

Friedmann with  $\Lambda \neq 0$ , I

⇒ Need to study cosmology with  $\Lambda \neq 0$ .

Reviews: Carroll, Press & Turner (1992), Carroll (2001)

Friedmann equation with  $\Lambda \neq 0$ :

$$H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k c^2}{R^2} + \frac{\Lambda c^2}{3} \quad (7.30)$$

And define the  $\Omega$ 's (Eqs. 4.58, 7.14):

$$\Omega_m = \frac{8\pi G\rho_m}{3H_0^2}, \quad \Omega_\Lambda = \frac{\Lambda c^4}{3H_0^2}, \quad \Omega_k = -\frac{k c^2}{R_0^2 H_0^2} \quad (8.50)$$

Because of Eq. (7.30),

$$\Omega_m + \Omega_\Lambda + \Omega_k = \Omega + \Omega_k = 1 \quad (8.51)$$

Friedmann with nonzero Lambda

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Friedmann with  $\Lambda \neq 0$ , II

It is easier to set  $c = 1$  and to work with the dimensionless scale factor,

$$a = \frac{R(t)}{R_0} \quad (4.29)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \frac{\rho_{m,0}}{a^3} - \frac{k}{a^2 R_0^2} + \frac{\Lambda}{3} \quad (8.52)$$

since  $\rho_m = \rho_{m,0} a^{-3}$  (Eq. 4.63).

Inserting the  $\Omega$ 's

$$\left(\frac{\dot{a}}{a} H_0\right)^2 = \Omega_m + \frac{1 - \Omega_m - \Omega_\Lambda}{a^2} + \Omega_\Lambda \quad (8.53)$$

Substituting the time in units of today's Hubble time,

$$\tau = H_0 \cdot t \quad (8.54)$$

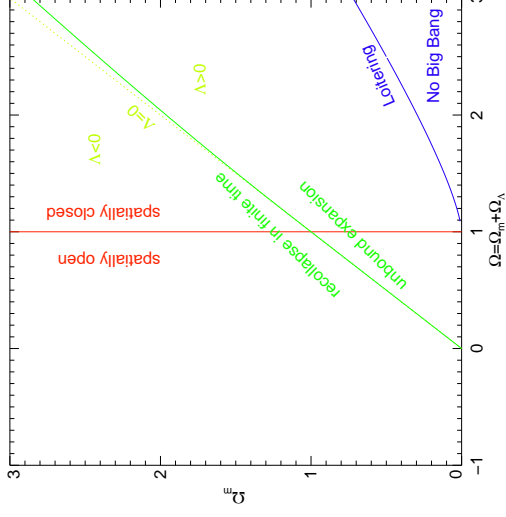
results in

$$\left(\frac{da}{d\tau}\right)^2 = 1 + \Omega_m \left(\frac{1}{a} - 1\right) + \Omega_\Lambda (a^2 - 1) \quad \text{where } a(\tau = 1) = 1 \quad \text{and} \quad \left.\frac{da}{d\tau}\right|_{\tau=1} = 1 \quad (8.55)$$

For most combinations of  $\Omega_m$  and  $\Omega_\Lambda$ , need to solve numerically.

Friedmann with nonzero Lambda

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Friedmann with  $\Lambda \neq 0$ , III

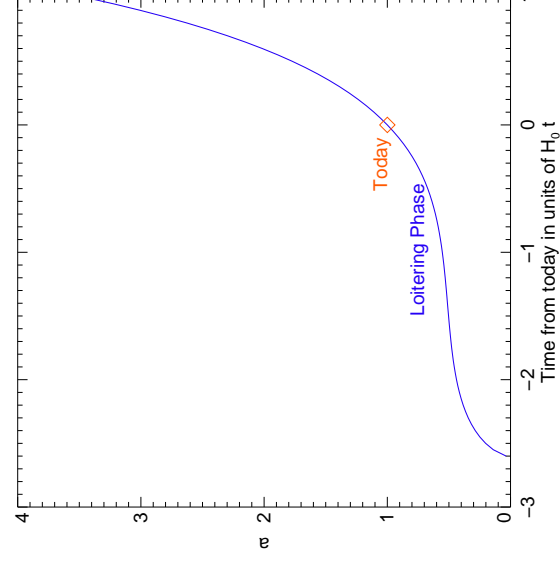
(after Carroll, Press & Turner, 1992, Fig. 1)

With  $\Lambda$ , evolution of universe is more complicated than without:

- unbound expansion possible for  $\Omega < 1$ ,
- For  $\Omega_\Lambda$  large: no big bang!
- For  $\Omega_\Lambda$  large: possible "loitering phase"

Friedmann with nonzero Lambda

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 $\Omega_\Lambda > 1$ , I

For large  $\Omega_\Lambda$ : contraction from  $+\infty$  and reexpansion  
 ⇒ no big bang.  
 For slightly smaller  $\Omega_\Lambda$ : phase where  $\dot{a} \sim 0$  in the past  
 ⇒ loitering universe.

"Loitering universe" with  $\Omega_m = 0.55$ ,  $\Omega_\Lambda = 2.055$

Friedmann with nonzero Lambda

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 $\Omega_\Lambda > 1, II$ 

QSO at  $z = 5.82$ , courtesy SDSS

Threshold for presence of a turning-point (Carroll, Press & Turner, 1992, Eq. 12):

$$\Omega_\Lambda \geq \Omega_{\Lambda, \text{thresh}} = 4\Omega_m \left\{ C_\kappa \left[ \frac{1}{3} C_\kappa^{-1} \left( \frac{1 - \Omega_m}{\Omega_m} \right) \right]^3 \right\} \quad (8.56)$$

where  $\kappa = \text{sgn}(0.5 - \Omega_m)$  and  $C_\kappa(\theta)$  was defined in Eq. (4.24).

