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# X-ray Continuum Emission and Broad Iron Lines



## Introduction

AGN have power law continua.

Purpose of this lecture: investigate physical origin of the continuum emission. Structure:

- 1. Compton Scattering and Comptonization
- 2. Source of hot electrons
- 3. X-ray Reflection
- 4. Relativistic Broadened Fe K $\alpha$  Lines



### AGN X-Ray Continua



$$\boldsymbol{F} = m_{\mathbf{e}} \boldsymbol{\dot{v}} = q E_0 \sin \omega_0 t \,\boldsymbol{\epsilon} \tag{6.1}$$

This neglects the B-field, i.e., assumes  $v \ll c.$ 

 $\Longrightarrow$  The electron feels an acceleration,  $\dot{v}$ , and therefore it radiates!



## Thomson Scattering, II

The power radiated by an accelerated charge in direction  $\Theta$  through the spherical angle  $d\Omega$  is given by Larmor's formula:

$$\frac{dP}{d\Omega}(\Theta) = \frac{1}{16\pi^2 c^3 \epsilon_0} q^2 \dot{v}^2 \sin^2 \Theta$$
(6.2)

Integrating Eq. (6.2) over  $4\pi$  sr gives

$$D = \frac{q^2 \dot{v}^2}{6\pi c^3 \epsilon_0} \tag{6.3}$$

For the case the charge is accelerated by an (sinusoidally varying) electric field E(t) one finds after a longish calculation:

$$\frac{dP}{d\Omega} = \frac{q^4 E_0^2}{\mathbf{16}\pi^2 m^2 c^3 \epsilon_0} \sin^2 \Theta \quad \text{and} \quad P = \frac{q^4 E_0^2}{\mathbf{12}\pi c^3 m^2 \epsilon_0} \tag{6.4}$$

**Compton Scattering** 

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Thomson Scattering, III

The incident flux on the electron (i.e.,  $c \times$  energy density for radiation) is

$$\langle \boldsymbol{S} \rangle = \frac{c\epsilon_0}{2} E_0^2 \tag{6.5}$$

Define the differential cross section for Thomson scattering,  $d\sigma/d\Omega$ , such that

$$\frac{dP}{d\Omega} = \langle \boldsymbol{S} \rangle \frac{d\sigma}{d\Omega} \quad \iff \quad \frac{q^4 E_0^2}{\mathbf{16}\pi^2 m^2 c^3 \epsilon_0} \sin^2 \Theta = \frac{c \epsilon_0^2}{\mathbf{2}} E_0^2 \frac{d\sigma}{d\Omega} \tag{6.6}$$

such that

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{polarized}} = \frac{q^4}{8\pi^2 m^2 c^4 \epsilon_0^2} \sin^2 \Theta = r_0^2 \sin^2 \Theta$$
(6.7)

with the classical electron radius

$$r_0 = \frac{e^2}{4\pi m_e c^2 \epsilon_0} = 2.82 \times 10^{-15} \,\mathrm{m} \tag{6.8}$$

## Thomson Scattering, IV

The differential cross section  $d\sigma/d\Omega$  is the area presented by the electron to a photon that is going to get scattered in direction  $d\Omega$ .

The total cross section for Thomson scattering,  $\sigma_{\rm T}$ , is then obtained from the differential cross section by integrating  $d\sigma/d\Omega$  from Eq. (6.7) over all angles:

$$P = \int \langle S \rangle \frac{d\sigma}{d\Omega} d\Omega = \langle S \rangle \int \frac{d\sigma}{d\Omega} d\Omega =: \langle S \rangle \sigma_{\mathsf{T}}$$
(6.9)

Performing the integration yields

$$\sigma_{\rm T} = \frac{8\pi}{3} r_0^2 = \frac{e^4}{6\pi m_{\rm e}^2 \epsilon_0^2 c^4} = 6.652 \times 10^{-25} \,{\rm cm}^2 \tag{6.10}$$

 $\sigma_{\rm T}$  is also called the Thomson cross section.

Compton Scattering



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$$E = \frac{1}{1 + \frac{E}{m_{e}c^{2}}(1 - \cos\theta)} \sim E \left(1 - \frac{1}{m_{e}c^{2}}(1 - \cos\theta)\right)$$

$$-\lambda = \frac{h}{m_{e}c}(1 - \cos\theta)$$
(6.14)

where  $h/m_{\rm e}c = 2.426 \times 10^{-12}$  m (Compton wavelength).

