Accretion and Accretion Disks

Introduction

AGN are powered by accretion $\Longrightarrow$ need to look at accretion as a physical mechanism.

Unfortunately, this will have to be somewhat theoretical, but this cannot be avoided...

Structure of this chapter:

1. Accretion Luminosity: Eddington luminosity
2. Accretion Disks: Theory
3. Accretion Disks: Confrontation with observations

Literature

- J. Frank, A. King, D. Raine, 2002, Accretion Power in Astrophysics, 3rd edition, Cambridge Univ. Press

  The standard textbook on accretion, covering all relevant areas of the field.

- T. Padmanabhan, 2001, Theoretical Astrophysics, II. Stars and Stellar Systems, Cambridge Univ. Press

  See introduction to this lecture.

- N.I. Shakura & R. Sunyaev, 1973, Black Holes in Binary Systems. Observational Appearance. Astron. Astrophys. **24**, 337

  The fundamental paper, which *really* started the field.

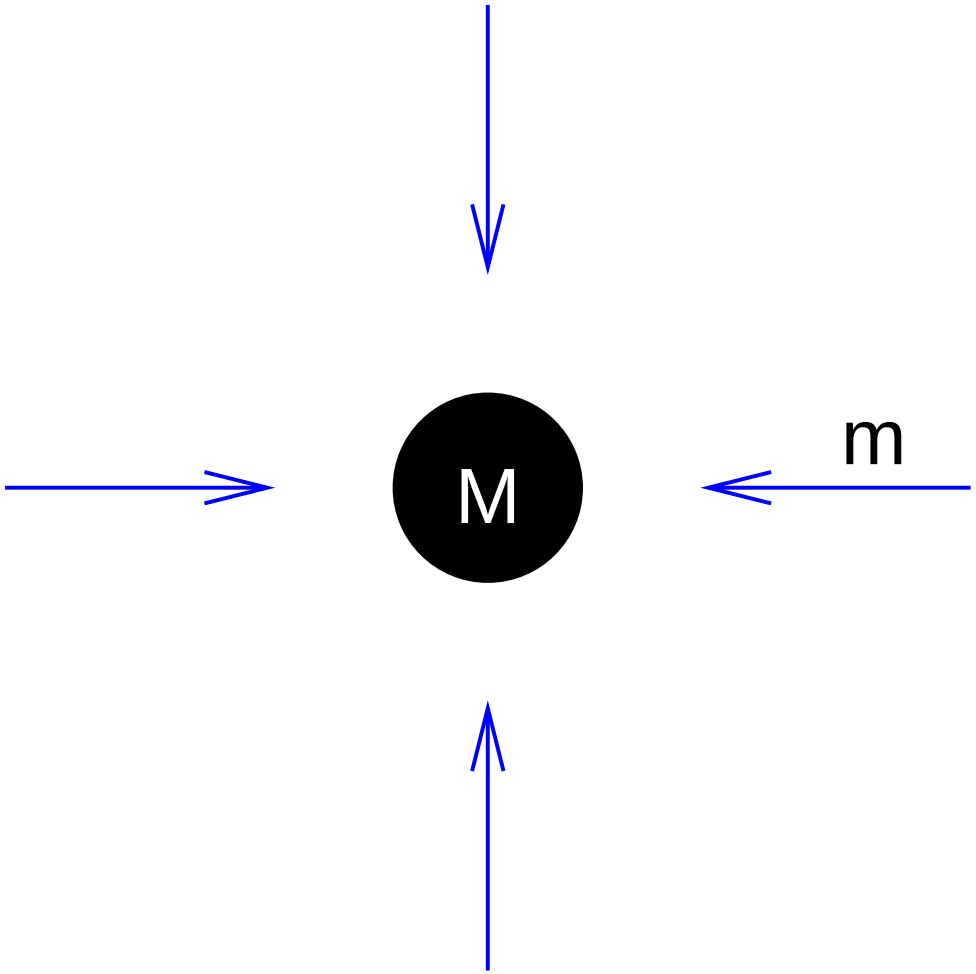
- J.E. Pringle, 1981, Accretion Disks in Astrophysics, Ann. Rev. Astron. Astrophys. **19**, 137

  Concise review of classical accretion disk theory.

Eddington luminosity, I

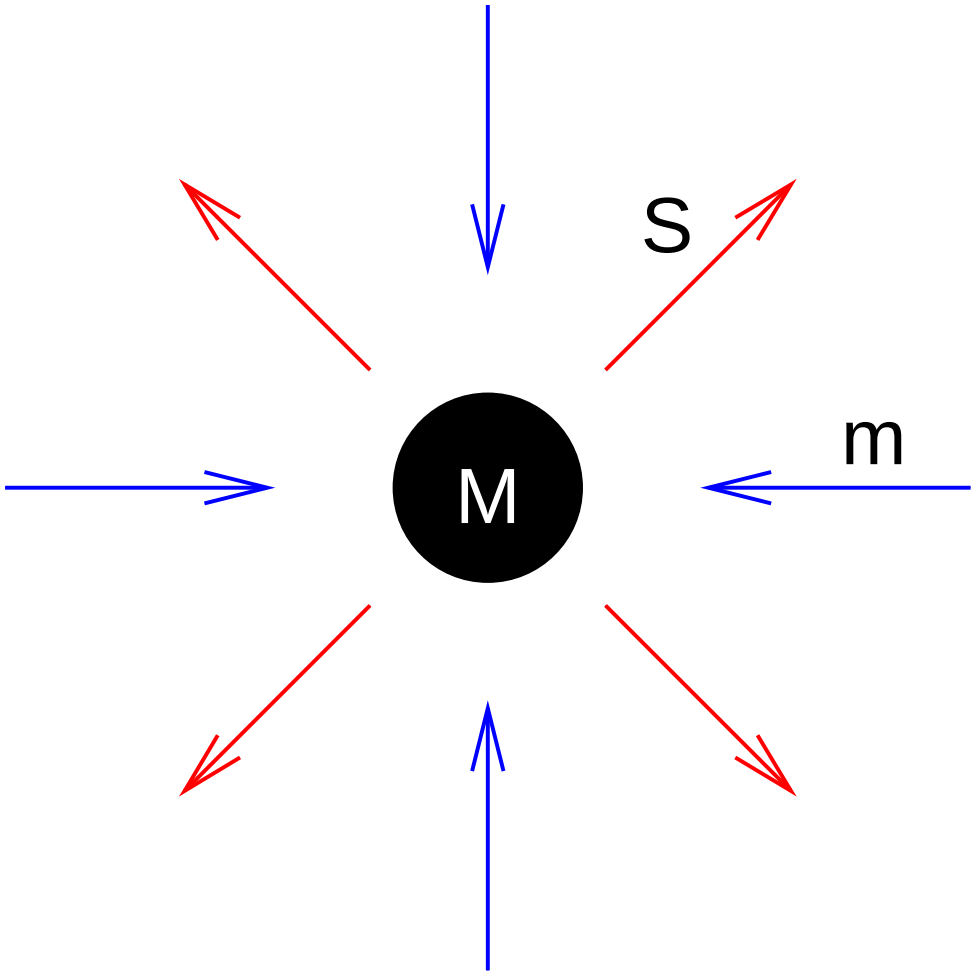
Assume mass M

Eddington luminosity, II



Assume mass M spherically symmetrically accreting ionized hydrogen gas.

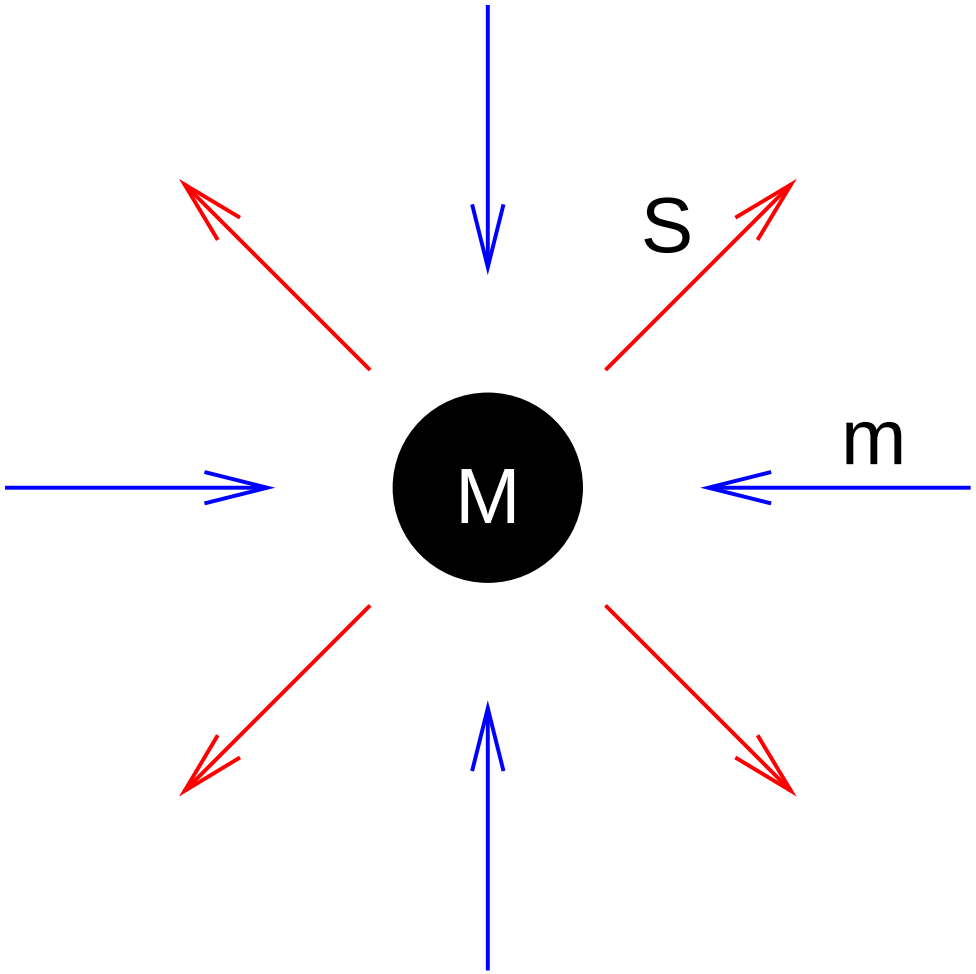
Eddington luminosity, III



Assume mass M spherically symmetrically accreting ionized hydrogen gas.