For  $\Omega_\Lambda = \Omega_{\Lambda, \text{thresh}}$ : turning-point, i.e., there is a minimal  $a$ .

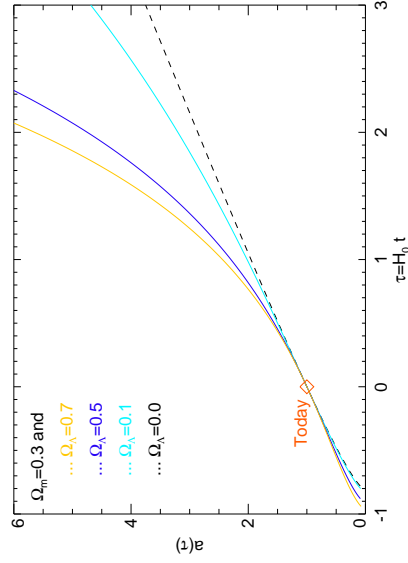
Since  $1 + z = 1/a$  (Eq. 4.40), existence of turning-point  $\implies$  maximal possible  $z$ :

$$z \leq 2C_\kappa \left( \frac{1}{3} C_\kappa^{-1} \left\{ \frac{1 - \Omega_m}{\Omega_m} \right\} - 1 \right) \quad (8.57)$$

(Carroll, Press & Turner, 1992, Eq. 14). Since quasars are observed with  $z > 5.82$ , this means that  $\Omega_m < 0.007$ , clearly not what is observed  $\implies \Omega_\Lambda < 1$ .

Friedmann with nonzero Lambda

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 $\Omega_\Lambda < 1$ 

For  $\Omega_\Lambda < 1$  evolution has two parts:

- matter domination, similar to earlier results
- $\Lambda$  domination, exponential rise.

Exponential rise called by some workers the "second inflationary phase"...

Friedmann with nonzero Lambda

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 $\Omega_\Lambda < 1$ 

Calculation of age of universe is similar to  $\Omega_\Lambda = 0$  case (see, e.g., Eq. 4.81), but generally only possible numerically.

Result:

Universes with  $\Omega_\Lambda > 0$  are *older* than those with  $\Omega_\Lambda = 0$ .

This solves the age problem, that some globular clusters have age comparable to age of universe if  $\Omega_\Lambda = 0$ .

Analytical formula for age (Carroll, Press & Turner, 1992, Eq. 17):

$$t = \frac{2}{3H_0} \frac{\sinh^{-1} \left( \sqrt{(1 - \Omega_a)/\Omega_a} \right)}{\sqrt{1 - \Omega_a}} \quad (8.58)$$

for  $\Omega_a < 1$ , where

$$\Omega_a = 0.7\Omega_m + 0.3(1 - \Omega_\Lambda) \quad (8.59)$$

For  $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$ ,  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ :  $t = 13.5 \text{ Gyr}$ .  
Remember that for  $\Omega_m = 1$ ,  $t = 3/2H_0$ .

Friedmann with nonzero Lambda

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## Luminosity Distance, I

Influence of  $\Lambda$  is most prominent at large distances!

- $\implies$  Expect influence on Hubble Diagram.
- $\implies$  Need to find relation between measured flux, emitted luminosity, and redshift.

Assume source with luminosity  $L$  at comoving coordinate  $r$ , emitting isotropically into  $4\pi \text{ sr}$ .

At time of detection today, photons are

- on sphere with proper radius  $R_0 r$ ,
- redshifted by factor  $1 + z$ ,
- spread in time by factor  $1 + z$ .

$\implies$  observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1 + z)^2} \quad (8.60)$$

Determination of  $\Omega_\Lambda$

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### Luminosity Distance, II

Because the observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} \quad (8.60)$$

in analogy to the inverse square law one defines the luminosity distance as

$$d_L = R_0 \cdot r \cdot (1+z) \quad (8.61)$$

The calculation of  $d_L$  is somewhat technical, one can show that (Carroll, Press & Turner, 1992):

$$d_L = \frac{c}{H_0} |\Omega_M|^{-1/2} \cdot S_{-\text{sgn}(\Omega_M)} \left\{ \int_0^z \left[ (1+z)^2 (1 + \Omega_M z) - z(2+z)\Omega_\Lambda \right]^{1/2} dz \right\} \quad (8.62)$$

Determination of  $\Omega_\Lambda$



### Supernovae

Best way to determine  $\Omega_\Lambda$ :

Type Ia supernovae

Remember: SN Ia = CO WD collapse... (Hoyle, Fowler, Colgate, Wheeler,...)

The distance modulus is

$$m - M = 5 \log \left( \frac{d_L}{1 \text{ Mpc}} \right) + 25 \quad (8.63)$$

Use SNe as standard candles  $\implies$  Deviations from  $d_L \propto z$  indicative of  $\Lambda$ .

Two projects:

- High- $z$  Supernova Team (STSCI, Riess et al.)
- Supernova Cosmology Project (LBNL, Perlmutter et al.)

Both find SNe out to  $z \sim 1$ .

Present mainly Perlmutter et al. results here, Riess et al. (1998) are similar.

Determination of  $\Omega_\Lambda$



### Supernovae

Basic observations: easy:

- Detect SN in rise  $\implies$  CTIO 4 m
- Follow SN for  $\sim 2-3$  months with 2-4 m class telescopes, HST, Keck...

