Determination of Ω and Λ

Inflation

Previous lectures: Inflation requires

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} = \Omega_{\text{m}} + \Omega_\Lambda = 1 \tag{8.1}$$

Here,

Ω_{m}: Ω due to gravitating stuff,

Ω_Λ: Ω due to vacuum energy or other exotic stuff.

To decide whether that is true:

- need inventory of gravitating material in the universe,
- need to search for evidence of non-zero Λ

Also search for evidence in structure formation $\Longrightarrow$ Later...

Inflation

Remember that

$$\Omega_{\text{m}} = \frac{\rho_{\text{m}}}{\rho_{\text{crit}}} = \frac{8\pi G\rho}{3H^2} \tag{4.58}$$

and

$$\Omega_\Lambda = \frac{\rho_{\text{vac}}}{\rho_{\text{crit}}} = \frac{\rho_{\text{vac}}}{3H^2/8\pi G} = \frac{c^4\Lambda}{3H^2} \tag{7.14}$$

As for a typical ensemble of stars,

$$\frac{M}{L} \approx \text{const.} \tag{8.2}$$

we often express Ω in terms of a mass to luminosity ratio:

Using canonical luminosity density of universe, one can show (Peacock, 1999, p. 368, for the B-band):

$$\left.\frac{M}{L}\right|_{\text{crit}} = 1390\, h \frac{M_\odot}{L_\odot} \tag{8.3}$$

...which means that there *must* be lots of dark matter.

Introduction

Constituents of Ω_{m}:

- Radiation (CMBR)
- Neutrinos
- Baryons ("normal matter", Ω_{b})
- Other, non-radiating, gravitating material ("dark matter")

Radiation: From temperature of CMBR, using $u = \rho c^2 = a_{\text{rad}} T^4$:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \tag{8.4}$$

for $h = 0.72$, $\Omega_\gamma = 4.8 \times 10^{-5}$

Massless Neutrinos have

$$\Omega_\nu = 3 \cdot \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \Omega_\gamma = 0.68\, \Omega_\gamma \tag{8.5}$$

Photons and massless neutrinos are unimportant for todays Ω.

Massive Neutrinos

Sudbury Neutrino Observator (SNO) and Super-Kamiokande: Neutrinos are not massless.

From neutrino decoupling and expansion:

Current neutrino density: 113 neutrinos cm^{-3} per neutrino family.

In terms of Ω:

$$\Omega_\nu h^2 = \frac{\sum m_i}{93.5\,\text{eV}} \qquad (8.6)$$

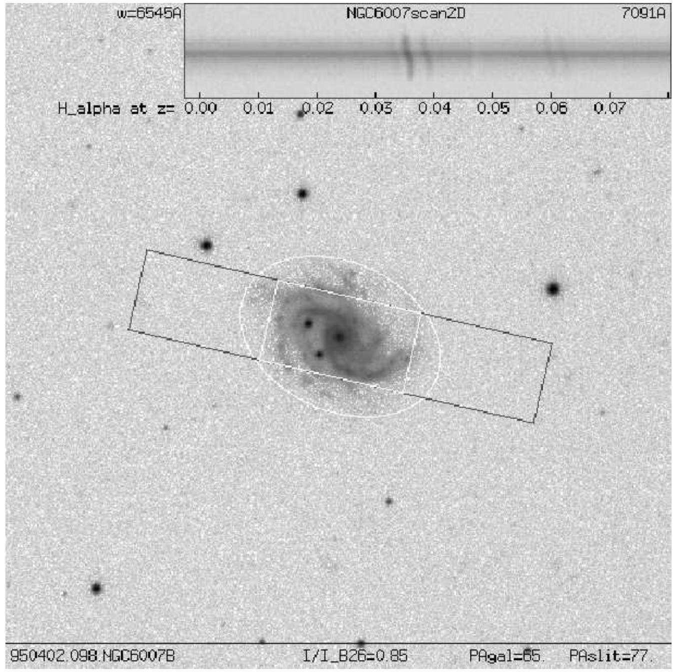
$\Longrightarrow$ For $h = 0.72$, $m \sim 16$ eV would be sufficient to close universe

Current mass limits: $m_{\nu_e} < 2.2$ eV, $m_{\nu_\mu} < 170$ keV, and $m_{\nu_\tau} < 15.5$ MeV

Source: `http://cupp.oulu.fi/neutrino/nd-mass.html`

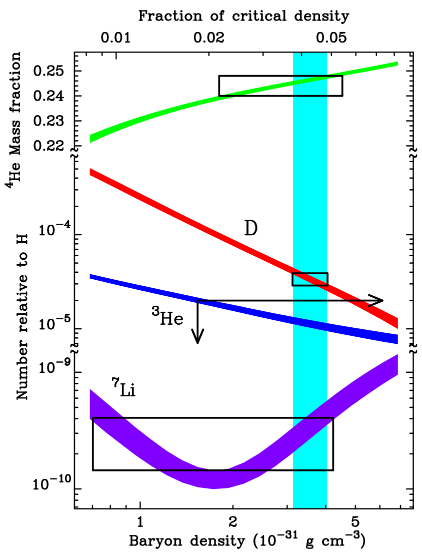
Note that solar neutrino oscillations imply Δm between ν_e and ν_μ is $\sim 10^{-4}$ eV, i.e., most probable mass for ν_μ is much smaller than the direct experimental limit.

Structure formation shows that $\sum m_\nu < 0.7$ eV (Spergel et al., 2007).



