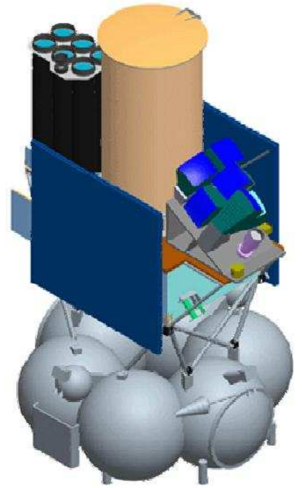
X-Ray Surveys



Spectrum-X-Γ in
Yamal-Configuration (outdated)

Cosmology: Black Holes play a significant role in structure formation.

"Structures": Galaxies and galaxy clusters

$\Longrightarrow$ *Spectrum-Roentgen-Gamma*:

- X-ray survey of the whole sky (3 years)
- most sensitive survey of black holes ever
- russian satellite bus, experiments:
  - *eROSITA* (CCDs): MPE, IAA Tübingen, FAU, Potsdam, Hamburg, industry ($\sim$50 Mio €)
  - *SXC* (Calorimeter): SRON/ISAS/GSFC (?)
  - *ART-XC* (CZT): Roscosmos (??)
- Launch: 2012, Operations at least until 2016

Correlation Function, I

Sky surveys show:

Galaxies are *not* evenly distributed: "cosmic web"!

- Structures at scales up to several 10 Mpc
- But: Over-density even in clusters not too dramatic ($\sim$ 100$\times$ denser than average)
- Voids on scales 50 h^{-1} Mpc

$\Longrightarrow$ Need quantitative description of structures.

$\Longrightarrow$ Need physical explanation of structures.

$\Longrightarrow$ Need to understand what we see (do galaxies trace matter distribution??).

Correlation Function, II

Mathematical description of clustering: Correlation function!

Assume *uniform* distribution of galaxies with galaxy density n galaxies Mpc^{-3}.

Probability to find galaxy in volume ΔV:

$$P \propto n\Delta V \tag{9.1}$$

Probability to find galaxies in *two* volumes 1 and 2:

$$P = P_1 \cdot P_2 \propto n^2 \Delta V_1 \Delta V_2 \tag{9.2}$$

Universe inhomogeneous: measure (distance dependent) deviation from mean:

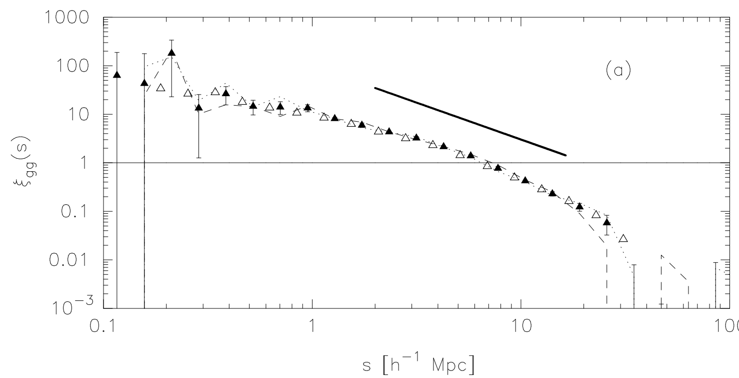
$$P \propto n^2(1 + \xi(r_{12}))\,\Delta V_1 \Delta V_2 \tag{9.3}$$

$\xi(r_{12})$ is called the two-point spatial correlation function.

For *small* r:

$$\xi(r) > 0 \Longrightarrow \text{clustering}$$

Correlation Function, III



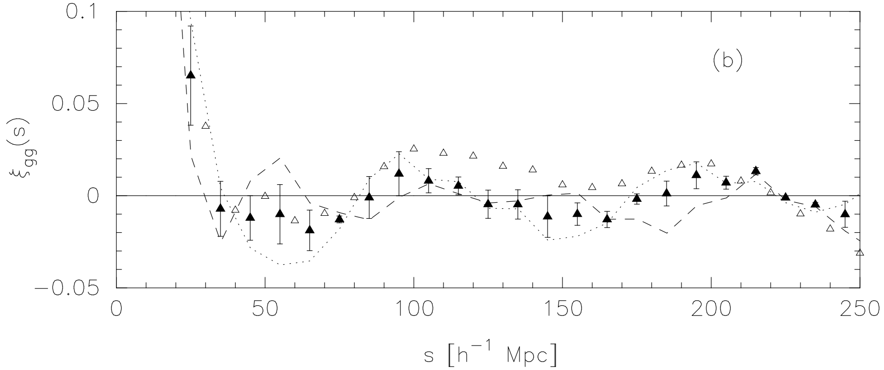
(galaxy-galaxy correlation function from the Las Campanas Redshift Survey; Tucker et al., 1997, Fig. 1)