More technical problems in data analysis: Conversion into source frame:

- Correction of photometric flux for redshift: "K-correction"
- Correct for time dilatation in SN light curve

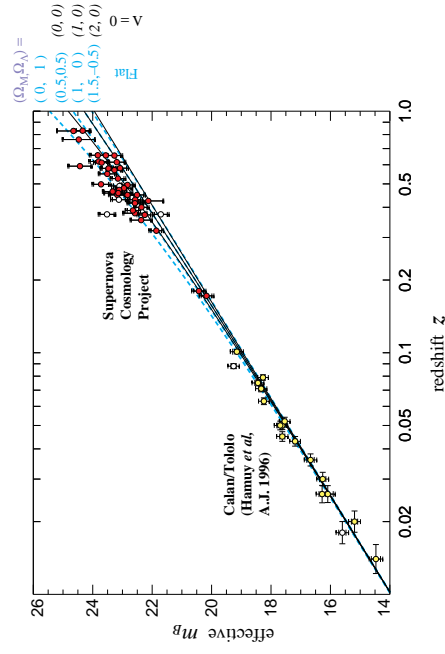
Further things to check

- SN internal extinction
- Galactic extinction
- Galactic reddening
- Photometric cross calibration
- Peculiar motion of SN

Determination of  $\Omega_\Lambda$



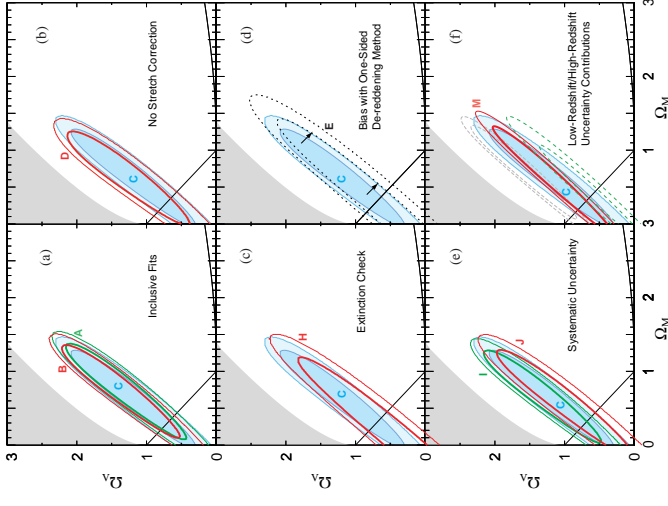
### Supernovae



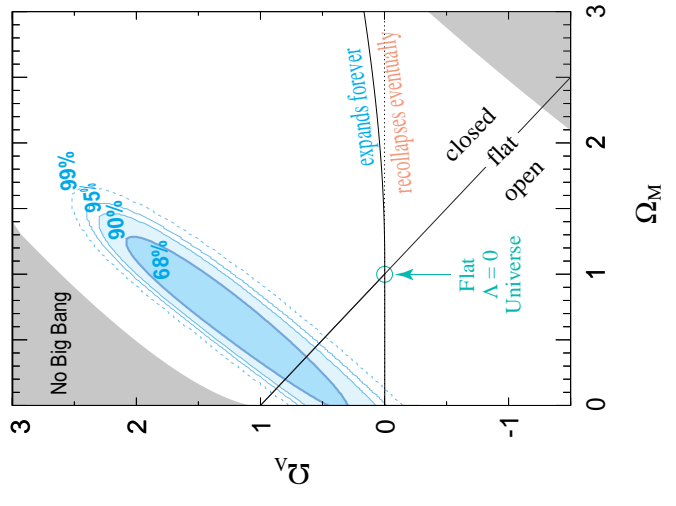
(Perlmutter et al., 1999, Fig. 1)

42 SNe from SCP, 18 low redshift from Calán/Tololo SN Survey

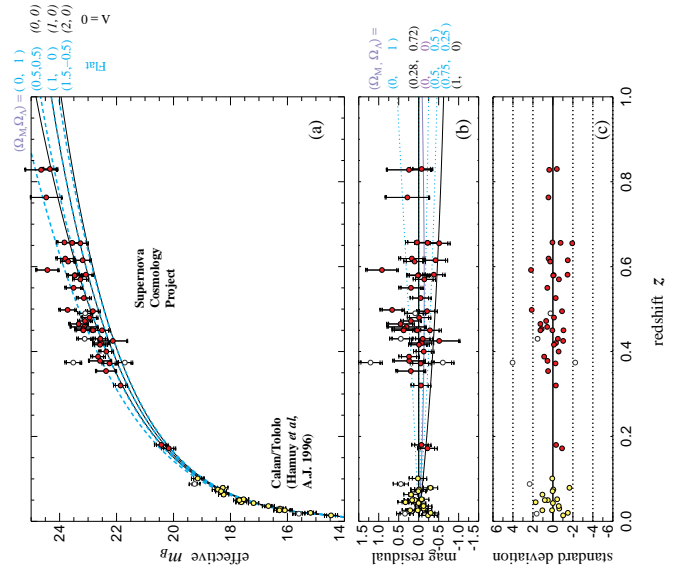
Determination of  $\Omega_\Lambda$



(68% and 90% confidence contours for sources of systematic error, Perlmutter et al., 1999, Fig. 5)