NGC 6007 (Jansen; `http://www.astro.rug.nl/~nfgs/`)

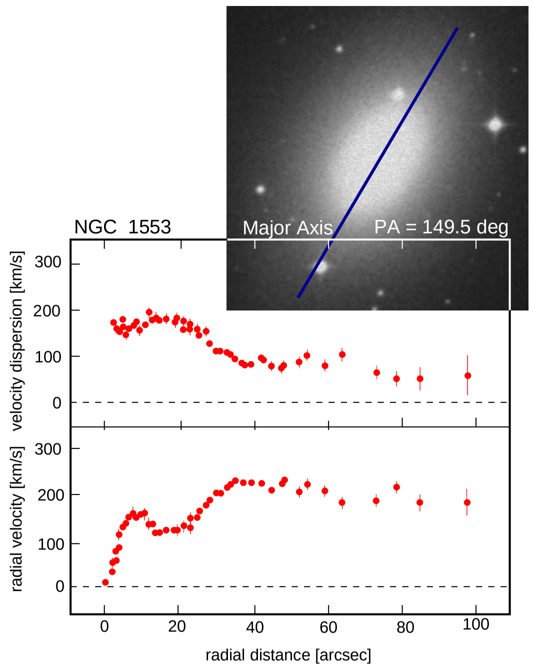
Baryons



Best evidence for mass in baryons, Ω_b: primordial nucleosynthesis.

$$\Omega_b h^2 = 0.02 \pm 0.002 \qquad (8.7)$$

(Burles, Nollett & Turner, 1999, Fig. 1)



NGC 1553 (S0) after Kormendy (1984, ApJ 286, 116)

Galaxy Rotation Curves, IV

Stellar motion due to mass within r:

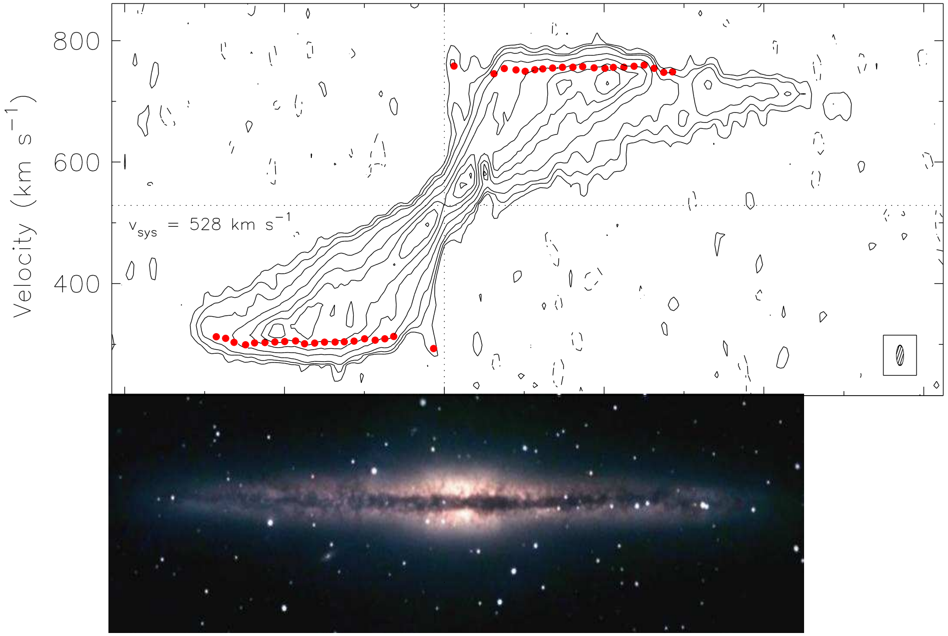
$$\frac{GM(\leq r)}{r^2} = \frac{v_{\text{rot}}^2(r)}{r}$$

$$\Longrightarrow M(\leq r) = \frac{v_{\text{rot}}^2 r}{G}$$

therefore:

$v \sim$ const. $\Longrightarrow M(\leq r) \propto r$.

NGC 891, KPNO 1.3 m
Barentine & Esquerdo



NGC 891 (Swaters et al., 1997, ApJ 491, 140 / Paul LeFevre, S&T Nov. 2002)

Galaxy Rotation Curves, V

For disk in spiral galaxies, $I(r) = I_0 \exp(-r/h)$ such that

$$L(r < r_0) = I_0 \int_0^{r_0} 2\pi r \exp(-r/h)\,\mathrm{d}r \propto h^2 - h(r+h)\exp(-r/h) \qquad (8.8)$$

such that for $r \longrightarrow \infty$:

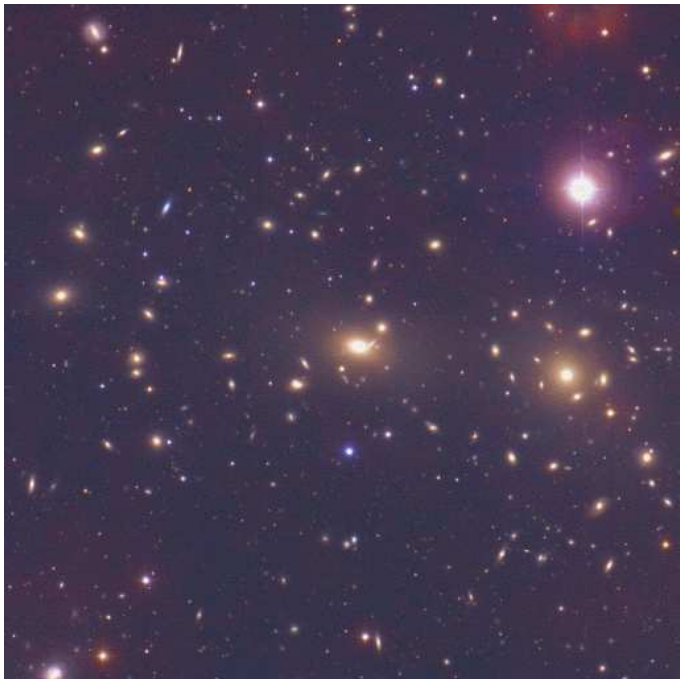
$$L(r < r_0) \rightarrow \text{const.} \qquad (8.9)$$

If $M/L \sim$ const. $\Longrightarrow$ contradiction with observations! (we would expect $v \propto r^{-1/2}$)

Result for galaxies compared to stars

$$\left.\frac{M}{L}\right|_{\text{galaxies}} = 10\ldots20\,\frac{M_\odot}{L_\odot} \quad \text{vs.} \quad \left.\frac{M}{L}\right|_{\text{stars}} = 1\ldots3\,\frac{M_\odot}{L_\odot}$$

Only about 10% of the gravitating matter in universe radiates.



Coma Cluster (O. Lopez-Cruz, I. K. Shelton, & KPNO)

Galaxy Clusters, II

For mass of galaxy clusters, make use of the virial theorem:

$$E_{\text{kin}} = -E_{\text{pot}}/2 \tag{8.10}$$

in statistical equilibrium.

