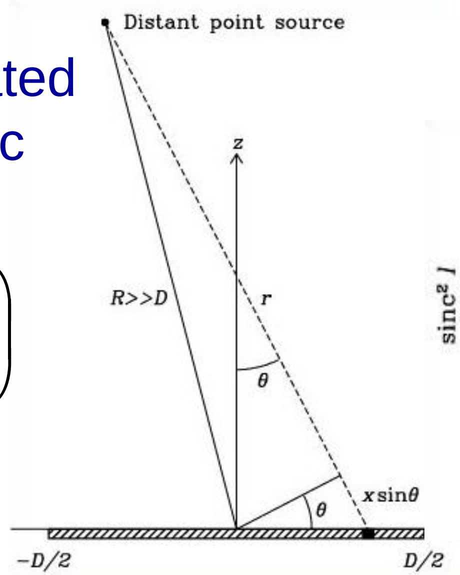
$$f(l) = \int_{aperture} g(u) e^{-2\pi i l u} du$$

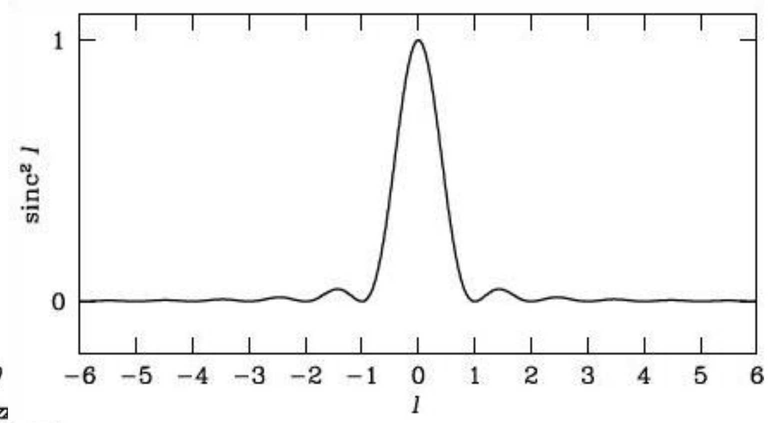
- $u = x/\lambda$
- $l = \sin\,\theta$
- Uniformly-illuminated aperture gives sinc function response

$$P(\theta) = \mathrm{sinc}^2\left(\frac{\theta D}{\lambda}\right)$$

- $\theta_{HPBW} \;\alpha\; \lambda/D$



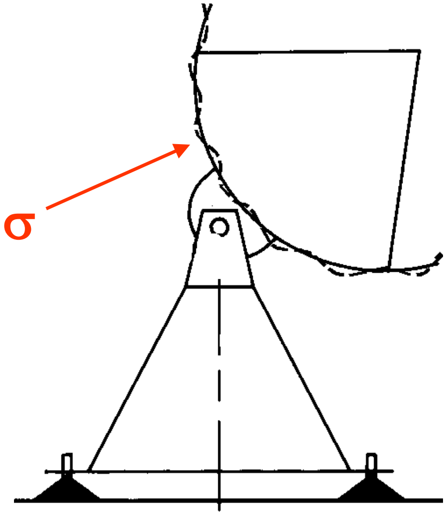




Antenna efficiency

On axis response: $A_0 = \eta A$

Efficiency: $\eta = \eta_{sf} \cdot \eta_{bl} \cdot \eta_s \cdot \eta_t \cdot \eta_{misc}$



η_{sf} = Reflector surface efficiency

Due to imperfections in reflector surface

$\eta_{sf} = \exp[-(4\pi\sigma/\lambda)^2]$ e.g., $\sigma = \lambda/16$, $\eta_{sf} = 0.5$

η_{bl} = Blockage efficiency

Caused by subreflector and its support structure

η_s = Feed spillover efficiency

Fraction of power radiated by feed intercepted by subreflector

η_t = Feed illumination efficiency

Outer parts of reflector illuminated at lower level than inner part

η_{misc}= Reflector diffraction, feed position phase errors, feed match and loss

Interferometry

- Largest fully-steerable dishes have D~100m
  - $\lambda/D \leq 10^{-4}$ at cm wavelengths
    - Sub-arcsecond resolution impossible
    - Confusion limits sensitivity
  - Collecting area limited to $\pi D^2/4$
- Tracking accuracy limited to σ~1"
  - Gravitational deformation
  - Differential heating
  - Wind torques
  - Must keep $\sigma < \lambda/10D$ for accurate imaging/photometry
- Interferometers consisting of multiple small dishes mitigate these problems

Interferometry

- Interferometry = Aperture Synthesis
  - Combine signals from multiple small apertures
  - Technique developed in the 1950s in England and Australia.
  - Martin Ryle (University of Cambridge) earned a Nobel Prize for his contributions.
- Basic operation:

  Correlation = multiply and average separate voltage outputs

Interferometry

- We seek a relation between the characteristics of the product of the voltages from two separated antennas and the distribution of the brightness of the originating source emission.

- To establish the basic relations, consider first the simplest interferometer:
  - Fixed in space – no rotation or motion
  - Quasi-monochromatic
  - Single frequency throughout – no frequency conversions
  - Single polarization (say, RCP).
  - No propagation distortions (no ionosphere, atmosphere …)
  - Idealized electronics (perfectly linear, perfectly uniform in frequency and direction, perfectly identical for both elements, no added noise, …)

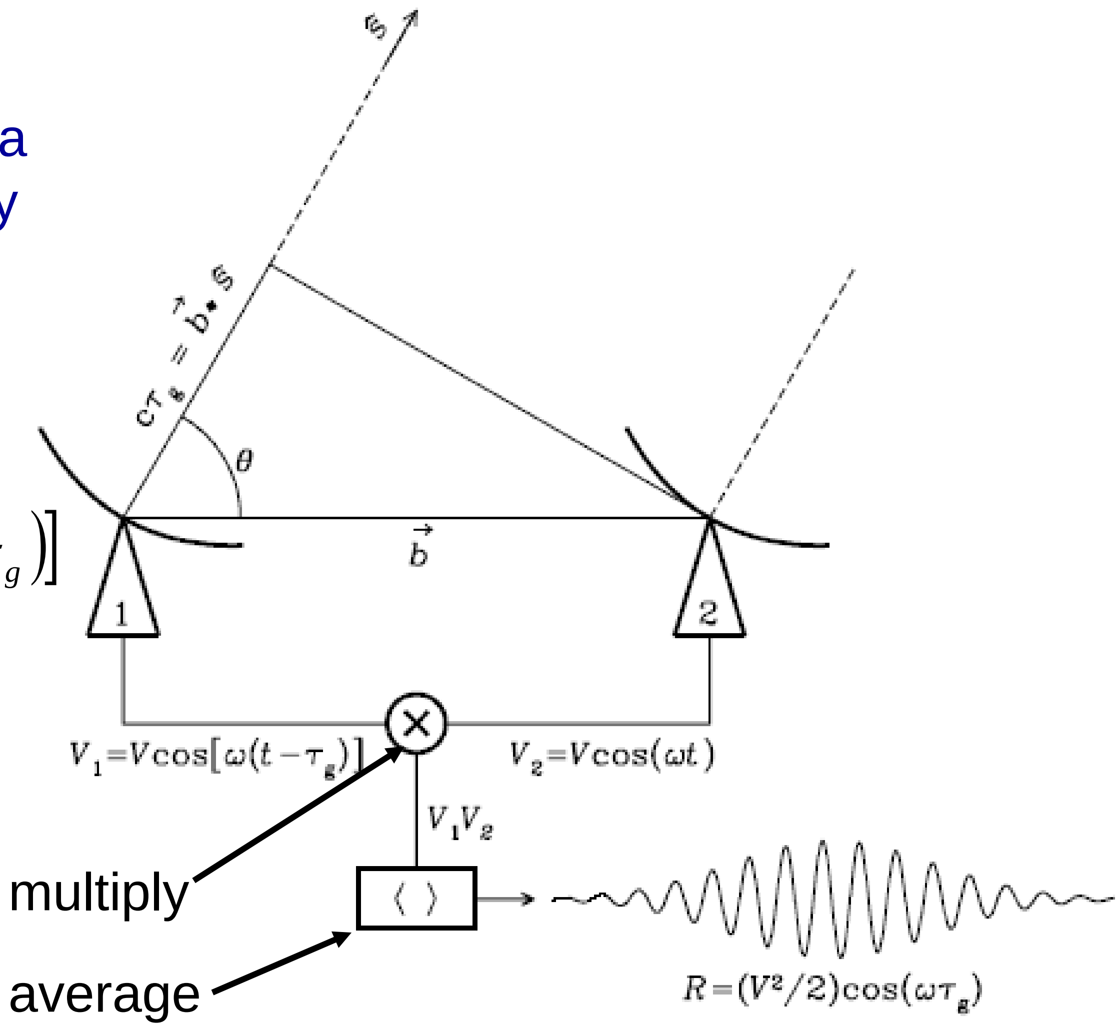
The two-element interferometer

- Radiation reaching 1 is delayed relative to 2
- Output voltage of antenna 1 lags by geometric delay τ_g = **b.s**/c
- Multiply outputs:

$$V_1V_2 = V^2 \cos\omega t \cos(\omega t - \tau_g)$$
$$= \frac{V^2}{2}\left[\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)\right]$$