Rough description: power law

$$\xi(r) = (r/r_0)^{-\gamma} \tag{9.4}$$

where $r_0 \sim 6\,h^{-1}$ Mpc (correlation length), and $\gamma \sim 1.5 \ldots 1.8$.

Correlation Function, IV



(galaxy-galaxy correlation function from the Las Campanas Redshift Survey; Tucker et al., 1997, Fig. 1)

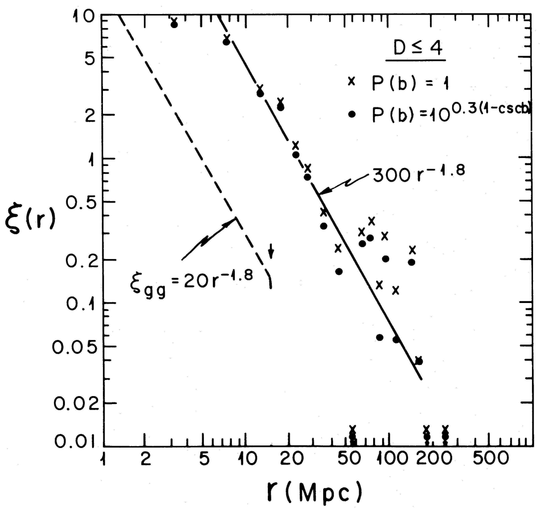
Rough description: power law

$$\xi(r) = (r/r_0)^{-\gamma} \tag{9.5}$$

where $r_0 \sim 6\,h^{-1}$ Mpc (correlation length), and $\gamma \sim 1.5 \ldots 1.8$.

Above $r = 30\,h^{-1}$ Mpc: oscillation due to voids.

Correlation Function, V



The spatial correlation function for rich (Abell) galaxy clusters is similar to the one for galaxies,

$$\xi_{\text{clusters}}(r) = 360(rh)^{-1.8} \tag{9.6}$$

for $r \leq 150h^{-1}$ Mpc.

For $r > 150h^{-1}$ Mpc no correlations are observed.

Note that richer clusters show stronger correlations.

(Bahcall & Soneira, 1983, Fig. 9a)

Correlation Function, VI

ξ is related to the density contrast $\Delta(x)$:

Write the density n as

$$n(\boldsymbol{x}) = n_0(1 + \Delta(\boldsymbol{x})) \quad \Longleftrightarrow \quad \Delta(\boldsymbol{x}) = \delta n/n \tag{9.7}$$

Average joint probability to have galaxies at $\boldsymbol{x}$ and $\boldsymbol{x} + \boldsymbol{r}$:

$$P = \langle n(\boldsymbol{x})\mathrm{d}V_1 \cdot n(\boldsymbol{x}+\boldsymbol{r})\mathrm{d}V_2\rangle \tag{9.8}$$

$$= \left\langle n_0^2(1+\Delta(\boldsymbol{x}))(1+\Delta(\boldsymbol{x}+\boldsymbol{r}))\ \mathrm{d}V_1\mathrm{d}V_2\right\rangle \tag{9.9}$$

Since $\langle\Delta\rangle = 0$, only the cross product survives the averaging:

$$= n_0^2\left(1 + \langle\Delta(\boldsymbol{x})\Delta(\boldsymbol{x}+\boldsymbol{r})\rangle\right)\ \mathrm{d}V_1\mathrm{d}V_2 \tag{9.10}$$

where $\langle\ldots\rangle$ denotes averaging over an appropriate volume, i.e.,

$$\langle f(\boldsymbol{r})\rangle = \frac{1}{V}\int_V f(\boldsymbol{r})\,\mathrm{d}^3r \tag{9.11}$$

Correlation Function, VII

The two-point correlation function was defined via

$$P \propto n^2(1+\xi(r_{12}))\Delta V_1\Delta V_2 \tag{9.3}$$

while we just found

$$P = n_0^2\left(1 + \langle\Delta(\boldsymbol{x})\Delta(\boldsymbol{x}+\boldsymbol{r})\rangle\right)\ \mathrm{d}V_1\mathrm{d}V_2 \tag{9.10}$$

Comparing these two equations shows:

$$\xi(r) = \langle\Delta(\boldsymbol{x})\Delta(\boldsymbol{x}+\boldsymbol{r})\rangle \tag{9.12}$$

$\xi(r)$ is a measure for the average density contrast at places separated by distance r.