Measurement: assume isotropy, such that

$$\left\langle v^2 \right\rangle = \left\langle v_x^2 \right\rangle + \left\langle v_y^2 \right\rangle + \left\langle v_z^2 \right\rangle = 3 \left\langle v_\parallel^2 \right\rangle \tag{8.11}$$

Assuming that the velocity dispersion is independent of m_i gives:

$$E_{\text{kin}} = \frac{1}{2} \sum_i m_i \boldsymbol{v}_i^2 = \frac{3}{2} M \left\langle v_\parallel^2 \right\rangle \tag{8.12}$$

where M is the total mass.

If the cluster is spherically symmetric $\Longrightarrow$ Define weighted mean separation R_{cl}, such that

$$E_{\text{pot}} = \frac{GM^2}{R_{\text{cl}}} \tag{8.13}$$

From Eqs. (8.12) and (8.13):

$$M = \frac{3}{G} \left\langle v_\parallel^2 \right\rangle R_{\text{cl}} \tag{8.14}$$

E.g.: $v_\parallel \sim 1000\,\text{km}\,\text{s}^{-1}$, $R \sim 1\,\text{Mpc} \Longrightarrow M = 1.4 \times 10^{48}\,\text{g} = 7 \times 10^{14}\,M_\odot$ (MW: $6 \times 10^{11}\,M_\odot$).

8–13

Derivation of the Virial Theorem

Assume system of particles, each with mass m_i. Acceleration on particle i:

$$\ddot{\mathbf{r}} = \sum_{j \neq i} \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{8.15}$$

…scalar product with $m_i \mathbf{r}_i$

$$m_i \mathbf{r}_i \cdot \ddot{\mathbf{r}}_i = \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{8.16}$$

…since

$$\frac{1}{2}\frac{\text{d}^2 \mathbf{r}_i^2}{\text{d}t^2} = \frac{\text{d}}{\text{d}t}(\dot{\mathbf{r}}_i \cdot \mathbf{r}_i) = \ddot{\mathbf{r}}_i \cdot \mathbf{r}_i + \dot{\mathbf{r}}_i \cdot \mathbf{r}_i \tag{8.17}$$

…therefore Eq. (8.16)

$$\frac{1}{2}\frac{\text{d}^2}{\text{d}t^2}(m_i \mathbf{r}_i^2) - m_i \dot{\mathbf{r}}_i{}^2 = \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{8.18}$$

Summing over all particles in the system gives

$$\frac{1}{2}\sum_i \frac{\text{d}^2}{\text{d}t^2}(m_i \mathbf{r}_i^2) - \sum_i m_i \dot{\mathbf{r}}_i{}^2 = \sum_i \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{8.19}$$

$$= \frac{1}{2}\left(\sum_i \sum_{j \neq i} Gm_i m_j \frac{\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j \sum_{i \neq j} Gm_j m_i \frac{\mathbf{r}_j \cdot (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \right) \tag{8.20}$$

$$= \frac{1}{2}\left(\sum_i \sum_{j \neq i} Gm_i m_j \frac{\mathbf{r}_i \cdot \mathbf{r}_j - \mathbf{r}_i^2}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j \sum_{i \neq j} Gm_j m_i \frac{\mathbf{r}_j \cdot \mathbf{r}_i - \mathbf{r}_j^2}{|\mathbf{r}_j - \mathbf{r}_i|^3} \right) \tag{8.21}$$

$$= -\frac{1}{2} \sum_{\substack{i,j \\ i \neq j}} \frac{Gm_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|} \tag{8.22}$$

8–13

Thus, identifying the total kinetic energy, T, and the gravitational potential energy, U, gives

$$2T - U = \frac{1}{2}\frac{\text{d}^2}{\text{d}t^2}\sum_i m_i \mathbf{r}_i^2 = 0 \tag{8.23}$$

in statistical equilibrium.

Thus we find the virial theorem: $T = \frac{1}{2}|U|$

Galaxy Clusters, III

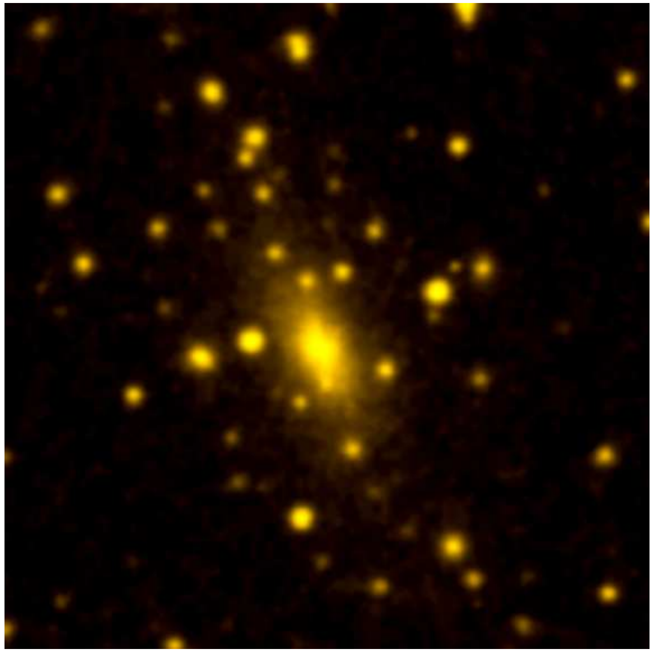
More detailed analysis using more complicated mass models gives (Merritt, 1987):

$$\frac{M}{L} \sim 350 h^{-1} \frac{M_\odot}{L_\odot} \tag{8.24}$$

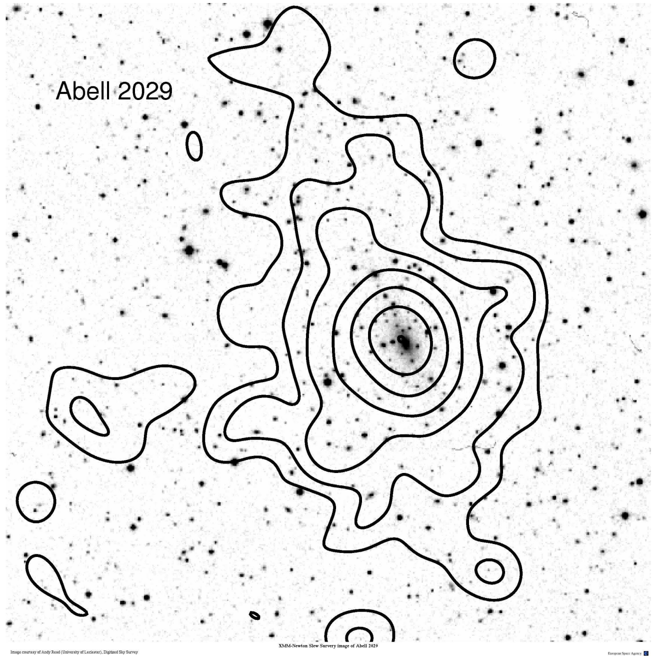
while we would have expected $M/L = 10 \ldots 20$ as for galaxies