At radius r, accretion produces energy flux S.

Eddington luminosity, IV



Assume mass M spherically symmetrically accreting ionized hydrogen gas.

At radius r, accretion produces energy flux S.

Important: Interaction between accreted material and radiation!

Eddington luminosity, V

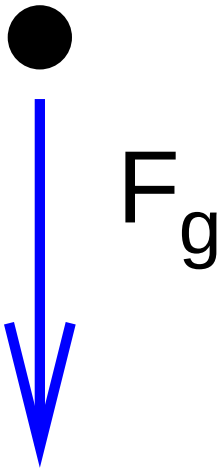
Force balance on accreted electrons and protons:

Eddington luminosity, VI

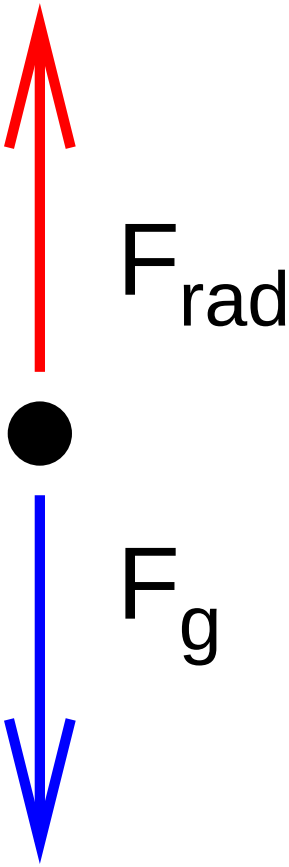
Force balance on accreted electrons and protons:

Inward force: gravitation:

$$F_{\mathrm{g}} = \frac{GMm_{\mathrm{p}}}{r^2}$$



Eddington luminosity, VII



Force balance on accreted electrons and protons:

Inward force: gravitation:

$$F_{\mathrm{g}} = \frac{GMm_{\mathrm{p}}}{r^2}$$

Outward force: radiation force:

$$F_{\mathrm{rad}} = \frac{\sigma_{\mathrm{T}} S}{c}$$

where energy flux S is given by

$$S = \frac{L}{4\pi r^2}$$

where L: luminosity.

Eddington luminosity, VIII

Force balance on accreted electrons and protons:

Inward force: gravitation:

$$F_{\mathrm{g}} = \frac{GMm_{\mathrm{p}}}{r^2}$$

Outward force: radiation force:

$$F_{\mathrm{rad}} = \frac{\sigma_{\mathrm{T}} S}{c}$$

where energy flux S is given by

$$S = \frac{L}{4\pi r^2}$$

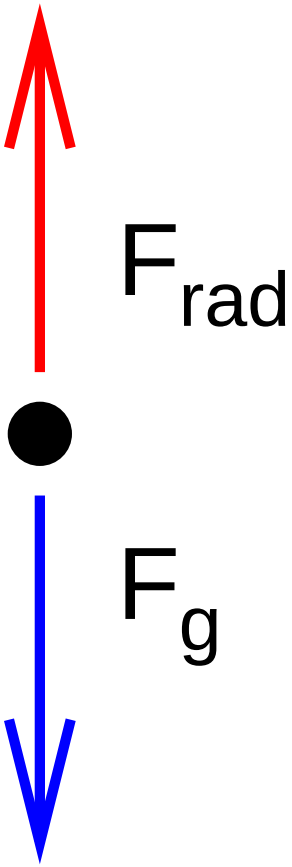
where L: luminosity.

Note: $\sigma_{\mathrm{T}} \propto (m_{\mathrm{e}}/m_{\mathrm{p}})^2$, so negligable for protons.

But: strong Coulomb coupling between electrons and protons

$\Longrightarrow F_{\mathrm{rad}}$ also has effect on protons!

Eddington luminosity, IX



Accretion is only possible if gravitation dominates:

$$\frac{GMm_\mathrm{p}}{r^2} > \frac{\sigma_\mathrm{T} S}{c} = \frac{\sigma_\mathrm{T}}{c} \cdot \frac{L}{4\pi r^2}$$

and therefore

$$L < L_\mathrm{Edd} = \frac{4\pi GMm_\mathrm{p}c}{\sigma_\mathrm{T}}$$

or, in astronomically meaningful units

$$L < 1.3 \times 10^{38}\,\mathrm{erg\,s^{-1}} \cdot \frac{M}{M_\odot}$$

where L_Edd is called the Eddington luminosity.

But remember the assumptions entering the derivation: spherically symmetric accretion of fully ionized pure hydrogen gas.

Eddington luminosity, X

Characterize accretion process through the accretion effi ciency, η:

$$L = \eta \cdot \dot{M} c^2$$

where $\dot{M}$: mass accretion rate (e.g., g s^{-1} or $M_\odot$ yr^{-1}).

Therefore maximum accretion rate ("Eddington rate"):

$$\dot{m} = \frac{L_{\rm Edd}}{\eta c^2} \sim 2 \cdot \left(\frac{M}{10^8\, M_\odot} \right)\ M_\odot\, {\rm yr}^{-1}$$

(for $\eta = 0.1$)

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{\text{rad}} \quad \Longrightarrow \quad T_{\text{rad}} = h\nu/k$$

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{\text{rad}} \quad \Longrightarrow \quad T_{\text{rad}} = h\nu/k$$

Optically thick medium: blackbody radiation

$$T_{\text{b}} = \left(\frac{L}{4\pi R^2 \sigma_{\text{SB}}} \right)^{1/4}$$

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{\text{rad}} \quad \Longrightarrow \quad T_{\text{rad}} = h\nu/k$$

Optically thick medium: blackbody radiation

$$T_{\text{b}} = \left(\frac{L}{4\pi R^2 \sigma_{\text{SB}}} \right)^{1/4}$$

Optically thin medium: L directly converted into radiation without further interactions $\Longrightarrow$ mean particle energy

$$T_{\text{th}} = \frac{GMm_{\text{p}}}{3kR}$$

Emitted spectrum

Characterize photon by its radiation temperature, T_{rad}:

$$h\nu \sim kT_{\text{rad}} \quad \Longrightarrow \quad T_{\text{rad}} = h\nu/k$$

Optically thick medium: blackbody radiation

$$T_{\text{b}} = \left(\frac{L}{4\pi R^2 \sigma_{\text{SB}}} \right)^{1/4}$$

Optically thin medium: L directly converted into radiation without further interactions $\Longrightarrow$ mean particle energy

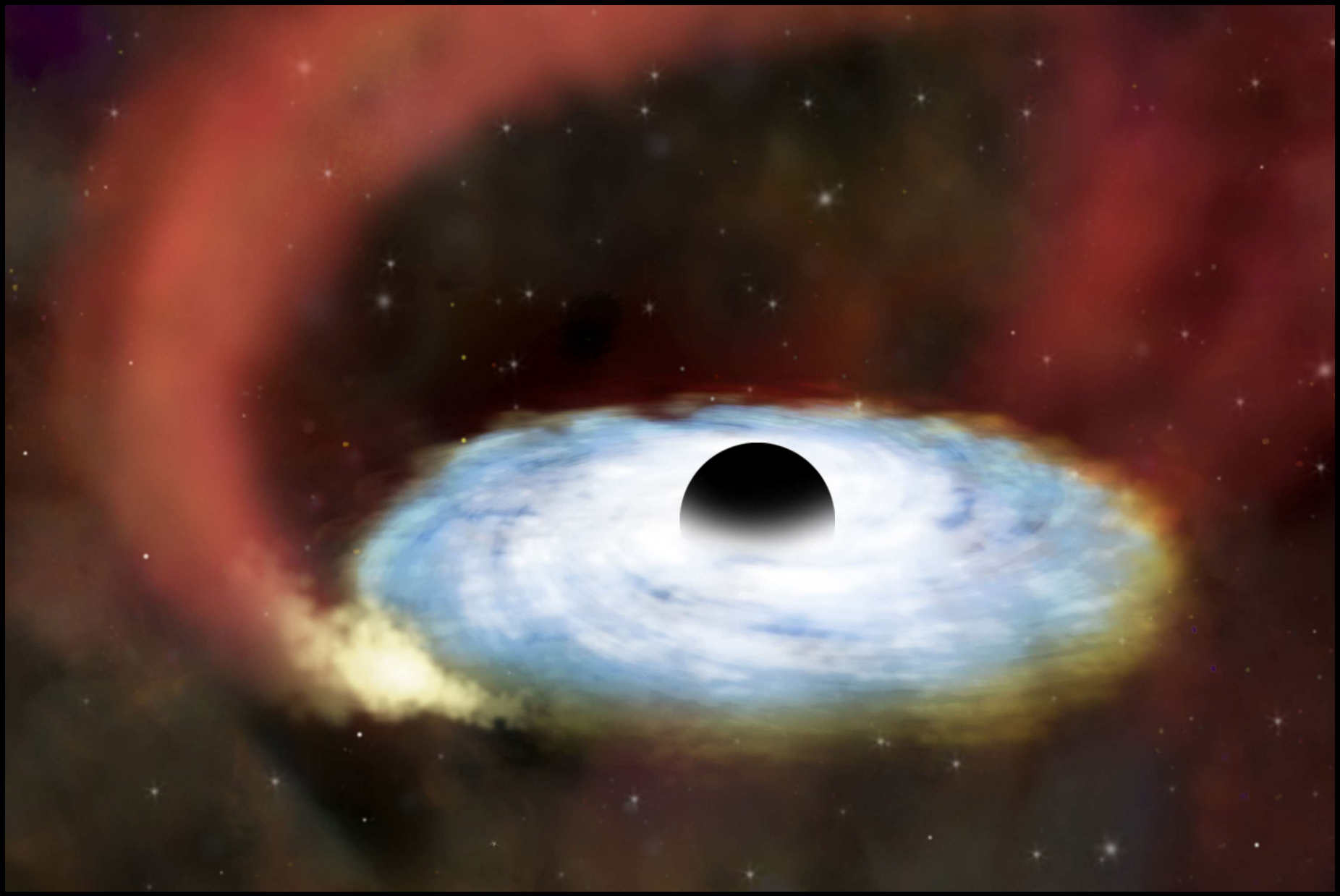
$$T_{\text{th}} = \frac{GMm_{\text{p}}}{3kR}$$

Plugging in numbers for a typical solar mass compact object (NS/BH):

$T_{\text{rad}} \sim 1\,\text{keV}$ and $T_{\text{bb}} \sim 50\,\text{MeV}$

Accreting objects are broadband emitters in the X-rays and gamma-rays.

