

(PG 1416-129: de Kool et al., 1994, Williams et al., 1992, Staubert \& Maisack, 1996; NGC 4151: Maisack 1991, 1993)

Spectral shape of AGN very similar to galactic Black Holes $\Longrightarrow$ Same physical mechanism (=Comptonization) responsible!

Note: NGC 4151 not corrected for interstellar absorption.

AGN X-Ray Continua


The power radiated by an accelerated charge in direction $\Theta$ through the spherical angle $d \Omega$ is given by Larmor's formula:

$$
\begin{equation*}
\frac{d P}{d \Omega}(\Theta)=\frac{1}{16 \pi^{2} c^{3} \epsilon_{0}} q^{2} \dot{v}^{2} \sin ^{2} \Theta \tag{6.2}
\end{equation*}
$$

Integrating Eq. (6.2) over $4 \pi$ sr gives

$$
\begin{equation*}
P=\frac{q^{2} \dot{v}^{2}}{6 \pi c^{3} \epsilon_{0}} \tag{6.3}
\end{equation*}
$$

For the case the charge is accelerated by an (sinusoidally varying) electric field $E(t)$ one finds after a longish calculation:

$$
\begin{equation*}
\frac{d P}{d \Omega}=\frac{q^{4} E_{0}^{2}}{16 \pi^{2} m^{2} c^{3} \epsilon_{0}} \sin ^{2} \Theta \quad \text { and } \quad P=\frac{q^{4} E_{0}^{2}}{12 \pi c^{3} m^{2} \epsilon_{0}} \tag{6.4}
\end{equation*}
$$

## Compton Scattering

The differential cross section $d \sigma / d \Omega$ is the area presented by the electron to a photon that is going to get scattered in direction $d \Omega$.
The total cross section for Thomson scattering, $\sigma_{\mathrm{T}}$, is then obtained from the differential cross section by integrating $d \sigma / d \Omega$ from Eq. (6.7) over all angles:

$$
\begin{equation*}
P=\int\langle S\rangle \frac{d \sigma}{d \Omega} d \Omega=\langle S\rangle \int \frac{d \sigma}{d \Omega} d \Omega=:\langle S\rangle \sigma_{T} \tag{6.9}
\end{equation*}
$$

Performing the integration yields

$$
\begin{equation*}
\sigma_{\mathrm{T}}=\frac{8 \pi}{3} r_{0}^{2}=\frac{e^{4}}{6 \pi m_{\mathrm{e}}^{2} \epsilon_{0}^{2} c^{4}}=6.652 \times 10^{-25} \mathrm{~cm}^{2} \tag{6.10}
\end{equation*}
$$

$\sigma_{\mathrm{T}}$ is also called the Thomson cross section.

## Thomson Scattering,


after Rybicki \& Lightman, Fig. 3.7

$$
\begin{equation*}
\left.\frac{d \sigma}{d \Omega}\right|_{\text {unpol }}=\frac{1}{2}\left(\left.\frac{d \sigma(\Theta)}{d \Omega}\right|_{\mathrm{pol}}+\left.\frac{d \sigma(\pi / 2)}{d \Omega}\right|_{\mathrm{pol}}\right) \tag{6.11}
\end{equation*}
$$

Let $\theta=\angle(\boldsymbol{k}, \boldsymbol{n})$ to obtain

$$
\begin{equation*}
\left.\frac{d \sigma}{d \Omega}\right|_{\text {unpol }}=\frac{r_{0}^{2}}{2}\left(1+\cos ^{2} \theta\right)=\frac{3 \sigma_{T}}{16 \pi}\left(1+\cos ^{2} \theta\right) \quad \text { and } \int \frac{d \sigma}{d \Omega} d \Omega=\sigma_{T} \tag{6.12}
\end{equation*}
$$

