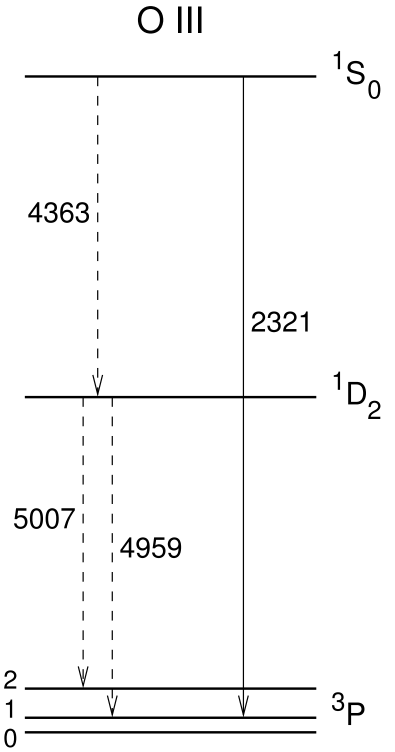
Friedrich-Alexander-Universität Erlangen-Nürnberg

Active Galactic Nuclei
Sommersemester 2010
Übungsaufgaben 2
J. Wilms/M. Kadler
02 June 2010

Question 1: *Temperature Diagnostics*



In this exercise we will be looking at the atomic physics behind density diagnostics, using the [O III] multiplets as an example. The relevant term scheme is shown at the left. In the following we will call the ^{3}P ground state level 1, while $^1D_2$ is level 2 and $^1S_0$ is level 3.

a) In plasmas of sufficiently low density, higher levels are excited via collisions but collisional de-excitation can usually be ignored, i.e., for two levels 1, and 2, we can assume $n_e C_{21} \ll A_{21}$. Using this information write down the rate equations connecting levels 1, 2, and 3 and solve them for the density ratio n_3/n_2. You may assume that $C_{23} = 0$.

b) For an optically thin plasma, the relation between the line intensity of a line at frequency ν_{ul} and the Einstein-A-coefficient is

$$4\pi I_\nu = n_u \nu_{ul} A_{ul} \tag{1.1}$$

where u denotes the upper and l denotes the lower level. Derive an equation for the intensity ratio of the 4636 Å and the sum of the 5007 Å and the 4959 Å lines (the latter shall be described by A_{21}), i.e., for $I(4363\,\text{Å})/(I(5007\,\text{Å}) + I(4959\,\text{Å}))$.

c) The collisional excitation rate, C_{ji}, is given by

$$C_{ji}(T) = \int_{v_{min}}^{\infty} v\sigma_{ji} f(v)\, dv \tag{1.2}$$

where the minimum excitation velocity is

$$v_{min} = \sqrt{2E_{ij}/m_e} \tag{1.3}$$

and where the cross section is given in terms the collision strength, Ω_{ji}, as

$$\sigma_{ji}(v) = \left(\frac{\pi h^2}{4\pi^2 m_e^2 v^2}\right) \frac{\Omega_{ji}}{g_j} \tag{1.4}$$

i) For a thermal plasma with a Maxwellian velocity distribution

$$f(v) = \sqrt{\frac{2}{\pi}\left(\frac{m_\mathrm{e}}{kT}\right)^3} \cdot v^2 \cdot \exp\left(-\frac{mv^2}{2kT}\right) \tag{1.5}$$

show that

$$C_{ji} = \sqrt{\frac{h^4}{8\pi^3 m_\mathrm{e}^3 k}} \cdot \frac{\Omega_{ji}}{g_j} \cdot T^{-1/2} \cdot \exp\left(-\frac{E_{ij}}{kT}\right) \tag{1.6}$$

- It will be useful to know that

$$\int x \exp(-kx^2)\, dx = -\frac{1}{2k} \exp(-kx^2) \tag{1.7}$$

- Inserting all constants, the square-root evaluates to $8.629 \times 10^{-6}\,\mathrm{cm^3\,s^{-1}}$.

ii) Show that $I(4363\,\text{Å})/(I(5007\,\text{Å}) + I(4959\,\text{Å}))$ depends only on temperature and on atomic constants.

d) *(To be done at home with a computer)* For [O III], the relevant atomic constants are $E_{32} = 2.84\,\mathrm{eV}$, $A_{32} = 1.8\,\mathrm{s^{-1}}$, $A_{31} = 0.221\,\mathrm{s^{-1}}$, $\Omega_{13} = 0.28$, and $\Omega_{12} = 2.17$. Plot the line ratio.

Note: that for densities around the critical density, de-excitation cannot be ignored, which will result in more complicated equations (see lecture notes). For [O III] the critical density of the $^1D_2$–$^3P_2$-transition is $8.6 \times 10^4\,\mathrm{cm^{-3}}$.