CXC/MPE/S. Komossa



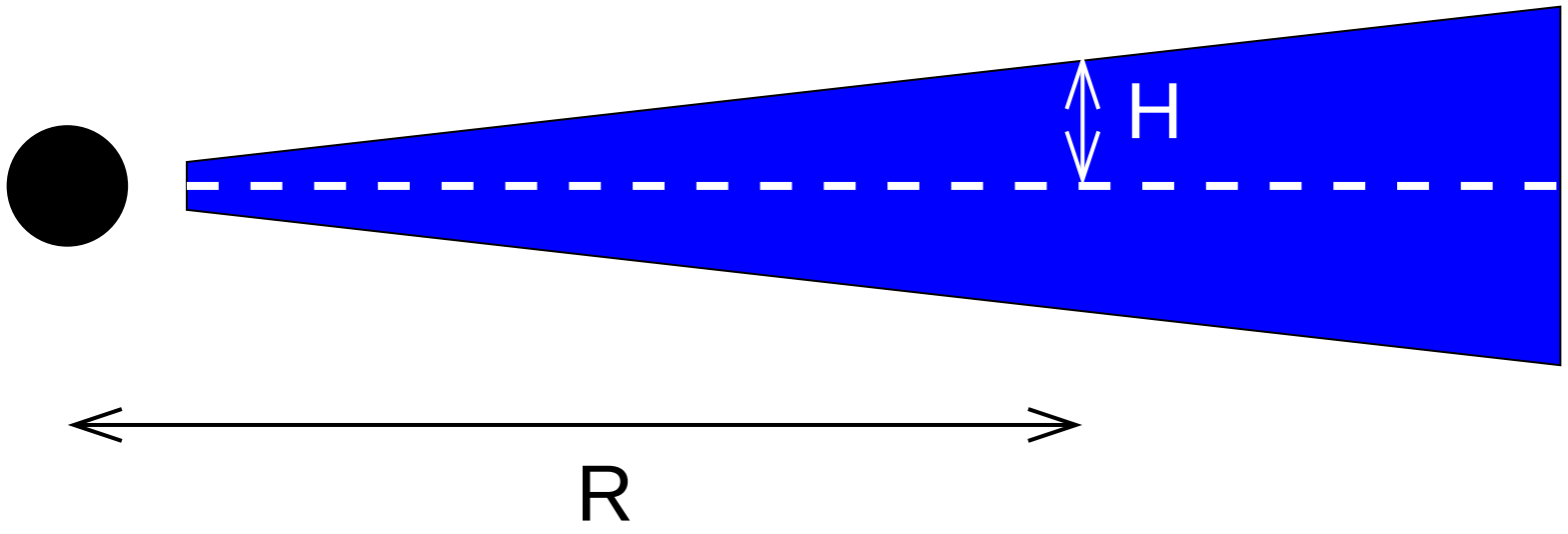
NASA/CXC/SAO

Source of matter: probably disrupted stars

$\Longrightarrow$ accreted matter has angular momentum

$\Longrightarrow$ accretion disk forms.

Thin Disks, I



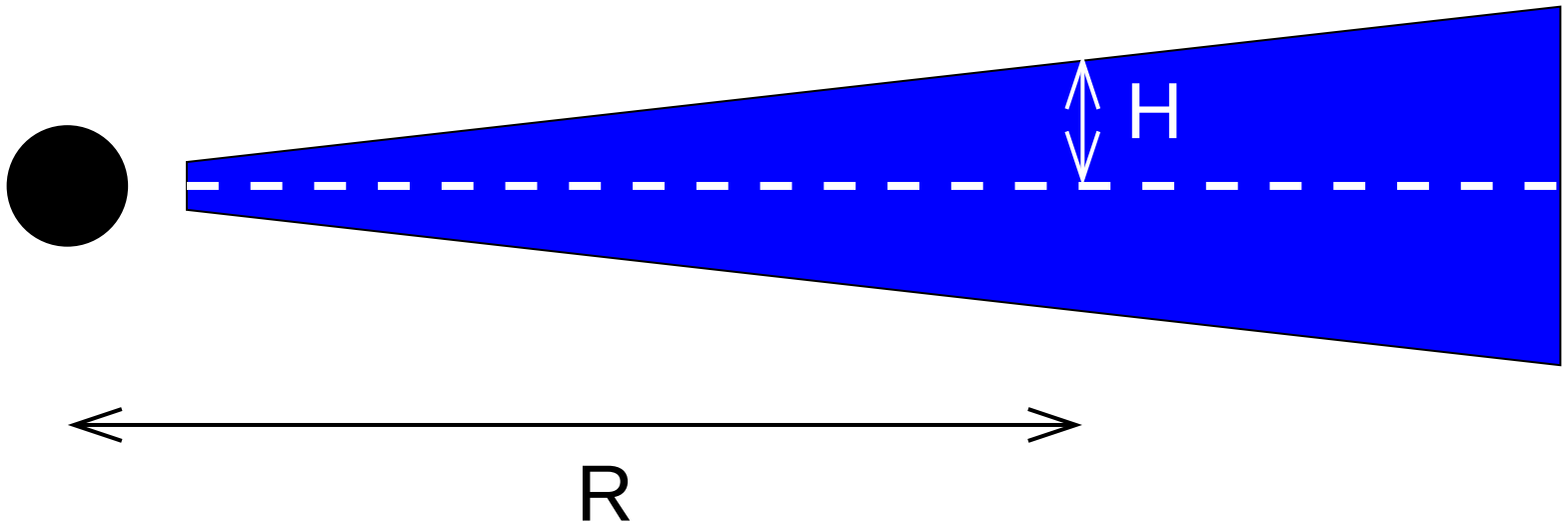
Most important case: thin accretion disks, i.e., vertical thickness, H, much smaller than radius R:

$$H \ll R$$

$\Longrightarrow$ Requires that radiation pressure is negligable

$\Longrightarrow$ $L \ll L_{\text{Edd}}$

Thin Disks, II



Thin assumption: no radiation pressure

$\Longrightarrow$ gas pressure must support disk vertically against gravitation:

$$\frac{GM}{R^2}\frac{H}{R} = \frac{1}{\rho}\left|\frac{\partial P}{\partial z}\right| \sim \frac{P_\mathrm{c}}{\rho_\mathrm{c} H}$$

where P_c characteristic pressure, ρ_c characteristic density.

Thin Disks, III

Because the speed of sound is

$$c_s^2 = \frac{P}{\rho}$$

the condition for vertical support can be written as

$$\frac{GM}{R^2}\frac{H}{R} \sim \frac{P_c}{\rho_c H} = \frac{c_s^2}{H}$$

Therefore

$$c_s^2 = \frac{GM}{R}\frac{H^2}{R^2} = v_\phi^2 \cdot \frac{H^2}{R^2}$$

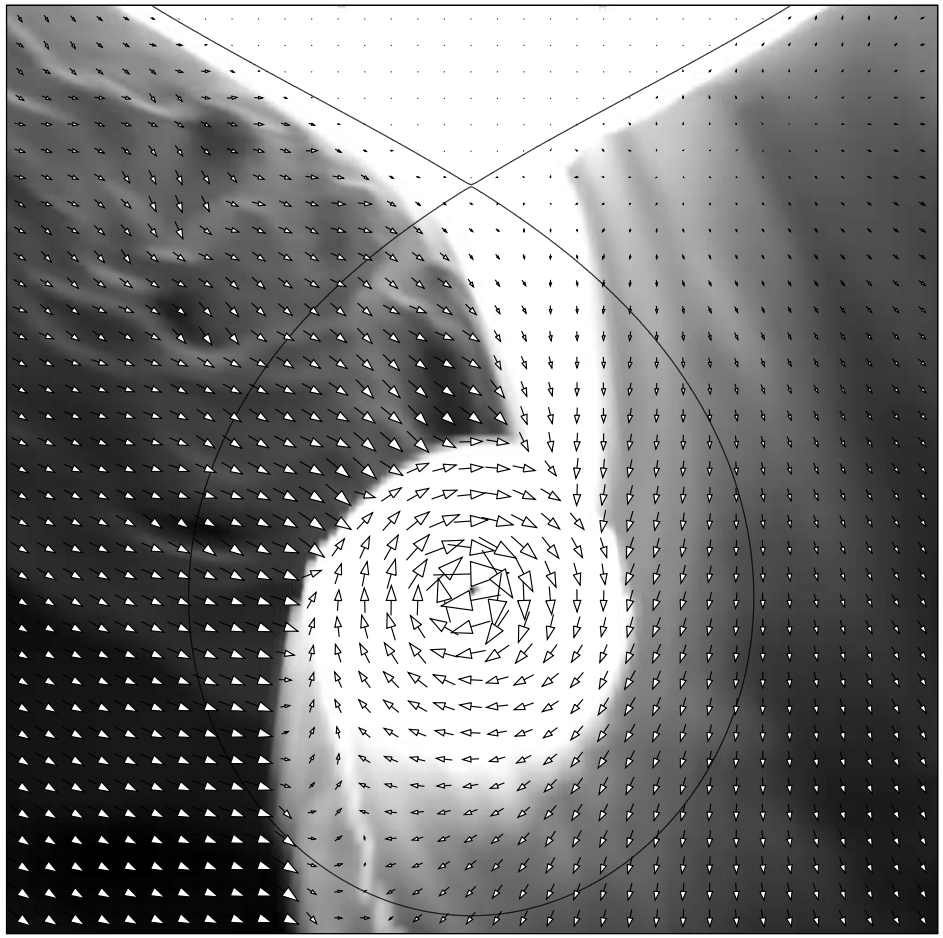
where $v_\phi = \sqrt{GM/R}$: Kepler speed.