- Time-average the result

$$R = \langle V_1V_2 \rangle = \frac{V^2}{2}\cos(\omega\tau_g)$$



The two-element interferometer

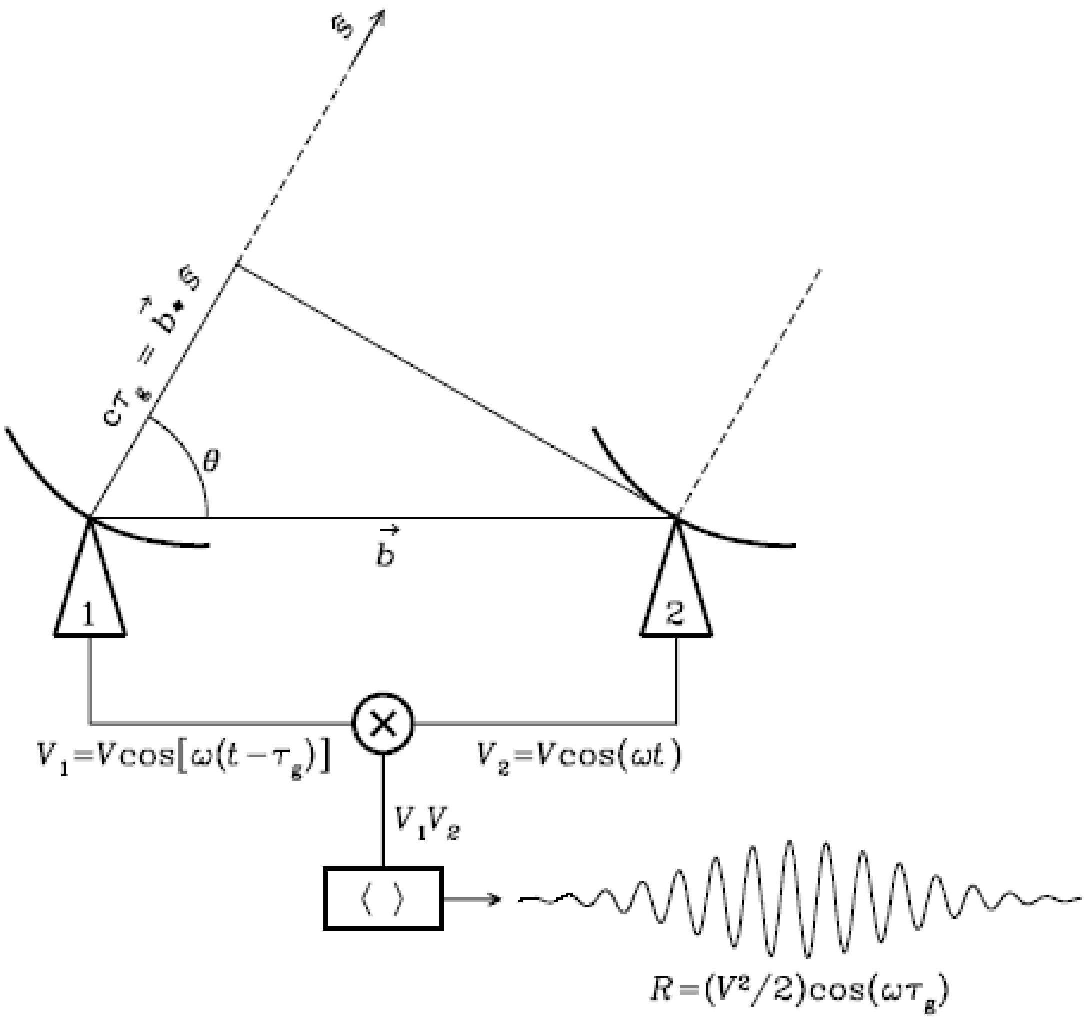
$$R = \langle V_1 V_2 \rangle = \frac{V^2}{2}\cos(\omega\tau_g)$$

- Sinusoidal variation with changing source direction: fringes

$$\phi = \omega\tau_g = \frac{\omega b\cos\theta}{c}$$

$$\frac{d\phi}{d\theta} = 2\pi\left(\frac{b\sin\theta}{\lambda}\right)$$

- Fringe period $\lambda/b\sin\theta$
- Phase measures source position (for large *b*)

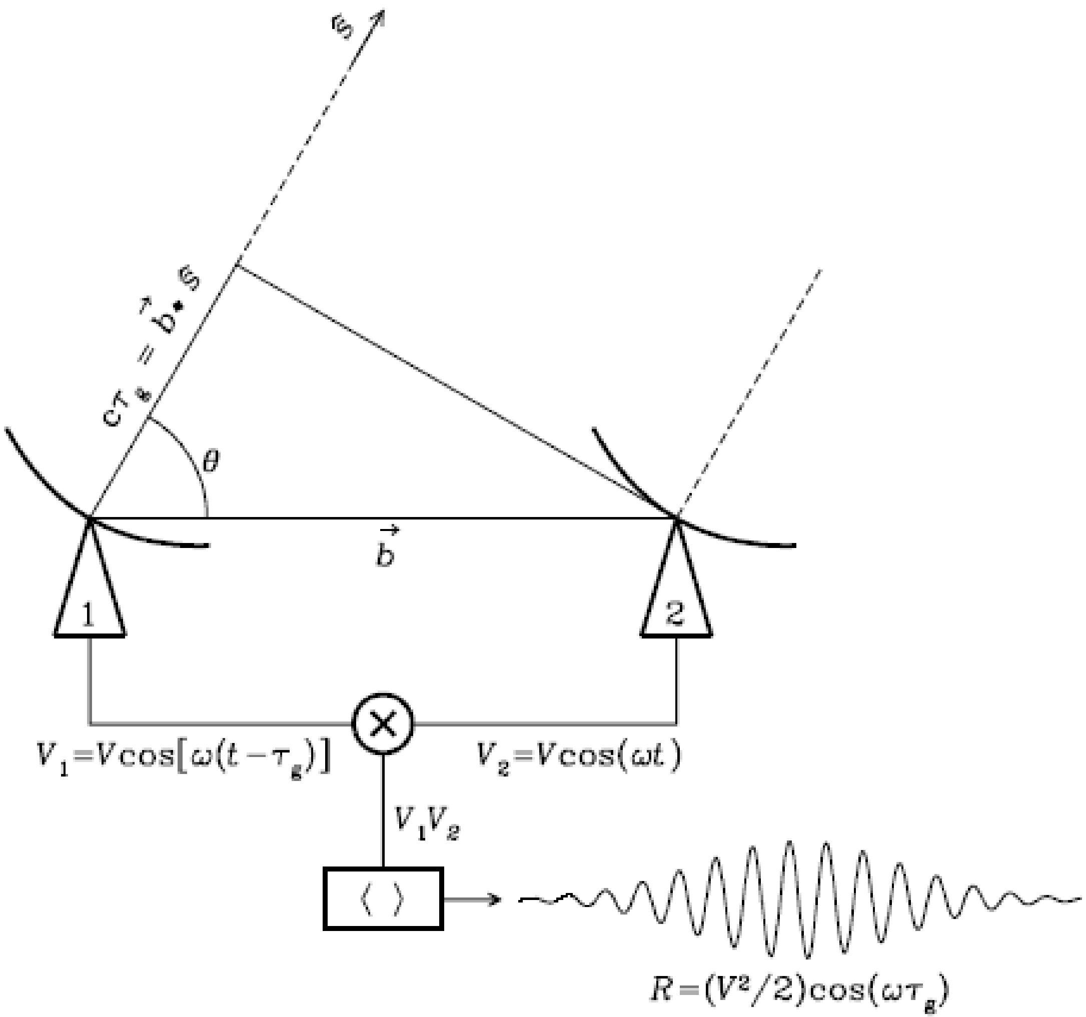


The two-element interferometer

$$R = \langle V_1 V_2 \rangle = \frac{V^2}{2}\cos(\omega\tau_g)$$

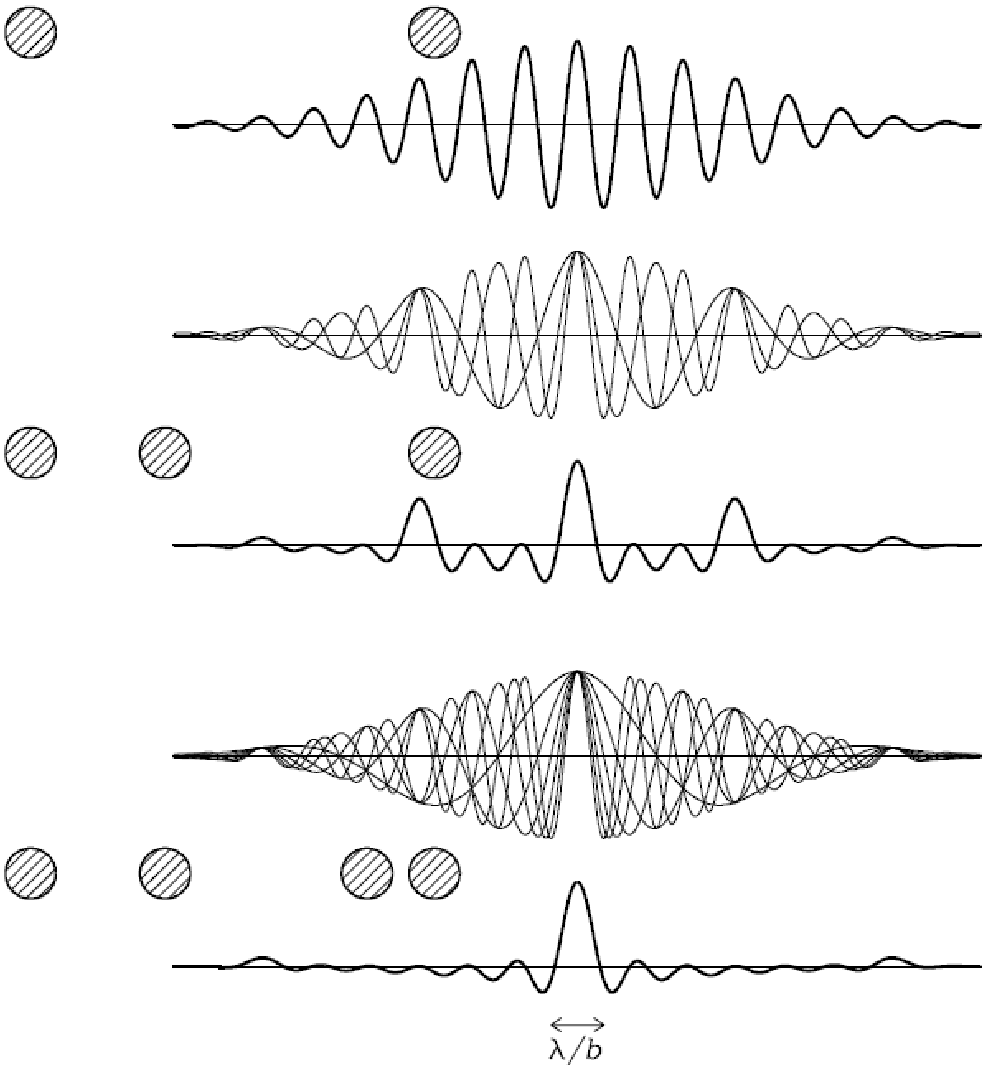
- Sensitivity to a single Fourier component of sky brightness, of period $\lambda/b\sin\theta$