Power Spectrum, I

To describe the strength of fluctuations, a good measure is the variance of the density fluctuations.

Let's calculate this around an arbitrary location, $\boldsymbol{x} = 0$.

$$\mathrm{Var}(n) = \frac{1}{V}\int (n(\boldsymbol{r}) - \langle n\rangle)^2\,dV \tag{9.13}$$
$$= \frac{1}{V}\int (n_0(1+\Delta(\boldsymbol{r})) - n_0)^2\,dV \tag{9.14}$$
$$= \frac{1}{V}\int \Delta(\boldsymbol{r})^2 dV \tag{9.15}$$

To evaluate the integral, it is more convenient for most calculations to work in Fourier space than in "normal" space.

Define the Fourier transform in spatial coordinates through:

$$\Delta_r(\boldsymbol{r}) = \frac{V}{(2\pi)^3}\int \Delta_k(\boldsymbol{k})\exp(-i\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3k \tag{9.16}$$
$$\Delta_k(\boldsymbol{k}) = \frac{1}{V}\int \Delta(\boldsymbol{r})\exp(+i\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3r \tag{9.17}$$

Power Spectrum, II

Therefore

$$\mathrm{Var}(n) = \frac{1}{V}\int \Delta_r(\boldsymbol{r})^2 dV \tag{9.18}$$
$$= \frac{1}{V}\int\left(\frac{V}{(2\pi)^3}\int \Delta_k \exp(-i\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3k\right)^2 dV \tag{9.19}$$
$$= \frac{V}{(2\pi)^6}\int\left(\int \Delta_k \exp(-i\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3k\right)\left(\int \Delta_{k'} \exp(-i\boldsymbol{k}'\cdot\boldsymbol{r})\,\mathrm{d}^3k'\right) dV \tag{9.20}$$
$$= \frac{V}{(2\pi)^6}\int\int\int \Delta_k\Delta_{k'}\exp\left(-i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}\right)\mathrm{d}^3k\,\mathrm{d}^3k'\,\mathrm{d}^3V \tag{9.21}$$
$$= \frac{V}{(2\pi)^6}\int\int \Delta_k\Delta_{k'}(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}')\,\mathrm{d}^3k'\mathrm{d}^3k \tag{9.22}$$
$$= \frac{V}{(2\pi)^3}\int \Delta_k^2\mathrm{d}^3k \tag{9.23}$$

This result is called Parseval's theorem

$$\frac{1}{V}\int \Delta_r^2(\boldsymbol{r})\,\mathrm{d}^3x = \frac{V}{(2\pi)^3}\int \Delta_k^2(\boldsymbol{k})\,\mathrm{d}^3k \tag{9.24}$$

(from signal theory: the power in a time series is the same as the power in the associated Fourier transform)

Power Spectrum, III

Therefore

$$\mathrm{Var}(n) = \left\langle\Delta^2\right\rangle = \frac{V}{(2\pi)^3}\int \Delta_k^2\mathrm{d}^3k = \frac{V}{(2\pi)^3}\int P(k)\,\mathrm{d}^3k \tag{9.25}$$

where the power spectrum is defined by

$$P(k) = \Delta_k^2 = \left(\frac{1}{V}\int \Delta(\boldsymbol{r})\exp(+i\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3r\right)^2 \tag{9.26}$$

The power spectrum is a measure for the strength of density fluctuations at wave number k.

Power Spectrum, IV

How are $\left\langle\Delta^2\right\rangle$ and ξ related?

$\Longrightarrow$ Use brute force computation or make use of the correlation theorem.

For functions g, h, the correlation theorem states that the Fourier transform of the correlation,

$$\mathrm{Corr}(g,h) = \int g(x+r)h(r)\,\mathrm{d}x \tag{9.27}$$

is given by

$$\mathrm{FT}\left(\mathrm{Corr}(g,h)\right) = G\,H^* \tag{9.28}$$

where $G = \mathrm{FT}(g)$, etc.