Dark matter is an important constituent in galaxy clusters

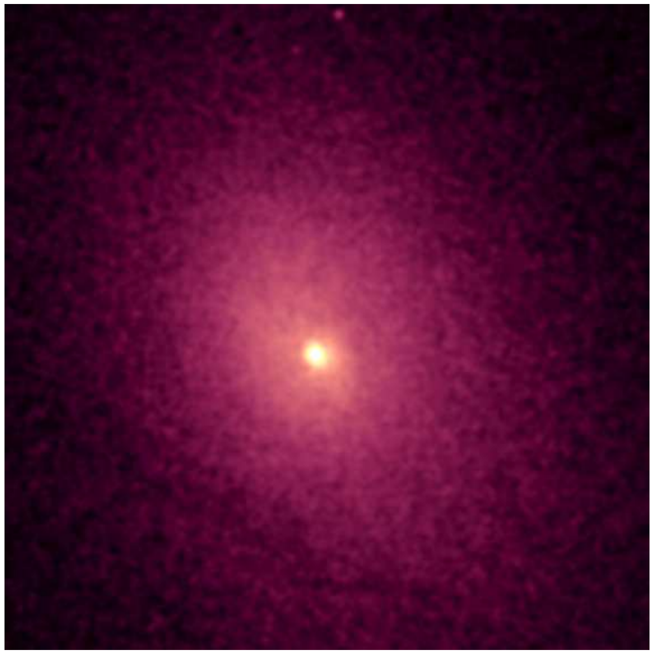
Abell 370 (VLT UT1+FORS)



Abell 2029, Palomar Schmidt [DSS]



Abell 2029, Optical and X-rays (*XMM-Newton*; Andy Read [Leicester]/DSS/ESA; larger FoV)



Abell 2029, Soft X-rays (*Chandra*; NASA/CXC/UCI/A.Lewis et al.)

X-ray emission, IV

X-ray emission from galaxy clusters gives mass to higher precision:

Assume gas in potential of galaxy cluster. If gas is in hydrostatic equilibrium:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2} \tag{8.25}$$

where the pressure P can be determined from the equation of state:

$$P = nkT = \frac{\rho kT}{\mu m_\mathrm{H}} \tag{8.26}$$

where m_H: mass of H-atom, μ mean molecular weight of gas ($\mu = 0.6$ for fully ionized).

Differentiating Eq. (8.26) wrt r gives

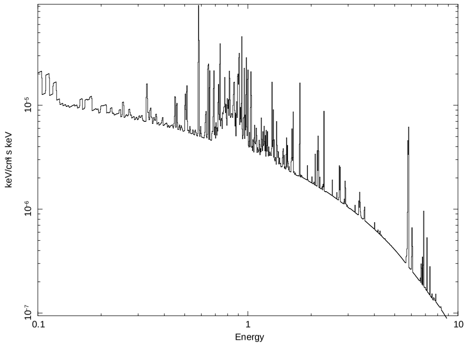
$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{k}{\mu m_\mathrm{H}}\left(T\frac{\mathrm{d}\rho}{\mathrm{d}r} + \rho\frac{\mathrm{d}T}{\mathrm{d}r}\right) = \frac{\rho kT}{\mu m_\mathrm{H}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{8.27}$$

Inserting $\mathrm{d}P/\mathrm{d}r$ into Eq. (8.25) and solving for M_r gives

$$M_r = -\frac{kTr^2}{G\mu m_\mathrm{H}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{8.28}$$

X-ray emission, V

To determine M_r, we need to measure $T(r)$ and $\rho(r)$. These quantities can be obtained from the observed X-ray spectrum:



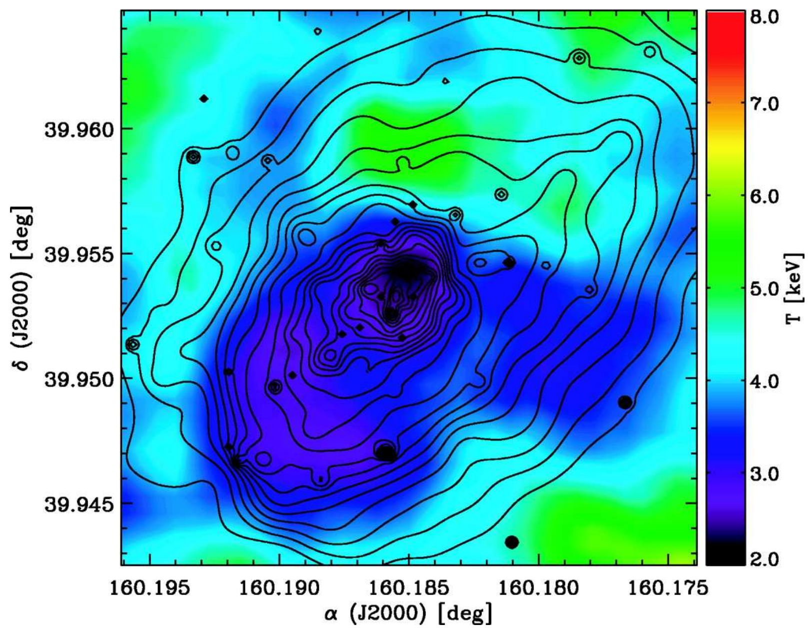
Theoretical X-ray spectrum of a cluster.