Since $H/R \ll 1$:

$$c_s \ll v_\phi$$

Thin accretion disks are highly supersonic.

Thin Disks: Radial Structure



J. Blondin (priv. comm.; calculations for stellar accretion)

Radial acceleration due to pressure:

$$\frac{1}{\rho}\frac{\partial P}{\partial R} \sim \frac{P_c}{\rho_c R} \sim \frac{c_s^2}{R} \sim \frac{GM}{R^2}\frac{H^2}{R^2} \ll \frac{GM}{R^2}$$

$\Longrightarrow$ radial acceleration due to pressure negligable compared to gravitational acceleration

Thin disk: fluid motion is Keplerian to very high degree of precision.

$\Longrightarrow$ for the radial velocity, v_R: $v_R \ll v_\phi$

Thin Disks: Vertical Structure and Mass Conservation

Amount of mass crossing radius R:

$$\dot{M} = -2\pi R \cdot \Sigma \cdot v_R$$

where Σ: surface density of disk,

$$\Sigma(R) = \int n(r) dz$$

and where $\dot{M}$: mass accretion rate

Since acceleration $\perp z$

$$F_z \propto \frac{GM}{R^2}\frac{z}{R} \propto z$$

vertical density profi le

$$n(z) \propto \exp\left(-\frac{z}{H}\right)$$

where H: scale height (depends on details of accretion disk theory).

Thin Disks: Angular Momentum Transport, I

Most important question: angular momentum transport

Angular velocity in Keplerian disk:

$$\Omega(R) = \left(\frac{GM}{R^3}\right)^{1/2}$$

("differential rotation")

$\Longrightarrow$ angular momentum per mass ("specifi c angular momentum"):

$$\mathcal{L} = R \cdot v = R \cdot R\Omega(R) = R^2\,\Omega(R) \propto R^{1/2}$$

$\Longrightarrow$ decreases with decreasing R!

Total angular momentum lost when mass moves in unit time from $R + dR$ to R:

$$\frac{dL}{dR} = \dot{M} \cdot \frac{d(R^2\Omega(R))}{dR}$$

Thin Disks: Angular Momentum Transport, II

Since L changes: accreting matter needs to lose angular momentum. This is done by viscous forces excerting torques:

Force due to viscosity per unit length:

$$\mathcal{F} = \nu\Sigma \cdot \Delta v = \nu\Sigma \cdot R\frac{d\Omega}{dR}$$

where ν: coeffi cient of kinematic viscosity

Therefore total torque

$$G(R) = 2\pi R\mathcal{F} \cdot R = \nu\Sigma 2\pi R^3 \left(\frac{d\Omega}{dR}\right)$$

and the net torque acting on a ring is

$$\frac{dG}{dR}dR$$

$\Longrightarrow$ This net torque needs to balance change in specifi c angular momentum in disk.

Thin Disks: Angular Momentum Transport, III

Balancing net torque and angular momentum loss gives:

$$\dot{M}\frac{d(R^2\Omega)}{dR} = -\frac{d}{dR}\left(\nu\Sigma 2\pi R^3\frac{d\Omega}{dR}\right)$$

Insert $\Omega(R) = (GM/R^3)^{1/2}$ and integrate:

$$\nu\Sigma R^{1/2} = \frac{\dot{M}}{3\pi}R^{1/2} + \text{const.}$$

const. obtained from no torque boundary condition at inner edge of disk at $R = R_*$: $dG/dR(R_*) = 0$, such that

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

Therefore the viscous dissipation rate per unit area is

$$D(R) = \nu\Sigma\left(R\frac{d\Omega}{dR}\right)^2 = \frac{3GM\dot{M}}{4\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

Thin Disks: Temperature Profile, I

The viscous dissipation rate was

$$D(R) = \nu\Sigma \left(R\frac{d\Omega}{dR} \right)^2 = \frac{3GM\dot{M}}{4\pi R^3} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right]$$

If disk is optically thick: Thermalization of dissipated energy

$\Longrightarrow$ Temperature from Stefan-Boltzmann-Law:

$$2\sigma_{\mathrm{SB}} T^4 = D(R)$$

(disk has *two* sides!) and therefore

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma_{\mathrm{SB}}} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

Thin Disks: Temperature Profile, II

Inserting astrophysically meaningful numbers:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma_{\mathrm{SB}}} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

$$= 6.8 \times 10^5\,\mathrm{K} \cdot \eta^{-1/4} \left(\frac{L}{L_{\mathrm{Edd}}} \right)^{1/2} L_{46}^{-1/4} \mathcal{R}^{1/4} x^{-3/4}$$

where $\eta = L_{\mathrm{Edd}}/\dot{M}_{\mathrm{Edd}}c^2$, $x = c^2 R/2GM$, $\mathcal{R} = (1 - (R_*/R)^{1/2})$.

Thin Disks: Temperature Profile, III

Inserting astrophysically meaningful numbers:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma_{\rm SB}} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

$$= 6.8 \times 10^5\,{\rm K} \cdot \eta^{-1/4} \left(\frac{L}{L_{\rm Edd}} \right)^{1/2} L_{46}^{-1/4} \mathcal{R}^{1/4} x^{-3/4}$$

where $\eta = L_{\rm Edd}/\dot{M}_{\rm Edd}c^2$, $x = c^2 R/2GM$, $\mathcal{R} = (1 - (R_*/R)^{1/2})$.

Radial dependence of T:

$$T(R) \propto R^{-3/4}$$

Thin Disks: Temperature Profile, IV

Inserting astrophysically meaningful numbers:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma_{\mathrm{SB}}} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

$$= 6.8 \times 10^5\,\mathrm{K} \cdot \eta^{-1/4} \left(\frac{L}{L_{\mathrm{Edd}}} \right)^{1/2} L_{46}^{-1/4} \mathcal{R}^{1/4} x^{-3/4}$$

where $\eta = L_{\mathrm{Edd}}/\dot{M}_{\mathrm{Edd}}c^2$, $x = c^2 R/2GM$, $\mathcal{R} = (1 - (R_*/R)^{1/2})$.

Radial dependence of T:

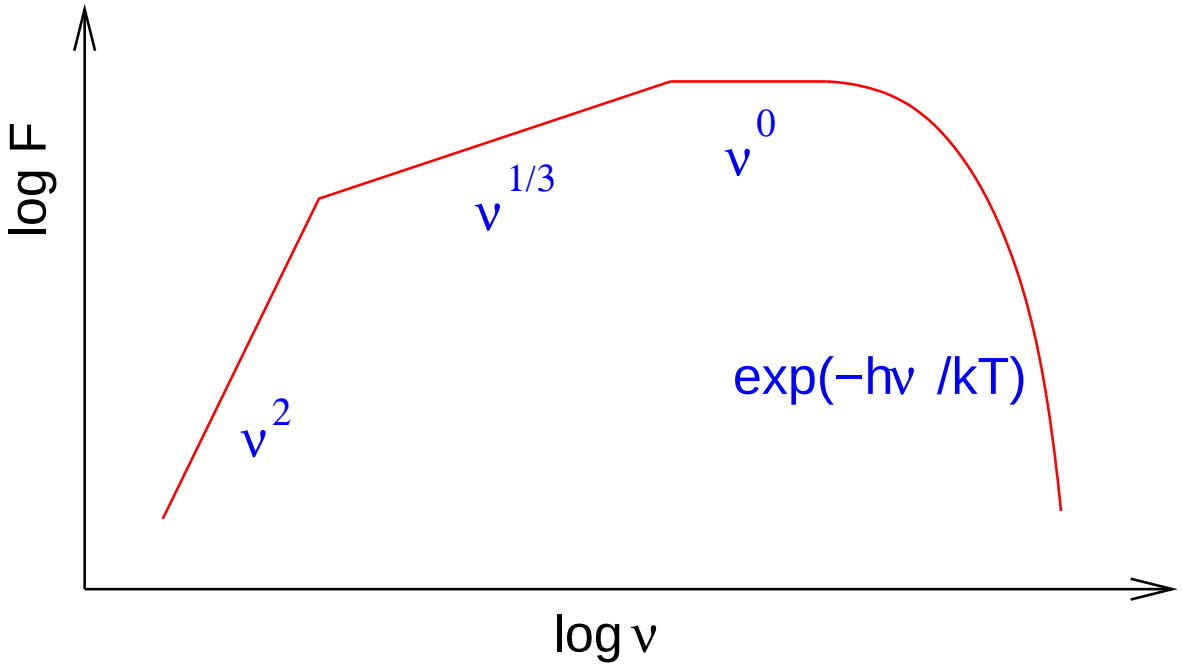
$$T(R) \propto R^{-3/4}$$

Dependence on mass (note: for NS/BH inner radius $R_* \propto M$!):

