Department of Physics
$3^{rd}$ Year Modules 2004/2005
Term 1, Week 01–05

THE UNIVERSITY OF WARWICK

# PX318: Astrophysics from Space

## Academic Week 03: Synchrotron Radiation

**Question 1:** *Synchrotron radiation: Derivation*
Using the outline shown in the lectures, show explicitly that the synchrotron spectrum of a power-law distribution of electrons is a power law and verify that the equation for the total synchrotron power emitted at frequency $\nu$ given in the lectures,

$$P_\nu = \frac{2}{3} c \sigma_\mathrm{T} n_0 \frac{U_\mathrm{B}}{\nu_\mathrm{L}} \left(\frac{\nu}{\nu_\mathrm{L}}\right)^{-\frac{p-1}{2}} \tag{1.1}$$

is correct.

**Question 2:** *Synchrotron radiation: Supernova Remnants*
A supernova remnant has a mean density of 1 electron per cubic-centimetre and a volume of 2 cubic parsecs. The mean magnetic field in the cluster is $10^{-7}$ T. The remnant contains electrons with $\gamma$-values between 1 and $10^6$, which have a power-law energy distribution $n(\gamma)d\gamma = n_0\gamma^{-5}d\gamma$.

a) Determine $n_0$, using that $\int_{\gamma_\mathrm{min}}^{\gamma_\mathrm{max}} n(\gamma)d\gamma$ gives the total number of electrons in the remnant.

b) Determine the power emitted at frequency $\nu$ by the remnant using Eq. (1.1).

c) By integrating $P_\nu$ over $\nu$ determine the total synchrotron luminosity of the remnant.

d) Compute the total energy content of the supernova remnant. Note that the energy content has two contributors: the energy in particles and the energy stored in the magnetic field. Compare this total energy content with the luminosity emitted by the remnant in synchrotron radiation. Estimate how long the remnant can radiate at its present level. Discuss.

The arguments in this question can be turned around to estimate the minimum $B$-field strength of a synchrotron source: the total energy content of the source better be such that it can produce its observed luminosity.

**Question 3:** *Synchrotrons on Earth*
One of the world's largest electron positron synchrotrons, the Large Electron-Positron Collider (LEP) at CERN in Geneva, Switzerland was essentially a ring of 4500 m radius containing electrons with a typical energy of 50 GeV (the LEP is currently being replaced with the Large Hadron Collider, LHC, which will provide energies of several TeV).

a) Estimate the magnetic field strength needed to keep the electrons on their circular path.

b) Assuming that the energy loss of the electrons in the storage ring is solely due to synchrotron radiation, determine the power radiated by one electron in synchrotron radiation. In what waveband does the electron radiate most of its energy, i.e., what is its characteristic frequency and the corresponding photon energy?

c) Determine the number of electrons in the storage ring, assuming that the electrons are responsible for a current of $I = 6$ mA.

d) Using these results, determine the power emitted in synchrotron radiation by the electrons.

*Remark:* The numbers you will find, although already impressive, will grossly underestimate the total power requirements of a modern synchrotron (by a factor of about 60). The reason for this is that the major cause for accelerations of the electrons is not the acceleration to keep the electrons on a circular orbit, but small scale disturbances, e.g., in the $B$-field of the synchrotron, resulting in extremely large accelerations.

**Question 4:** *Comments on this week's lectures*

In order to improve the teaching and to enable myself to react to problems you might have with the module, I would like to hear your opinion on my teaching as early as possible. I would appreciate it if you would voice any problems and criticisms as soon as possible, e.g., on the speed with which I talk about the subjects of the lectures, the overall difficulty level of the class and the homework, the quality and contents of the handouts, and so on.

Please write these comments on a separate sheet of paper and give them to me: Either put the paper on the lectern before class or put it in my "pigeon hole" in the mailboxes on the 5th floor of the physics building, close to the physics undergraduate office. Feel free to remain anonymous, if you deem this necessary. You can also ask questions by sending email to `j.wilms@warwick.ac.uk`.

*Solutions to all questions can be found at `http://pulsar.astro.warwick.ac.uk/wilms/teach/astrospace/handouts.html`.*