Cluster gas mainly radiates by bremsstrahlung emission, with a spectral continuum shape

$$\epsilon(E) \propto \left(\frac{m_e}{kT}\right)^{1/2} g(E,T)\, n\, n_e \exp\left(-\frac{E}{kT}\right) \tag{8.29}$$

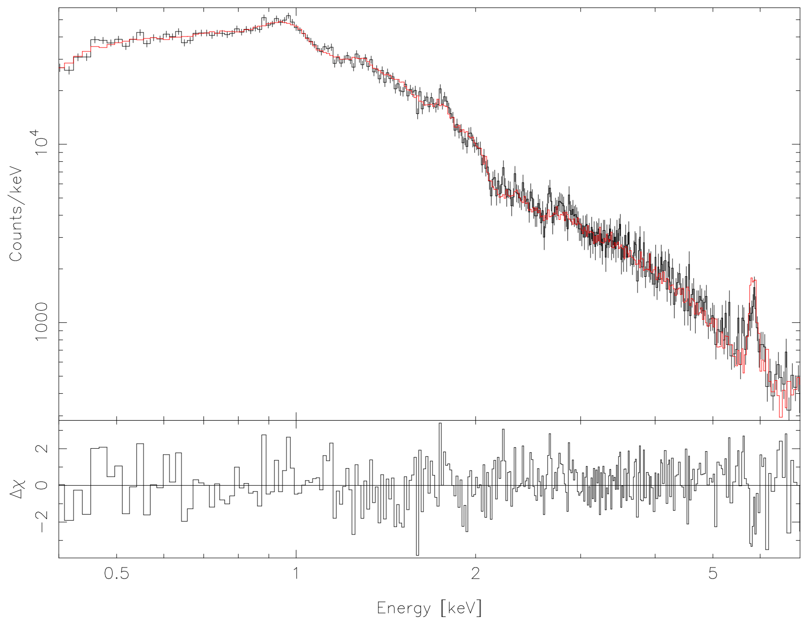
where
n: number density of nuclei,
n_e: number density of electrons,
$g(E,T)$: Gaunt factor (QM correction factor, roughly constant).
plus emission lines...

$\Longrightarrow T(r)$ can be obtained from the X-ray spectral shape, n and n_e from the measured flux
$\Longrightarrow M_r$.



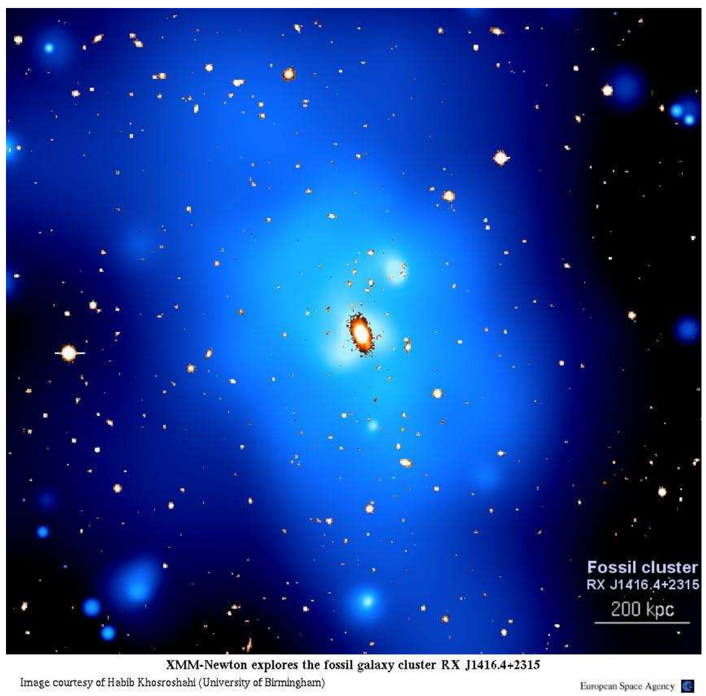
(Wise, McNamara & Murray, 2004, Fig. 8)

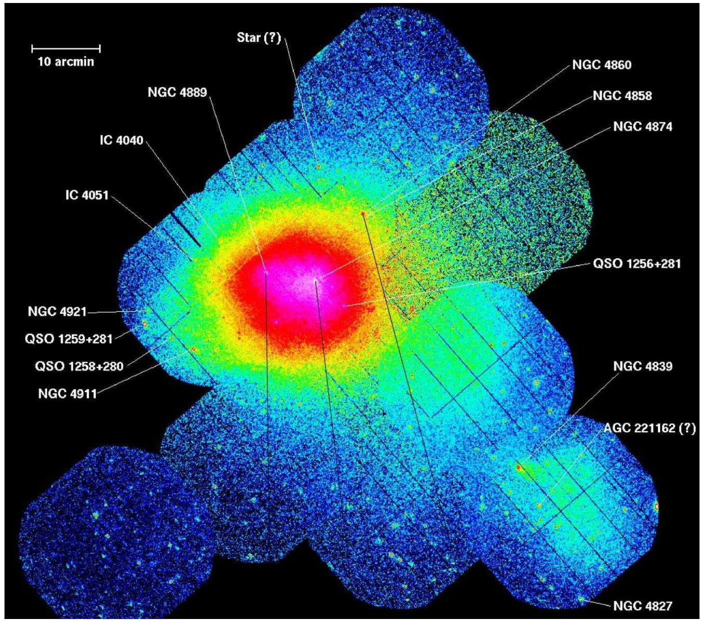
Temperature distribution in A1068 obtained with *Chandra*



(Wise, McNamara & Murray, 2004, Fig. 2)

X-ray spectrum of A1068 obtained from *Chandra*





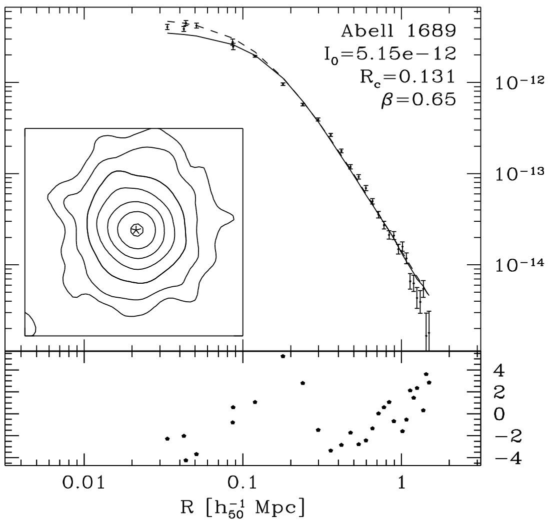
Technical problems:
- see through cluster $\Longrightarrow$ integrate over line of sight, assuming spherical geometry.
- spherical geometry is assumed
- it is unclear whether gas is in hydrostatic equilibrium (cooling flows? – but note, there is sparse evidence for a "flow")

XMM-Newton, EPIC-pn

Result for Coma:

$$\frac{M_\mathrm{B}}{M_\mathrm{tot}} = 0.01 + 0.05\,h^{-3/2} \qquad (8.30)$$

X-ray emission, X



(Mohr, Mathiesen & Evrard, 1999)

Generally: assume intensity profile from β-model,

$$\frac{I(r)}{I_0} = \left(1 + \left(\frac{r}{R_\mathrm{c}}\right)^2\right)^{-3\beta+\frac{1}{2}} \qquad (8.31)$$

and obtain T from fitting X-ray spectra to "shells" $\Longrightarrow$ technically complicated...