$$T_{\mathrm{in}} \propto (\dot{M}/M^2)^{1/4}$$

$\Longrightarrow$ AGN disks are colder than disks around galactic BH

Thin Disks: Emitted Spectrum, I



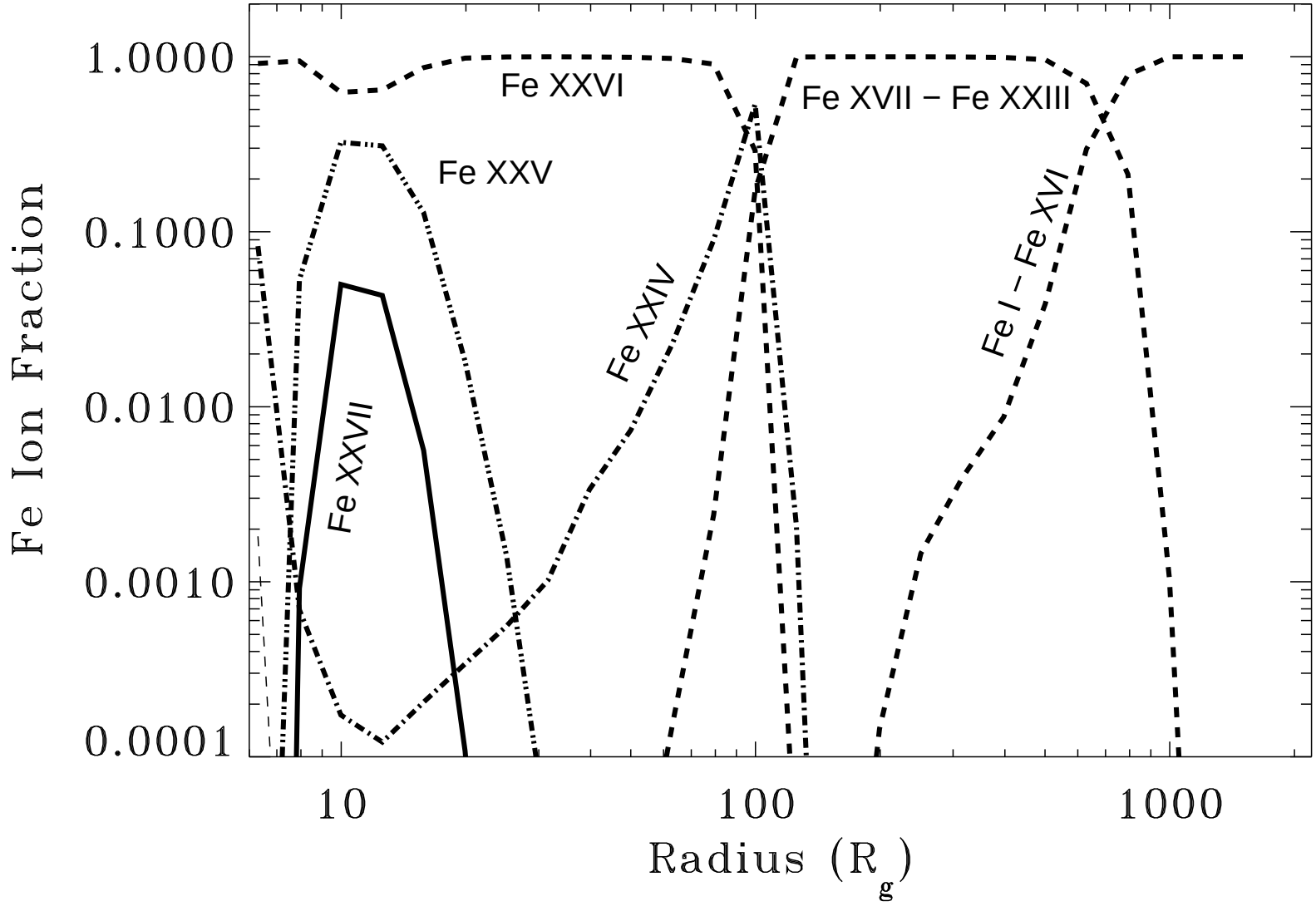
If disk is optically thick, then locally emitted spectrum is black body.

Total emitted spectrum obtained by integrating over disk

$$F_\nu = \int_{R_*}^{R_{\rm out}} B(T(R))\, 2\pi R\, dR$$

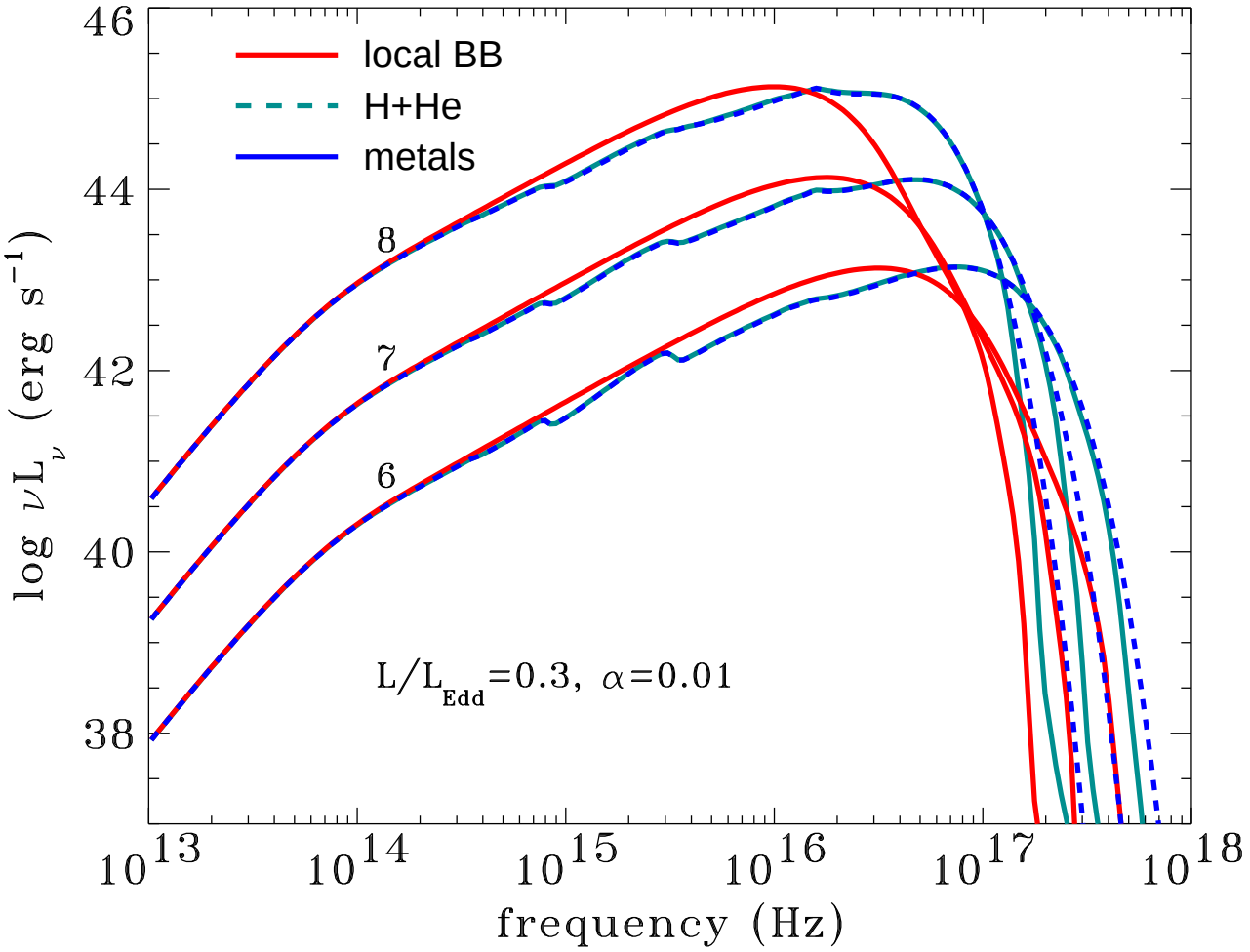
Resulting spectrum looks essentially like a stretched black body.

Thin Disks: Emitted Spectrum, II



Fe species in a disk around a Galactic BH (Davis et al., 2005, Fig. 6)

Thin Disks: Emitted Spectrum, III



Hubeny et al., 2001, Fig. 13

In reality: accretion disk spectrum depends on

- elemental composition (“metallicity”)
- viscosity (“α-parameter”)
- ionization state and luminosity of disk ($\dot{M}$)
- properties of compact object

and many further parameters

Until today: no really satisfactory disk model available.

Viscosity

Most important unknown in accretion disk theory: viscosity

even though it dropped out of $T(R)$!

Earth: viscosity of fluids typically due to molecular interactions (molecular viscosity).

Kinematic viscosity:

$$\nu_{\mathrm{mol}} \sim \lambda_{\mathrm{mfp}} c_{\mathrm{s}}$$

where the mean free path

$$\lambda_{\mathrm{mfp}} \sim \frac{1}{n\sigma} \sim 6.4 \times 10^4 \left(\frac{T^2}{n}\right) \mathrm{cm}$$

and the speed of sound

$$c_{\mathrm{s}} \sim 10^4 T^{1/2} \,\mathrm{cm\,s^{-1}}$$

such that

$$\nu_{\mathrm{mol}} \sim 6.4 \times 10^8 \, T^{5/2} n^{-1} \,\mathrm{cm^2\,s^{-1}}$$

Viscosity

Viscosity important when Reynolds number small ("laminar flow"), where

$$\mathrm{Re} = \frac{\text{inertial force}}{\text{viscous force}} \sim \frac{\rho R v}{\rho \nu} = \frac{R v}{\nu}$$

Follows from Navier-Stokes Equations

Using typical accretion disk parameters:

$$\mathrm{Re}_{\mathrm{mol}} \sim 2 \times 10^{14} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{10^{10}\,\mathrm{cm}}\right)^{1/2} \left(\frac{n}{10^{15}\,\mathrm{cm}^{-3}}\right) \left(\frac{T}{10^{4}\,\mathrm{K}}\right)^{-5/2}$$