Averaging over  $\theta$ , for  $E \ll m_e c$ :

 $\lambda'$ 

$$\frac{\Delta E}{E} \approx -\frac{E}{m_{\rm e}c^2} \tag{6.15}$$

E.g., at 6.4 keV,  $\Delta E \approx$  0.2 keV.

#### **Compton Scattering**

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(6.20)



The derivation of Eq. (6.13) is most simply done in special relativity using four-vectors. In the following, we will use capital letters for four-vectors and small letters for three-vectors. Furthermore, we will adopt the convention  $P \cdot Q = P_0 Q_0 - P_1 Q_1 - P_2 Q_2 - P_3 Q_3$ (6.16)

for the product of two four vectors, following, e.g., the convention of Rindler (1991, Introduction to Special Relativity). The four-momentum of a particle with non-zero rest-mass, m<sub>0</sub>, e.g., an electron, is

$$Q = m_0 \gamma \begin{pmatrix} c \\ v \end{pmatrix} = \begin{pmatrix} m_0 \gamma c \\ q \end{pmatrix}$$
(6.17)

where v is the velocity of the particle and q its momentum. As usual,  $\gamma = (1 - (v/c)^2)^{-1/2}$ . The square of Q is

$$\mathbf{Q}^2 = m_0^2 \gamma^2 c^2 - m_0^2 \gamma^2 v^2 = m_0^2 c^2 \gamma^2 \left( \mathbf{1} - \left(\frac{v^2}{c^2}\right) \right) = m_0^2 c^2$$
(6.18)

Obviously,  $Q^2$  is relativistically invariant.

$$P = \frac{E}{c} \begin{pmatrix} 1 \\ \hat{u} \end{pmatrix}$$
(6.19)

where  $\hat{u}$  is an unit-vector pointing into the direction of motion of the photon. Note that for photons

$$P^2 = 0$$

as the photon's rest-mass is zero.

We will now look at the collision between a photon and an electron. We will denote the four-momenta after the collision with primed quantities.

 $(\mathbf{P}$ 

$$P+Q=P'+Q' \tag{6.21}$$

We now use a trick from Lightman et al. (1975, Problem Book in Relativity and Gravitation), solving this equation for Q' and squaring the resulting expression:

$$+Q - P')^2 = (Q')^2$$
(6.22)

Since the collision is elastic, i.e., the rest mass of the electron is not changed by the collision,

$$Q^2 = (Q')^2 \tag{6.23}$$

$$oldsymbol{P}^2=(oldsymbol{P}')^2=0$$
, such that

 $P \cdot Q - P \cdot P' - Q \cdot P' = 0 \quad \iff \quad P \cdot P' = Q \cdot (P - P') \tag{6.24}$ 

But in the frame where the electron is initially at rest,

furthermore

$$Q \cdot (P - P') = m_e c \left(\frac{E}{c} - \frac{E'}{c}\right) = m(E - E')$$
  
(6.25)

$$\boldsymbol{P} \cdot \boldsymbol{P}' = \frac{E}{c} \frac{E'}{c} (1 - \hat{\boldsymbol{u}} \cdot \hat{\boldsymbol{u}}') = \frac{EE'}{c^2} (1 - \cos\theta)$$
(6.26)

where  $\theta = \angle(\hat{u}, \hat{u}')$ . Inserting into Eq. (6.24) and solving for E' gives Eq. (6.13).



For unpolarized radiation,

$$\frac{d\sigma_{\rm es}}{d\Omega} = \frac{3}{16\pi} \sigma_{\rm T} \left(\frac{E'}{E}\right)^2 \left(\frac{E}{E'} + \frac{E'}{E} - \sin^2\theta\right) \tag{6.27}$$

(Klein-Nishina formula).



Compton Scattering

## Energy Exchange

For non-stationary electrons, use previous formulae and Lorentz transform photon into electron's frame of rest (FoR):

1. Lab system  $\Rightarrow$  electron's frame of rest:

$$E_{\text{FoR}} = E_{\text{Lab}}\gamma(1 - \beta\cos\theta) \tag{6.29}$$

- 2. Scattering occurs, gives  $E'_{\text{FoR}}$ .
- 3. Electron's frame of rest  $\Rightarrow$  Lab system:

$$E'_{\text{Lab}} = E'_{\text{FoR}}\gamma(1 + \beta\cos\theta') \tag{6.30}$$

Therefore, if electron is relativistic:

$$E'_{\text{Lab}} \sim \gamma^2 E_{\text{Lab}}$$
 (6.31)

since (on average) 
$$heta,\, heta'$$
 are  $\mathcal{O}(\pi/2)$  (beaming!)