E, p


Thomson scattering: initial and final photon
energy are identical.
But: in QM: light consists of photons
$\Longrightarrow$ Scattering: photon changes direction
$\Longrightarrow$ Momentum change
$\Longrightarrow$ Energy change!
This process is called Compton scattering.
Energy/wavelength change in scattering (see handout):

$$
\begin{align*}
E^{\prime} & =\frac{E}{1+\frac{E}{m_{\mathrm{e}} c^{2}}}(1-\cos \theta) \tag{6.13}
\end{align*} E\left(1-\frac{E}{m_{\mathrm{e}} c^{2}}(1-\cos \theta)\right) .
$$

where $h / m_{\mathrm{e}} c=2.426 \times 10^{-12} \mathrm{~m}$ (Compton wavelength).
Averaging over $\theta$, for $E \ll m_{\mathrm{e}} c$ :

$$
\frac{\Delta E}{E} \approx-\frac{E}{m_{\mathrm{e}} c^{2}}
$$

E.g., at $6.4 \mathrm{keV}, \Delta E \approx 0.2 \mathrm{keV}$.

## Compton Scattering

The derivation of Eq. (6.13) is most simply done in special relativity using four-vectors. In the following, we will use capital letters for four-vectors and small letters for
three-vectors. Furthermore, we will sdopt the cone in specia
for the product of two four vectors, following, e.g., the convention of Rindler (1991, Introduction to Special Relativity).
The four-momentum of a particle with non-zero rest-mass, $m_{0}$, e.g., an electron, is

$$
Q=m_{0 \gamma}\binom{c}{v}=\binom{m_{0} \gamma c}{q}
$$

where $v$ is the velocity of the particle and $q$ its momentum. As usual, $\gamma=\left(1-(v / c)^{2}\right)^{-1 / 2}$. The square of $Q$ is

$$
Q^{2}=m_{0}^{2} \gamma^{2} c^{2}-m_{0}^{2} \gamma^{2} v^{2}=m_{0}^{2} c^{2} \gamma^{2}\left(1-\left(\frac{v^{2}}{c^{2}}\right)\right)=m_{0}^{2} c^{2}
$$

Obviously, $Q^{2}$ is relativistically invariant.
In the same spirit, the four-momentum of a photon is

$$
P=\frac{E}{c}\binom{1}{\hat{u}}
$$

where $\hat{u}$ is an unit-vector pointing into the direction of motion of the photon. Note that for photons
$P^{2}=0$
as the photon's rest-mass is zero.
We will now look at the collision between a photon and an electron. We will denote the four-momenta after the collision with primed quantities.
Conservation of four-momentum requires

$$
P+Q=P^{\prime}+Q^{\prime}
$$

We now use a trick from Lightman et al. (1975, Problem Book in Relativity and Gravitation), solving this equation for $Q^{\prime}$ and squaring the resulting expression:

$$
\left(P+Q-P^{\prime}\right)^{2}=\left(Q^{\prime}\right)^{2}
$$

Since the collision is elastic, i.e., the rest mass of the electron is not changed by the collision,
furthermore, $\boldsymbol{P}^{2}=\left(\boldsymbol{P}^{\prime}\right)^{2}=0$, such that
But in the frame where the electron is initially at rest.

$$
\begin{aligned}
Q \cdot\left(\boldsymbol{P}-\boldsymbol{P}^{\prime}\right) & =m_{\mathrm{e}}\left(\left(\frac{E}{c}-\frac{E^{\prime}}{c}\right)=m\left(E-E^{\prime}\right)\right. \\
P \cdot \boldsymbol{P}^{\prime} & =\frac{E}{c} \frac{E^{\prime}}{c}\left(1-\hat{\boldsymbol{u}} \cdot \hat{\boldsymbol{u}}^{\prime}\right)=\frac{E E^{\prime}}{c^{2}}(1-\cos \theta)
\end{aligned}
$$



Integrating over $d \sigma_{\text {es }} / d \Omega$ gives total cross-section:

$$
\sigma_{\text {es }}=\frac{3}{4} \sigma_{\mathrm{T}}\left[\frac{1+x}{x^{3}}\left\{\frac{2 x(1+x)}{1+2 x}-\ln (1+2 x)\right\}+\frac{1}{2 x} \ln (1+2 x)-\frac{1+3 x}{(1+2 x)^{2}}\right]
$$

where $x=E / m_{\mathrm{e}} c^{2}$
Compton Scattering

## Energy Exchange

For non-stationary electrons, use previous formulae and Lorentz transform photon into electron's frame of rest (FoR):