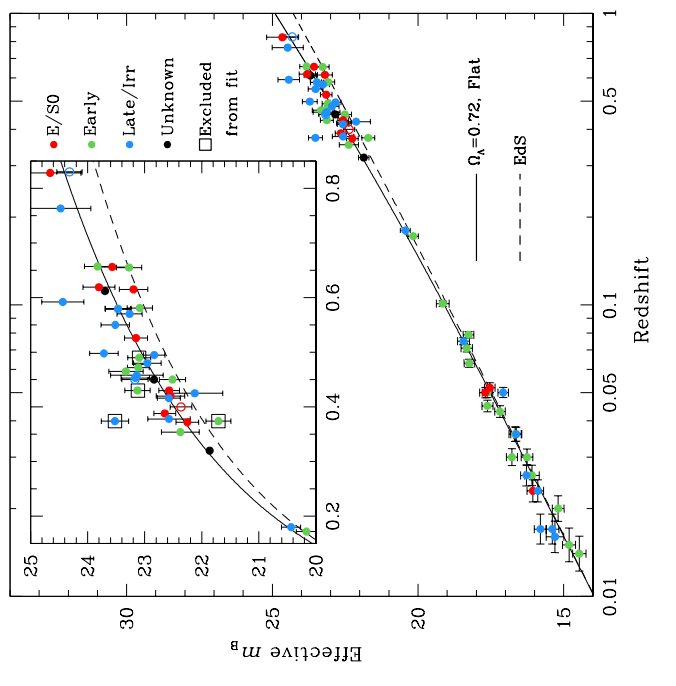


Combined confidence region (Perlmutter et al., 1999, Fig. 7; lower right: universes that are younger than oldest heavy elements)



Best fit:  $\Omega_{m, flat} = 0.28^{+0.09}_{-0.08}$ ,  
 $\chi^2/DOF = 56/50$   
 corresponding best free fit:  
 $(\Omega_m, \Omega_\Lambda) = (0.73, 1.32)$ .

(Perlmutter et al., 1999, Fig. 2)



Updated 2002 Hubble diagram for SN Iae confirms Perlmutter 1999.

Sullivan et al., 2002



### Outlook

What is physical reason for  $\Omega_\Lambda \neq 0$ ?

Currently discussed: quintessence: "rolling scalar field", corresponding to very lightweight particle ( $\lambda_{\text{de Broglie}} \sim 1 \text{ Mpc}$ ), looks like time varying cosmological "constant".

Why?  $\implies$  More naturally explains why  $\Omega_\Lambda$  so close to 0 (i.e., why matter and vacuum have so similar energy densities)

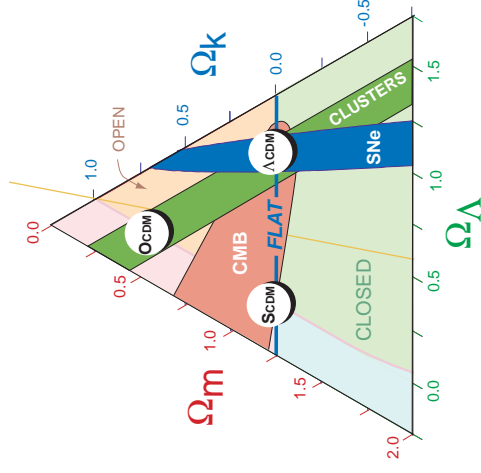
Motivated by string theory and M theory...

Still VERY SPECULATIVE, decision  $\Delta$  vs. quintessence should be possible in next 5... 10 years when new instruments become available.

Summary



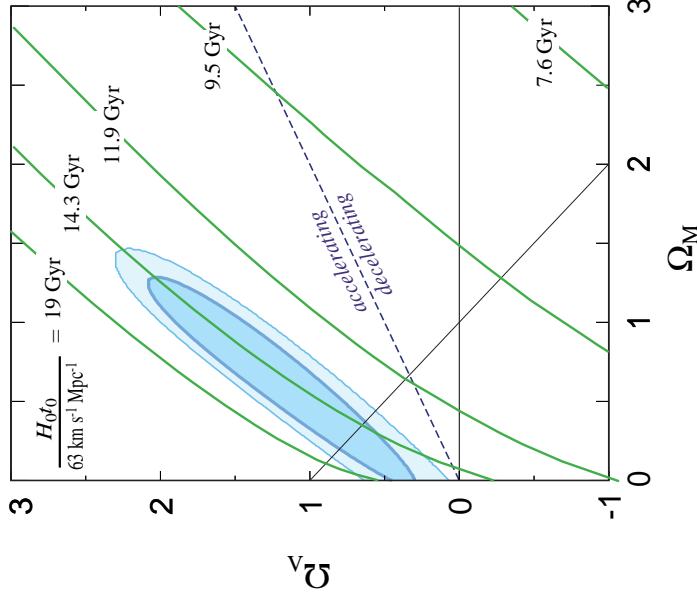
### Outlook



Bahcall et al.

Even better constraints come from combination of SNe data with structure formation.

Summary



Isochrones for age of universe for  $H_0 = 63 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (for  $h = 0.7$ : age 10% smaller).  $\implies$  Consistent with globular cluster ages!

(Perlmutter et al., 1999, Fig. 9)



### Summary

For all practical purposes, currently the best values of  $\Omega_m$  and  $\Omega_\Lambda$  are

$\Omega_m \sim 0.3$  and  $\Omega_\Lambda = 0.7$

Even if  $\Omega \neq 1$ :

$\Omega_\Lambda \neq 0$

And therefore

Baryons are an energetically unimportant constituent of the universe.

"The dark side of the force..." :-)

Summary



## The Lumpy Universe

So far: treated universe as smooth universe.

In reality:

Universe contains structures!