Therefore, setting $g = \Delta(r)$ and $h = \Delta(r)$,

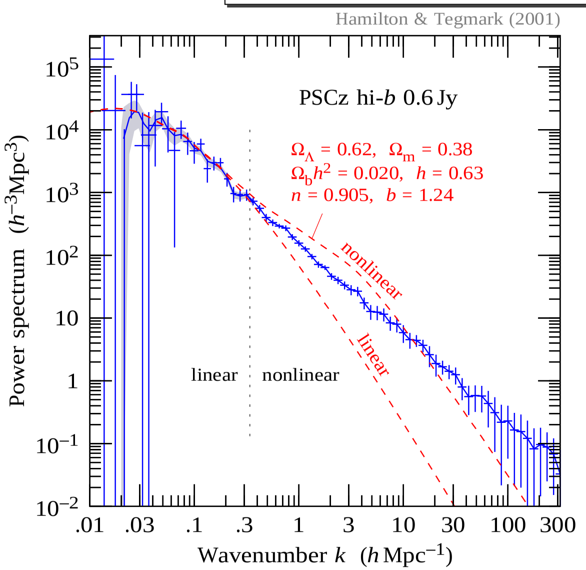
$$\xi(r) = \langle\Delta(\boldsymbol{x})\Delta(\boldsymbol{x}+\boldsymbol{r})\rangle = \frac{V}{(2\pi)^3}\int |\Delta_k|^2\exp(i\,\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3k \tag{9.29}$$

The power spectrum and ξ are Fourier transform pairs.

(remember Eq. 9.26, $P(k) = \Delta_k^2$!)

See Peebles (1980, sect. 31) for 100s of pages of the properties of ξ, P, etc.

Power Spectrum: Measurements, I



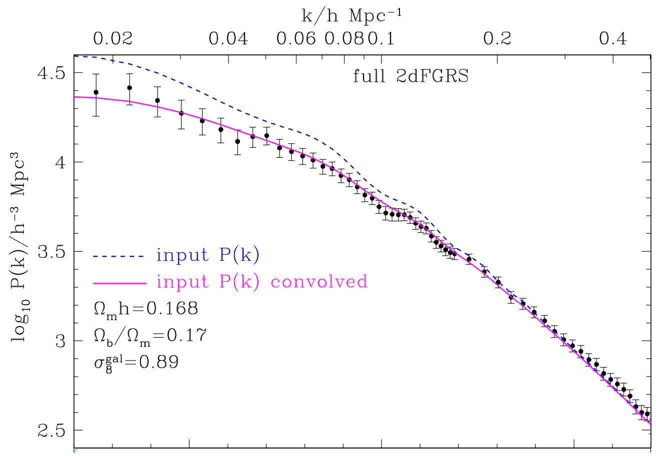
The power spectrum of high galactic latitude IR galaxies is a power-law, roughly described by

$$P(k) \sim 150 k^{-1.46} h^{-3}\,\mathrm{Mpc}^3 \qquad (9.30)$$

As we will see later, this result gives problems with results of theory of structure formation, see dashed lines in Figure.

(Hamilton & Tegmark, 2002, Fig. 6)

Power Spectrum: Measurements, II



(Cole et al., 2005, Fig. 12)

The galaxy-galaxy-power spectrum flattens towards small k.

Power Spectrum: Interpretation

To understand ξ and P better, let's assume an isotropic universe...

Since

$$\xi(\boldsymbol{r}) \propto \int P(\boldsymbol{k}) \exp(i\,\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}^3k \qquad (9.29)$$

using spherical coordinates in k space, one finds:

$$\boldsymbol{k}\cdot\boldsymbol{r} = kr\cos\theta \qquad (9.31)$$
$$\mathrm{d}V = k^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{d}k \qquad (9.32)$$

such that

$$\xi(r) \propto \int_0^\infty \int_0^\pi \int_0^{2\pi} P(k)\exp(ikr\cos\theta)k^2\sin\theta\,\mathrm{d}\phi\,\mathrm{d}\theta\,\mathrm{d}k \qquad (9.33)$$
$$= 2\pi \int_0^\infty \int_0^\pi \xi(r)\,\exp(ikr\cos\theta)\,r^2\,\mathrm{d}(\cos\theta)\,\mathrm{d}r \qquad (9.34)$$
$$= \frac{V}{2\pi^2}\int_0^\infty P(k)\frac{\sin kr}{kr}\,\mathrm{d}r \qquad (9.35)$$

(the last eq. is exact).

