Question 1: *Detectors*

(a) The index of refraction for X-rays, n, is given by

$$n = 1 - \frac{N_A}{2}\frac{r_e}{2\pi}\rho\lambda^2 = 1 - \delta \quad (1.1)$$

where N_A is Avogadro's number, $r_e = 2.8 \times 10^{-15}$ m, ρ the density and λ the wavelength.

(i) Using Snell's law, show that the critical angle for reflection of X-rays is given approximately by

$$\theta_c = \sqrt{2\delta} \quad (1.2)$$

where θ_c is measured with respect to the surface tangent.

Solution: Snell's law of refraction is

$$\frac{\sin\alpha_1}{\sin\alpha_2} = n \quad (s1.1)$$

where α is measured with respect to the surface normal. Total reflection occurs for $\alpha_2 = 90°$, i.e., for

$$\sin\alpha_1 = n \quad (s1.2)$$

in terms of the critical angle θ_c, measured with respect to the surface

$$\cos\theta_c = n = 1 - \delta \quad (s1.3)$$

Because δ is very close to 0, we can use a Taylor series for $\cos\theta$ to find

$$\cos\theta_c \sim 1 - \frac{\theta_c^2}{2} = 1 - \delta \quad (s1.4)$$

such that

$$\theta_c = \sqrt{2\delta} \quad (s1.5)$$

(ii) Determine the critical angle for reflection of X-rays with an energy of $E = 5\,\text{keV}$ off gold ($\rho = 19.3\,\text{g}\,\text{cm}^{-3}$).

Solution: For using the above equations, we need the wavelength of the photons. Therefore, because of

$$E = h\nu \quad (s1.6)$$

and

$$\lambda = \frac{c}{\nu} \quad (s1.7)$$

we find

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34}\,\text{J}\,\text{s} \cdot 3 \times 10^8\,\text{m}\,\text{s}^{-1}}{5000 \times 1.6 \times 10^{-19}\,\text{J}} = 2.49 \times 10^{-10}\,\text{m} \quad (s1.8)$$

Inserting into Eq. (1.1) then gives

$$\delta = N_A \frac{Z}{A}\frac{r_e}{2\pi}\rho\lambda^2 = 6 \times 10^{23} \cdot 0.5 \cdot \frac{2.8 \times 10^{-15}\,\text{m}}{2\pi} \cdot 19300\,\text{kg}\,\text{m}^{-3} \cdot 6.25 \times 10^{-20}\,\text{m}^2 = 1.6 \times 10^{-7} \quad (s1.9)$$

such that

$$\theta_c = 5.66 \times 10^{-4} = 0.032° \quad (s1.10)$$

(iii) Sketch the major components of a Wolter telescope.

Solution: See slide 4-8 of the lecture notes. Crucial points are that two reflecting surfaces are used to obtain focus and that several mirror shells are stacked into each other to increase the collecting area of the telescope.

(b) The major detectors used in X-ray astronomy are proportional counters and Silicon based Charge Coupled Devices. Describe how these detectors work and compare their general properties such as the energy resolution, imaging capabilities and useful energy range.

Solution: Proportional counters are gas filled counters (e.g., Ar or Xe) and operate by accelerating electrons produced by the interaction of an X-ray with the detector gas towards their anode. The acceleration of these electrons leads to a cascade which greatly amplifies the primary signal. Imaging is not attainable with one proportional counter (although wire grids in proportional counters have been used to obtain imaging, so called position sensitive proportional counters, but they were not mentioned in the lectures). The energy resolution of these counters is moderate, on the order of $\Delta E/E \sim 10$–20% at 6 keV and they can operate from a few 0.1 keV to energies of several 10 keV. Because proportional counters have a large volume, they have rather high background rates and generally need to be shielded.

Charge coupled devices are silicon based. They operate by collecting the energy deposited by an X-ray in a minimum of the potential. The number of electrons produced by the X-ray is $N = E/\delta E$ where E is the energy of the X-ray and δE the band gap (a few eV). Because of this, the number of electrons deposited is proportional to E, which means that CCDs can be used as spectrometers. The charge collected in the collecting area is moved over the Si chip and then digitized and read out. Therefore, the read out speed of a X-ray sensitive CCD must be faster than the arrival rate of the observed X-rays. CCDs are pixelized detectors, which means that they have a good imaging resolution. Their energy resolution is also better than that of proportional counters, better than 1% at 6 keV. The detector volume is small, and therefore the background produced by cosmic rays in the detector is much smaller than for proportional counters.

(c) In order to understand the physics of Active Galactic Nuclei, why is it important to perform space measurements?

Solution: Active Galactic Nuclei emit a large amount of radiation in all energy bands, from the radio to the X-rays and gamma-rays. Because the Earth's atmosphere absorbs radiation in the infra-red and in the UV and X-rays, it is necessary to perform measurements from outside the Earth's atmosphere.

The following constants might be useful for solving this question.

$$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$$
$$c = 3 \times 10^{8}\,\mathrm{m\,s^{-1}}$$
$$1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$$
$$N_\mathrm{A} = 6 \times 10^{23}$$