1. Lab system $\Rightarrow$ electron's frame of rest:

$$
\begin{equation*}
E_{\mathrm{FoR}}=E_{\mathrm{Lab}} \gamma(1-\beta \cos \theta) \tag{6.29}
\end{equation*}
$$

2. Scattering occurs, gives $E_{\text {FoR }}^{\prime}$.
3. Electron's frame of rest $\Rightarrow$ Lab system:

$$
\begin{equation*}
E_{\mathrm{Lab}}^{\prime}=E_{\mathrm{FoR}}^{\prime} \gamma\left(1+\beta \cos \theta^{\prime}\right) \tag{6.30}
\end{equation*}
$$

Therefore, if electron is relativistic:

$$
\begin{equation*}
E_{\mathrm{Lab}}^{\prime} \sim \gamma^{2} E_{\mathrm{Lab}} \tag{6.31}
\end{equation*}
$$

since (on average) $\theta, \theta^{\prime}$ are $\mathcal{O}(\pi / 2)$ (beaming!).
Thus: Energy transfer is very efficient.

As shown in the following, in Compton scattering the radiation field is also amplified by a factor $\gamma^{2}$.
We first look at the energy budget of one single scattering.
The total power emitted in the frame of rest of the electron is given by

$$
\left.\frac{d E_{\text {or }}^{\prime}}{d t_{\text {FRR }}}\right|_{\mathrm{em}}=\int c \sigma_{\mathrm{T}} E_{\mathrm{FOR}}^{\prime} V^{\prime}\left(E_{\mathrm{FoR}}^{\prime}\right) d E_{\mathrm{FOR}}^{\prime}
$$

where $V^{\prime}\left(E^{\prime}\right)$ is the photon energy density distribution (number of photons per cubic metre with an energy between $E^{\prime}$ and $E^{\prime}+d E^{\prime}$ ) This power is Lorentz invariant

$$
\frac{V_{\text {Lab }}\left(E_{\text {Lab }}\right) d E_{\text {Lab }}}{E_{\text {Lab }}}=\frac{V_{\text {FoR }}\left(E_{\text {FoR }}\right) d E_{\text {FoR }}}{E_{\text {For }}}
$$

In the "Thomson limit" one assumes that the energy change of the photon in the rest frame of the electron is small, $E_{\text {FoR }}^{\prime}=E_{\text {For }}$
(this limit was also used in the derivation of Eq. (6.31)). Furthermore one can show that the power is Lorentz invariant: $\frac{d E_{\text {For }}}{d t_{\text {ooR }}}=\frac{d E_{\text {Lab }}}{d t_{\text {Lab }}}$
. Therefore

$$
\begin{aligned}
\left.\frac{d E_{\text {Lab }}}{d t_{\text {Lab }}}\right|_{\text {em }} & =c \sigma_{\mathrm{T}} \int E_{\text {FoR }}^{2} \frac{V_{\text {For }} d E_{\text {FoR }}}{E_{\text {ori }}} \\
& =c \sigma_{\mathrm{T}} \int E_{\text {Foo }}^{2} \frac{V_{\text {Lad }} d E_{\text {Lab }}}{E_{\text {Lab }}}
\end{aligned}
$$

.. Lorentz transtorming $E_{\text {Foi }}$
.averaging over angles $\left(\langle\cos \theta\rangle=0,\left\langle\cos ^{2} \theta\right\rangle=\frac{1}{3}\right)$

$$
=c \sigma_{T} \gamma^{2}\left(1+\frac{\beta^{2}}{3}\right) U_{\mathrm{rad}}
$$

6-12
where

$$
U_{\text {rad }}=\int E V(E) d E
$$

(initial photon energy density).
To determine the power gain of the photons, we need to subtract the power irradiated onto the electron

$$
\left.\frac{d E_{\text {Lab }}}{d t_{\text {Lab }}}\right|_{\text {inc }}=c \sigma_{T} \int E V(E) d E=\sigma_{\mathrm{T} C} C U_{\text {rad }}
$$