- Antennas are directional
- Multiply correlator output by antenna response (primary beam)



Adding more elements

- Treat as *N(N-1)/2* independent two-element interferometers
- Average responses over all pairs
- Synthesized beam approaches a Gaussian
- For unchanging sources, move antennas to make extra measurements



Extended sources

- Point source response of our cosine correlator: $R = \langle V_1 V_2 \rangle = \frac{V^2}{2}\cos(\omega\tau_g)$

$$\frac{V^2}{2} \propto S_\nu (A_1 A_2)^{1/2}$$

- Treat extended sources as the sum of individual point sources
  - Sum responses of each antenna over entire sky, multiply, then average

$$R_c = \left\langle \int V_1 d\Omega_1 \int V_2 d\Omega_2 \right\rangle$$

- If emission is spatially incoherent $\left\langle \varepsilon_\nu(\mathbf{R}_1)\varepsilon_\nu^*(\mathbf{R}_2) \right\rangle = 0$ for $\mathbf{R}_1 \neq \mathbf{R}_2$

- Then we can exchange the integrals and the time averaging

$$R_c = \int I_\nu(\mathbf{s})\cos\left(2\pi\nu \frac{\mathbf{b.s}}{c}\right) d\Omega$$

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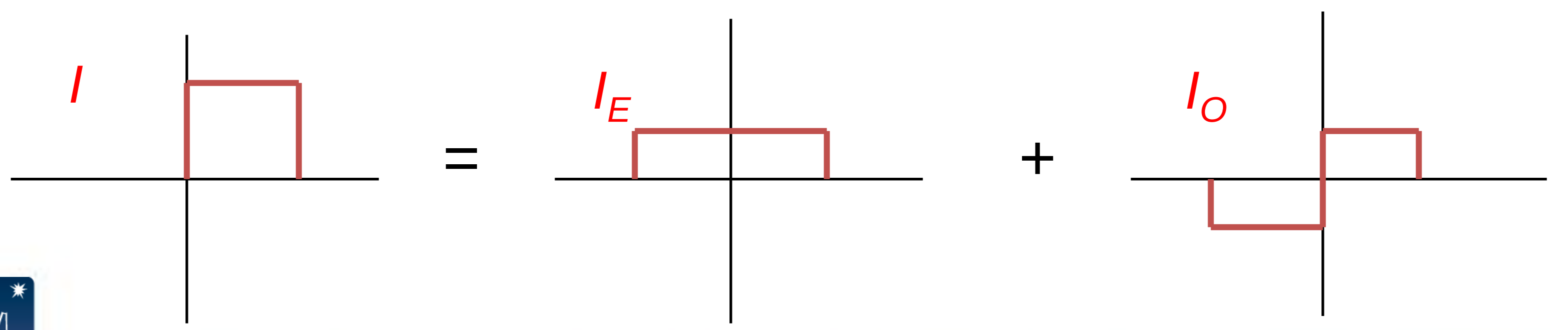
Odd and even brightness distributions

- Response of our cosine correlator: $R_c = \int I_\nu(\mathbf{s}) \cos\left(2\pi\nu \frac{\mathbf{b.s}}{c}\right) d\Omega$
- This is only sensitive to the even (inversion-symmetric) part of the source brightness distribution $I = I_E + I_O$
- Add a second correlator following a 90° phase shift to the output of one antenna

$$R_s = \langle V_1 V_2 \rangle = \frac{V^2}{2}\sin(\omega\tau_g)$$

$$R_s = \int I_\nu(\mathbf{s}) \sin\left(2\pi\nu \frac{\mathbf{b.s}}{c}\right) d\Omega$$



Complex correlators

$A = (R_c^2+R_s^2)^{1/2}$

$\phi = \tan^{-1}(R_s/R_c)$

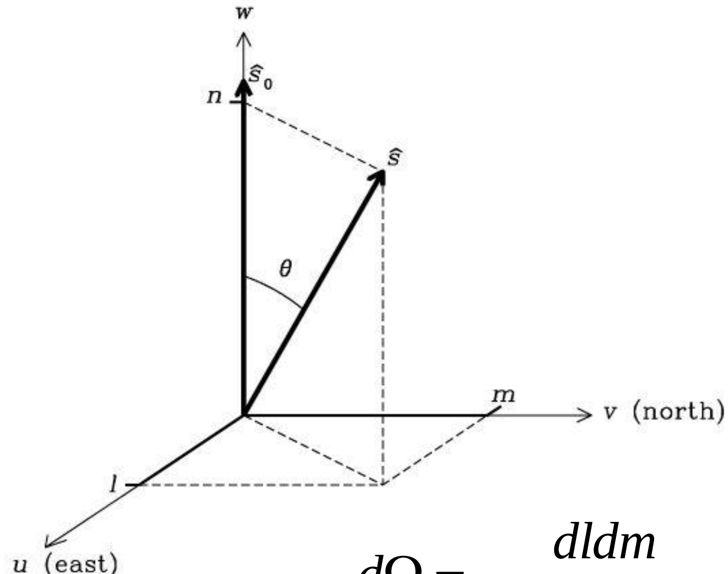
- Define complex visibility $V = R_c - iR_s = Ae^{i\phi}$
- Thus response to extended source is

$$V_\nu = \int I_\nu(\mathbf{s}) \exp\left(-2\pi i \frac{\mathbf{b.s}}{\lambda}\right) d\Omega$$

- Under certain circumstances, this is a Fourier transform, giving us a well established way to recover $I(\mathbf{s})$ from V
  - *1) Confine measurements to a plane*
  - *2) Limit radiation to a small patch of sky*
- Use an interferometer to measure the spatial coherence function V_ν and invert to measure the sky brightness distribution

Digression: co-ordinate systems

- Baseline **b** measured in wavelengths: the (*u,v,w*) co-ordinate system
  - *w* points along reference direction $\mathbf{s_o}$
  - (*u,v*) point east and north in plane normal to *w*-axis
  - *(u,v,w)* are the components of **b**/λ along these directions



- In this co-ordinate system, **s** = (*l,m,n*)
  - *l,m,n* are direction cosines
  - $n = \cos\theta = (1-l^2-m^2)^{1/2}$

$$V_\nu = \int I_\nu(\mathbf{s}) \exp\left(-2\pi i \frac{\mathbf{b.s}}{\lambda}\right) d\Omega$$

$$d\Omega = \frac{dldm}{\sqrt{1-l^2-m^2}}$$

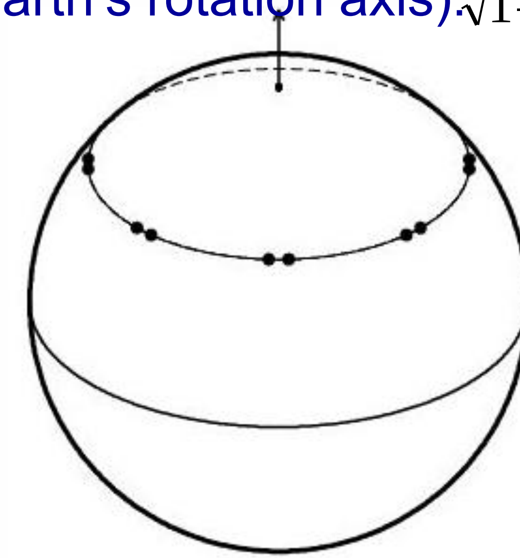
$$V_\nu(u,v,w) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} \exp\left[-2\pi i(ul+vm+wn)\right] dldm$$

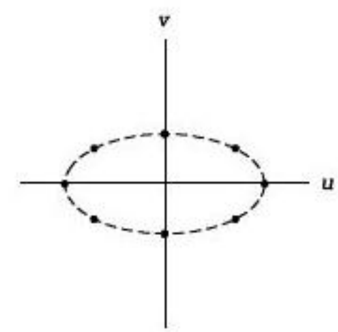
Measurements confined to a plane

- All baselines **b** lie in a plane, so **b** = $\mathbf{r_1}$-$\mathbf{r_2}$ = $\lambda(u,v,0)$
- Components of **s** are then $\left(l, m, \sqrt{1-l^2-m^2}\right)$

$$V_\nu(u,v,w\equiv 0) = \iint I_\nu(l,m)\frac{e^{-2\pi i(ul+vm)}}{\sqrt{1-l^2-m^2}}\,dldm$$

