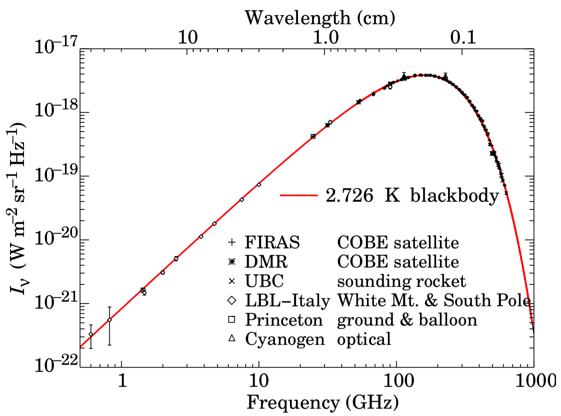
The Hot Big Bang

CMBR



(after Smoot, 1997, Fig. 1)

Penzias & Wilson (1965): "Measurement of Excess Antenna Temperature at 4080 Mc/s"

$\Longrightarrow$ Cosmic Microwave Background Radiation (CMBR)

The CMBR spectrum is fully consistent with a pure Planckian with temperature $T_{\text{CMBR}} = 2.728 \pm 0.004\,\text{K}$: a relict of the hot big bang.

CMBR

Assumption: Early universe was hot and dense

$\Longrightarrow$ Equilibrium between matter and radiation.

Generation of radiation, e.g., in pair equilibrium,

$$\gamma + \gamma \longleftrightarrow \text{e}^- + \text{e}^+ \tag{6.1}$$

Equilibrium with electrons, e.g., via Compton scattering:

$$\text{e}^- + \gamma \longrightarrow \text{e}^- + \gamma \tag{6.2}$$

where the electrons are linked to protons via Coulomb interaction.

Once density low and temperature below photoionization for Hydrogen,

$$\text{H} + \gamma \longleftrightarrow \text{p} + \text{e}^- \tag{6.3}$$

Decoupling of radiation and matter $\Longrightarrow$ Adiabatic cooling of photon field.

Proof for these assumptions, and lots of gory details: this and the next few lectures!

CMBR

Reminder: Planck formula for energy density of photons:

$$B_\lambda = \frac{\mathrm{d}u}{\mathrm{d}\lambda} = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp(hc/k_\mathrm{B}T\lambda) - 1} \tag{6.4}$$

(units: erg cm^{-3} Å^{-1}), where

$$k_\mathrm{B} = 1.38\times10^{-16}\,\text{erg K}^{-1} \quad \text{(Boltzmann)} \quad \text{and} \quad h = 6.625\times10^{-27}\,\text{erg s} \quad \text{(Planck)} \tag{6.5}$$

For $\lambda \gg hc/k_\mathrm{B}T$: Rayleigh-Jeans formula:

$$B_\lambda \sim \frac{8\pi k_\mathrm{B}T}{\lambda^4} \tag{6.6}$$

(classical case, diverges for $\lambda \longrightarrow 0$, "Jeans catastrophe").

The wavelength of maximum emission is given by Wien's displacement law:

$$\lambda_{\max} = 0.201\frac{hc}{k_\mathrm{B}T} \tag{6.7}$$

CMBR

The total energy density of the CMB is obtained by integration:

$$u = \int_0^\infty B_\lambda \, \mathrm{d}\lambda = \frac{8\pi^5 (kT)^4}{15 h^3 c^3} = \frac{4\sigma_{\mathrm{SB}}}{c} T^4 = a_{\mathrm{rad}} T^4 \qquad (6.8)$$

where

$$\sigma_{\mathrm{SB}} = 5.670 \times 10^{-5}\,\mathrm{erg\,cm^{-3}\,K^{-4}} \quad \text{Stefan-Boltzmann} \qquad (6.9)$$

$$a_{\mathrm{rad}} = 7.566 \times 10^{-15}\,\mathrm{erg\,cm^{-2}\,K^{-4}\,s^{-1}} \quad \text{radiation density constant} \qquad (6.10)$$

Since the energy of a photon is $E_\gamma = h\nu = hc/\lambda$, the total number density of photons is

$$n = \int_0^\infty \frac{B_\lambda \, \mathrm{d}\lambda}{hc/\lambda} = 20.28\, T^3\,\mathrm{photons\,cm^{-3}} \qquad (6.11)$$

Thus, for today's CMBR:

$$n_{\mathrm{CMBR}} = 400\,\mathrm{photons\,cm^{-3}} \qquad (6.12)$$

CMBR

For the CMBR today:

$$n_{\mathrm{CMBR}} = 400\,\mathrm{photons\,cm^{-3}} \qquad (6.12)$$

Compare that to gravitating matter (protons for now).

$\Longrightarrow$ critical density:

$$\rho_{\mathrm{c}} = \frac{3H^2}{8\pi G} = 1.88 \times 10^{-29} h^2\,\mathrm{g\,cm^{-3}} = 1.13 \times 10^{-5}\, h^2\,\mathrm{protons\,cm^{-3}} \qquad (3.58)$$

since $m_{\mathrm{p}} = 1.67 \times 10^{-24}$ g.