Last part of this class:

1. What are structures?
2. How can we quantify them?
3. How do structures form?
4. How do structures evolve?

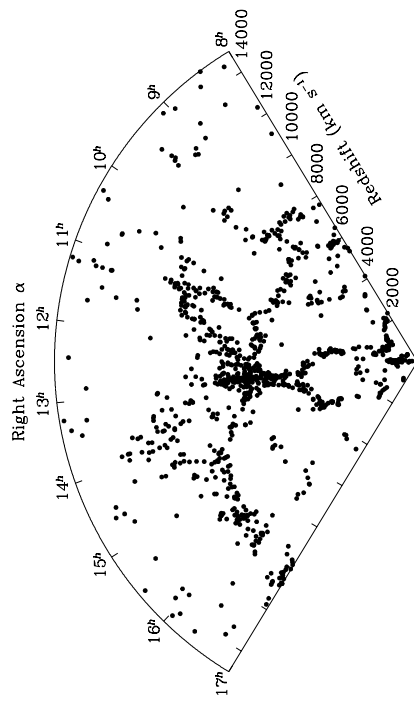
Will see that all these questions are deeply connected with parameters of the universe seen so far:

1.  $H_0$
2.  $\Omega_0, \Omega_b, \Omega_m, \Omega_\Lambda, \dots$
3. Existence and Nature of Dark Matter

The Lumpy Universe



## Introduction, I



$26.5^\circ < \delta < 32.5^\circ$

(de Lapparent, Geller & Huchra, 1986, limiting mag  $m_B = 15.6$ )

Lumpy universe: spatial distribution of galaxies and greater structures.

Redshift Surveys

- Alcock, C., et al., 2001, ApJ, in press (astro-ph/0012163)
- Burles, S., Nollett, K. M., & Turner, M. S., 1999, Big-Bang Nucleosynthesis: Linking Inner Space and Outer Space, AFS Centennial Exhibit, astro-ph/9903300
- Carlfstrom, J. E., Joy, M. K., Grego, L., Holder, G. P., Holzappel, W. L., Mohr, J. J., Pabel, S., & Reese, E. D., 2000, Phys. Scr., T85, 148
- Carroll, S. M., 2001, Living Rev. Rel., 4, 2001
- Carroll, S. M., Press, W. H., & Turner, E. L., 1992, ARA&A, 50, 489
- Drell, P. S., Loredo, T. J., & Wasserman, I., 2000, ApJ, 530, 593
- Merritt, D., 1987, ApJ, 313, 121
- Mohr, J. J., Malmgren, B., & Evrard, A. E., 1989, ApJ, 517, 627
- Peacock, J. A., 1999, Cosmological Physics, (Cambridge: Cambridge Univ. Press)
- Perlmutter, S., et al., 1999, ApJ, 517, 565
- Riess, A. G., et al., 1998, ApJ, 116, 1009
- Spiegel, D. N., et al., 2007, Astrophys. J. Suppl. Ser., 170, 377
- Turner, M. S., 1999, in The Third Stromlo Symposium: The Galactic Halo, ed. B. K. Gibson, T. S. Axelrod, M. E. P. Urmann, ASP, in press (astro-ph/9811454)
- Wambsganss, J., 1998, Living Rev. Rel., 1, 12
- Wise, M. W., McNamara, B. R., & Murray, S. S., 2004, ApJ, 601, 184



## Large Scale Structures and Structure Formation





### Introduction, II

How do we study the structure of the Universe?

- ⇒ We need distance information for many ( $10^4 \dots 10^7$ ) objects
- ⇒ Large redshift surveys

Review: Strauss & Willick (1995)

Redshift survey: Survey of (patch of) sky determining galaxy  $z$  and position to predefined magnitude or  $z$ .

First larger survey: de Lapparent, Geller & Huchra (1986)

Classification:

**1D-surveys:** very deep exposures of small patch of sky, e.g. HST Deep Field, Lockman Hole Survey, Marano Field.

**2D-surveys:** cover long strip of sky, e.g., CfA-Survey ( $1.5 \times 100^\circ$ ), 2dF-Survey ("2 degree Field").

**3D-surveys:** cover part of the sky, e.g., Sloan Digital Sky Survey.

These surveys attempt to go to certain limit in  $z$  or  $m$ .

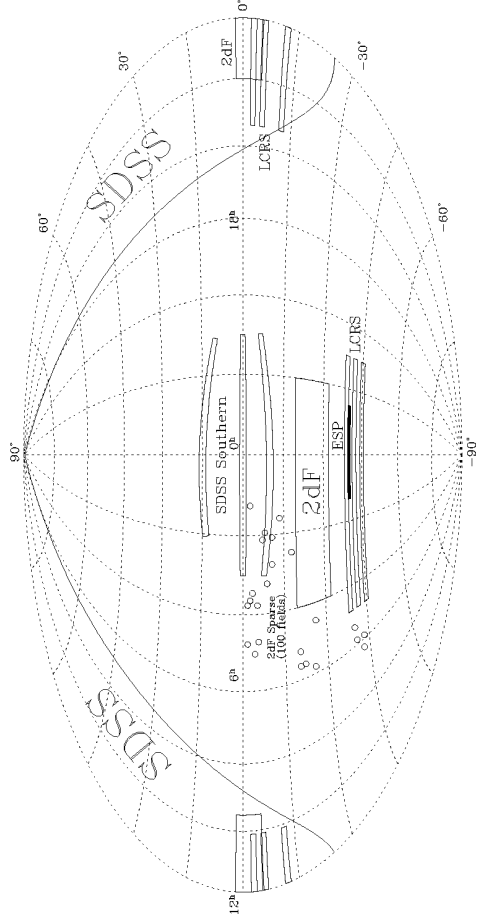
Other approaches: use pre-existing galaxy catalogues (e.g., QDOT Survey [IRAS galaxies], APM survey,...).

We will concentrate here on the larger surveys based on no other catalogue.