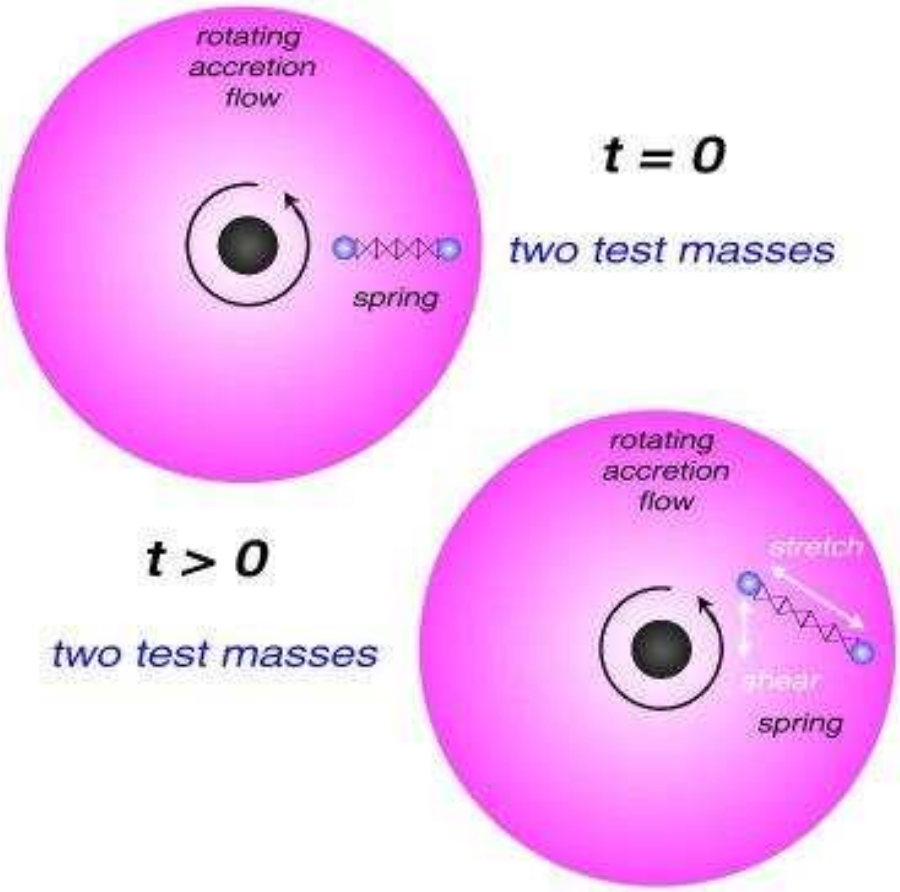
$\Longrightarrow$ Molecular viscosity is irrelevant for astrophysical disks!

since $\mathrm{Re} \gtrsim 10^3$: turbulence $\Longrightarrow$ Shakura & Sunyaev posit turbulent viscosity

$$\nu_{\mathrm{turb}} \sim v_{\mathrm{turb}} \ell_{\mathrm{turb}} \sim \alpha\, c_{\mathrm{s}} \cdot H$$

where $\alpha \lesssim 1$ and $\ell_{\mathrm{turb}} \lesssim H$ typical size for turbulent eddies.

Viscosity



R. Müller

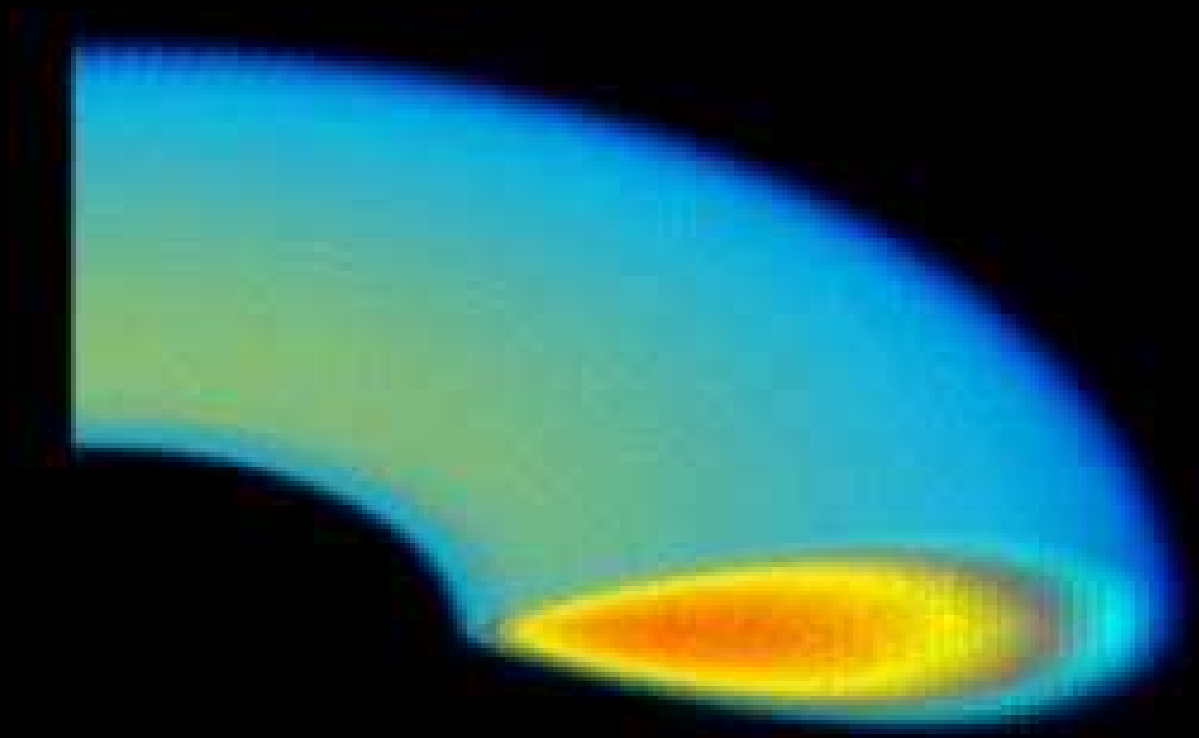
Mechanical analogy of MRI: spring in differentially rotating medium.

Physics of turbulent viscosity is unknown, however, α prescription yields good agreement between theory and observations.

Possible origin: Magnetorotational instability (MRI): MHD instability amplifying B-fi eld inhomogeneities caused by small initial radial displacements in accretion disk

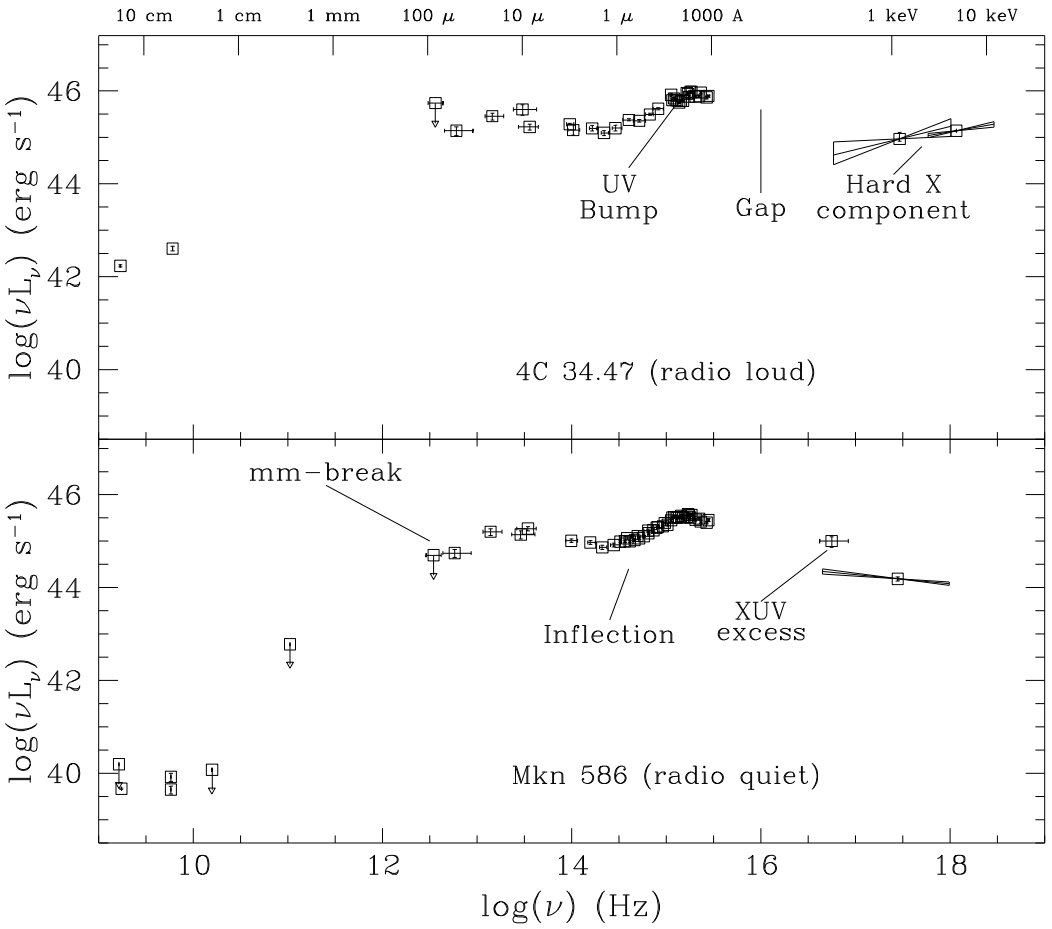
$\Longrightarrow$ angular momentum transport

(Balbus & Hawley 1991, going back to Velikhov 1959 and Chandrasekhar (1961).