$\Longrightarrow$ photons dominate the particle number:

$$\frac{n_{\mathrm{CMBR}}}{n_{\mathrm{baryons}}} = \frac{3.54 \times 10^7}{\Omega h^2} \qquad (6.13)$$

$\Longrightarrow$ baryons dominate the energy density:

$$\frac{u_{\mathrm{CMBR}}}{u_{\mathrm{baryons}}} = \frac{a_{\mathrm{rad}} T^4}{\Omega \rho_{\mathrm{c}} c^2} = \frac{4.20 \times 10^{-13}}{1.69 \times 10^{-8} \Omega h^2} = \frac{1}{40260 \Omega h^2} \qquad (6.14)$$

That's why we talk about the matter dominated universe.

CMBR

The Universe was not always matter dominated:

Remember the scaling laws for the (energy) density of matter and radiation:

$$\begin{array}{l} \rho_{\mathrm{m}} \propto R^{-3} \\ \rho_{\mathrm{r}} \propto R^{-4} \end{array} \Longrightarrow \frac{\rho_{\mathrm{r}}}{\rho_{\mathrm{m}}} \propto \frac{1}{R} \qquad (3.63, 3.64)$$

$\Longrightarrow$ Photons dominate for large z, i.e., early in the universe!

Since $1 + z = R_0/R$ (Eq. 3.40), matter-radiation equality was at

$$1 + z_{\mathrm{eq}} = 40260\,\Omega h^2 \qquad (6.15)$$

(for $h = 0.75$, $1 + z_{\mathrm{eq}} = 22650$)

The above definition of z_{eq} is not entirely correct: neutrino background, which increases the background energy density, is ignored ($u_\nu \sim 68\% u_\gamma$, see later).

Formally, matter-radiation equality defined from $n_{\mathrm{baryons}} = n_{\text{relativistic particles}}$, giving

$$1 + z_{\mathrm{eq}} = 23900\,\Omega h^2 \qquad (6.16)$$

(for $h = 0.75$, $1 + z_{\mathrm{eq}} = 13440$).

CMBR

What happened to the temperature of the CMBR?

Compare CMBR spectrum today with earlier times.

(Differential) Energy density in $[\lambda, \lambda + \mathrm{d}\lambda]$:

$$\mathrm{d}u = B_\lambda \mathrm{d}\lambda \qquad (6.17)$$

Cosmological redshift:

$$\frac{\lambda'}{\lambda} = \frac{R'}{R} = \frac{1}{1+z} = a \qquad (3.47)$$

Taking the expansion into account:

$$\mathrm{d}u' = \frac{\mathrm{d}u}{a^4} = \frac{8\pi hc}{a^4 \lambda^5} \frac{\mathrm{d}\lambda}{\exp(hc/kT\lambda) - 1} = \frac{8\pi hc}{a^5 \lambda^5} \frac{a\,\mathrm{d}\lambda}{\exp(hc/kT\lambda) - 1}$$
$$= \frac{8\pi hc}{\lambda'^5} \frac{\mathrm{d}\lambda'}{\exp(hca/kT\lambda') - 1} = B_{\lambda'}(T/a) \qquad (6.18)$$

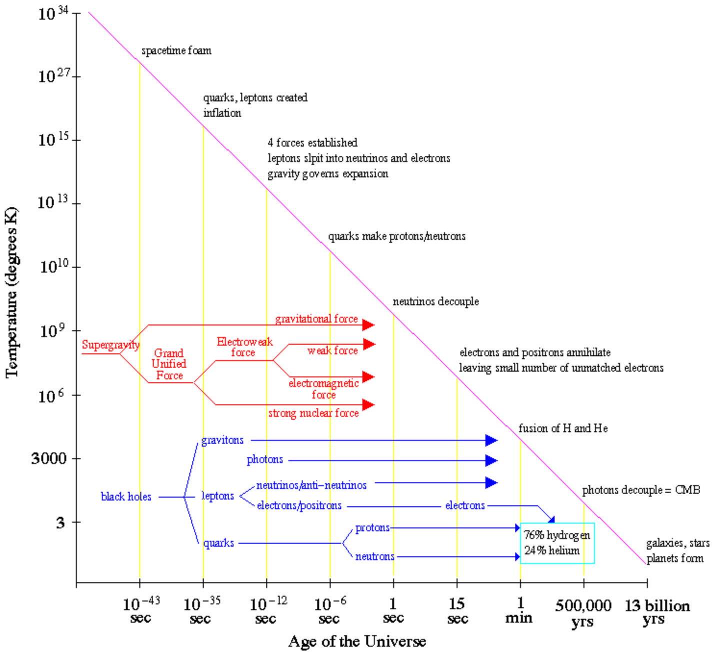
Therefore, the Planckian remains a Planckian, and the temperature of the CMBR scales as

$$T(z) = (1 + z) T_0 \qquad (6.19)$$

The early universe was hot $\Longrightarrow$ Hot Big Bang Model!