For $kr < \pi$: $\sin kr/kr > 0$, while oscillation for $kr > \pi$

$\Longrightarrow$ only wavenumbers $k \lesssim r^{-1}$ contribute to amplitude on scale r.

Power Spectrum: Interpretation

For a power law spectrum,

$$P(k) \propto k^n \qquad (9.36)$$

the correlation function is

$$\xi(r) \propto \int_0^\infty \frac{\sin kr}{kr} k^{n+2}\,\mathrm{d}k \sim \int_0^{1/r} k^{n+2}\,\mathrm{d}k \propto r^{-(n+3)} \qquad (9.37)$$

But the mass within a fluctuation is $M \sim \rho r^3$, i.e., the mass fluctuation spectrum is

$$\xi(M) \propto M^{-(n+3)/3} \qquad (9.38)$$

and therefore the rms density fluctuation at mass scale M is

$$\frac{\delta\rho}{\rho} = \xi(M)^{1/2} \propto M^{-(n+3)/6} \qquad (9.39)$$

For $n > -3$, the rms mass fluctuations decrease with $M \Longrightarrow$ isotropic universe on largest scales

Power Spectrum: Interpretation

What spectra do we expect?

Two simple cases:

Poisson noise: Random statistical fluctuations in number of particles on scale r:

$$\frac{\delta N}{N} = \frac{1}{N} \quad \Longrightarrow \quad \frac{\delta M}{M} = \frac{1}{M} \tag{9.40}$$

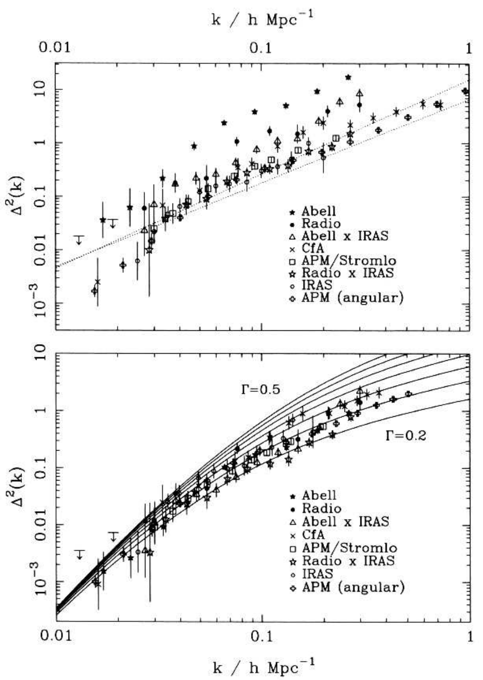
and therefore $n = 0$ ($\rho \propto M!$) ("white noise").

Zeldovich spectrum: defined by $n = 1$. Thus

$$\frac{\delta\rho}{\rho} \propto M^{-2/3} \tag{9.41}$$

. . . will be important later

The Zeldovich spectrum is the spectrum expected for the case when initial density fluctuations coming through the horizon had the same amplitude.



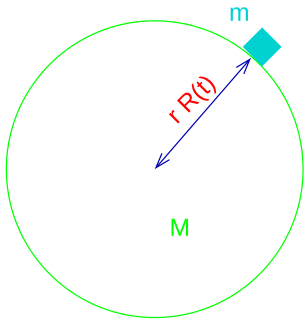
The measured power spectrum is more complicated

$\Longrightarrow$ Structure formation to understand details!

(Peacock, 1999, Fig. 16.4)

Linear Theory, I

Structure formation = evolution of overdensity in universe with time.



Describe density and scale factor with respect to normal expansion:

$$\rho(t) = \bar{\rho}(t) \cdot (1 + \delta(t)) \tag{9.42}$$

$$a(t) = \bar{a}(t) \cdot (1 - \epsilon(t)) \tag{9.43}$$

Sign:
$\delta > 0 \Longrightarrow$ *Overdensity*
$\epsilon > 0 \Longrightarrow$ *collapse*
Seek mathematical model for collapse of gravitating material in expanding universe

Question is similar to that used when deriving Friedmann equations.