Thus: Energy transfer is very efficient.

We first look at the energy budget of one single scattering.

The total power emitted in the frame of rest of the electron is given by

$$\frac{dE_{\text{FoR}}^{\prime}}{dt_{\text{FoR}}}\Big|_{\text{em}} = \int c\sigma_{\text{T}}E_{\text{FoR}}^{\prime}V^{\prime}(E_{\text{FoR}}^{\prime})dE_{\text{FoR}}^{\prime}$$
(6.32)

where V'(E') is the photon energy density distribution (number of photons per cubic metre with an energy between E' and E' + dE').

$$\frac{V_{\text{Lab}}(E_{\text{Lab}})dE_{\text{Lab}}}{E_{\text{Lab}}} = \frac{V_{\text{FoR}}(E_{\text{FoR}})dE_{\text{FoR}}}{E_{\text{FoR}}}$$
(6.33)

In the "Thomson limit" one assumes that the energy change of the photon in the rest frame of the electron is small

(this limit was also used in the derivation of Eq. (6.31)). Furthermore one can show that the power is Lorentz invariant:

$$\frac{dE_{\text{FoR}}}{dt_{\text{FoR}}} = \frac{dE_{\text{Lab}}}{dt_{\text{Lab}}}$$
(6.35)

(this follows from the fact that energy and time are both "time-like quantities", i.e., the formulae for the Lorentz transform of energy and time are the same). Therefore

 $E'_{\rm FoR}$ 

$$\frac{dE_{\text{Lab}}}{dt_{\text{Lab}}}\Big|_{\text{em}} = c\sigma_{\text{T}} \int E_{\text{FoR}}^2 \frac{V_{\text{FoR}} dE_{\text{FoR}}}{E_{\text{FoR}}}$$
(6.36)

$$= c\sigma_{\rm T} \int E_{\rm FoR}^2 \frac{V_{\rm Lab} dE_{\rm Lab}}{E_{\rm Lab}}$$
(6.37)

... Lorentz transforming  $E_{\text{FoR}}$ 

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This power is Lorentz invariant

$$= c\sigma_{\rm T}\gamma^2 \int (1 - \beta \cos\theta)^2 E_{\rm Lab} V_{\rm Lab} dE_{\rm Lab}$$
(6.38)

... averaging over angles ( $\langle \cos \theta \rangle = 0$ ,  $\langle \cos^2 \theta \rangle = \frac{1}{3}$ )

$$= c\sigma_{\rm T}\gamma^2 \left(1 + \frac{\beta^2}{3}\right) U_{\rm rad} \tag{6.39}$$

$$U_{\rm find} = \int EV(E)dE$$

(initial photon energy density).

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To determine the power gain of the photons, we need to subtract the power irradiated onto the electron,

$$\frac{dE_{Lab}}{dt_{Lab}}\Big|_{lnc} = c\sigma_{\rm T} \int EV(E)dE = \sigma_{\rm T}cU_{\rm rad}$$
(6.41)

Therefore, since

$$\gamma^2 - 1 = \gamma^2 \beta^2 \tag{6.42}$$

the net power gain of the photon field is

$$P_{\text{compt}} = \frac{dE_{\text{Lab}}}{dt}\Big|_{\text{em}} - \frac{dE_{\text{Lab}}}{dt}\Big|_{\text{inc}}$$

$$= \frac{4}{2}\sigma_{7}c_{7}^{2}\beta^{2}U_{\text{rad}}$$
(6.43)
(6.44)

(6.40)



### Amplification factor, I

As shown before, in the electron frame of rest,

$$\frac{\Delta E}{E} = -\frac{E}{m_{\rm e}c^2} \tag{6.15}$$

Assuming a thermal (Maxwell) distribution of electrons (i.e., they're not at rest), using the equations from the previous slides one can show that the relative energy change is given by

$$\frac{\Delta E}{E} = \frac{4kT - E}{m_{\bullet}c^2} = A \tag{6.45}$$

where A is the Compton amplification factor. Thus:

 $E \lesssim 4kT_e \Longrightarrow$  Photons gain energy, gas cools down.  $E \gtrsim 4kT_e \Longrightarrow$  Photons loose energy, gas heats up.