Overview

$a(t)$	t since BB	T[K] [K]	ρ_{matter} [g cm^{-3}]	Major Events
	10^{-42}	10^{30}		Planck era, "begin of physics"
	$10^{-40\ldots-30}$	10^{25}		Inflation?
10^{-13}	$\sim 10^{-5}$ s	$\sim 10^{13}$	$\sim 10^{9}$	generation of p-p^{-}, and baryon anti-baryon pairs from radiation background
3×10^{-9}	1 min	10^{10}	0.03	generation of e^{+}-e^{-} pairs out of radiation background
10^{-9}	10 min	3×10^{9}	10^{-3}	nucleosynthesis
$10^{-4}\ldots10^{-3}$	$10^{6\ldots7}$ yr	$10^{3\ldots4}$	$10^{-21\ldots-18}$	End of radiation dominated epoch
7×10^{-4}	10^{7} yr	4000	10^{-20}	Hydrogen recombines, decoupling of matter and radiation
1	15×10^{9} yr	3	10^{-30}	now



Thermodynamics, I

Density in early universe is very high.

Physical processes (e.g., photon-photon pair creation, electron-positron annihilation etc.) all have reaction rates

$$\Gamma \propto n\sigma v \tag{6.20}$$

where

n: number density (cm^{-3})
σ: interaction cross-section (cm^{2})
v: velocity (cm s^{-1})

Thermodynamic equilibrium reached if reaction rate much faster than "changes" in the system,

$$\Gamma \gg H \tag{6.21}$$

Where the Hubble parameter, H, is a good measure for $(\text{typical timescale of the Universe})^{1}$.

If thermodynamic equilibrium holds, then we can assume evolution of universe as sequence of states of local thermodynamic equilibrium, and use standard thermodynamics.

Before looking at real universe, first need to derive certain useful formulae from relativistic thermodynamics.

Thermodynamics, II

For ideal gases, thermodynamics shows that number density $f(\boldsymbol{p})\,\mathrm{d}p$ of particles with momentum in $[p, p+\mathrm{d}p]$ is given by

$$f(\boldsymbol{p}) = \frac{1}{\exp\left((E-\mu)/k_{\mathrm{B}}T\right) + a} \tag{6.22}$$

where

$$a = \begin{cases} +1 & : \text{Fermions (spin=1/2, 3/2,...)} \\ -1 & : \text{Bosons (spin=1, 2,...)} \\ 0 & : \text{Maxwell-Boltzmann} \end{cases}$$

and where the energy includes the rest-mass:

$$E^2 = |\boldsymbol{p}|^2 c^2 + m^2 c^4 \tag{6.23}$$

μ is called the "chemical potential". It is preserved in chemical equilibrium:

$$i + j \leftrightarrow k + l \quad \Longrightarrow \quad \mu_i + \mu_j = \mu_k + \mu_l \tag{6.24}$$

photons: multi-photon processes exist $\Longrightarrow \mu_\gamma = 0$.
particles in thermal equilibrium: $\mu = 0$ as well because of the first law of thermodynamics,

$$\mathrm{d}E = T\,\mathrm{d}S - P\,\mathrm{d}V + \mu\,\mathrm{d}N \tag{6.25}$$

and in equilibrium system stationary with respect to changes in particle number N.

Thermodynamics, III

In addition to number density: different particles have internal degrees of freedom, g.

Examples:

photons: two polarization states $\Longrightarrow g = 2$
neutrinos: one polarization state $\Longrightarrow g = 1$
electrons, positrons: spin=$1/2 \Longrightarrow g = 2$

Knowing g and $f(p)$, it is possible to calculate interesting quantities:

particle number density: $$n = \frac{g}{(2\pi\hbar)^3} \int f(\boldsymbol{p})\, \mathrm{d}^3 p \qquad (6.26)$$

energy density: $$u = \rho c^2 = \frac{g}{(2\pi\hbar)^3} \int E(\boldsymbol{p})\, f(\boldsymbol{p})\, \mathrm{d}^3 p \qquad (6.27)$$

To calculate the pressure, remember that kinetic theory shows:

$$P = \frac{n}{3}\langle pv \rangle = \frac{n}{3} \left\langle \frac{p^2 c^2}{E} \right\rangle \qquad (6.28)$$

such that

$$P = \frac{g}{(2\pi\hbar)^3} \int \frac{p^2 c^2}{3E} f(\boldsymbol{p})\, \mathrm{d}^3 p \qquad (6.29)$$

To derive the number density, the energy density, and the equation of state, note that Eq. (6.23) shows

$$E = \sqrt{p^2 c^2 + m^2 c^4} \qquad (6.23)$$

such that

$$p = \sqrt{E^2 - m^2 c^4}/c \qquad (6.30)$$

Therefore

$$\frac{\mathrm{d}E}{\mathrm{d}p} = \frac{pc^2}{\sqrt{p^2 c^2 + m^2 c^4}} \qquad (6.31)$$

from which it follows that

$$E\, \mathrm{d}E = pc^2\, \mathrm{d}p \qquad (6.32)$$

Thus the following holds

$$\iiint_{-\infty}^{+\infty} \mathrm{d}^3 p = \int_0^\infty 4\pi p^2\, \mathrm{d}p = \int_{mc^2}^\infty \frac{4\pi}{c^3} \left(E^2 - m^2 c^4\right)^{1/2} E\, \mathrm{d}E \qquad (6.33)$$