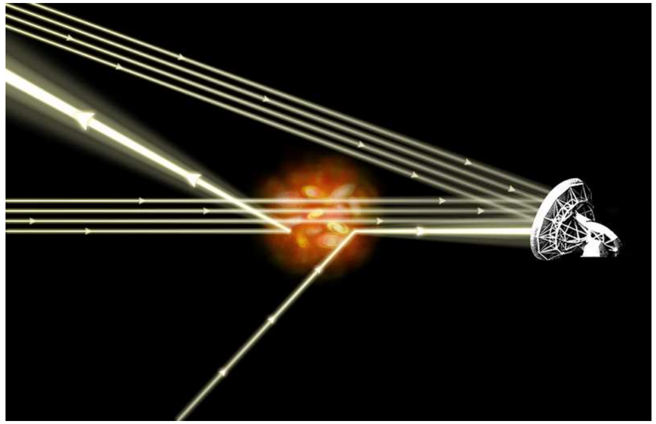
Summary for X-ray mass determination for 45 clusters (Mohr, Mathiesen & Evrard, 1999):

$$f_\mathrm{gas} = (0.07 \pm 0.002)h^{-3/2} \qquad (8.32)$$

resulting in

$$\Omega_\mathrm{m} = \Omega_\mathrm{b}/f_\mathrm{gas} = (0.3 \pm 0.05)\,h^{-1/2} \qquad (8.33)$$

Sunyaev-Zeldovich, I

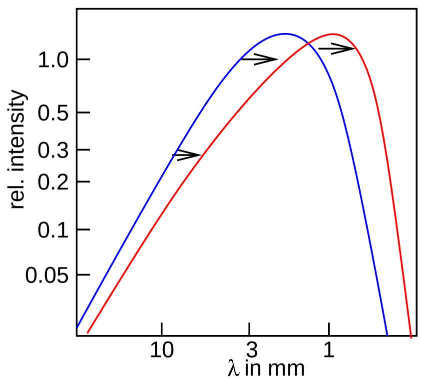


NASA/CXC/M.Weiss

Gas in cooling flow influences CMBR by Compton upscattering
$\Longrightarrow$ Sunyaev-Zeldovich effect (1970).

Sunyaev-Zeldovich, II

The quantitative derivation of the SZ-effect is difficult, basically, one sets up the Fokker-Planck equation for the photon gas and from this derives the so-called Kompaneets equation, see, e.g., Peacock (1999, p. 375ff.).



after Schneider

The basic ingredients are the optical depth for Compton scattering (Compton y-parameter):

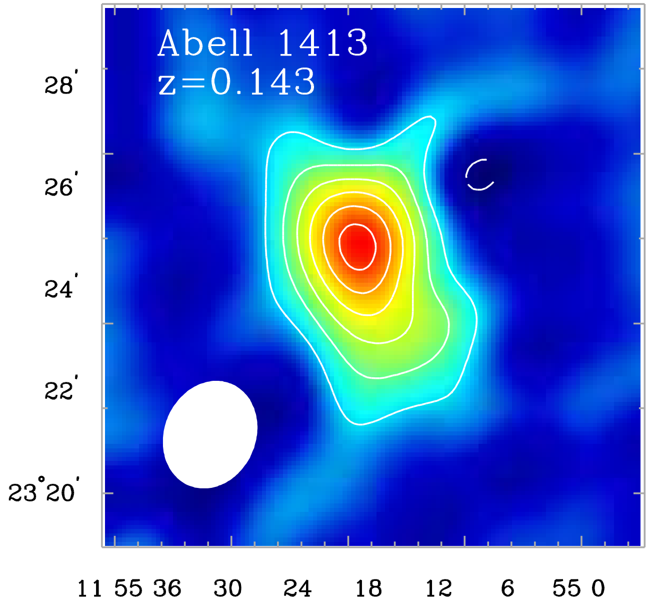
$$y = \int \left(\frac{kT_\mathrm{e}}{m_\mathrm{e}c^2}\right) \sigma_\mathrm{T} N_\mathrm{e}\,\mathrm{d}l \qquad (8.34)$$

From this follows in the Rayleigh-Jeans regime that the intensity due to Compton upscattering changes as follows:

$$\frac{\Delta I}{I} = -2y \sim 10^{-4} \qquad (8.35)$$

(for typical parameters).
$\Longrightarrow$ ΔI allows to measure of $\int N_\mathrm{e} T_\mathrm{e}\,\mathrm{d}l$
$\Longrightarrow$ Mass!

T is known from X-ray spectrum.



(temperature decrement from 3 K background, Carlstrom et al., 2000, Fig. 3)

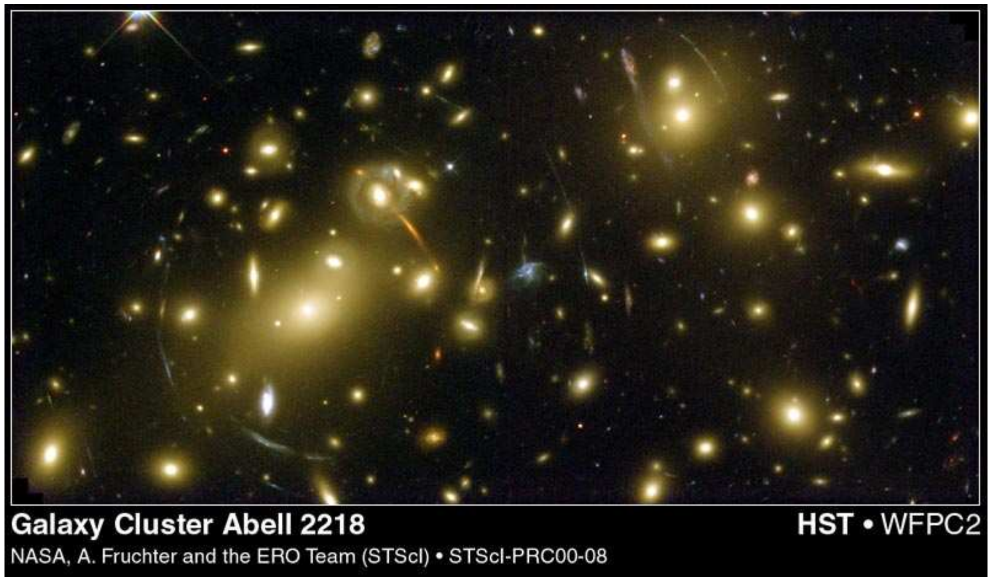
SZ analysis gives gas fraction for 27 clusters