Redshift Surveys

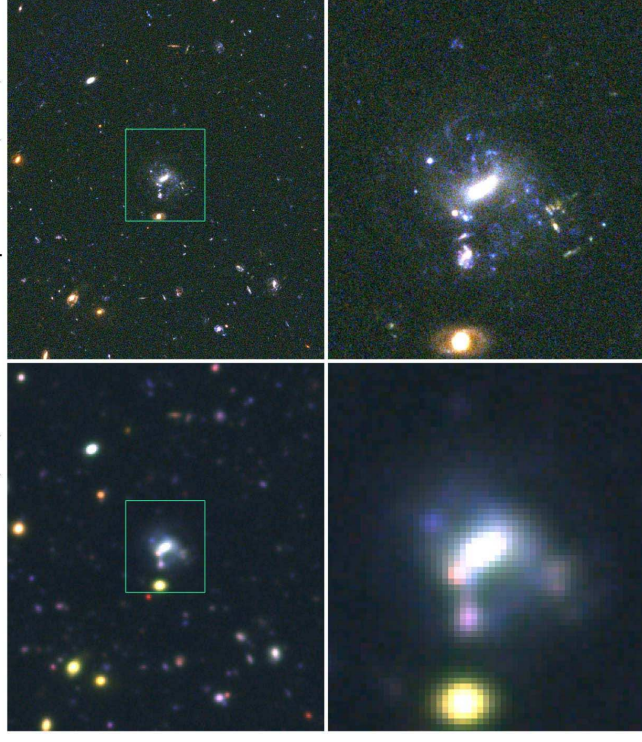


### Introduction, III



(Strauss, 1999)

Redshift Surveys



To go deep one needs to go to space

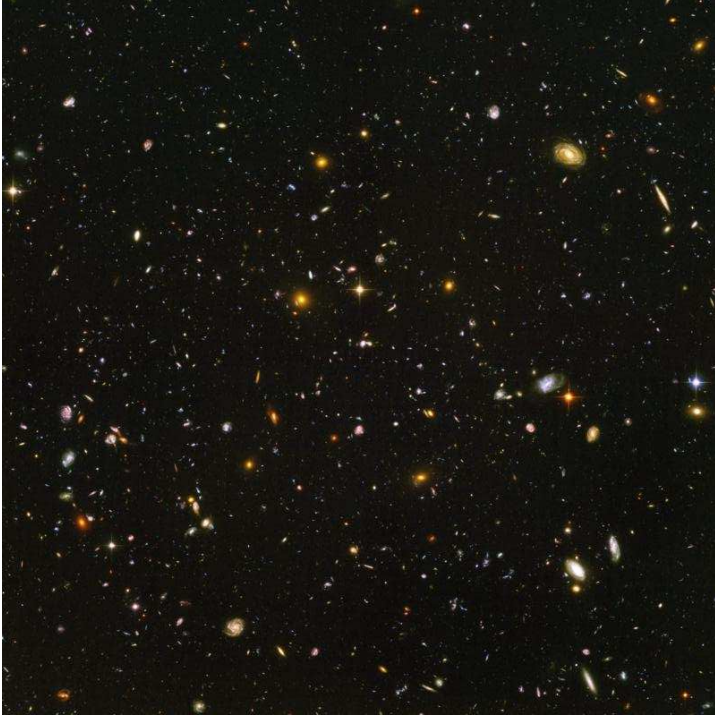


1995 December: Hubble  
 Deep Field:  
 ~ 150 ksec/Filter for four  
 HST Filters  
 Many galaxies with weird  
 shapes  $\implies$  protogalaxies!  
 Redshifts:  $z \in [0.5, 5.3]$   
 (Fernández-Soto et al.,  
 1999)

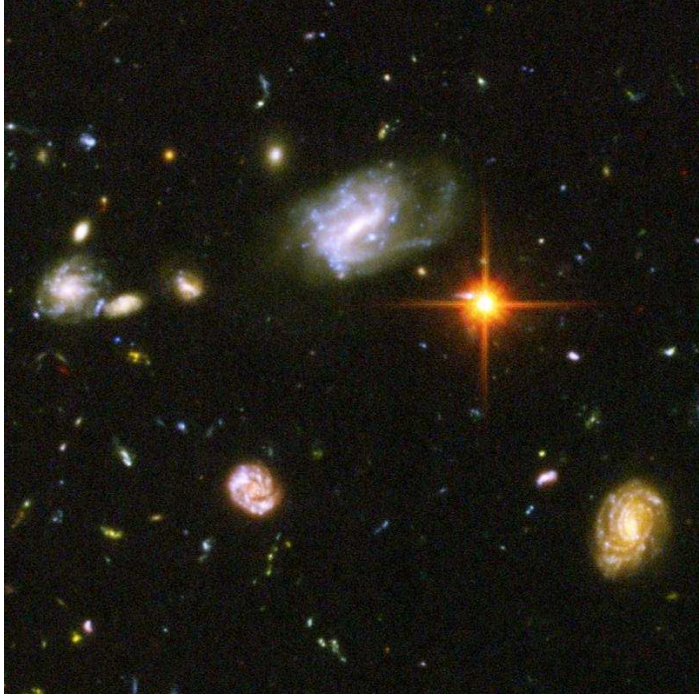




1998: Hubble Deep Field South, 10 d of total observing time!