Going to a system of units where

$$c = k_\mathrm{B} = \hbar = 1 \qquad (6.34)$$

to save me some typing, substitute these equations into Eqs. (6.26)–(6.29) to find

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E\, \mathrm{d}E}{\exp((E-\mu)/T) \pm 1} \qquad (6.35)$$

$$\rho = \frac{g}{2\pi^2} \int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E^2\, \mathrm{d}E}{\exp((E-\mu)/T) \pm 1} \qquad (6.36)$$

$$P = \frac{g}{6\pi^2} \int_m^\infty \frac{\left(E^2 - m^2\right)^{3/2}\, \mathrm{d}E}{\exp((E-\mu)/T) \pm 1} \qquad (6.37)$$

which can in some limiting cases be expressed in a closed form (Kolb & Turner, 1990, eq. 3.52 ff.) (see following viewgraphs).

Thermodynamics, IV

Generally, we are interested in knowing n, u, and P in two limiting cases:

1. the ultra-relativistic limit, where $k_\mathrm{B}T \gg mc^2$, i.e., kinetic energy dominates the rest-mass
2. the non-relativistic limit, where $k_\mathrm{B}T \ll mc^2$

Transitions between these limits (i.e., what happens during "cooling") are usually much more complicated $\Longrightarrow$ ignore...

Thermodynamics, V

In the ultra-relativistic limit, $k_\mathrm{B}T \gg mc^2$, and assuming $\mu = 0$,

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2}\, g \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 & \text{Bosons} \\ \frac{3}{4}\frac{\zeta(3)}{\pi^2}\, g \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 & \text{Fermions} \end{cases} \qquad (6.38)$$

$$u = \begin{cases} \frac{\pi^2}{30}\, g\, k_\mathrm{B}T \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 & \text{Bosons} \\ \frac{7}{8}\frac{\pi^2}{30}\, g\, k_\mathrm{B}T \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 & \text{Fermions} \end{cases} \qquad (6.39)$$

$$P = \rho c^2/3 = u/3 \qquad (6.40)$$

where $\zeta(3) = 1.202\ldots$, and $\zeta(s)$ is Riemann's zeta-function (see handout, Eq. 6.48).

Eq. (6.40) is a simple result of the fact that in the relativistic limit, $E \sim pc$. Inserting this and $v = c$ into Eq. (6.28) gives the desired result.

As expected, we find the T^4 proportionality from the Stefan Boltzmann law!

6–15

Obtaining the previous formulae is an exercise in special functions. For example, the $T \gg m$, $T \gg \mu$ case for ρ for Bosons (Eq. 6.39) is obtained as follows (setting $c = k_B = \hbar = 1$):

$$\rho_{\text{Boson}} = \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{1/2}\, E^2 \, dE}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.41}$$

because of $T \gg \mu$

$$\approx \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{1/2}\, E^2 \, dE}{\exp(E/T) \pm 1} \tag{6.42}$$

for *Bosons*, choose -1, and substitute $x = E/T$:

$$= \frac{g}{2\pi^2} \int_{m/T}^\infty \frac{(x^2T^2 - m^2)^{1/2}\, x^2 T^3 \, dx}{\exp(x) - 1} \tag{6.43}$$

Since $T \gg m$,

$$\approx \frac{g}{2\pi^2} \int_0^\infty \frac{x^3 T^4 \, dx}{\exp(x) - 1} \tag{6.44}$$

$$= \frac{gT^4}{2\pi^2} \int_0^\infty \frac{x^3 \, dx}{\exp(x) - 1} \tag{6.45}$$

$$= \frac{gT^4}{2\pi^2} \cdot 6\zeta(4) \tag{6.46}$$

$$= \frac{\pi^2}{30} g T^4 \tag{6.47}$$

where $\zeta(s)$ is *Riemann's zeta-function*, which is defined by (Abramowitz & Stegun, 1964)

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{\exp(x) - 1}\, dx \quad \text{for } \mathscr{R}e\, s > 1 \tag{6.48}$$

where $\Gamma(x)$ is the Gamma-function. Note that $\zeta(4) = \pi^4/90$.

6–15

For *Fermions*, everything is the same except for that we now have to choose the $+$ sign. The equivalent of Eq. (6.45) is then

$$\rho_{\text{Fermi}} = \frac{gT^4}{2\pi^2} \int_0^\infty \frac{x^3 \, dx}{\exp(x) + 1} \tag{6.49}$$

Now we can make use of formula 3.411.3 of Gradstein & Ryshik (1981),

$$\int_0^\infty \frac{x^{\nu-1}\, dx}{\exp(\mu x) + 1} = \frac{1}{\mu^\nu}(1 - 2^{1-\nu})\Gamma(\nu)\zeta(\nu) \quad \text{for } \mathscr{R}e\, \mu, \nu > 1 \tag{6.50}$$

to see where the additional factor of $7/8$ in Eq. (6.39) comes from.

