Astronomy from Space
Example Exam Question

Question 1: *Neutron Star Accretion*

(a) Typical magnetic fields of neutron stars are $B = 10^7\,\mathrm{T}$. Because of this strong magnetic field, matter accreting on a neutron star will not form an accretion disk, but fall along the magnetic field lines onto the magnetic poles of the neutron star.

(i) Derive a formula for the velocity of matter accreting from infinity onto the surface of a neutron star with mass M and radius R. You may assume free fall. Ignore relativistic effects.

(ii) Using your result from subquestion ai, calculate the free fall velocity for a $1.4\,M_\odot$ neutron star with $10\,\mathrm{km}$ radius and express it $\mathrm{m\,s^{-1}}$ and as a fraction of the speed of light.

(iii) At the surface of the neutron star, the accreted material comes to an abrupt halt. Due to energy conservation, the kinetic energy of the accreted material is converted into thermal motion. By setting the kinetic energy of the accreted particles equal to $\frac{3}{2}kT$, determine the temperature of the accreted gas at the magnetic pole of the $1.4\,M_\odot$ neutron star and give it in both, Kelvins and keV. You may assume that the accreted gas is pure hydrogen. Note that because of the short timescales involved, electrons and protons will not be in thermodynamic equilibrium, i.e., they will have different temperatures.

(iv) Due to the deceleration of the accreted material, soft photons will be emitted. Name the formula which describes the emission of radiation due to the deceleration.

(v) The radiation resulting from the deceleration described above is called *bremsstrahlung* (breaking radiation). In neutron stars, typical *bremsstrahlung* photon energies are on the order of a few keV. These photons must propagate through the hot plasma formed by the accreting matter. Describe the expected spectral shape expected, including an estimate the maximum photon energy emitted by the accretion column. Explain your reasoning. Why can you ignore the hot protons?

(b) We now look at the physical processes in the strong magnetic field at the polar cap in greater detail.

(i) A nonrelativistic electron is moving with velocity v perpendicular to a magnetic field of strength B. Why is the resulting orbit a circle and what is its radius?

(ii) For sufficiently high B-fields, the Larmor radius will be small enough that quantum mechanical effects become important. This is generally the case once the characteristic length of the electron's orbit becomes comparable to the de Broglie wavelength, $\lambda = h/p$, of the electron. In analogy to the Bohr atom, this leads to a quantization of the electron's energy. In quantum mechanics it is shown that the proper quantization condition is that the circumference of the electron's orbit must be an even integer multiple of the de Broglie wavelength. Show that this leads to quantized electron energies which are proportional to B.

$$E_n = n\frac{\hbar q}{m_\mathrm{e}}B \tag{1.1}$$

where n is an integer. These energies are called Landau levels.

(iii) The continuum photons described in part (a) of this question can scatter off the electrons in the Landau levels. Analoguous to photons interacting with the quantized electrons around a proton in a hydrogen atom, energy can be transferred from the photons to the electrons. This is only possible if the photon has an energy corresponding to the difference between two energy levels of the electron. As the scattering photon looses energy, an absorption line is observed in the spectrum. In the source Hercules X-1, such a line was discovered at a photon energy of $40\,\mathrm{keV}$, corresponding to the energy between the $n = 1$ and the $n = 2$ Landau level. Determnine the magnetic field at the poles of the neutron star in this system.

The following are constants that might be useful for solving the example questions.

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\begin{aligned}
h &= 6.63 \times 10^{-34}\,\mathrm{J\,s} \\
c &= 3 \times 10^{8}\,\mathrm{m\,s^{-1}} \\
1\,\mathrm{eV} &= 1.6 \times 10^{-19}\,\mathrm{J} \\
q &= 1.6 \times 10^{-19}\,\mathrm{C} \\
m_{\mathrm{e}} &= 9.1 \times 10^{-31}\,\mathrm{kg} \\
m_{\mathrm{p}} &= 1.67 \times 10^{-27}\,\mathrm{kg} \\
M_{\odot} &= 2 \times 10^{30}\,\mathrm{kg} \\
G &= 6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}} \\
k &= 1.38 \times 10^{-23}\,\mathrm{m^{2}\,kg\,s^{-2}\,K^{-1}}
\end{aligned}
$$