$\Longrightarrow$ Equation describing structure formation:

$$\dot{a}(t) = \frac{8\pi G}{3}\rho(t)a^2(t) + H_0^2(1 - \Omega_0) \tag{9.44}$$

We will drop the explicit t dependency in the following

Linear Theory, II

Onset of structure formation: $\delta(t), \epsilon(t) \ll 1$ ("linear regime") $\Longrightarrow$ Ignore all higher orders of δ and ϵ.

Left hand side of Friedmann:

$$\dot{a}^2 = (\dot{\bar{a}} - \dot{\bar{a}}\epsilon - \bar{a}\dot{\epsilon})^2 = \dot{\bar{a}}^2 - 2\dot{\bar{a}}^2\epsilon - 2\bar{a}\dot{\bar{a}}\,\dot{\epsilon} = \dot{\bar{a}}^2 - 2\dot{\bar{a}}\frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\epsilon) \tag{9.45}$$

Right hand side of Friedmann:

$$\frac{8\pi G}{3}\bar{\rho}(1+\delta)\bar{a}^2(1-\epsilon)^2 + H_0^2(1-\Omega_0) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(1+\delta)(1-2\epsilon) + H_0^2(1-\Omega_0) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(1+\delta-2\epsilon) + H_0^2(1-\Omega_0) \tag{9.46}$$

Therefore Eq. (9.45)=Eq. (9.46):

$$\dot{\bar{a}}^2 - 2\dot{\bar{a}}\frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\epsilon) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(1+\delta-2\epsilon) + H_0^2(1-\Omega_0) \tag{9.47}$$

Use the Friedmann Equation for $\bar{a}$ (Eq. 9.44) to simplify this to

$$2\dot{\bar{a}} \cdot \frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\epsilon) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(\delta - 2\epsilon) \tag{9.48}$$

Linear Theory, III

To solve Eq. (9.48): Assume for simplicity $\Omega = 1$, matter-dominated universe.

Matter domination $\Longrightarrow \rho a^3 = \text{const.} \Longrightarrow$

$$\bar{\rho}(1+\delta)\bar{a}^3(1-\epsilon)^3 \sim \bar{\rho}\bar{a}^3(1-3\epsilon+\delta) \stackrel{!}{=} \text{const.} \tag{9.49}$$

and therefore

$$\epsilon = \delta/3 \tag{9.50}$$

$\Longrightarrow$ Eq. (9.48) becomes

$$2\dot{\bar{a}} \cdot \frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\delta) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2\delta \tag{9.51}$$

In a $k = 0$ universe,

$$\bar{a}(t) = \left(\frac{3H_0}{2}t\right)^{2/3} =: a_0 t^{2/3} \tag{4.72}$$

and because of $\rho a^3 = \text{const.}$,

$$\bar{\rho}(t) \propto t^{-2} =: \rho_0 t^{-2} \tag{9.52}$$

Linear Theory, IV

Insert $\bar{a}$, $\bar{\rho}$ into Eq. (9.51):

$$\frac{4a_0}{3}t^{-1/3}\left(\frac{2a_0}{3}t^{-1/3}\delta + a_0 t^{2/3}\dot{\delta}\right) = \frac{8\pi G}{3}\rho_0 t^{-2}a_0^2 t^{4/3}\delta \tag{9.53}$$

and simplify

$$t^{-2/3}\delta + t^{1/3}\dot{\delta} = 2\pi G\rho_0 t^{-2/3}\delta \tag{9.54}$$

which gives

$$t\dot{\delta} + (1 - 2\pi G\rho_0)\delta = 0 \tag{9.55}$$

The general solution of Eq. (9.55) is a power-law

$\Longrightarrow$ Growth of structure!

Since also *negative* PL indexes possible

$\Longrightarrow$ Some initial perturbations can be damped out!

We need a better theory to do that in detail...

Linear Theory, V

Better linear theory: Use linearized equations of motion from hydrodynamics.

Detailed theory very difficult, see handout for a few ideas of what is going on...

Classical approach, considering a collapsing sphere:

Potential energy and kinetic energy content of sphere:

$$U = -\frac{1}{2}\int \rho(x)\Phi(x)\,\mathrm{d}^3x \sim -\frac{16\pi^2}{15}G\rho^2 r^5 \quad \text{and} \quad T \sim \frac{c_\mathrm{s}^2}{2}\frac{4\pi r^3\rho}{3} \tag{9.56}$$

c_s: speed of sound; for neutral Hydrogen, $c_\mathrm{s} = \sqrt{5T/3m_\mathrm{p}}$.