Thermodynamics, VI

In the non-relativistic limit: $k_B T \ll mc^2$

$\Longrightarrow$ can ignore the $\pm$1 term in the denominator

$\Longrightarrow$ Same formulae for Bosons and Fermions!

$$n = \frac{2g}{(2\pi\hbar)^3}(2\pi m k_B T)^{3/2} e^{-mc^2/k_B T} \tag{6.51}$$

$$u = nmc^2 \tag{6.52}$$

$$P = nk_B T \tag{6.53}$$

Therefore:

- density dominated by rest-mass ($\rho = u/c^2 = mn$)
- $P \ll \rho c^2/3$, i.e., *much* smaller than for relativistic particles.
- Particle pressure only important if particles are relativistic.

Obviously, relativistic particles with $m = 0$ (or very close to 0) will never get nonrelativistic. Still, they can "decouple" from the rest of the universe when the interaction rates go to 0.

Equation of State

Pressure of ultra-relativistic particles $\gg$ Pressure of nonrelativistic particles

$\Longrightarrow$ Nonrelativistic particles unimportant for equation of state.

For relativistic particles:

$$u_{\text{bosons}} = \frac{\pi^2}{30}\, g\, k_B T \left(\frac{k_B T}{\hbar c}\right)^3 \quad \text{and} \quad u_{\text{fermions}} = \frac{7}{8} u_{\text{bosons}} \tag{6.39}$$

$\Longrightarrow$ Total energy density for mixture of particles:

$$u = g_* \cdot \frac{\pi^2}{30} k_B T \left(\frac{k_B T}{\hbar c}\right)^3 \tag{6.54}$$

where the effective degeneracy factor

$$g_* = \sum_{\text{bosons}} g_B \left(\frac{T_B}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_F \left(\frac{T_F}{T}\right)^4 \tag{6.55}$$

g_* counts total number of internal degrees of freedom of *all* relativistic bosonic and fermionic species, i.e., all relativistic particles which are in thermodynamic equilibrium

The pressure is obtained from Eq. (6.54) via $P = u/3$.

Early Expansion, I

Knowing the equation of state, we can now use Friedmann equations to determine the early evolution of the universe.

Friedmann:

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 \qquad (3.55)$$

or, dividing by R^2

$$\frac{\dot{R}^2}{R^2} = H(t)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} \qquad (3.56)$$

But: The early universe is dominated by relativistic particles

$\Longrightarrow \rho \propto R^{-4}$

$\Longrightarrow$ Density-term dominates

$\Longrightarrow$ we can set $k = 0$.

Early universe is asymptotically flat!

This will prove to be one of the most crucial problems of modern cosmology…

Early Expansion, II

To obtain the evolution of the early universe, insert the Equation of State (Eq. 6.54) into Eq. (3.56):

$$H(t)^2 = \frac{8\pi G}{3} g_* \frac{\pi^2}{30}\frac{(k_B T)^4}{(\hbar c)^3} = \frac{4\pi^3 G}{45(\hbar c)^3} g_* (k_B T)^4 \qquad (6.56)$$

such that

$$H(t) = \left(\frac{4\pi^3 G}{45(\hbar c)^3}\right)^{1/2} g_*^{1/2} (k_B T)^2 \qquad (6.57)$$

On the other hand, since $\rho \propto R^{-4}$ (relativistic background),

$$\rho = \rho_0 \left(\frac{R_0}{R}\right)^4 \qquad (6.58)$$

Friedmann:

$$\frac{dR}{dt} = \sqrt{\frac{8\pi G\rho_0}{3}}\frac{R_0^2}{R} \qquad (6.59)$$

Introducing the dimensionless scale factor, $a = R/R_0$ (Eq. 3.29), gives

$$\frac{da}{dt} = \sqrt{\frac{8\pi G\rho_0}{3}}\frac{1}{a} =: \xi a^{-1} \qquad (6.60)$$

Early Expansion, III

And using separation of variables gives

$$\int_0^{a(t)} a\, da = \int_0^t \xi\, dt \quad \Longrightarrow \quad a(t) = \xi^{1/2}\cdot t^{1/2} \qquad (6.61)$$

Therefore, the Hubble constant evolves as

$$H(t) = \frac{\dot{a}}{a} = \frac{1}{2t} \qquad (6.62)$$

Equating Eqs. (6.57) and (6.62) gives the time-temperature relationship:

$$t = \left(\frac{45(\hbar c)^3}{16\pi^3 G}\right)^{1/2}\frac{1}{g_*^{1/2}}\frac{1}{(k_B T)^2} \qquad (6.63)$$

Inserting all constants and converting to more useful units gives

$$t = \frac{2.4\ \mathrm{sec}}{g_*^{1/2}}\cdot\left(\frac{k_B T}{1\ \mathrm{MeV}}\right)^{-2} \qquad (6.64)$$

…one of the most useful equations for the early universe.

Elementary Particles, I

Behavior of universe depends on $g_* \Longrightarrow$ Strong dependency on elementary particle physics.