**Thermal Comptonization** 

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Amplification factor, II  
In reality, photons will scatter more than once before leaving the hot electron medium.  
The *total* relative energy change of photons by traversal of a hot (
$$E \ll kT_e$$
) medium with electron density  $n_e$  and size  $\ell$  is then approximately  
(rel. energy change  $y$ ) =  $\frac{\text{rel. energy change}}{\text{scattering}} \times (\# \text{ scatterings})$  (6.46)  
The number of scatterings is  $\max(\tau_e, \tau_e^2)$ , where  $\tau_e = n_e \sigma_T \ell$  ("optical depth"), such that  
 $y = \frac{4kT_e}{m_e c^2} \max(\tau_e, \tau_e^2)$  (6.47)

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## Spectral shape, I

Photon spectra can be found by analytically solving the "Kompaneets equation", but this is very difficult.

Approximate spectral shape from the following arguments:

After k scatterings, the energy of a photon with initial energy  $E_i$  is approximately

$$E_k = E_i A^k \tag{6.48}$$

But the probability to undergo k scatterings in a cloud with optical depth  $\tau_e$  is  $p_k(\tau_e) = \tau_e^k$ 

(follows from theory of random walks, note that the mean free path is  $\ell=1/\tau_{\rm e}$ ).

Therefore, if there are  $N(E_i)$  photons initially, then the number of photons emerging at energy  $E_k$ is

$$N(E_k) \sim N(E_i) A^k \sim N(E_i) \left(\frac{E_k}{E_i}\right)^{-\alpha} \quad \text{with} \quad \alpha = -\frac{\ln \tau_e}{\ln A}$$
(6.49)

Comptonization produces power-law spectra.

General solution: Possible via the Monte Carlo method.

#### **Thermal Comptonization**



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t<sub>e</sub>=5 t.=2  $y \ll$  1: pure power-law. =0 y < 1: power-law with =0.05 exponential cut-off

1000.0

10.0 E [keV]

Sphere with  $kT_{\rm e} = 0.7 m_{\rm e}c^2$  ( $\sim 360 \, \rm keV$ ), seed photons come from

100.0

1.0

Ξ

10

10-4

0.1

center of sphere.

 $y \gg 1$ : "Saturated Comptonization".

Saturated Comptonization has never been observed.



- grav. Redshift
- Light bending

[arbi

\_2

4.5

5.0

5.5

6.0

Energy [keV]

6.5

7.0

7.5

8.0

• rel. Doppler shift



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6-28 Other Sources  $Flux (10^{-3} keV/cm^{2/s})$ 1 0.1 0.01 0.01 2.0 data/model 1.2 10 0.5 5 5 2 10 Energy [keV] Energy [keV] (Porquet & Reeves, 2003, Fig. 3) (Matt et al., 2005, Fig. 1) XMM data from 2001 comparison 2003 vs. 2001 data Q0056-363 (broad line radio-quiet quasar,  $L_X > 10^{45} \text{ erg s}^{-1}$ ): Fe K $\alpha$  has FWHM 24500 km s<sup>-1</sup>, EW 275 eV Q0056–363 is highest luminosity radio-quiet QSO with broad Fe K $\alpha$  line. Broad Lines with XMM 5







Fender et al. (2004), Markoff, Nowak & Wilms (2005) for galactic BHs



### Conclusions, II

To be successful, models will have to consider:

- Broad Fe K $\alpha$  lines are rare:
  - Truncated Disks?

e.g., invoked by Zdziarski et al. (1999) to explain  $\Omega/2\pi$ - $\Gamma$ -correlation

- Disk ionization (but needs fine tuning!)
- And what about the Unified Model? Is the viewing angle really edge on?
- Narrow lines are ubiquitous:
  - Are they formed in the torus? but narrow lines often have FWHM $\sim$ 4000–7000 km s<sup>-1</sup>  $\implies$  too large for torus! (expect ~ 760 km s<sup>-1</sup>  $(M_8/r_{pc})^{1/2}$ )
  - Do they originate in the BLR or an ionized disk?

... and we should not forget the observational constraints: Strong Fe K $\alpha$ variability  $\implies$  we need a larger collecting area (XEUS!)

#### Conclusions

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