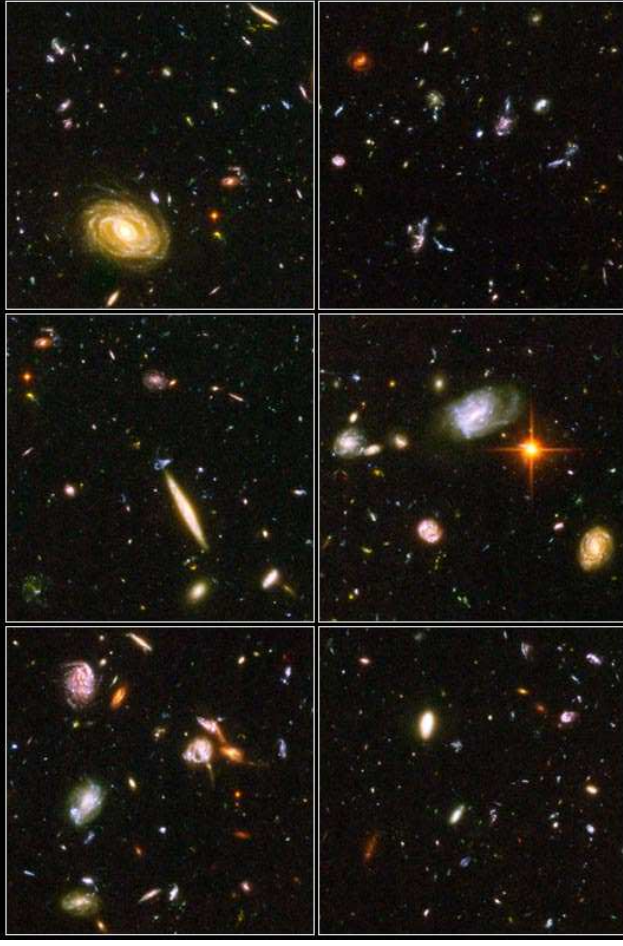


2004: Hubble Ultra Deep Field, 1 Msec long exposure of field in Fornax. Uses updated HST with Advanced Camera for Surveys (ACS) and Near Infrared Camera and Multi-Object Spectrometer (NICMOS); diameter: 3' (2x older HDF) Limiting magnitude: 30 mag, ~10000 galaxies visible, up to  $z \gtrsim 7$  IR reveals many reddened objects





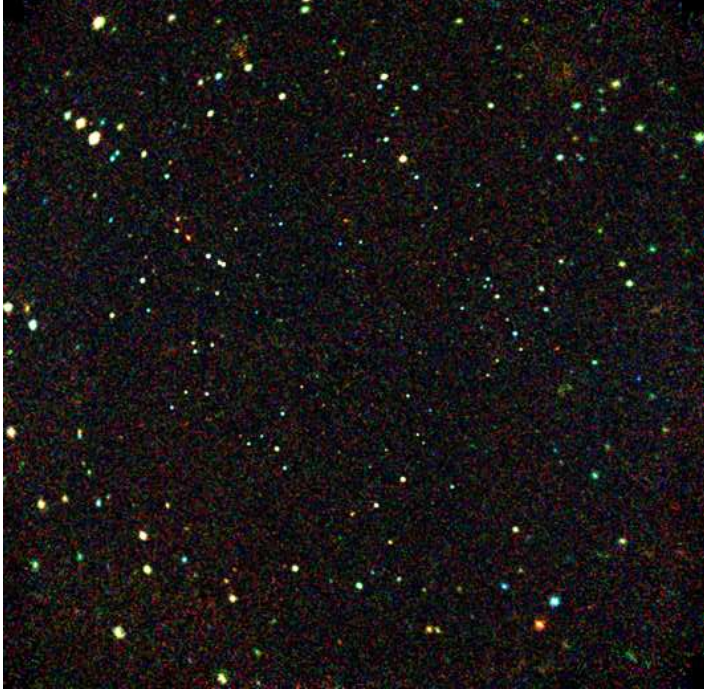
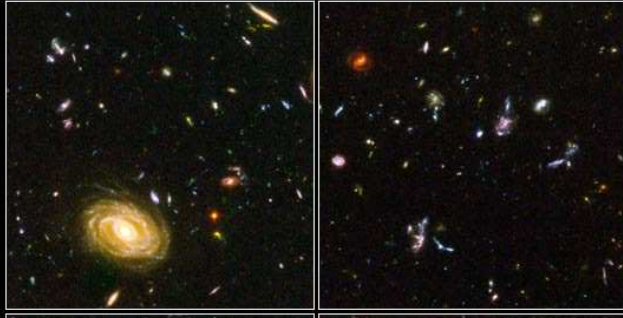
Hubble Ultra Deep Field Details



NASA, ESA, S. Beckwith (STScI) and The HUDF Team

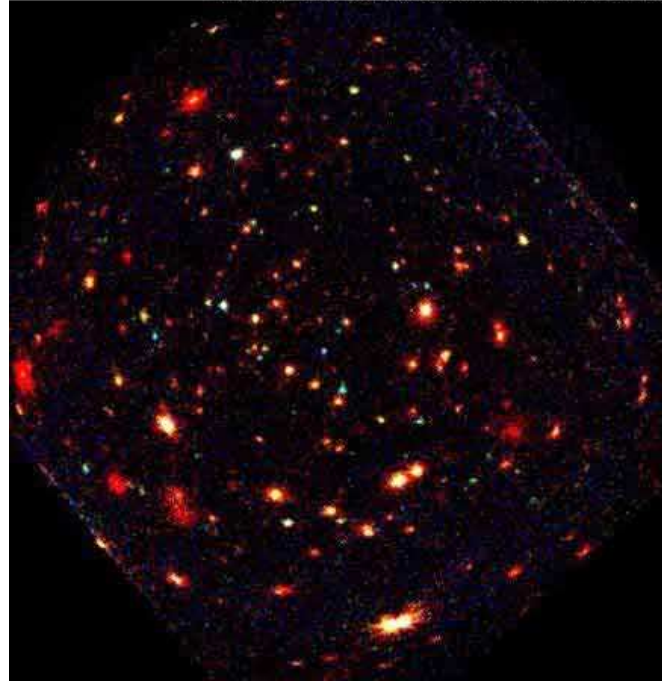
STScI-PRC04-07c

HST - ACS



Chandra Deep Field South:  
1 Msec (10.8 days) on one  
region in Fornax  
( $\alpha_{J2000.0} = 3^h 32^m 28.0^s$ ,  
 $\delta_{J2000.0} = -27^\circ 48' 30''$ )  
Deepest X-ray field ever  
color code: spectral hardness

