Department of Physics
3rd Year Modules 2004/2005
Term 1, Week 01–05

THE UNIVERSITY OF WARWICK

PX318: Astrophysics from Space

Academic Week 04: Comptonization

Question 1: *Power law slope for Compton scattering*
NB: This question is rather lengthy, but given that it is currently rainy and windy, it is still well worth your time...

a) A slab of thickness ℓ and electron number density n (measured in units of electrons m^{-3}) is irradiated by light with initial intensity N_0 (where N is the number of photons per second and square-metre). Convince yourself that due to scattering, the decrease in photon number over infinitesimal distance dx is given by

$$dN = -n\sigma dx \tag{1.1}$$

where σ is the Thomson cross section. Use Eq. 1.1 to show that the number of photons emerging in the original direction of the photons on the other side of the slab is

$$N(\ell) = N_0 \exp(-\tau) \tag{1.2}$$

where $\tau = n\sigma\ell$.

b) Using Eq. (1.2), convince yourself that the probability of a photon to scatter at least an optical depth τ is

$$p(\tau) = \exp(-\tau) \tag{1.3}$$

and that the mean optical depth traveled before the photon scatters, $\langle\tau\rangle = 1$.

c) Use the result from the previous section to show that the mean physical distance traveled in the slab, the *mean free path* l, is

$$l = \frac{1}{n\sigma} \tag{1.4}$$

d) Show that for small τ the probability of a photon undergoing k scatterings before escaping the medium is approximately

$$p_k(\tau) \sim \tau^k \tag{1.5}$$

e) For Compton scattering and a seed photon energy $E_\mathrm{s} \ll kT$, the amplification factor is

$$A \sim \frac{4kT}{mc^2} \tag{1.6}$$

Show that after k scatterings the energy of the seed photon, E_k, is approximately

$$E_k \sim E_\mathrm{s} A^k \tag{1.7}$$

f) Using Eqs. 1.5 and 1.7, show that the emergent intensity at energy E_k is a power law

$$N(E_k) = N(E_\mathrm{s}) \left(\frac{E_k}{E_\mathrm{s}}\right)^{-\alpha} \quad \text{where} \quad \alpha = -\frac{\ln\tau}{\ln A} \tag{1.8}$$

Question 2: *Even More Compton Scattering*

a) Show that the formula for the energy change of the electron,

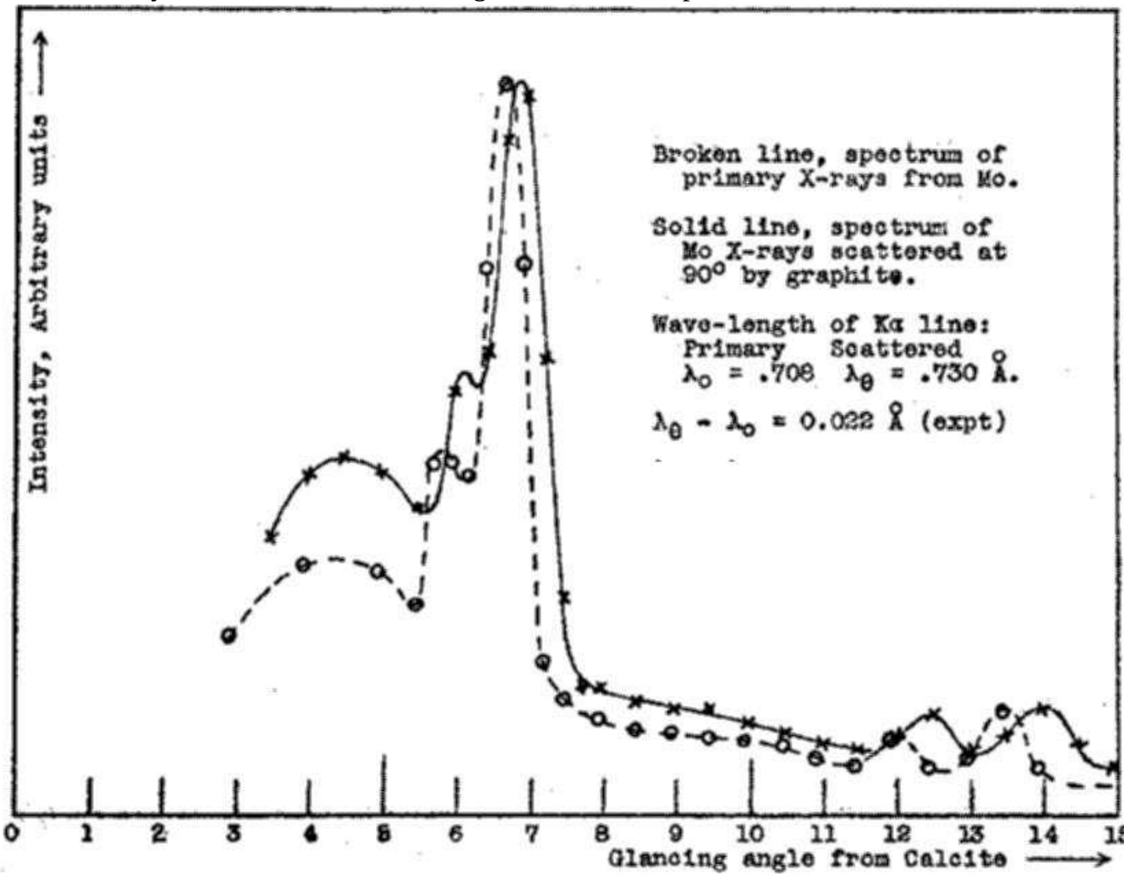
$$E' = \frac{E}{1 + \frac{E}{m_e c^2}(1 - \cos\theta)} \tag{2.1}$$