Sphere collapses for $|U| > T$, i.e., when

$$2r \gtrsim \sqrt{\frac{5}{2\pi}}\sqrt{\frac{c_\mathrm{s}^2}{G\rho}} \sim c_\mathrm{s}\sqrt{\frac{\pi}{G\rho}} =: \lambda_\mathrm{J} \tag{9.57}$$

λ_J is called the Jeans length, the corresponding mass is the Jeans mass,

$$M_\mathrm{J} = \frac{\pi}{6}\rho\lambda_\mathrm{J}^3 \tag{9.58}$$

Structures with $m < M_\mathrm{J}$ cannot grow.

Note that c_s is time dependent $\Longrightarrow M_\mathrm{J}$ can change with time!

A better derivation of the Jeans length comes from considering the evolution of a fluid in an expanding universe. Assuming that the initial density perturbations were small, we can use perturbation theory for obtaining deviations from homogeneity (=structures).

In a Friedmann universe, for length scales $< 1/H$, dynamical equations are Newtonian to first order, but we need to still use the scale factor, $a(t)$ in the fluid equations.

Continuity equation:

$$\dot{\rho} + \nabla\cdot(\rho\boldsymbol{v}) = 0 \tag{9.59}$$

Euler's equation:

$$\dot{\boldsymbol{v}} + (v\cdot\nabla)v = -\nabla\left(\Phi + \frac{\rho}{c}\right) \tag{9.60}$$

Poisson's equation:

$$\nabla^2\Phi = 4\pi G\rho \tag{9.61}$$

Without perturbations (i.e., the zeroth order solution) is given by the normal Friedmann solutions:

$$\rho_0(t,\boldsymbol{r}) = \frac{\rho_0}{a^3(t)} \quad \text{(dilution by expansion)} \tag{9.62}$$

$$\boldsymbol{v}_0(t,\boldsymbol{r}) = \frac{\dot{a}(t)}{a(t)}\boldsymbol{r} \quad \text{(Hubble law)} \tag{9.63}$$

$$\Phi_0(t,\boldsymbol{r}) = \frac{2\pi G\rho_0 r^2}{3} \quad \text{(soln. of Poisson with } \rho = \text{const.)} \tag{9.64}$$

Convert into comoving coordinates ($\boldsymbol{x} = \boldsymbol{r}/a(t)$) to get rid of the $a(t)$'s and write down perturbation equations:

$$\rho(t,\boldsymbol{x}) = \rho_0(t) + \rho_1(t) =: \rho_0(t)\,(1 + \delta(t,\boldsymbol{x})) \tag{9.65}$$

$$\boldsymbol{v}(t,\boldsymbol{x}) = \boldsymbol{v}_0(t,\boldsymbol{x}) + \boldsymbol{v}_1(t,\boldsymbol{x}) \tag{9.66}$$

$$\Phi(t,\boldsymbol{x}) = \Phi_0(t,\boldsymbol{x}) + \Phi_1(t,\boldsymbol{x}) \tag{9.67}$$

where $|\delta|$, $|\boldsymbol{v}_1|$, $|\Phi_1|$ small (δ is called density perturbation field).

Since the equations are spatially homogeneous, we can Fourier transform them to search for plane wave solutions. The general perturbation solution can then later be found by performing linear combinations of these plane waves.

$$\delta(t,\boldsymbol{x}) = \frac{1}{(2\pi)^3}\int \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}\delta(t,\boldsymbol{k})\mathrm{d}^3k \Longleftrightarrow \delta(t,\boldsymbol{k}) = \int \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\delta(t,\boldsymbol{x})\mathrm{d}^3x \tag{9.68}$$

Inserting into hydro equations gives

$$\ddot{\delta}(t,\boldsymbol{k}) + 2\frac{\dot{a}(t)}{a(t)}\dot{\delta}(t,\boldsymbol{k}) + \left(\frac{k^2c_\mathrm{s}^2}{a^2(t)} - 4\pi G\rho_0\right)\delta(t,\boldsymbol{k}) = 0 \tag{9.69}$$

where the sound speed is $c_s^2 = (\partial p/\partial \rho)_{\text{adiabatic}}$.

