





BH Masses

Regardless of the detailed interpretation of the BLR, measurements of the BLR allow for a (statistical) determination of the mass of the Black Hole: The virial theorem of mechanics states:

$$2T = \frac{1}{2}m\Delta V^2 = \eta \cdot G\frac{mM_{\mathsf{BH}}}{r_0} = U \tag{8.28}$$

where *m* mass of a test particle, r_0 : characteristic BLR radius, ΔV : velocity dispersion, and η : geometry dependent factor.

Since r_0 and ΔV can be measured from reverberation mapping and the line width:

$$M_{\mathsf{BH}} = \frac{fr\Delta V^2}{G} \tag{8.29}$$

where f is a geometry dependent normalization factor, obtained from calibration measurements.

Note: For virial theorem to apply, motion of BLR must be dominated by gravity.

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To derive the virial theorem, we look at a system of particles of mass m_i . The acceleration on particle i by all other particles is

$$\ddot{\mathbf{r}} = \sum_{j \neq i} \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3}$$
(8.30)

... scalar product with miri

$$m_i \mathbf{r}_i \cdot \ddot{\mathbf{r}}_i = \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3}$$

(8.31)

. since

$$\frac{1}{2}\frac{d^2\mathbf{r}_i^2}{dt^2} = \frac{d}{dt}(\dot{\mathbf{r}}_i \cdot \mathbf{r}_i) = \ddot{\mathbf{r}}_i \cdot \mathbf{r}_i + \dot{\mathbf{r}}_i \cdot \mathbf{r}_i$$
(8.32)

... therefore Eq. (8.31)

$$\frac{1}{2}\frac{d^2}{dt^2}(m_i \mathbf{r}_i^2) - m_i \dot{\mathbf{r}}_i^2 = \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3}$$
(8.33)

Summing over all particles in the system gives

$$\frac{1}{2}\sum_{i}\frac{\mathrm{d}^{2}}{\mathrm{d}t^{2}}(m_{i}\mathbf{r}_{i}^{2})-\sum_{i}m_{i}\mathbf{r}_{i}^{2}=\sum_{i}\sum_{j\neq i}\frac{Gm_{i}m_{j}\mathbf{r}_{i}\cdot(\mathbf{r}_{j}-\mathbf{r}_{i})}{|\mathbf{r}_{j}-\mathbf{r}_{i}|^{3}}$$
(8.34)

$$= \frac{1}{2} \left(\sum_{i} \sum_{j \neq i} Gm_i m_j \frac{\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_{j} \sum_{i \neq j} Gm_j m_i \frac{\mathbf{r}_j \cdot (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \right)$$

$$= \frac{1}{2} \left(\sum_{i} Gm_i m_j \frac{\mathbf{r}_i \cdot \mathbf{r}_j - \mathbf{r}_i^2}{|\mathbf{r}_i - \mathbf{r}_i|^3} + \sum_{i} Gm_j m_i \frac{\mathbf{r}_j \cdot \mathbf{r}_i - \mathbf{r}_j^2}{|\mathbf{r}_i - \mathbf{r}_i|^3} \right)$$
(8.36)

$$2\left(\frac{1}{i}\frac{j\neq i}{j\neq i} |\mathbf{r}_{i} - \mathbf{r}_{j}| - \frac{1}{j}\frac{Gm_{i}m_{j}}{i\neq j} |\mathbf{r}_{j} - \mathbf{r}_{i}|\right) = -\frac{1}{2}\sum_{\substack{i,j \\ i\neq j}} \frac{Gm_{i}m_{j}}{|\mathbf{r}_{i} - \mathbf{r}_{j}|}$$
(8.37)

$$|\mathbf{r}_j - \mathbf{r}_i|^{\circ}$$

Thus, identifying the total kinetic energy, T, and the gravitational potential energy, U, gives

$$2T - U = \frac{1}{2} \frac{d^2}{dt^2} \sum_i m_i \mathbf{r}_i^2 = \mathbf{0}$$
 (8.38)

in statistical equilibrium.

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Thus we find the virial theorem: $T = \frac{1}{2}|U|$



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Since $M \propto r \Delta V^2$, for a single object (M = const.), we expect $r \propto \Delta V^{-1/2}$.

Observations: Line width versus lag scales as

$$\log V_{\rm rms} = a + b \log c\tau \qquad (8.39)$$

with $b=-{\rm 1/2},$ as expected! solid lines in the figure

Masses obtained from lag and V (Peterson & Wandel, 2000):

• NGC 7469: 8.4
$$imes$$
 10° M_{\odot}

• NGC 5548: 5.9
$$imes$$
 10' M_{\odot}

 \bullet 3C 390.3: 3.2 \times 10 $^{8}\,M_{\odot}$

(Peterson, 2001)

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