- This is a Fourier-transform relationship between V_ν and I_ν, which we know how to invert
- Earth-rotation synthesis ($\mathbf{s_o}$ is Earth's rotation axis): $\sqrt{1-l^2-m^2} = \cos\theta = \sec\delta$
  - E-W interferometers
  - ATCA, WSRT
- VLA snapshots





All sources in a small region of sky

- Radiation from a small part of celestial sphere: $\mathbf{s} = \mathbf{s_o} + \boldsymbol{\sigma}$
- $n = \cos\theta = 1 - (\theta^2/2)$

$$V_\nu(u,v,w) = e^{-2\pi i w} \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} \exp\left[-2\pi i\left(ul+vm+w\theta^2/2\right)\right] dl\,dm$$

- If $w\theta^2 << 1$, i.e. $\theta \sim (\lambda/b)^{1/2}$ then we can ignore last term in brackets

$$V'_\nu(u,v,w) = e^{-2\pi i w} \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm)} dl\,dm$$

- Let $V_\nu(u,v,w) = e^{2\pi i w} V'_\nu(u,v,w)$

$$V_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm)} dl\,dm$$

- Again, we have recovered a Fourier transform relation
- Here the *endpoints* of the vectors **s** lie in a plane
- Break up larger fields into multiple facets each satisfying $w\theta^2 << 1$

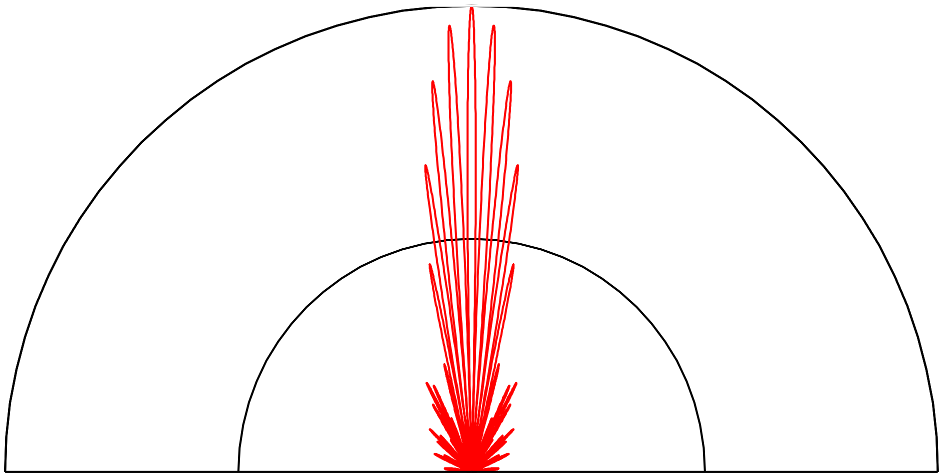
NRAO

All sources in a small region of sky

- This assumption is valid since antennas are *directional*

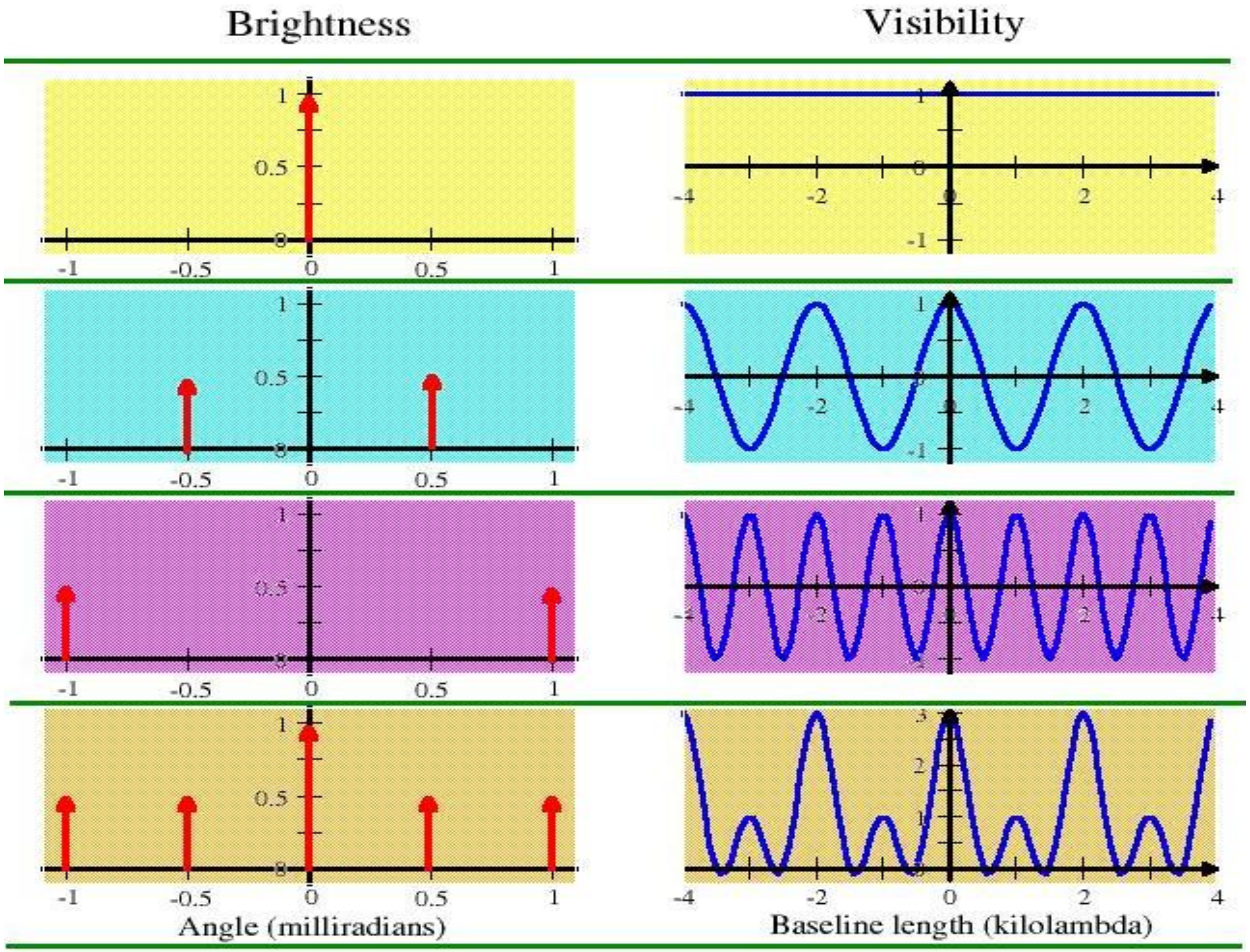
$$V_\nu(u,v) = \iint A_\nu(l,m) I_\nu(l,m) e^{-2\pi i(ul+vm)} dl dm$$

- Primary beam of interferometer elements A_ν gives sensitivity as a function of direction
- Falls rapidly to 0 away from pointing centre $\mathbf{s_o}$
- Holds where field of view is not too large
  - centimetre-wavelength interferometers
    - EVLA
    - MERLIN
    - VLBA
    - EVN
    - GMRT



Visibility functions

- Brightness and Visibility are Fourier pairs.
- Some simple and illustrative examples make use of ‘delta functions’ – sources of infinitely small extent, but finite total flux.