can be written as

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{2.2}$$

where λ and λ' are the photon's wavelength before and after the scattering and where $h/m_e c$ is called the Compton wavelength.

b) The following figure is from A.H. Compton's discovery paper on the effect of electron scattering on photons, which eventually resulted in this effect being called the "Compton effect":



Compton irradiated a block of graphite with X-rays from a Molybdenium source. As shown in the figure, he found that the observed spectrum was shifted by 0.022 Åwhen looking at the graphite block under an angle of 90°. This small shift is due to Compton scattering in graphite (wavelength is measured in degrees in the figure above, since Compton used a crystal spectrometer to do his spectroscopy).

Compute the wavelength shift using the equations above and compare it to the measured value.

After doing this you might want to read Compton's original 1923 paper (Phys. Rev. 21, 483-502), available at `http://prola.aps.org/abstract/PR/v21/i5/p483_1/p483` from all computers within the `warwick.ac.uk` domain. This paper is a beautiful piece of work, which finally convinced many physicists in the 1920s of the reality of the quantum nature of radiation. It is also one of the papers which eventually gained Compton his nobel prize in 1927, at age 35, "for the discovery of the effect named after him" (see `http://nobelprize.org/physics/laureates/1927/compton-bio.html`).

Question 3: *Synchrotron Self-Compton Radiation*
The broad-band spectra of radio-loud AGN are dominated by two humps: a low energy hump thought to be due to synchrotron radiation and a high energy hump, going up to TeV energies.

a) Compute the energy and γ-factor needed for an electron in a 10^{-7} T magnetic field to emit photons with an energy of 10 keV.

b) What is the typical energy of such 10 keV photons after one Compton scattering with the electrons which produced them?

Question 4: *Comments on this week's lectures*
In order to improve the teaching and to enable myself to react to problems you might have with the module, I would like to hear your opinion on my teaching as early as possible. I would appreciate it if you would voice any problems and criticisms as soon as possible, e.g., on the speed with which I talk about the subjects of the lectures, the overall difficulty level of the class and the homework, the quality and contents of the handouts, and so on.
Please write these comments on a separate sheet of paper and give them to me: Either put the paper on the lectern before class or put it in my "pigeon hole" in the mailboxes on the 5th floor of the physics building, close to the physics undergraduate office. Feel free to remain anonymous, if you deem this necessary. You can also ask questions by sending email to `j.wilms@warwick.ac.uk`.

Solutions to all questions can be found at `http://pulsar.astro.warwick.ac.uk/wilms/teach/astrospace/handouts.html`.