Generally, particles present when energy in other particles allows generation of particle–antiparticle pairs, i.e., when $k_B T \gtrsim mc^2$ (threshold temperature)

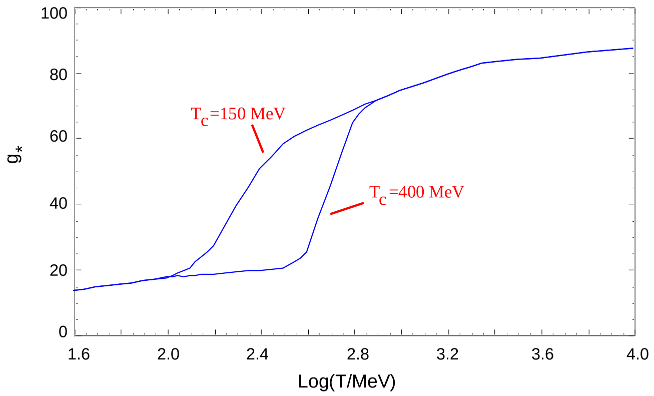
Current particle physics provides the following picture (Olive, 1999, Tab. 1):

Temp.	New Particles	$4g_*$
$k_B T < m_e c^2$	γ's and ν's	29
$m_e c^2 < k_B T < m_\mu c^2$	$e^\pm$	43
$m_\mu c^2 < k_B T < m_\pi c^2$	$\mu^\pm$	57
$m_\pi c^2 < k_B T < k_B T_c$	π's	69
$k_B T_c < k_B T < m_{strange} c^2$	$-\pi$'s+u, $\bar{u}$, d, $\bar{d}$, gluons	205
$m_s c^2 < k_B T < m_{charm} c^2$	s, $\bar{s}$	247
$m_c c^2 < k_B T < m_\tau c^2$	c, $\bar{c}$	289
$m_\tau c^2 < k_B T < m_{bottom} c^2$	$\tau^\pm$	303
$m_b c^2 < k_B T < m_{W,Z} c^2$	b, $\bar{b}$	345
$m_{W,Z} c^2 < k_B T < m_{top} c^2$	$W^\pm$, Z	381
$m_t c^2 < k_B T < m_{Higgs} c^2$	t, $\bar{t}$	423
$m_H c^2 < k_B T$	H^0	427

T_c: energy of confinement-deconfinement for transitions quarks $\Longrightarrow$ hadrons, somewhere between 150 MeV and 400 MeV.

Example: photons (2 polarization states, i.e., $g = 2$) and three species of neutrinos ($g = 1$, but with distinguishable anti-particles) $\Longrightarrow$ $g_* = 2 + (7/8)\cdot 2\cdot 3 = 58/8 = 29/4$.

Elementary Particles, II



(Olive, 1999, Fig. 1)

Will now consider times when only Neutrinos and Electron/Positrons present (after baryogenesis, see next lecture for that).

Interlude

Previous (abstract) formulae allow to estimate quantities like

1. The existence and energy of primordial neutrinos,
2. The formation of neutrons,
3. The formation of heavier elements.

Detailed computations require solving nonlinear differential equations
$\Longrightarrow$ difficult, only numerically possible.

Essentially, need to self-consistently solve Boltzmann equation in expanding universe for evolution of phase space density with time, using the correct QCD/QED reaction rates $\Longrightarrow$ too complicated (at least for me...).

Will use approximate analytical way here, which gives surprisingly exact answers.

Neutrinos, I

Neutrino equilibrium caused by weak interactions such as

$$\mathrm{e}^- + \mathrm{e}^+ \longleftrightarrow \nu + \bar{\nu} \quad \text{or} \quad \mathrm{e}^- + \nu \longleftrightarrow \mathrm{e}^- + \nu \quad etc. \tag{6.65}$$

Reaction rate for these processes:

$$\Gamma = n \langle \sigma v \rangle \tag{6.66}$$

where the thermally averaged interaction cross-section is

$$\langle \sigma v \rangle \approx \left\langle \frac{\alpha^2 p}{m_\mathrm{W}^4} \cdot p \right\rangle \sim 10^{-2} \frac{(k_\mathrm{B} T)^2}{m_\mathrm{W}^4} \tag{6.67}$$

m_W: mass of W-boson (exchange particle of weak interaction), $\alpha \approx 1/137$: fine structure constant.

But in the ultra-relativistic limit, $n \propto T^3$ (Eq. 6.38), such that

$$\Gamma_\mathrm{weak} \propto \frac{\alpha^2 T^5}{m_\mathrm{W}^4} \tag{6.68}$$

Neutrinos, II

Because of Eqs. (6.62) and (6.63), the temperature dependence of the Hubble constant is

$$H(T) = 1.66 g_*^{1/2} \cdot \frac{T^2}{m_\mathrm{P}} \tag{6.69}$$

where m_P is the Planck mass, $m_\mathrm{P} c^2 = 1.22 \times 10^{19}$ GeV (see later, Eq. 6.122).

Neutrino equilibrium possible as long as $\Gamma_\mathrm{weak} > H$, i.e., (inserting exact numbers)

$$k_\mathrm{B} T_\mathrm{dec} \gtrsim \left(\frac{500\, c^6\, m_\mathrm{W}^4}{m_\mathrm{P}} \right)^{1/3} \sim 1\,\mathrm{MeV} \tag{6.70}$$

Neutrinos decouple ~ 1 s after the big bang.

This follows from Eq. (6.64), remembering that for this phase, $g_* \sim 10$.

Since decoupling, primordial neutrinos just follow expansion of universe, virtually no interaction with "us" anymore.