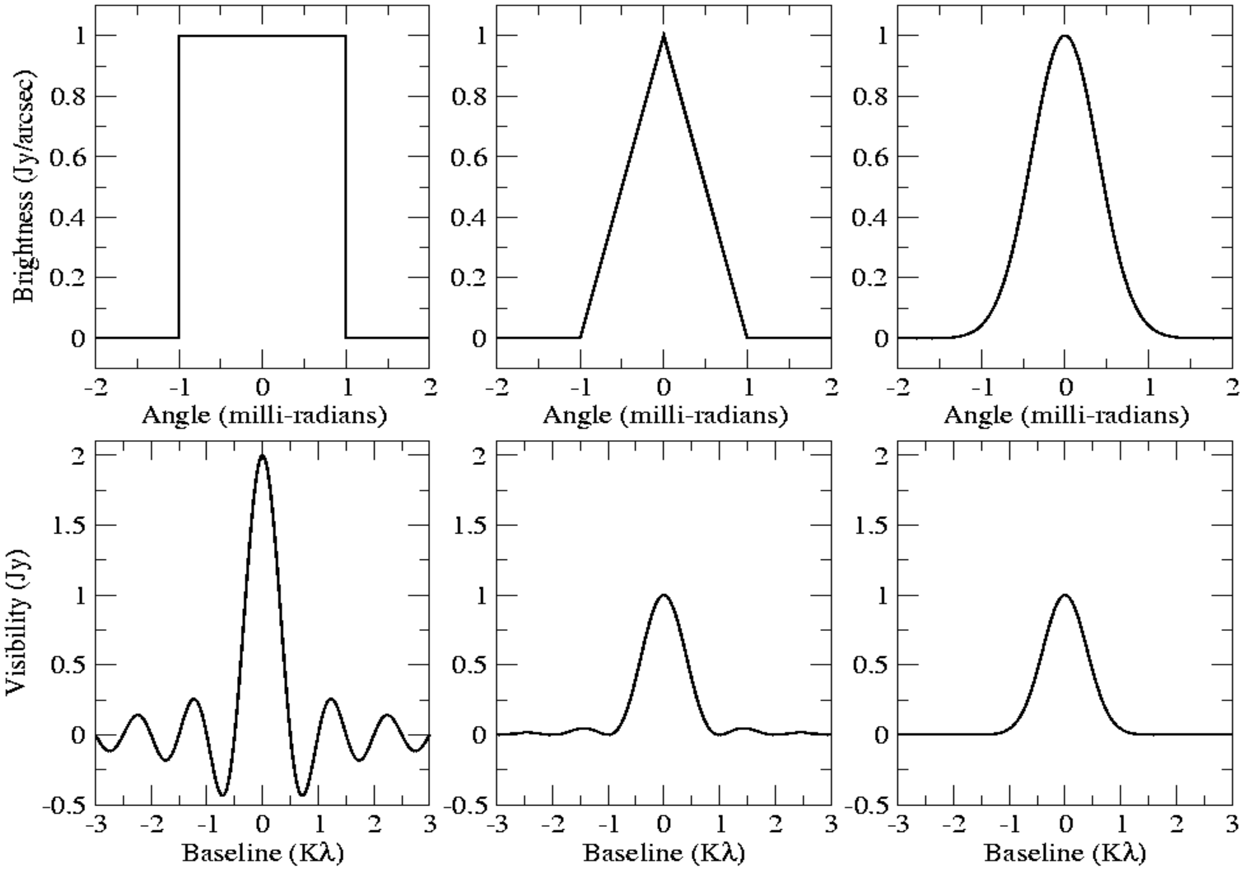


Visibility functions

- Top row: 1-dimensional even brightness distributions.
- Bottom row: The corresponding real, even, visibility functions.

$I_\nu(\theta)$

$V_\nu(u)$



Inverting the Fourier transform

- Fourier transform relation between measured visibilities and sky brightness

$$V_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm)} dldm$$

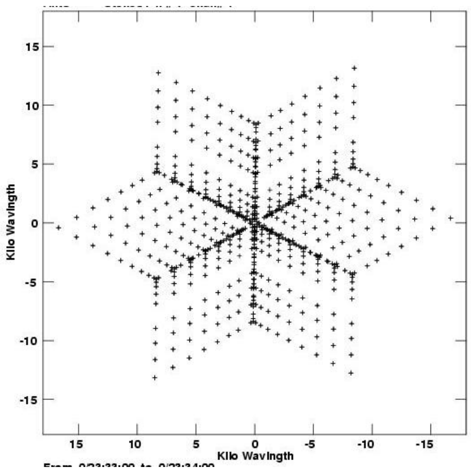
- Invert FT relation to recover sky brightness

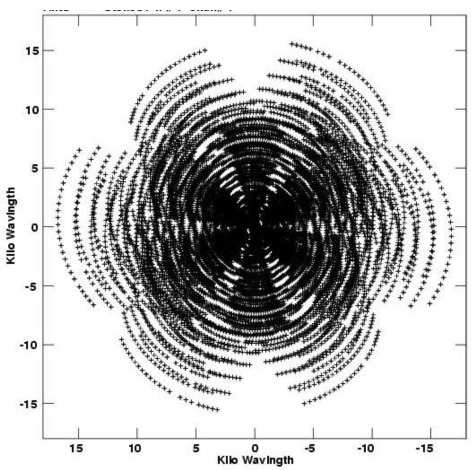
$$\frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} = \iint V_\nu(u,v) e^{-2\pi i(ul+vm)} dudv$$

- This assumes complete sampling of the (*u*,*v*) plane
- Introduce sampling function *S(u,v)*

$$\frac{I_\nu^D(l,m)}{\sqrt{1-l^2-m^2}} = \iint V_\nu(u,v) S_\nu(u,v) e^{-2\pi i(ul+vm)} dudv$$

- Must deconvolve to recover I_ν: $I_\nu^D = I_\nu * B$
- See Katherine Blundell's lecture for details





Seems simple?

- We have discovered a beautiful, invertible relation between the visibility measured by an interferometer and the sky brightness
- BUT:
- Between the source and the correlator lie:
  - Atmosphere
  - Man-made interference (RFI)
  - Instrumentation
- *The visibility function we measure is not the true visibility function*
- Therefore we rely on:
  - Editing: remove corrupted visibilities
  - Calibration: remove the effects of the atmosphere and instrumentation

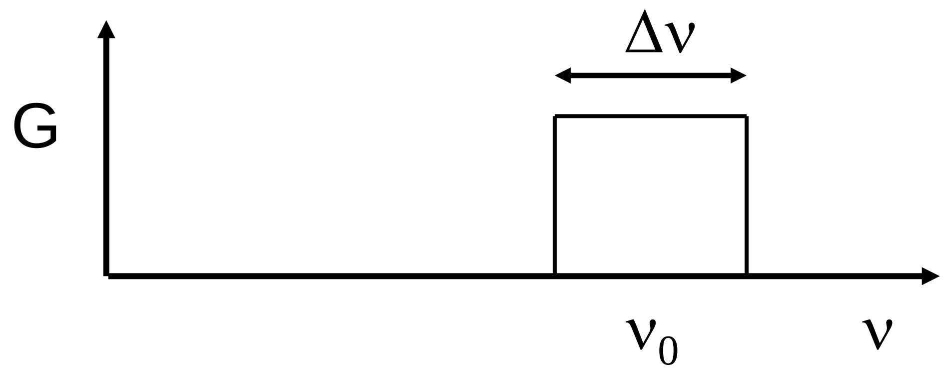
$$V_{ij}^{obs} = \left(G_{ij}\right) V_{ij}^{true} = \left(G_i G_j^*\right) V_{ij}^{true}$$

NRAO

What about bandwidth?

- Practical instruments are not monochromatic
- Assume:
  - constant source brightness
  - constant interferometer response

  over the finite bandwidth, then:



$$V_\nu = \int \left[(\Delta\nu)^{-1} \int_{\nu_c - \Delta\nu/2}^{\nu_c + \Delta\nu/2} I_\nu(\mathbf{s}) \exp\left(-2\pi i \nu \tau_g\right) d\nu \right] d\Omega$$

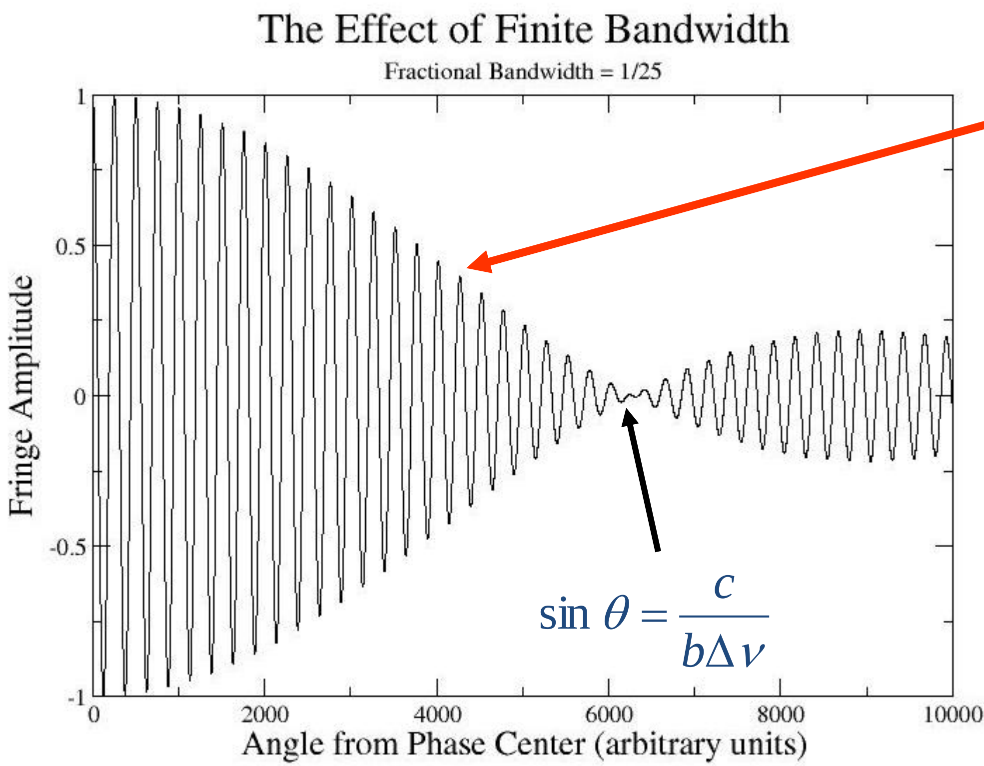
- The integral of a rectangle function is a sinc function:

$$V_\nu = \int I_\nu(\mathbf{s}) \operatorname{sinc}(\Delta\nu\tau_g) \exp(-2\pi i \nu_c \tau_g) d\Omega$$

- Fringe amplitude is reduced by a sinc function envelope
- Attenuation is small for $\Delta\nu\tau_g << 1$, i.e. $\Delta\nu\Delta\theta << \theta_s \nu$

What about bandwidth?