(Hawley & Krolik, 2002)

Accretion Disks in AGN, I

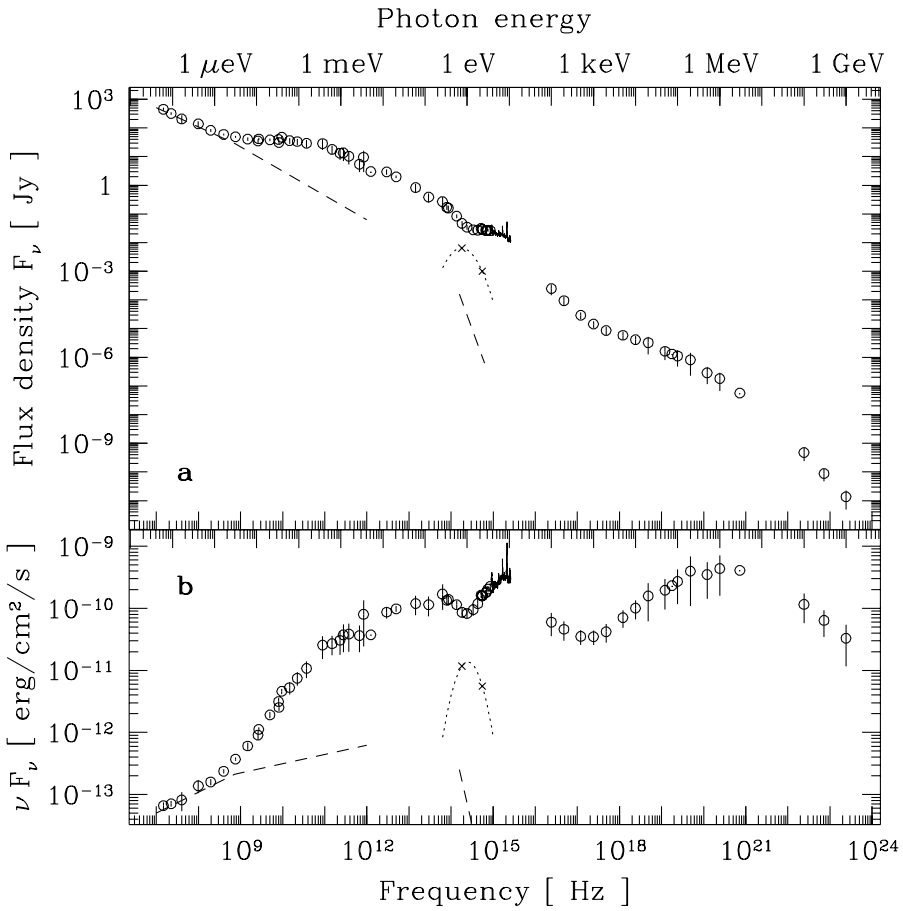


Spectral Energy Distribution of radio-loud and radio-quiet AGN (Elvis et al., 1994)

Big Blue Bump: Excess radiation in ∼UV range $\Longrightarrow$ disk?

IR Bump: Excess radiation in ∼IR range $\Longrightarrow$ dust? (peak T: 2000 K; dust sublimation?)

Accretion Disks in AGN, II

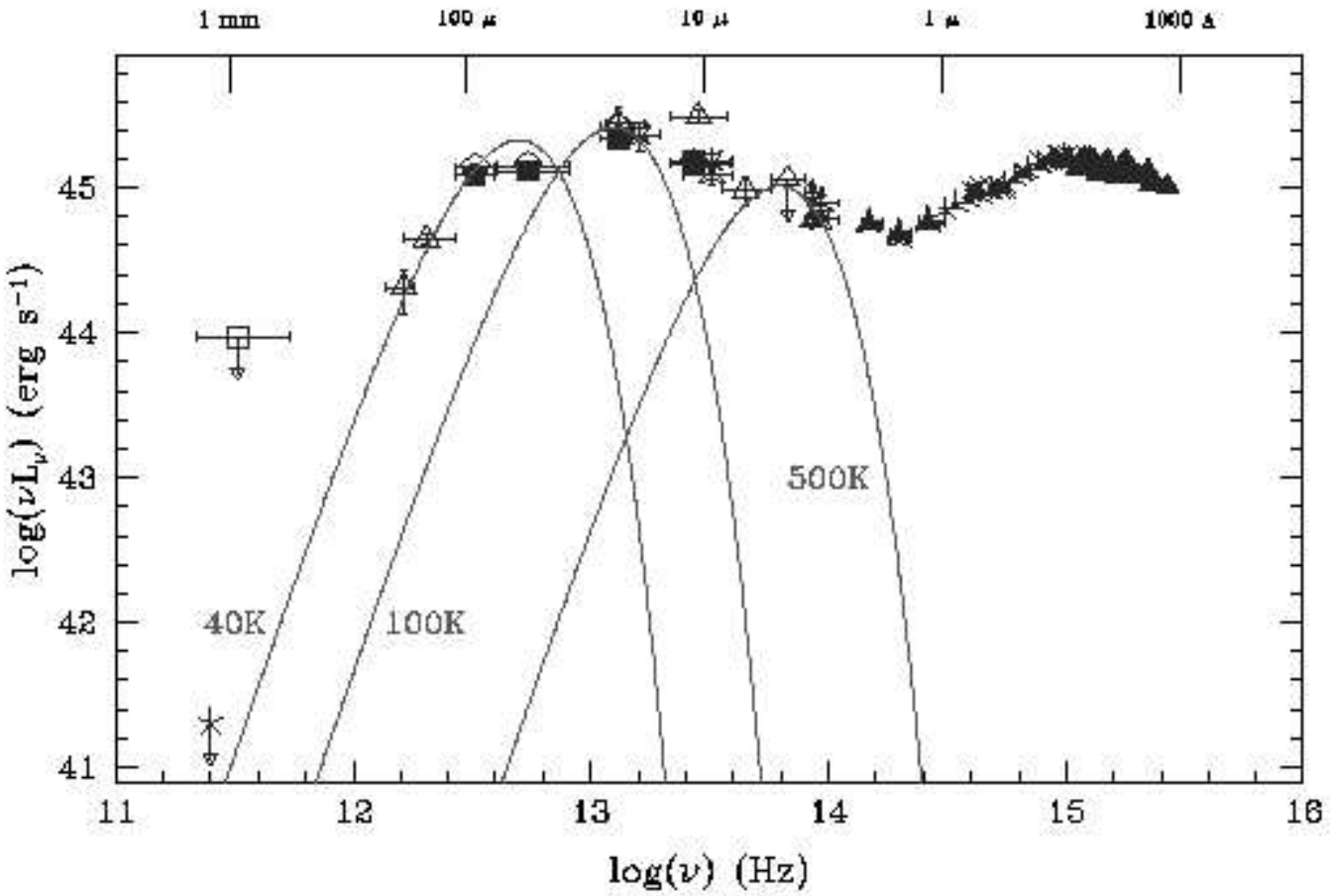


Spectral Energy Distribution of 3C273 (Türler et al., 1999)

Big Blue Bump: Excess radiation in ∼UV range $\Longrightarrow$ disk?

IR Bump: Excess radiation in ∼IR range $\Longrightarrow$ dust? (peak T: 2000 K; dust sublimation?)

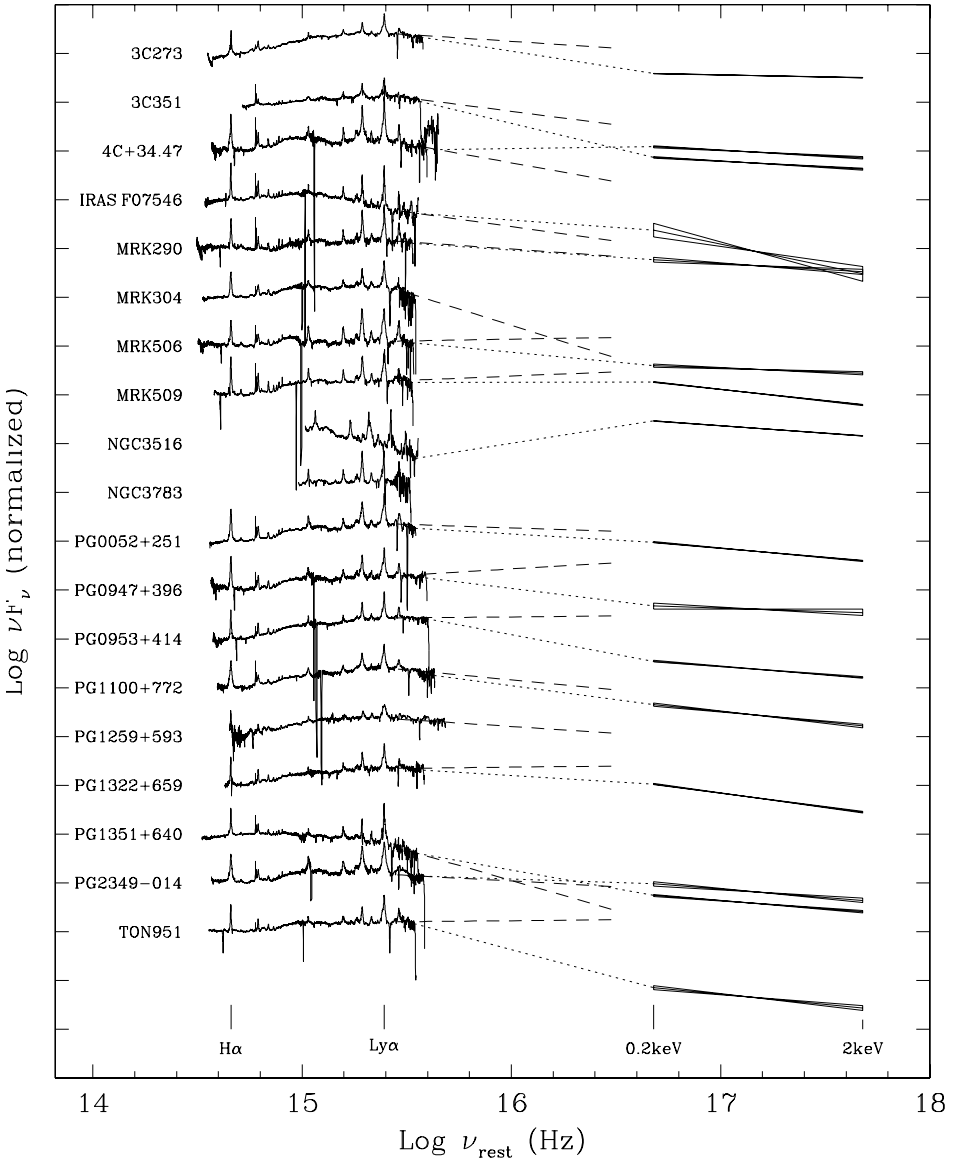
IR Bump



mm–optical SED of PG1351+640: dust has wide range of temperatures (Wilkes, 2004).