Entropy, I

The entropy of particles is defined through

$$S = \frac{E + PV}{T} \tag{6.71}$$

Important for cosmology: relativistic limit. Define the entropy density,

$$s = \frac{S}{V} = \frac{E/V + P}{T} = \frac{u + P}{T} \approx \frac{4}{3}\frac{u}{T} \tag{6.72}$$

(last step for relativistic limit; Eq. 6.40)

Inserting Eq. (6.39) ($u \propto (7/8)T^4$; 7/8 for Fermions only) gives

$$s = \frac{7}{8}\frac{2\pi^2}{45} g k_\mathrm{B} \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 = \frac{7}{8}\frac{2\pi^4}{45\,\zeta(3)} k_\mathrm{B}\, n \tag{6.73}$$

Since $s \propto n$ for backgrounds, $\eta = n_\mathrm{CMBR}/n_\mathrm{baryons}$ is often called "entropy per baryon".

Entropy, II

For a mixture of backgrounds, Eq. (6.73) gives

$$\frac{s}{k_\mathrm{B}} = g_{*,S} \cdot \frac{2\pi^2}{45}\left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 \tag{6.74}$$

where $g_{*,S}$ is the analogue to g_* (Eq. 6.55),

$$g_{*,S} = \sum_\mathrm{bosons} g_\mathrm{B}\left(\frac{T_\mathrm{B}}{T}\right)^3 + \frac{7}{8}\sum_\mathrm{fermions} g_\mathrm{F}\left(\frac{T_\mathrm{F}}{T}\right)^3 \tag{6.75}$$

Note that if the species are not at the same temperature, $g_* \neq g_{*,S}$.

Entropy per mass today:

$$\frac{S}{M} = \frac{10^{16}}{\Omega h^2}\,\mathrm{erg\,K^{-1}\,g^{-1}} \tag{6.76}$$

while the entropy gain of heating water at 300 K by 1 K is $\sim 1.4 \times 10^5\,\mathrm{erg\,K^{-1}\,g^{-1}}$.

$\Longrightarrow$ "Human attempts to obey 2nd law ... are swamped by ... microwave background" (Peacock, 1999, p. 277).

$\Longrightarrow$ $S =$ const. for universe to very good approximation.

$\Longrightarrow$ Universe expansion is adiabatic!

Reheating

After decoupling of neutrinos, neutrino distribution just gets redshifted (similar to CMBR, Eq. 6.19):

$$\frac{T_\nu}{T_\mathrm{dec}} = \frac{R_\mathrm{dec}}{R(t)} \quad\Longrightarrow\quad T_\nu \propto R^{-1} \tag{6.77}$$

On the other hand, the temperature of the universe is

$$T \propto g_{*,S}^{1/3} R^{-1} \tag{6.78}$$

This follows from $S/V \propto T^3$ (Eq. 6.74), $V \propto R^3$, and $S =$ const. (adiabatic expansion of the universe).

$\Longrightarrow$ as long as $g_{*,S} =$ const. we have $T_\nu = T$

$\Longrightarrow$ Immediately after decoupling, neutrino background appears as if it is still in equilibrium.

However: Temperature for neutrino decoupling $\sim 2m_\mathrm{e}c^2$. But, for $kT_\mathrm{BB} < 2\,m_\mathrm{e}c^2$, pair creation,

$$\gamma + \gamma \longleftrightarrow \mathrm{e}^- + \mathrm{e}^+ \tag{6.79}$$

is kinematically impossible.

$\Longrightarrow$ Shortly after neutrino decoupling: $\mathrm{e}^\pm$ annihilation

$\Longrightarrow$ $g_{*,S}$ changes!

$\Longrightarrow$ We expect that $T_\mathrm{CMBR} \neq T_\nu$.

Reheating

Difference in $g_{*,S}$:

- before annihilation: $\mathrm{e}^-, \mathrm{e}^+, \gamma \Longrightarrow g_{*,S} = 2 + 2\cdot 2\cdot (7/8) = 11/2$.
- after annihilation: $\gamma \Longrightarrow g_{*,S} = 2$

But: the total entropy for particles in equilibrium conserved ("expansion is adiabatic"):

$$g_{*,S}(T_\mathrm{before}) \cdot T^3_\mathrm{before} = g_{*,S}(T_\mathrm{after}) \cdot T^3_\mathrm{after} \tag{6.80}$$

such that

$$T_\mathrm{after} = \left(\frac{11}{4}\right)^{1/3} T_\mathrm{before} \sim 1.4 \cdot T_\mathrm{before} \tag{6.81}$$

Since $T_\mathrm{after} > T_\mathrm{before}$: "reheating".

Note that in reality the annihilation is not instantaneous and T decreases (albeit less rapidly) during "reheating"...

$\Longrightarrow$ Since neutrino-background does not "see" annihilation

$\Longrightarrow$ just continues to cool

$\Longrightarrow$ current temperature of neutrinos is

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\mathrm{CMBR} \sim 1.95\,\mathrm{K} \tag{6.82}$$

History

After reheating: universe consists of p, n, γ (and e^- to preserve charge neutrality)

$\Longrightarrow$ Ingredients for Big Bang Nucleosynthesis (BBN).