$$f_{\text{gas}} = (0.06 \pm 0.006)h^{-3/2} \tag{8.36}$$

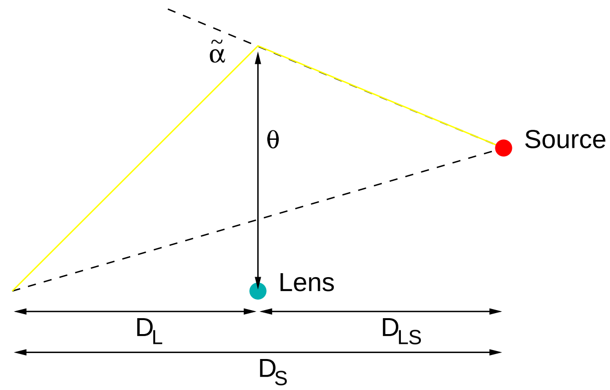
remarkably similar to X-ray result $\Longrightarrow$ clumping of gas does not influence results! (SZ only traces real gas...)

f_{gas} translates to

$$\Omega_{\text{m}} = (0.25 \pm 0.04)\,h^{-1} \tag{8.37}$$



Gravitational Lenses, II



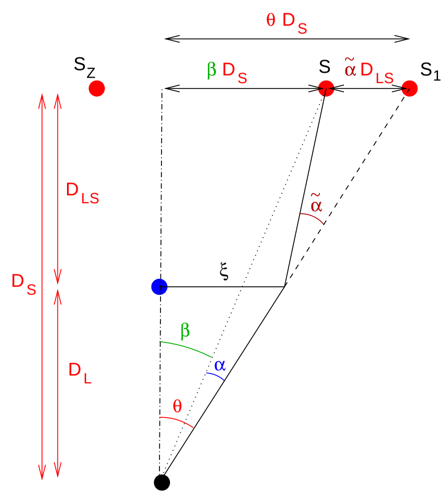
(after Longair, 1998, Fig. 4.8a)

GR: Angular deflection of light due to presence of mass M:

$$\tilde{\alpha} = \frac{4GM}{\theta c^2} = \frac{2}{c^2} \cdot \frac{2GM}{\theta} \tag{8.38}$$

where θ: distance of closest approach (*twice* the classical result).

Gravitational Lenses, III



after Wambsganss (1998, Fig. 3)

In the small angle approximation:

$$\theta D_{\text{S}} = \beta D_{\text{S}} + \tilde{\alpha} D_{\text{LS}} \tag{8.39}$$

such that

$$\beta = \theta - \frac{D_{\text{LS}}}{D_{\text{S}}}\tilde{\alpha} \tag{8.40}$$

Defining the reduced deflection angle,

$$\alpha = \frac{D_{\text{LS}}}{D_{\text{S}}}\tilde{\alpha} = \frac{D_{\text{LS}}}{D_{\text{S}}} \cdot \frac{2}{c^2} \cdot \frac{2GM}{\theta} \tag{8.41}$$

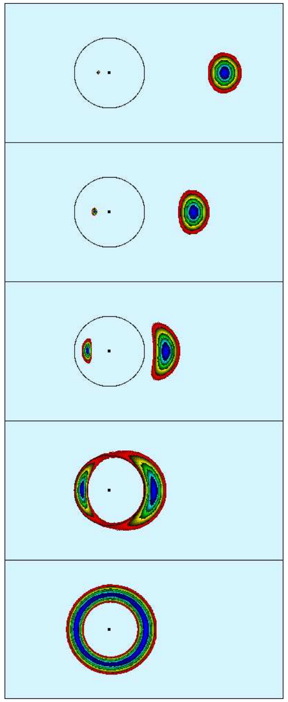
then gives the lens equation

$$\beta = \theta - \alpha = \theta - \frac{D_{\text{LS}}}{D_{\text{L}} D_{\text{S}}} \cdot \frac{4GM}{c^2\theta} = \theta - \frac{1}{D} \cdot \frac{4GM}{c^2\theta} \tag{8.42}$$

where

$$D = \frac{D_{\text{L}} D_{\text{S}}}{D_{\text{LS}}} \tag{8.43}$$

(last expression valid for a point-mass)



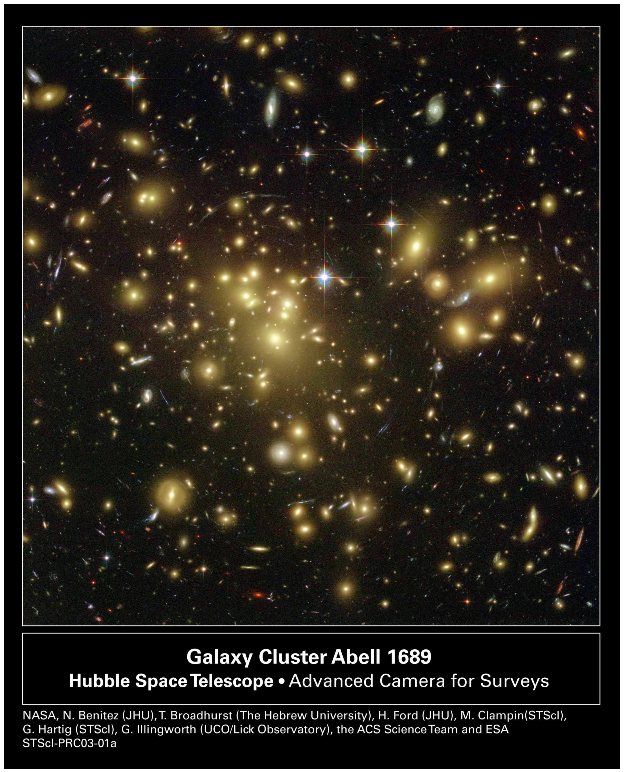
Einstein ring: source directly behind lens,
Obtain radius by setting $\beta = 0$ in lens-equation (Eq. 8.42):

$$\theta_{\mathrm{E}}^2 = \frac{4GM}{c^2}\frac{1}{D} \tag{8.44}$$

i.e.,

$$\theta_{\mathrm{E}} = 98.9'' \left(\frac{M}{10^{15}\,M_\odot}\right)^{1/2} \frac{1}{(D/1\,\mathrm{Gpc})^{1/2}} \tag{8.45}$$

Mass measurements possible by observing "giant luminous arcs" and Einstein rings.