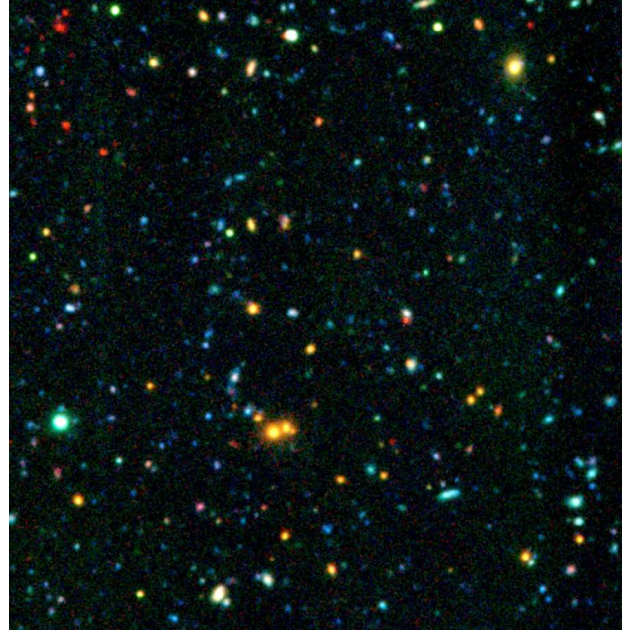
scale:  $15' \times 15'$ ; courtesy  
NASA/JHU/AUI/R.Giacconi et  
al.



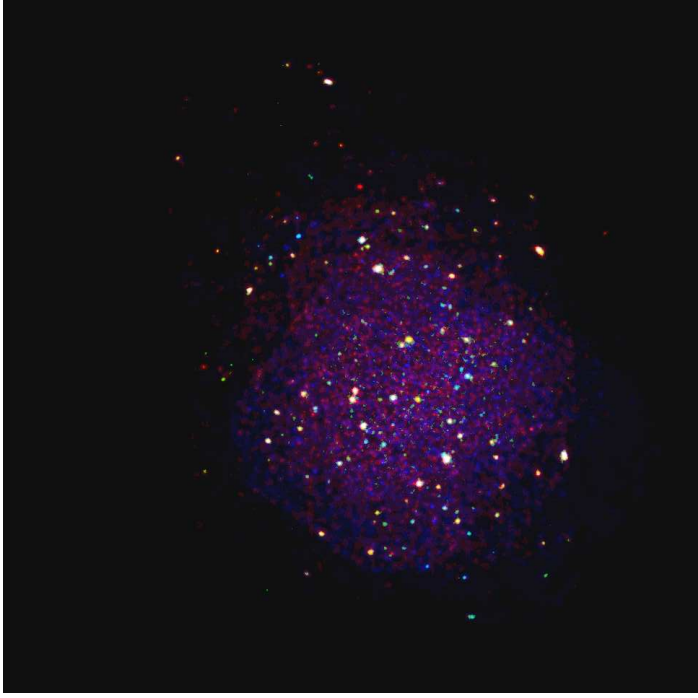
GUNTHER HASINGER/ASTROPHYSICS INSTITUT FÜR POTSDAM

Lockman Hole: Northern  
Sky region with very low  
 $N_H \Rightarrow$  low interstellar  
absorption  
 $\Rightarrow$  "Window in the sky"  
 $\Rightarrow$  X-rays: evolution of  
active galaxies with  $z$ !

XMM-Newton, Hasinger et al.,  
2001,  
blue: hard X-ray spectrum,  
red: soft X-ray spectrum

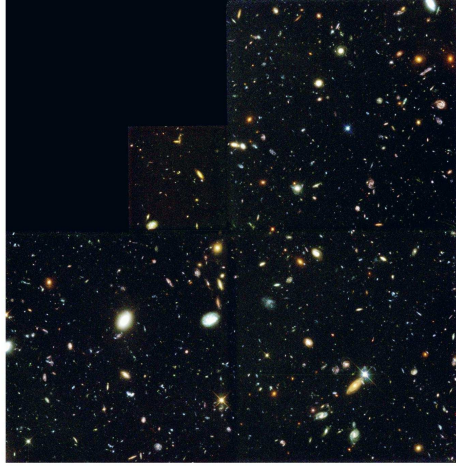


Distant Galaxies in "AXAF Deep Field" (VLT ANTU/ISAAC + NTT/SUSI-2)

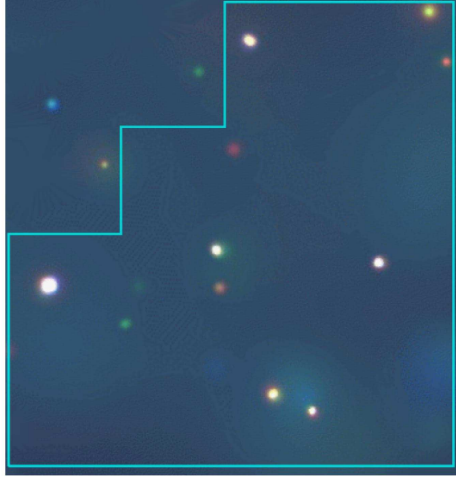


1D Surveys ("Deep Exposures") give snapshot of evolution of galaxies over large  $z$ .

Deep *XMM-Newton* image of the Marano Field (IAAT/AIP/MPE)



HST



Chandra

Chandra/HST Image of Hubble Deep Field North; 500 ksec

Joint multi-wavelength campaigns allow the measurement of broad-band spectra of sources in the early universe!

⇒ GOODS-Survey (Great Observatories Origins Deep Survey)