- For a square bandpass, the bandwidth attenuation reaches a null at an angle equal to the fringe separation divided by the fractional bandwidth: $\Delta\nu/\nu_0$.
- Depends only on baseline and bandwidth, not frequency

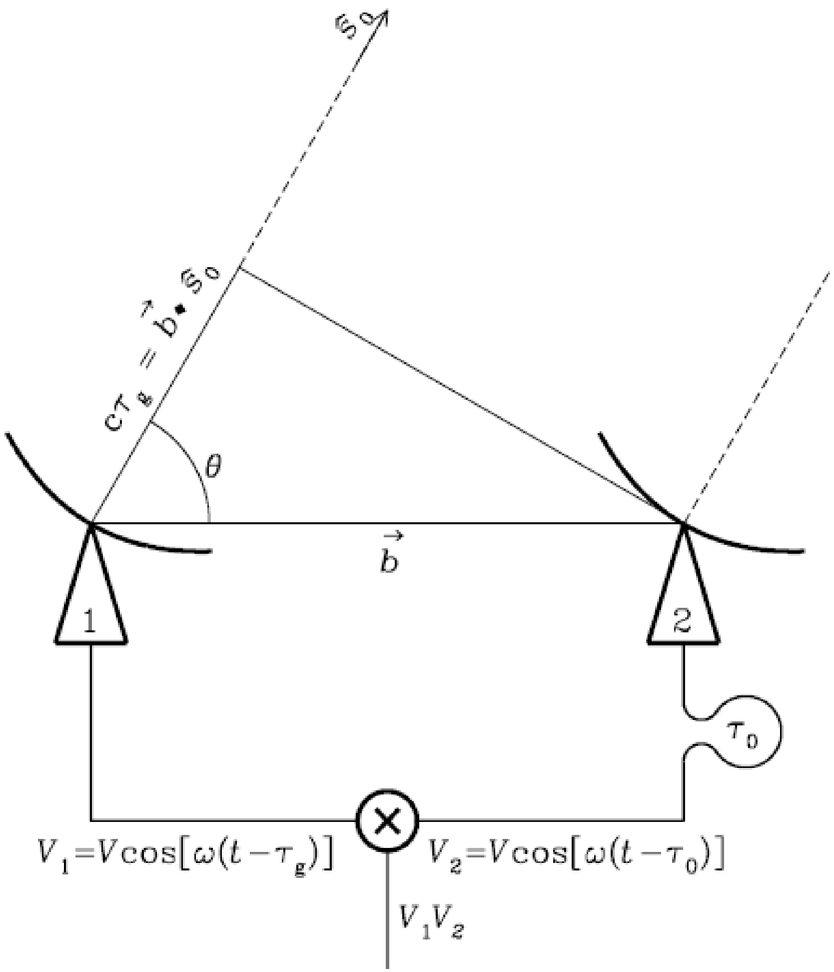


Fringe Attenuation function:

$$\mathrm{sinc}\left(\frac{b}{\lambda}\frac{\Delta\nu}{\nu}\theta\right)$$

Delay tracking

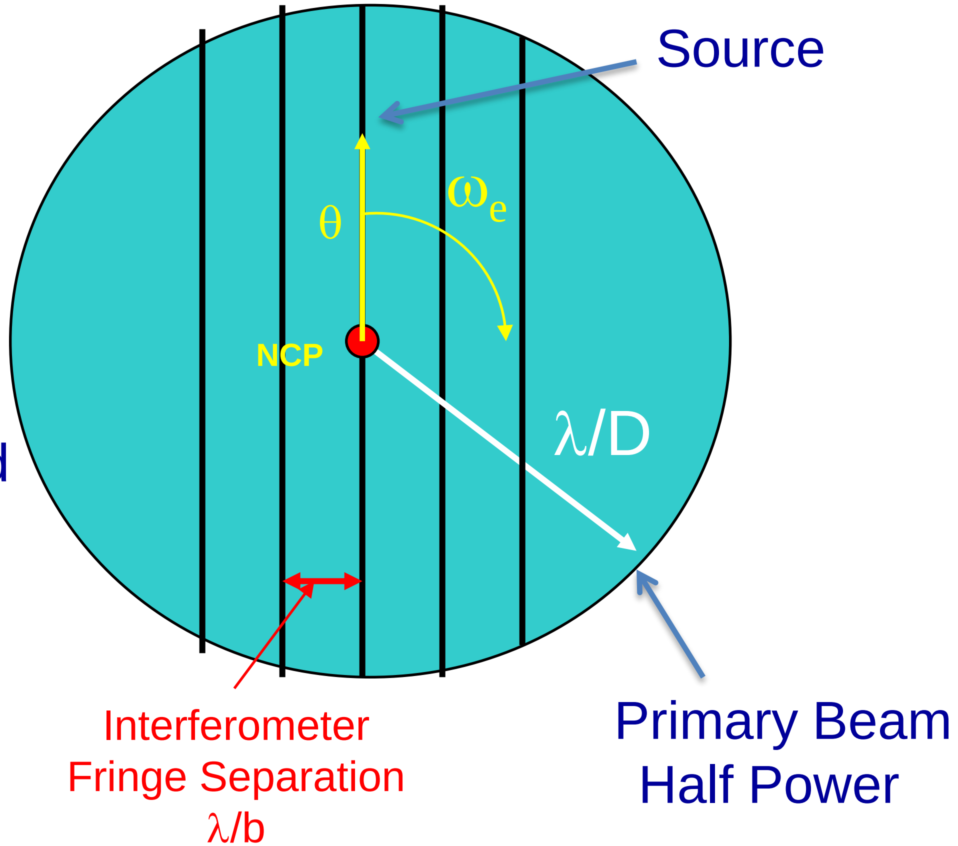
- Fringe amplitude reduced by sinc function envelope
- Eliminate the attenuation in any one direction $\mathbf{s_o}$ by adding extra delay $\tau_0 = \tau_g$
- Shifts the fringe attenuation function across by $\sin\theta = c\tau_0/b$ to centre on source of interest
- As Earth rotates, adjust τ_0 continuously so $|\tau_0 - \tau_g| < (\Delta\nu)^{-1}$
- $\Delta\nu\Delta\theta << \theta_s\nu$
- Away from this, bandwidth smearing: radial broadening
  - Convolve brightness with rectangle $\Delta\nu\Delta\theta/\nu$



Field of view

- Bandwidth smearing
  - Field of view limited by bandwidth smearing to

    $\Delta\theta << \theta_s \nu/\Delta\nu$
  - Split bandwidth into many narrow channels to widen FOV
- Time smearing
  - Earth rotation should not move source by 1 synthesized beam in correlator averaging time
  - Tangential broadening
  - $\Delta\theta << P\theta_s/2\pi\Delta t$ where P=86164s (sidereal day)

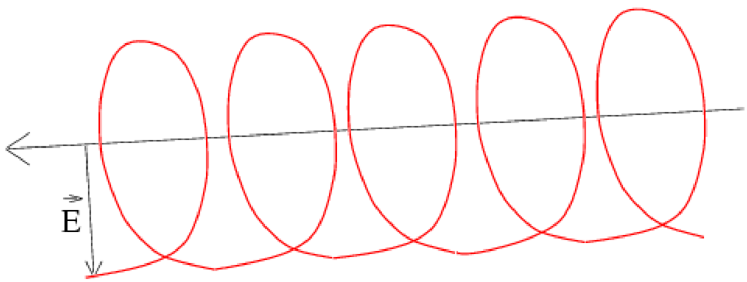


What about polarization?

- All the above was derived for a scalar electric field (single polarization)
- Electromagnetic radiation is a vector phenomenon:
  - EM waves are intrinsically polarized
  - monochromatic waves are fully polarized
- Polarization state of radiation can tell us about:
  - the origin of the radiation
    - intrinsic polarization, orientation of generating B-field
  - the medium through which it traverses
    - propagation and scattering effects
  - unfortunately, also about the purity of our optics
    - you may be forced to observe polarization even if you do not want to!

The Polarization Ellipse

- From Maxwell's equations E•B=0 (E and B perpendicular)
  - By convention, we consider the time behavior of the E-field in a fixed perpendicular plane, from the point of view of the receiver.



$$\mathbf{k} \bullet \mathbf{E} = 0$$

transverse wave

- For a monochromatic wave of frequency ν, we write

$$E_x = A_x \cos(2\pi\nu t + \varphi_x)$$
$$E_y = A_y \cos(2\pi\nu t + \varphi_y)$$

  - These two equations describe an ellipse in the (x-y) plane.
- The ellipse is described fully by three parameters:
  - A_X, A_Y, and the phase difference, $\delta = \phi_Y - \phi_X$.