General results of mass determinations from lensing agree with other methods.

Summary

So far, we have seen:

Photons:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \tag{8.46}$$

Neutrinos:

$$\Omega_\nu h^2 = 1.69 \times 10^{-5} \tag{8.47}$$

Baryons (from nucleosynthesis):

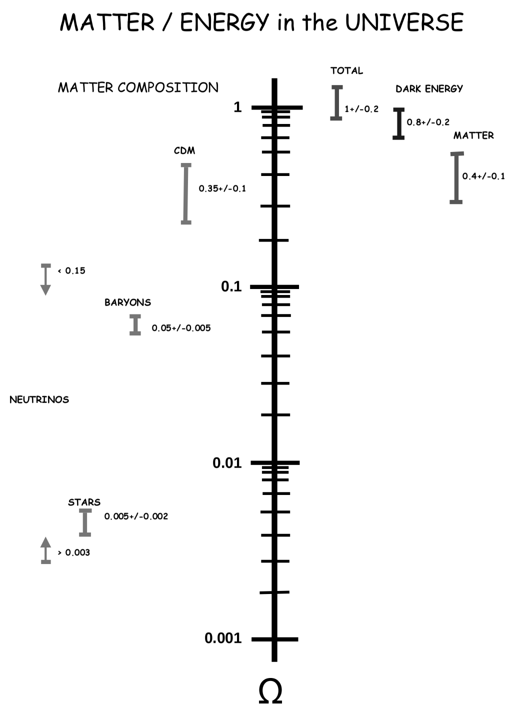
$$\Omega_{\mathrm{b}} h^2 = 0.02 \quad \text{where} \quad \Omega_{\mathrm{stars}} \sim 0.005\ldots0.01 \tag{8.48}$$

Baryons+dark matter (from clusters):

$$\Omega_{\mathrm{m}} \sim 0.25 \tag{8.49}$$

(of which $\sim 10\%$ in baryons)

If we believe in $\Omega_{\mathrm{total}} \equiv 1 \Longrightarrow \Omega_\Lambda \sim 0.7$.



(Turner, 1999, Fig. 1, numbers slightly different to ours. . .)

Introduction

Clusters and galaxies: $\Omega_\text{m} \sim 0.3$, but for baryons $\Omega_\text{b} \sim 0.02$

$\Longrightarrow$ Rest of gravitating material is dark matter.

$\Longrightarrow$ There are two dark matter problems:

$$\Omega_\text{m} \xleftarrow{\text{nonbaryonic dark matter}} \Omega_\text{b} \xleftarrow{\text{baryonic dark matter}} \Omega_\text{stars}$$

baryonic dark matter= undetected baryons:

- diffuse hot gas?
- MACHOs (Massive compact halo objects; white dwarfs, neutron stars, black holes, brown dwarfs, jupiters,. . .)

nonbaryonic dark matter= exotic stuff:

- massive neutrinos
- axions
- neutralinos

Baryonic Dark Matter, I

Intra Cluster Gas:

Pro:

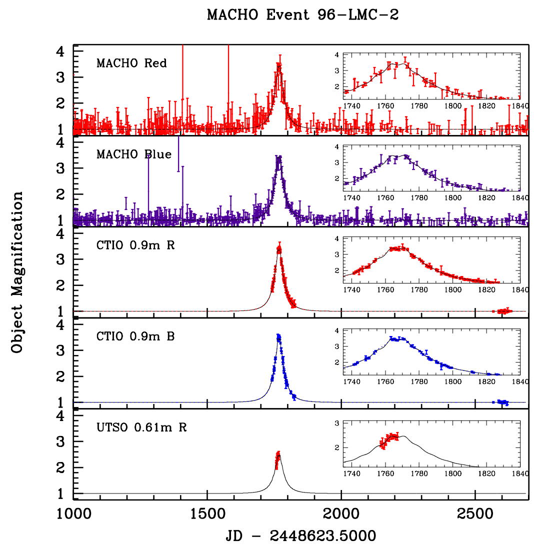
1. same location where the hot gas in clusters also found,
2. structure formation suggests most baryons are *not* in structures today

Contra:

1. 90% of the universe is *not* in clusters. . .
2. gas has not been detected at any wavelength

If gas cold enough, would not expect it to be detectable, so point 2 is not really valid.

Baryonic Dark Matter, II



MACHOS:

Pro:

1. detected by microlensing towards SMC and LMC (see figure) $\Longrightarrow$ MW halo consists of 50% WD

Contra:

1. possible "self-lensing" (by stars in MW or SMC/LMC; confirmed for a few cases)
2. where are white dwarfs?
3. WD formation rate too high (100 year^{-1} Mpc^{-3})

(Alcock et al., 2001, Fig. 2)

Nonbaryonic Dark Matter

Nonbaryonic dark matter:

Requirements: must be gravitating and non-interacting with baryons

$\Longrightarrow$ Grab-box of elementary particle physics:

1. Neutrinos with non-zero mass

   Pro: It exists, mass limits are a few eV, need only $\langle m_\nu \rangle \sim 10$ eV

   Contra: ν are relativistic $\Longrightarrow$ Hot dark matter $\Longrightarrow$ Forces top down structure formation, contrary to what is believed to have happened.

2. Axion

   (=Goldstone boson from QCD, invented to prevent strong CP violation in QCD; $m \sim 10^{-5...-2}$ eV)

   Pro: It *could* exist, would be in Bose-Einstein condensate due to inflation ($\Longrightarrow$ Cold dark matter!), might be detectable in the next 10 years

   Contra: We do not know it exists. . .

3. Neutralino or other WIMPs (weakly interacting massive particles; masses $m \sim$ GeV)

   Pro: Also is CDM

   Contra: We do not know they exist. . .