Therefore, since

$$
\gamma^{2}-1=\gamma^{2} \beta^{2}
$$

$$
\begin{aligned}
P_{\text {compt }} & =\left.\frac{d E_{\text {lab }}}{d t}\right|_{\mathrm{em}}-\left.\frac{d E_{\text {abd }}}{d t}\right|_{\text {inc }} \\
& =\frac{4}{2} \sigma \text { Tc鄉 } \beta^{2} U_{\text {rad }}
\end{aligned}
$$

$$
=\frac{4}{3} \sigma \operatorname{coc} \boldsymbol{c}^{2} \beta^{2} U_{\text {rad }}
$$

As shown before, in the electron frame of rest,

$$
\begin{equation*}
\frac{\Delta E}{E}=-\frac{E}{m_{\mathrm{e}} c^{2}} \tag{6.15}
\end{equation*}
$$

Assuming a thermal (Maxwell) distribution of electrons (i.e., they're not at rest), using the equations from the previous slides one can show that the relative energy change is given by

$$
\begin{equation*}
\frac{\Delta E}{E}=\frac{4 k T-E}{m_{\mathrm{e}} c^{2}}=A \tag{6.45}
\end{equation*}
$$

where $A$ is the Compton amplification factor.
Thus:
$E \lesssim 4 k T_{\mathrm{e}} \Longrightarrow$ Photons gain energy, gas cools down.
$E \gtrsim 4 k T_{\mathrm{e}} \Longrightarrow$ Photons loose energy, gas heats up.

## Thermal Comptonization

## Amplification factor, II

In reality, photons will scatter more than once before leaving the hot electron medium.

The total relative energy change of photons by traversal of a hot $\left(E \ll k T_{\mathrm{e}}\right)$
medium with electron density $n_{\mathrm{e}}$ and size $\ell$ is then approximately

$$
\begin{equation*}
(\text { rel. energy change } y)=\frac{\text { rel. energy change }}{\text { scattering }} \times(\# \text { scatterings }) \tag{6.46}
\end{equation*}
$$

The number of scatterings is $\max \left(\tau_{\mathrm{e}}, \tau_{\mathrm{e}}^{2}\right)$, where $\tau_{\mathrm{e}}=n_{\mathrm{e}} \sigma_{\mathrm{T}} \ell$ ("optical depth"), such that

$$
\begin{equation*}
y=\frac{4 k T_{\mathrm{e}}}{m_{\mathrm{e}} c^{2}} \max \left(\tau_{\mathrm{e}}, \tau_{\mathrm{e}}^{2}\right) \tag{6.47}
\end{equation*}
$$

## "Compton $y$-Parameter"



Photon spectra can be found by analytically solving the "Kompaneets equation", but this is very difficult.

Approximate spectral shape from the following arguments:
After $k$ scatterings, the energy of a photon with initial energy $E_{\mathrm{i}}$ is approximately

$$
\begin{equation*}
E_{k}=E_{\mathrm{i}} A^{k} \tag{6.48}
\end{equation*}
$$

But the probability to undergo $k$ scatterings in a cloud with optical depth $\tau_{\mathrm{e}}$ is $p_{k}\left(\tau_{\mathrm{e}}\right)=\tau_{\mathrm{e}}^{k}$
(follows from theory of random walks, note that the mean free path is $\ell=1 / \tau_{\mathrm{e}}$ ).
Therefore, if there are $N\left(E_{\mathrm{i}}\right)$ photons initially, then the number of photons emerging at energy $E_{k}$ is

$$
\begin{equation*}
N\left(E_{k}\right) \sim N\left(E_{\mathrm{i}}\right) A^{k} \sim N\left(E_{\mathrm{i}}\right)\left(\frac{E_{k}}{E_{i}}\right)^{-\alpha} \quad \text { with } \quad \alpha=-\frac{\ln \tau_{e}}{\ln A} \tag{6.49}
\end{equation*}
$$

Comptonization produces power-law spectra.
General solution: Possible via the Monte Carlo method.

Thermal Comptonization



Monte Carlo simulation shows: Spectrum is $\Longrightarrow$ Power law with exponential cutoff (here: with additional "Wien hump", see next slide)

Thermal Comptonization


Thermal Comptonization



Broad Lines with ASCA


ASCA: Average Seyfert Fe K $\alpha$ profile contains a narrow core and a red and blue wings, but they are much weaker than MCG-6-30-15.