Historical perspective: Cross section to make Deuterium:

$$\langle\sigma v\rangle(\mathrm{p}+\mathrm{n}\rightarrow\mathrm{D}+\gamma)\sim 5\times10^{-20}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \quad (6.83)$$

Furthermore, we need temperatures of $T_{\mathrm{BBN}}\sim 100\,\mathrm{keV}$, i.e., $t_{\mathrm{BBN}}\sim 200\,\mathrm{s}$ (Eq. 6.64).
By Eq. (6.20) this implies a particle density of

$$n\sim\frac{1}{\langle\sigma v\rangle\cdot t_{\mathrm{BBN}}}\sim 10^{17}\,\mathrm{cm}^{-3} \quad (6.84)$$

Today: Baryon density $n_{\mathrm{B}}\sim 10^{-7}\,\mathrm{cm}^{-3}$. Since $n\propto R^{-3}$,

$$T(\mathrm{today})=\left(\frac{n_{\mathrm{B}}}{n}\right)^{1/3}\cdot T_{\mathrm{BBN}}\sim 10\,\mathrm{K} \quad (6.85)$$

pretty close to the truth. . .

The above discussion was first asserted by George Gamov and coworkers in 1948, and was the first prediction of the cosmic microwave background radiation!

Observations: BBN is required by observations, since no other production region for Deuterium known, and since He-abundance $\sim 25\%$ by mass everywhere.

Proton/Neutron, I

Initial conditions for BBN: Set by Proton-Neutron-Ratio.

For $t\ll 1\,\mathrm{s}$, equilibrium via weak interactions:

$$\begin{aligned}\mathrm{n}&\longleftrightarrow p+\mathrm{e}^-+\bar\nu_{\mathrm{e}}\\ \nu_{\mathrm{e}}+\mathrm{n}&\longleftrightarrow p+\mathrm{e}^-\\ \mathrm{e}^++\mathrm{n}&\longleftrightarrow p+\bar\nu_{\mathrm{e}}\end{aligned} \quad (6.86)$$

Reactions fast as long as particles relativistic.

But once $T\sim 1\,\mathrm{MeV}$: n, p become non-relativistic

$\Longrightarrow$ Boltzmann statistics applies (or use Eq. 6.51):

$$\frac{n_{\mathrm{n}}}{n_{\mathrm{p}}}=\mathrm{e}^{-\Delta mc^2/k_{\mathrm{B}}T}=\mathrm{e}^{-1.3\,\mathrm{MeV}/k_{\mathrm{B}}T} \quad (6.87)$$

$\Longrightarrow$ Suppression of n with respect to p because of larger mass ($m_{\mathrm{n}}c^2=939.57\,\mathrm{MeV}$, $m_{\mathrm{p}}c^2=938.27\,\mathrm{MeV}$)

Proton/Neutron, II

As usual, the n, p abundance freezes out when $\Gamma\gg H$.

For the neutron, proton equilibrium, the reaction rate is

$$\Gamma(\nu_{\mathrm{e}}+\mathrm{n}\leftrightarrow\mathrm{p}+\mathrm{e}^-)\sim 2.1\left(\frac{T}{1\,\mathrm{MeV}}\right)^5\,\mathrm{s}^{-1} \quad (6.88)$$

The neutron abundance freezes out at $k_{\mathrm{B}}T\sim 0.8\,\mathrm{MeV}$ ($t=1.7\,\mathrm{s}$), such that $n_{\mathrm{n}}/n_{\mathrm{p}}=0.2$

After that: Neutron decay ($\tau_{\mathrm{n}}=886.7\pm1.2\,\mathrm{s}$).

$\Longrightarrow$ Nucleosynthesis has to be over before neutrons are decayed away!

$\Longrightarrow$ Nucleosynthesis only takes a few minutes at most!

Deuterium

The first step in nucleosynthesis is the formation of deuterium (binding energy $E_{\mathrm{B}}=2.225\,\mathrm{MeV}$, i.e., $1.7(m_{\mathrm{n}}-m_{\mathrm{p}})c^2$):

$$\mathrm{p}+\mathrm{n}\longleftrightarrow\mathrm{D}+\gamma \quad (6.89)$$

Note: Both fusion and photodisintegration are possible:

$$\Gamma_{\mathrm{fusion}}=n_{\mathrm{B}}\langle\sigma v\rangle \quad (6.90)$$

$$\Gamma_{\mathrm{photo}}=n_\gamma\langle\sigma v\rangle\mathrm{e}^{-E_{\mathrm{B}}/k_{\mathrm{B}}T} \quad (6.91)$$

At first: photodisintegration dominates since $\eta^{-1}=n_\gamma/n_{\mathrm{B}}\sim 10^{10}$ (see Eq. 6.73).

Build up of D is only possible once $\Gamma_{\mathrm{fusion}}>\Gamma_{\mathrm{photo}}$, i.e., when

$$\frac{n_\gamma}{n_{\mathrm{B}}}\mathrm{e}^{-E_{\mathrm{B}}/k_{\mathrm{B}}T}\sim 1 \quad (6.92)$$

Inserting numbers shows that

Deuterium production starts at $k_{\mathrm{B}}T\sim 100\,\mathrm{keV}$, or $t\sim 100\,\mathrm{s}$.