Department of Physics
3rd Year Modules 2004/2005
Term 1, Week 01–05

THE UNIVERSITY OF WARWICK

PX318: Astrophysics from Space – Example Solutions

Academic Week 02: Detectors

Question 1: *X-ray Detectors*
The aim of the following questions is to help you in revising the major properties of X-ray detectors.

a) A certain detector has a resolution of 12% at 5.9 keV. What is its resolution at an energy of 10 keV?

Solution: The energy resolution is

$$\frac{\Delta E}{E} \propto E^{-1/2} \tag{s1.1}$$

We can write this as

$$\frac{\Delta E}{E} = A\left(\frac{E}{5.9\,\mathrm{keV}}\right)^{-1/2} \tag{s1.2}$$

where $A = 0.12$. Therefore at 10 keV, $\Delta E/E = 0.092$, i.e. the resolution at 10 keV is 9.2%

b) Using the information given in the lectures, estimate the number of electron-hole pairs produced by a 7 keV photon in Silicon and in Germanium and, assuming Poisson statistics, estimate the energy resolution of Si and Ge detectors at that energy.

Solution: The band gap for Silicon is 1.12 eV, for Germanium it is 0.7 eV (this was mentioned in the lectures but is not in the handouts). Therefore the number of electron-hole pairs produced by a 7 keV photon is 6250 ± 80 for Si and 10000 ± 100 for Ge, where the uncertainties were determined assuming Poisson statistics. Since $\Delta E/E = \Delta N/N$, the energy resolution at 7 keV therefore is 1.28% for Si and 1% for Ge.

c) The electrons collected in a pixel have to be moved over the CCD before they can be read out. During each shift from one column to the next, a fraction f of electrons is lost, e.g., due to impurities in the semiconductor. The number f is called the *charge transfer inefficiency* (CTI), $1 - f$ is called the *charge transfer efficiency* (CTE). For a CCD with 400 columns, how many electrons are you allowed to loose per shift if you want to measure the charge deposited by a 7 keV photon in silicon in the column that is farthest away from the read out anode to a precision of 2% ? What does this imply for the CTE of the detector? **Note: The original question contained an unfortunate typo!**

Solution: If N_0 electrons are generated initially by the incident X-ray, the number of electrons remaining after one shift, N_1 is

$$N_1 = N_0(1 - f) \tag{s1.3}$$

and consequently after n shifts

$$N_n = N_0(1 - f)^n \tag{s1.4}$$

By Poisson statistics, the uncertainty of N_n is

$$\Delta N_n = N_n^{1/2} = N_0^{1/2}(1 - f)^{n/2} \tag{s1.5}$$

Therefore the relative uncertainty is

$$\frac{\Delta N_n}{N_n} = r = N_0^{-1/2}(1 - f)^{-n/2} \tag{s1.6}$$

such that

$$N_0^{-1/n} r^{-2/n} = (1 - f) = \mathrm{CTE} \tag{s1.7}$$

with $N_0 = 6250$ (see above), $n = 400$ and $r = 0.02$, we find CTE $= 0.9977$, i.e., in the first shift we loose 14 electrons (good detectors today loose $\ll$1 electron per shift!).

Question 2: *X-ray observations of AGN*

a) A supermassive black hole with a mass of $M = 10^7\,M_\odot$ is accreting at 5% of its maximum (Eddington) accretion rate. What is its luminosity?

Solution: The Eddington luminosity is

$$L_{\mathrm{Edd}} = \frac{4\pi c G m_{\mathrm{p}}}{\sigma_{\mathrm{T}}} M \tag{s2.1}$$

With $M = 10^7\,M_\odot = 2 \times 10^{37}$ kg, we find $L_{\mathrm{Edd}} = 1.26 \times 10^{38}$ W. Therefore the luminosity of the AGN is 6×10^{36} W.

b) Assuming 25% of the luminosity of the AGN is emitted as X-rays, and given a source distance of 15 Mpc, compute the source flux of this galaxy, i.e., the number of photons arriving from the galaxy per square metre and second. You may assume an average energy of 5 keV per X-ray photon.

Solution: The luminosity in the X-rays is 1.58×10^{36} W, and 15 Mpc corresponds to 4.6×10^{23} m. From the inverse square law, $F = L/4\pi r^2$, the energy flux is found to be $5.9 \times 10^{-13}\,\mathrm{J\,s^{-1}\,m^{-2}}$. Since 5 keV corresponds to 8×10^{-16} J, the photon flux is about 750 photons $\mathrm{s^{-1}\,m^{-2}}$.

c) You are observing the source with a telescope with a total collecting area of $2000\,\mathrm{cm}^2$. How many photons do you detect from the source in 10^3 s, 10^4 s, and 10^5 s?

Solution: The count rate in the detector is 150 counts s^{-1} (since $2000\,\mathrm{cm}^2$ is $0.2\,\mathrm{m}^2$. Therefore, the number of photons detected is 1.5×10^5, 1.5×10^6, 1.5×10^7, respectively.

d) The background count rate in the detector due to cosmic rays is 100 counts/sec, with the detector you can either only measure the background, or (when the telescope is pointed towards the source), the sum $T = S + B$ of the source (S) and background (B) counts. We say that a source is significantly detected at the $n\sigma$ level, if

$$\frac{T - B}{\sigma_{T-B}} > n$$

where σ_{T-B} is the standard deviation of $T - B$, which you can obtain from the standard deviation of T and B using standard error propagation. How long do you have to observe the source until you have detected it at the 5σ level? You can assume that Poisson statistics applies.

Solution: From error propagation we find

$$\sigma_{T-B} = \sqrt{\sigma_T^2 + \sigma_B^2} = \sqrt{T + B} \tag{s2.2}$$

assuming Poisson statistics. Writing s, b for the source and background count *rates*, then the requirement for detection above reads

$$\frac{T - B}{\sigma_{T-B}} = \frac{s\tau}{\sqrt{s\tau + 2b\tau}} = \frac{s\sqrt{\tau}}{\sqrt{s + 2b}} > n \tag{s2.3}$$

where τ is the exposure time (i.e., the total time one spends looking at the source – for simplicity here it is assumed that the source and background can be looked at at the same time, as is usually the case for an imaging telescope). Solving the above for τ gives

$$\tau = n^2 \frac{s + 2b}{s^2} \tag{s2.4}$$

For the case we have here ($s = 150$ counts per second, $b = 100$ cps, and $n = 5$), we find $\tau = 0.39$ s. The source is therefore very bright and immediately detected!

(compare this with a source with $s = 0.1$ cps, which is only detected significantly after 500 ksec, i.e., more than 5 days!).

Question 3: *Comments on this week's lectures*
In order to improve the teaching and to enable myself to react to problems you might have with the module, I would like to hear your opinion on my teaching as early as possible. I would appreciate it if you would voice any problems and criticisms as soon as possible, e.g., on the speed with which I talk about the subjects of the lectures, the overall difficulty level of the class and the homework, the quality and contents of the handouts, and so on.
Please write these comments on a separate sheet of paper and give them to me: Either put the paper on the lectern before class or put it in my "pigeon hole" in the mailboxes on the 5th floor of the physics building, close to the physics undergraduate office. Feel free to remain anonymous, if you deem this necessary. You can also ask questions by sending email to `j.wilms@warwick.ac.uk`.

Solutions to all questions can be found at `http://pulsar.astro.warwick.ac.uk/wilms/teach/astrospace/handouts.html`.