6-25
MCG-6-30-15, II



Better modeling of soft excess and reflection $\Longrightarrow \mathrm{Fe} \mathrm{K} \alpha$ line has extreme width and skewed profile.

Components of the final fit $\Longrightarrow$ Line emissivity is strongly concentrated towards the inner edge of the disk ( $\epsilon \propto r^{-4.6}$; cannot be explained with standard $\alpha$-disk)


2001 July/August: 315 ksec observation
(Fabian et al., 2002)

- Strong narrow line
- broad line clearly present
- emissivity profile very steep for radii close to $r_{\text {in }}$
$I_{\text {Fe K } \alpha} \propto r^{-5.5 \pm 0.3}$ for $r<6.1_{-0.5}^{+0.8} r_{\mathrm{g}}$, $\propto r^{-2.7 \pm 0.1}$ outside that;
Fabian \& Vaughan (2003); confirms Wilms et al. (2001)


Broad Lines with XMM


Line profile variability in NGC $3516 \Longrightarrow$ Corotating flare? $\left(7 r_{\mathrm{g}} \lesssim r \lesssim 16 r_{\mathrm{g}}\right)$ If interpretation is pushed further, gives $M \sim(1 \ldots 5) \times 10^{7} M_{\odot}$.

6-29

(Longinotti et al., 2003)

## IRAS 13349+2436:

- Model either 2 broad emission lines or
- relativistic line from Fe XXIII/XXIV plus narrow absorption feature
Line shape can be rather complex!

(1H0707-495; Fabian et al., 2004)


Energy [keV]
(IRAS 13224-3809; Boller et al., 2003)

NLSy1: Strong absorption or a relativistic line fron a reflection dominated spectrum both describe the data equally well!

Similar results have been found by Pounds et al. in a variety of sources. .
But: strong absorption models contradict observations where data $>10 \mathrm{keV}$ available.





$$
\begin{array}{lllllll}
5 & 5.5 & 6 & 6.5 & 7 & 7.5 & 8
\end{array}
$$

Rest frame energy [keV]

The majority of Seyfert galaxies and QSOs do not show evidence for broad Fe $K \alpha$ lines!
statistics for PG-QSO: 20/38 show Fe $\mathrm{K} \alpha$ line of these 3 have broad line (Jiménez-Bailón et al., 2005)

Bianchi et al. (2004, Fig. 4 ) [Sample of Seyferts with simultaneous BeppoSAX observations.]

Narrow Lines


## Conclusions,

Relativistically broadened $\mathrm{Fe} \mathrm{K} \alpha$ lines clearly do exist in a variety of different AGN

We need to rethink the details of the accretion process and the accretion geometry close to black hole:

## - Energy extraction for extremely broad lines?

Coupling BH - disk, structure of the inner disk (no torque condition?, structure of the infall region,...)


- "Lamppost model"?
(Petrucci \& Henri, 1997; Martocchia, Matt \& Karas, 2002; Miniutti \& Fabian, 2004)
$\Longrightarrow$ X-rays focused down from the jet base? $\Longrightarrow$ If true, is continuum Comptonization? Fender et al. (2004), Markoff, Nowak \& Wilms (2005) for galactic BH

To be successful, models will have to consider:

## - Broad Fe K $\alpha$ lines are rare:

- Truncated Disks?
e.g., invoked by Zdziarski et al. (1999) to explain $\Omega / 2 \pi-\Gamma$-correlation
- Disk ionization (but needs fine tuning!)
- And what about the Unified Model?

Is the viewing angle really edge on?

- Narrow lines are ubiquitous:
- Are they formed in the torus?
but narrow lines often have FWHM~4000-7000 $\mathrm{km} \mathrm{s}^{-1}$

$$
\left.\Longrightarrow \text { too large for torus! (expect } \sim 760 \mathrm{~km} \mathrm{~s}^{-1}\left(M_{8} / r_{\mathrm{pc}}\right)^{1 / 2}\right)
$$

- Do they originate in the BLR or an ionized disk?
.... and we should not forget the observational constraints: Strong Fe K $\alpha$ variability $\Longrightarrow$ we need a larger collecting area (XEUS!)

Conclusions

(Streblyanska et al., 2005) rest frame energy [keV]

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## 6-35

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