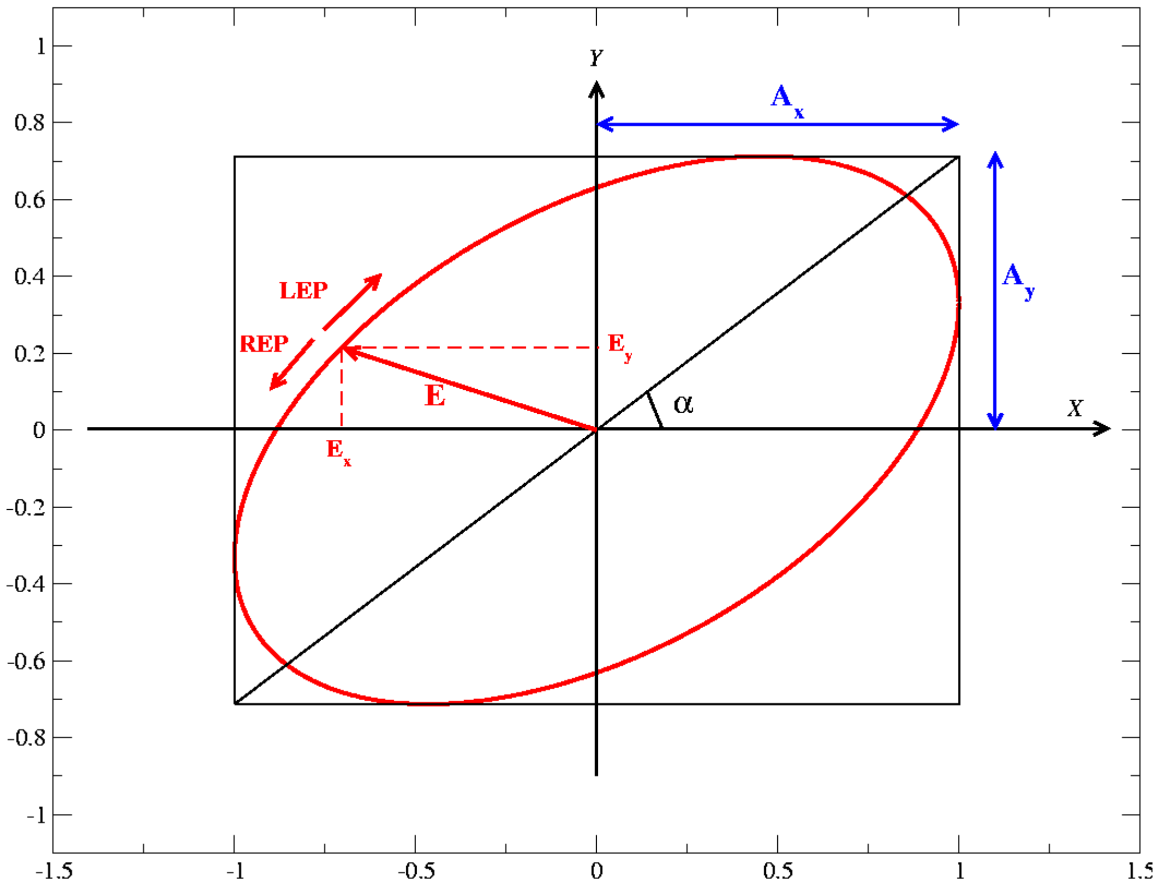
Elliptically Polarized Monochromatic Wave

The simplest description of wave polarization is in a Cartesian coordinate frame.

In general, three parameters are needed to describe the ellipse.

If the E-vector is rotating:

- clockwise, wave is 'Left Elliptically Polarized',
- counterclockwise, is 'Right Elliptically Polarized'.



equivalent to 2 independent E_x and E_y oscillators

Polarization Ellipse Ellipticity and P.A.

- A more natural description is in a frame (ξ,η), rotated so the ξ-axis lies along the major axis of the ellipse.
- The three parameters of the ellipse are then:

  A_η : the major axis length

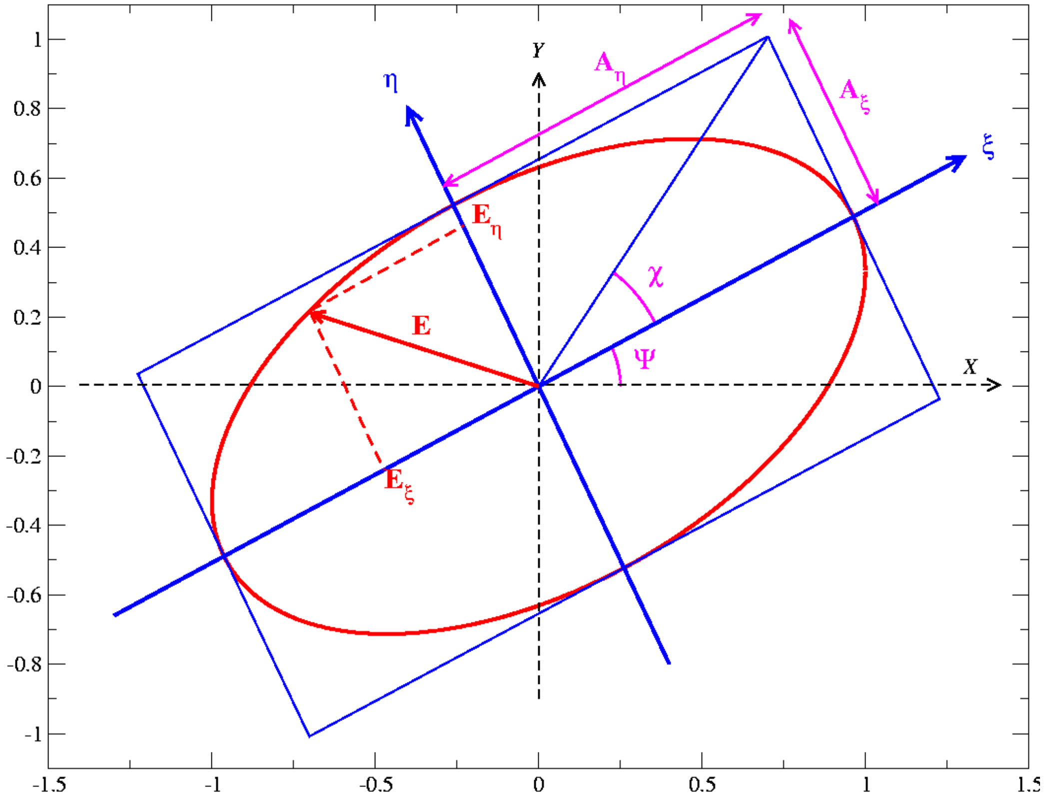
  $\tan \chi = A_\xi / A_\eta$: the axial ratio

  Ψ : the major axis p.a.

- The ellipticity χ is signed:

  $\chi > 0 \rightarrow$ REP

  $\chi < 0 \rightarrow$ LEP



$\chi = 0,90°$ $\rightarrow$ Linear (δ=0° ,180°)

$\chi = \pm 45°$ $\rightarrow$ Circular ($\delta = \pm 90$

Circular Basis

- We can decompose the E-field into a circular basis, rather than a (linear) Cartesian one:

$$\mathbf{E} = A_R \widehat{e}_R + A_L \widehat{e}_L$$

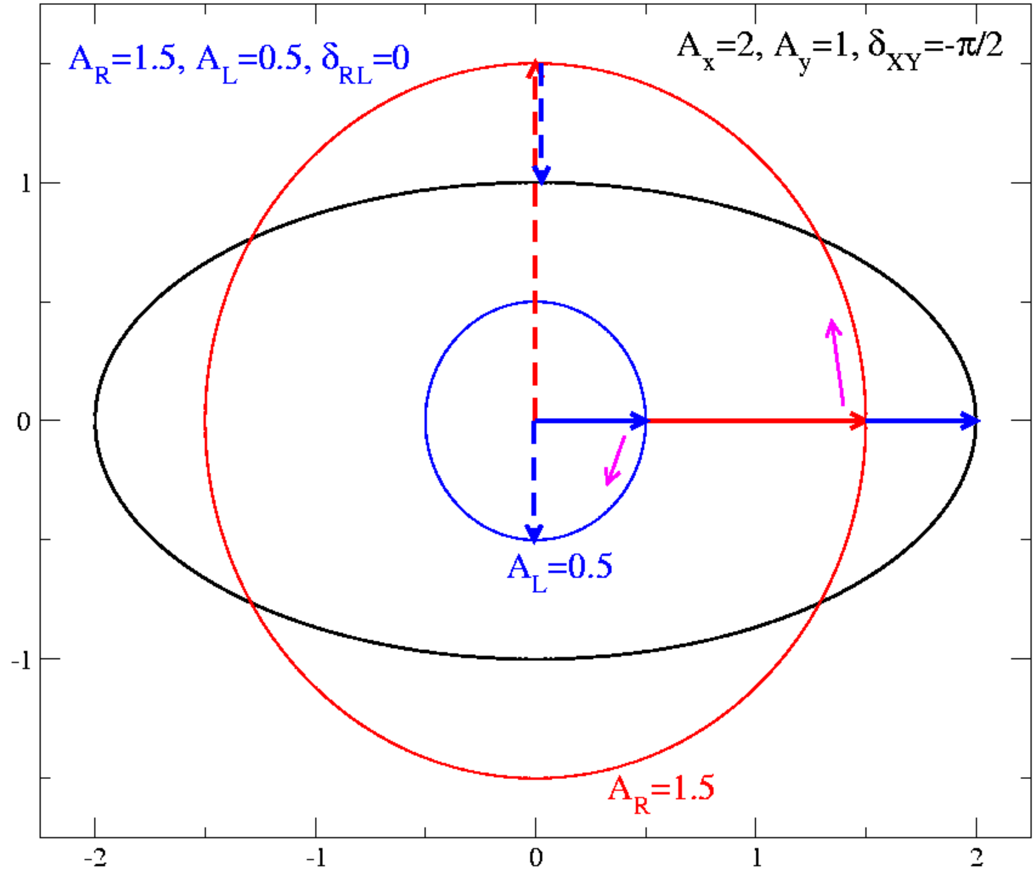
  - where A_R and A_L are the amplitudes of two counter-rotating unit vectors, e_R (rotating counter-clockwise), and e_L (clockwise)
  - NOTE: R,L are obtained from X,Y by a phase shift
- In terms of the linear basis vectors:

$$A_R = \frac{1}{2}\sqrt{A_X^2 + A_Y^2 - 2A_X A_Y \sin\delta_{XY}}$$

$$A_L = \frac{1}{2}\sqrt{A_X^2 + A_Y^2 + 2A_X A_Y \sin\delta_{XY}}$$

Circular Basis Example

- The black ellipse can be decomposed into an x-component of amplitude 2, and a y-component of amplitude 1 which lags by ¼ turn.
- It can alternatively be decomposed into a counterclockwise rotating vector of length 1.5 (red), and a clockwise rotating vector of length 0.5 (blue).