Solutions to eq. 9.69 grow or decrease depending on sign of

$$\kappa_J = \left(\frac{k^2 c_s^2}{a^2(t)} - 4\pi G\rho_0\right) \qquad (9.70)$$

Thus, growth is only possible for $k > k_J$ where

$$k_J = \sqrt{\frac{4\pi G\rho_0 a^2(t)}{c_s^2}} \qquad (9.71)$$

or, in terms of physical wavelengths,

$$\lambda_J = \frac{2\pi a(t)}{k_J} = c_s\sqrt{\frac{\pi}{G\rho_0}} \qquad (9.72)$$

the Jeans length.

Classical Structure Formation, I

Early universe: radiation dominates:

$$c_s = c/\sqrt{3} \quad \text{and} \quad \rho_r c^2 = \sigma T^4 \qquad (9.73)$$

and therefore

$$\lambda_{J,\text{rad}} = c^2\sqrt{\pi}3G\sigma T^4 \propto a^2 \quad \text{and} \quad M_J \propto \rho_m \lambda_{J,\text{rad}}^3 \propto a^3 \qquad (9.74)$$

In the early universe, the Jeans mass grows quickly.

At time of radiation – matter equilibrium,

$$\rho_m = \rho_{\text{rad}} = \sigma T_{\text{eq}}^4/c^2 \qquad (9.75)$$

and

$$M_J(t_{\text{eq}}) = \frac{\pi^{5/2}}{18\sqrt{3}}\frac{c^4}{G^{3/2}\sigma^{1/2}}\frac{1}{T_{\text{eq}}} \sim \frac{3.6\times 10^{16}(\Omega_0 h^2)^{-2}\,M_\odot}{(T/T_{\text{eq}})^3} \qquad (9.76)$$

assuming $1 + z_{\text{eq}} = 24000\,\Omega_0 h^2$.

$\Longrightarrow$ much larger than mass in galaxy cluster ($\sim$ mass of $(50\,\text{Mpc})^3$-cube)

Overdense regions with $m < M_{J,\text{rad}}$ are smoothed out by the radiation coupling to matter.

Much larger structures also cannot grow since λ is larger than horizon radius $\Longrightarrow$ Mass spectrum of possible structures.

Classical Structure Formation, II

After t_{eq} not much happens until $T_{\text{rec}} \sim 3000\,\text{K}$

$\Longrightarrow$ recombination

$\Longrightarrow$ Sound speed drops dramatically (radiation and matter decouple):

$$c_s \sim \frac{kT}{m_p} \sim 5\,\text{km}\,\text{s}^{-1} \qquad (9.77)$$

$\Longrightarrow$ M_J drops by 10^{11}:

$$M_{J,\text{eq}} = \frac{\pi\bar{\rho}}{6}\left(\frac{\pi k T_{\text{rec}}}{G\bar{\rho}m_p}\right)^{1/2} \sim 5\times 10^5(\Omega_0 h^2)^{-1/2}\,M_\odot \qquad (9.78)$$

after that, M_J drops because of expansion.

So, in a pure matter universe: huge structures (Zeldovich pancakes) form early, and then fragment at recombination. $\Longrightarrow$ “top-down model”

Problem: This is not really what has been observed (i.e., galaxy clusters are not yet fully formed, but galaxies are)

Solution: Dark matter

Structure Formation and DM

Structure formation with dark matter:

DM unaffected by radiation pressure $\Longrightarrow$ collapse of smaller structures possible $\Longrightarrow$ bottom-up model

As long as DM is relativistic:

$$M_{J,\text{HDM}} = \frac{\pi\rho_{\text{DM}}}{6}\left(\frac{\pi c_{\text{DM}}}{G\rho_{\text{DM}}}\right)^{3/2} \qquad (9.79)$$

Hot Dark Matter: $c_{\text{HDM}} \sim c/\sqrt{3}$

Cold Dark Matter: $c_{\text{CDM}} \ll c/\sqrt{3}$

Standard CDM Scenario:

- DM cools long before t_{rec}
- CDM structures form, M_J about galaxy mass, while baryons coupled to radiation $\Longrightarrow$ stays smooth
- t_{rec}: matter decouples, falls in DM gravity wells

CDM “seeds” structures!

Gives not exactly observed power spectrum

$\Longrightarrow$ Currently preferred: combination of CDM and ΛDM