IR-Bump: too cold for disk, has substructure $\Longrightarrow$ different emission regions.

UV Bump



In *some* AGN: extrapolated UV power law smoothly matches X-ray continuum.

Remember: $f_\nu \propto \nu^{-\alpha}$

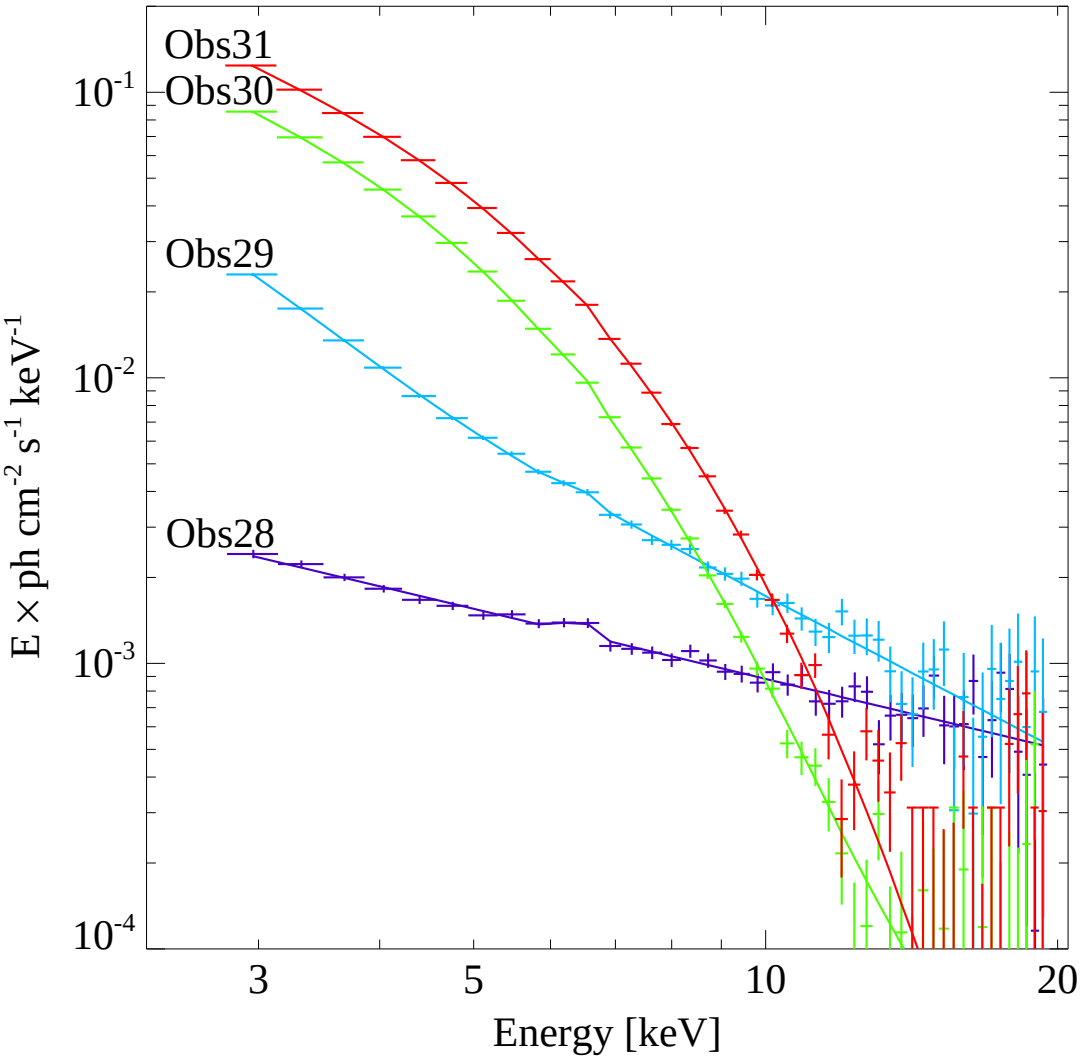
Break wavelengh between 800 and 1600 Å, in rough agreement with accretion disk models.

Theory of the break: H-Lyman edge, possibly smeared by Comptonization or relativistic effects.

However: no correlation between UV slope and BH mass as expected from accretion disk models?!?

(Shang et al., 2005)

Galactic Black Holes



LMC X-3, (Wilms et al., 2001)

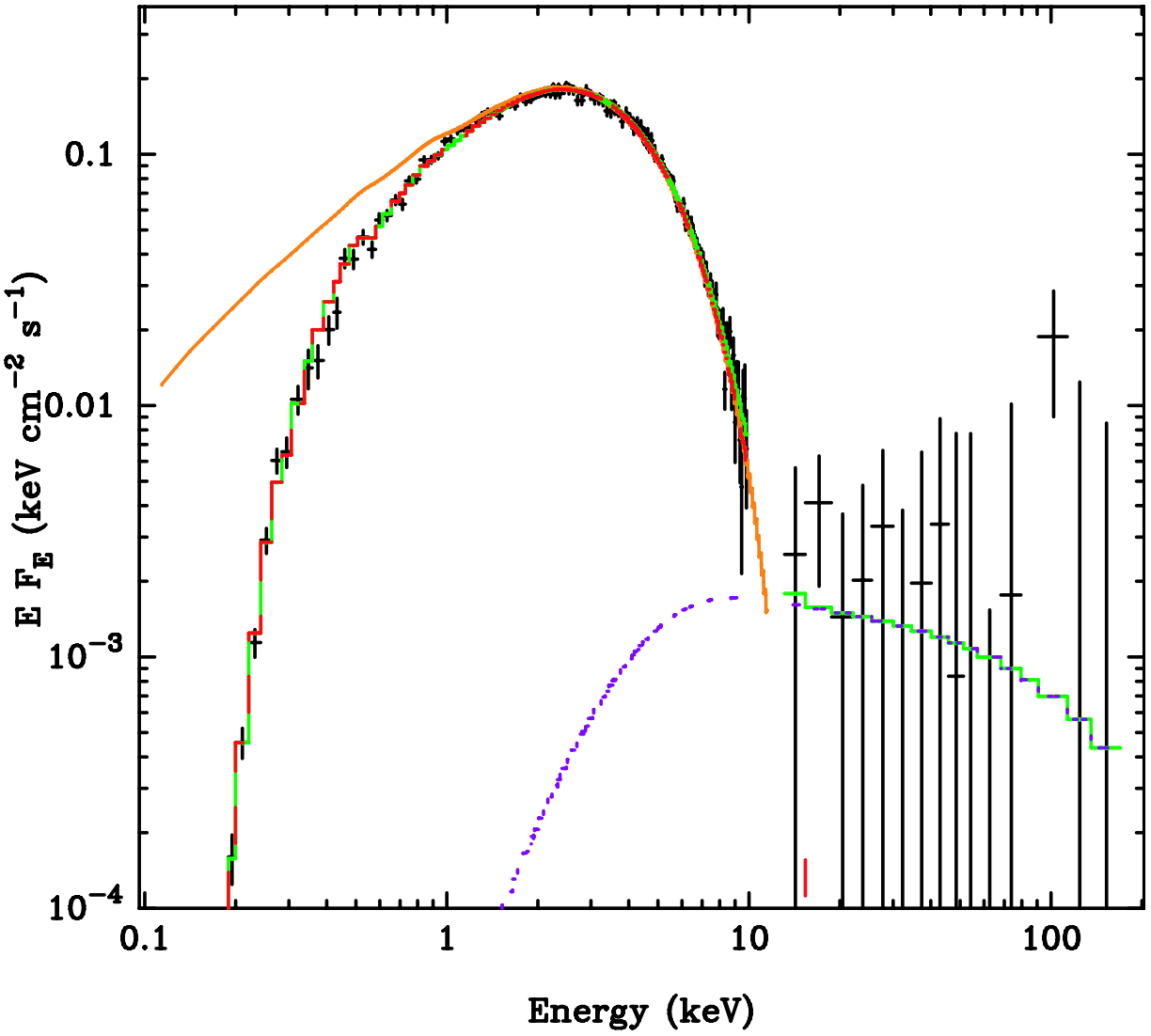
Problem with AGN: peak of disk in UV

$\Longrightarrow$ Galactic Black Holes: T is higher

Find ok agreement between accretion disk models and theory.

In general: models with just $T \propto r^{-3/4}$ and no additional (atomic) physics seem to work best?!?

Galactic Black Holes



(Davis, Done & Blaes, 2006)

Comparison of self-consistent accretion disk model with LMC X-3 data $\Longrightarrow$ good agreement, although values of α smaller than expected (fi ts fi nd $0.01 < \alpha < 0.1$ instead of 0.1–0.8).

Top red line: inferred accretion disk spectrum without interstellar absorption.

Balbus, S. A., & Hawley, J. F., 1991, ApJ, 376, 214

Chandrasekhar, S., 1961, Hydrodynamic and Hydromagnetic Stability, (Oxford: Oxford Univ. Press), (reprinted 1981 by Dover, New York)

Davis, S. W., Blaes, O. M., Hubeny, I., & Turner, N. J., 2005, ApJ, 621, 372

Davis, S. W., Done, C., & Blaes, O. M., 2006, ApJ, 647, 525

Elvis, M., et al., 1994, ApJS, 95, 1

Hawley, J. F., & Krolik, J. H., 2002, ApJ, 566, 164

Shang, Z., et al., 2005, ApJ, 619, 41

Türler, M., et al., 1999, A&AS, 134, 89

Velikhov, E. P., 1959, Sov. Phys. – JETP, 9, 995

Wilkes, B., 2004, in AGN Physics with the Sloan Digital Sky Survey, ed. G. T. Richards, P. B. Hall, 37

Wilms, J., Nowak, M. A., Pottschmidt, K., Heindl, W. A., Dove, J. B., & Begelman, M. C., 2001, MNRAS, 320, 327