Stokes parameters

- Stokes parameters I,Q,U,V
  - defined by George Stokes (1852)
  - scalar quantities independent of basis (XY, RL, etc.)
  - units of power (flux density when calibrated)
  - form complete description of wave polarization
  - Total intensity, 2 linear polarizations and a circular polarization
    - $I = E_X^2 + E_Y^2 = E_R^2 + E_L^2$
    - $Q = I \cos 2\chi \cos 2\psi = E_X^2 - E_Y^2 = 2 E_R E_L \cos \delta_{RL}$
    - $U = I \cos 2\chi \sin 2\psi = 2 E_X E_Y \cos \delta_{XY} = 2 E_R E_L \sin \delta_{RL}$
    - $V = I \sin 2\chi = 2 E_X E_Y \sin \delta_{XY} = E_R^2 - E_L^2$
- Only 3 independent parameters:
  - $I^2 = Q^2 + U^2 + V^2$ for fully-polarized emission
  - If not fully polarized, $I^2 > Q^2 + U^2 + V^2$

Stokes parameters

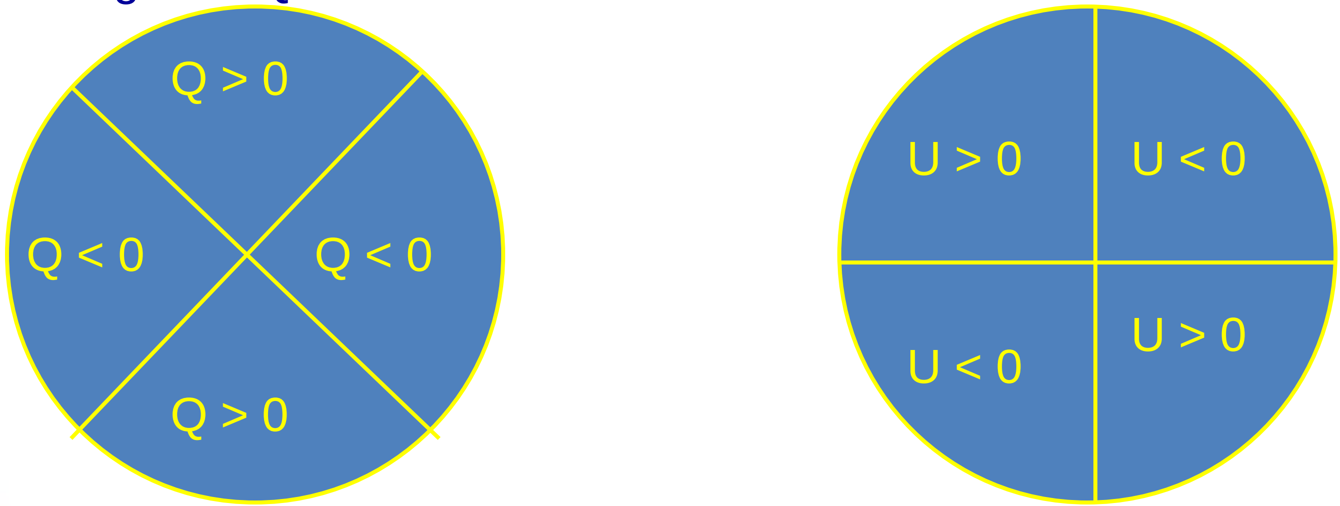
- Linearly Polarized Radiation: V = 0
  - Linearly polarized flux:

$$P = \sqrt{Q^2 + U^2}$$

  - Q and U define the linear polarization position angle:

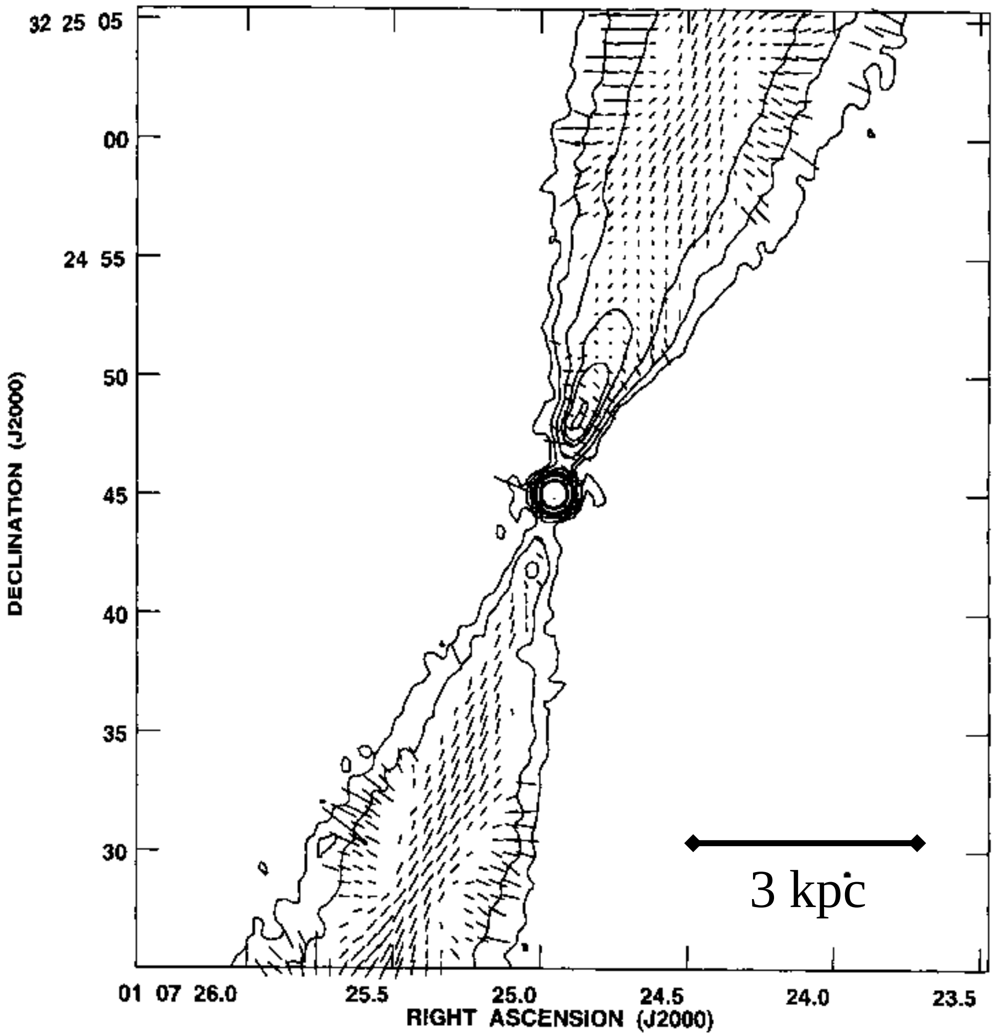
$$\tan 2\Psi = U / Q$$

  - Signs of Q and U:



Polarization example

- VLA @ 8.4 GHz
  - Laing (1996)
- Synchrotron radiation
  - relativistic plasma
  - jet from central "engine"
  - from pc to kpc scales
  - feeding >10kpc "lobes"
- E-vectors
  - along core of jet
  - radial to jet at edge



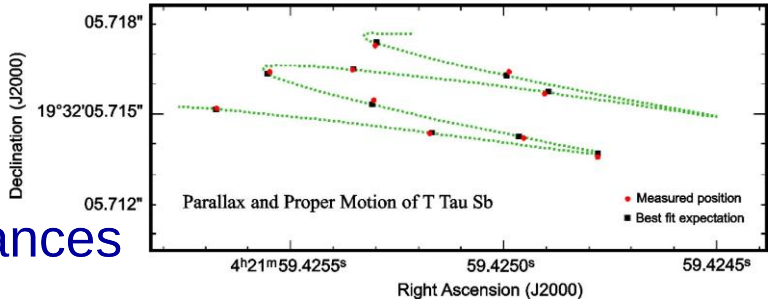
Sensitivity

- The radiometer equation for point source sensitivity:

$$\sigma_S = \frac{2k_B T_{sys}}{A_{eff}\sqrt{N(N-1)t_{\mathrm{int}}\Delta\nu}}$$

- Increase the sensitivity by:
  - Increasing the bandwidth
  - Increasing the integration time
  - Increasing the number of antennas
  - Increasing the size/efficiency of the antennas
- Surface brightness sensitivity is MUCH worse
  - synthesized beam << beam size of single dish of same effective area

Summary: why should I use radio telescopes?



- Unparalleled resolution
  - Astrometry: positions, velocities, distances
  - High-resolution imaging
- Unique probe
  - Radio waves not scattered by dust: probe inner Galaxy
  - Low-energy, long-lived non-thermal emission
  - Spectral lines probe physical conditions of gas
- Signature of high-energy processes; link to X-ray emission
- Exotic phenomena
  - Coherent emission (pulsar phenomenon)
- New "Golden Age"
  - New telescopes opening up new parameter space

Radio Astronomy: A Golden Age!

Equations to take away

- Resolution of an interferometer

$$\theta = \frac{\lambda}{b_{\max}}$$

- Field of view of an interferometer

$$\theta = \frac{\lambda}{D}$$

- Sensitivity of an interferometer

$$\sigma_S = \frac{2k_B T_{sys}}{A_{eff}\sqrt{N(N-1)t_{\mathrm{int}}\Delta\nu}}$$

- Relation between visibility and sky brightness

$$V_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i(ul+vm)} dl dm$$