## Classical Cosmology

To understand what universe we live in, we need to determine observationally the following numbers:

1. The Hubble constant, $H_{0}$
$\Longrightarrow$ Requires distance measurements.
2. The current density parameter, $\Omega_{0}$
$\Longrightarrow$ Requires measurement of the mass density.
3. The cosmological constant, $\Lambda$
$\Longrightarrow$ Requires acceleration measurements.
4. The age of the universe, $t_{0}$, for consistency checks
$\Longrightarrow$ Requires age measurements.
The determination of these numbers is the realm of classical cosmology.

First part: Distance determination and $H_{0}$ !

## Introduction, I

Distances are required for determination of $H_{0}$.
$\Longrightarrow$ Need to measure distances out to $\sim 200 \mathrm{Mpc}$ to obtain reliable values.
To get this far: cosmological distance ladder.

1. Trigonometric Parallax
2. Moving Cluster
3. Main Sequence Fitting
4. RR Lyr
5. Baade-Wesselink
6. Cepheids
7. Light echos
8. Luminosity function of planetary nebulae
9. Brightest Stars
10. Type la Supernovae
11. Tully-Fisher
12. $D_{n}-\sigma$ for ellipticals
13. Brightest Cluster Galaxies
14. Gravitational Lenses

The best reference is
Rowan-Robinson, M., 1985, The Cosmological Distance Ladder, New York: Freeman

(Jacoby et al., 1992, Fig. 1)

## Units

Basic unit of length in astronomy: Astronomical Unit (AU).
Colloquial Definition: $1 \mathrm{AU}=$ mean distance Earth-Sun.
Measurement: (Venus) radar ranging, interplanetary satellite positions,
$\chi^{2}$ minimization of $N$-body simulations of solar system

## $1 \mathrm{AU} \sim 149.6 \times 10^{6} \mathrm{~km}$

In the astronomical system of units (IAU 1976), the AU is defined via Gaussian gravitational constant $(k)$. Acceleration:

$$
\ddot{\mathbf{r}}=-\frac{k^{2}(1+m) \mathbf{r}}{r^{3}}
$$

where $k=0.01720209895$, leading to $a_{\text {万 }}=1.00000105726665$, and
$1 \mathrm{AU}=1.4959787066 \times 10^{11} \mathrm{~m}$ (Seidelmann, 1992).
Reason for this definition: $k$ much better known than $G$.

after Rowan-Robinson (1985, Fig. 2.1)
Motion of Earth around Sun $\Longrightarrow$ Parallax produces apparent motion by amount

$$
\begin{equation*}
\tan \pi \sim \pi=\frac{r_{\text {ठ }}}{d} \tag{5.1}
\end{equation*}
$$

$\pi$ is called the trigonometric parallax, and not 3.141 !

If $s t a r$ is at ecliptic latitude $b$, then ellipse with axes $\pi$ and $\pi \sin b$.
Measurement difficult: $\pi \lesssim 0.76^{\prime \prime}$ ( $\alpha$ Cen).
Define unit for distance:
Parsec: Distance where 1 AU has $\pi=1^{\prime \prime}$.

$$
1 \mathrm{pc}=206265 \mathrm{AU}=3.08 \times 10^{18} \mathrm{~cm}=3.26 \mathrm{ly}
$$

Best measurements to date: Hipparcos satellite (with Tübingen participation).

- systematic error of position: $\sim 0.1$ mas
- effective distance limit: 1 kpc
- standard error of proper motion: $\sim 1$ mas/yr
- broad band photometry
- narrow band: B-V,V-J
- magnitude limit: 12
- complete to mag: 7.3-9.0

Results available at
http://astro.estec.esa.nl/Hipparcos/:
Hipparcos catalogue: 120000 objects with milliarcsecond precision.
Tycho catalogue: $10^{6}$ stars with $20-30$ mas precision, two-band photometry

## UWarwick

Distance Determination

Plans for the future: GAIA (ESA mission, ~2010-2012):


GAIA: $\sim 4 \mu$ arcsec precision, 4 color to $V=20 \mathrm{mag}, 10^{9}$ objects.

## UWarwick

## 5-9

Moving Cluster, I


Perspective effect of
spatial motion towards convergent point:

$$
\begin{equation*}
\tan \lambda=\frac{v_{\mathrm{t}}}{v_{\mathrm{r}}}=\frac{\mu d}{v_{\mathrm{r}}} \tag{5.2}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{d}{1 \mathrm{pc}}=\frac{v_{\mathrm{r}} /(1 \mathrm{~km} / \mathrm{s}) \tan \lambda}{4.74 \mu /\left(1^{\prime \prime} / \mathrm{a}\right)} \tag{5.3}
\end{equation*}
$$

Problem: determination of convergent point Less error prone: moving cluster method = rate of variation of angular diameter of cluster:

$$
\begin{equation*}
\dot{\theta} d=\theta v_{\mathrm{r}} \tag{5.4}
\end{equation*}
$$

Observation of proper motions gives

$$
\begin{equation*}
\frac{\dot{\theta}}{\theta}=\frac{\mathrm{d} \mu_{\alpha}}{\mathrm{d} \alpha}=\frac{\mathrm{d} \mu_{\delta}}{\mathrm{d} \delta} \tag{5.5}
\end{equation*}
$$

where $\mu_{\alpha, \delta}$ proper motion in $\alpha$ and $\delta$, and from Eq. (5.4),

$$
\begin{equation*}
d=v_{\mathrm{r}} \frac{\dot{\theta}}{\theta} \tag{5.6}
\end{equation*}
$$

$v_{\mathrm{r}}$ from spectroscopical radial velocity measurements.


Source: ESA
Application: Distance to Hyades.
Tip of "arrow": Position of stars in 100000 a.
Moving cluster (Hanson): DM ~3.3.
Hipparcos: geometric distance to Hyades is
$d=46.34 \pm 0.27 \mathrm{pc}$, i.e., $\mathrm{DM}=3.33 \pm 0.01 \mathrm{mag} \Longrightarrow$ Moving cluster method only of historic interest.

## UWarwick

Distance Determination

## Interlude

Parallax and Moving Cluster: geometrical methods.

All other methods (exception: light echoes): standard candles.

Requirements for standard candles (Mould, Kennicutt, Jr. \& Freedman, 2000):

1. Physical basis should be understood.
2. Parameters should be measurable objectively.
3. No corrections ("fudges") required.
4. Small intrinsic scatter ( $\Longrightarrow$ requiring small number of measurements!).
5. Wide dynamic range in distance.

Assuming isotropic emission, distance and luminosity are related ("inverse square law")
$\Longrightarrow$ luminosity distance:

$$
\begin{equation*}
F=\frac{L}{4 \pi d_{\mathrm{L}}^{2}} \tag{5.7}
\end{equation*}
$$

where $F$ is the measured flux ( $\mathrm{erg} \mathrm{cm}^{-2} \mathrm{~s}^{-1}$ ) and $L$ the luminosity ( $\mathrm{erg} \mathrm{s}^{-1}$ ).
Definition also true for flux densities, $I_{\nu}\left(\operatorname{erg~cm}^{-2} \mathrm{~s}^{-1} \AA^{-1}\right)$.
The magnitude is defined by

$$
\begin{equation*}
m=A-2.5 \log _{10} F \tag{5.8}
\end{equation*}
$$

where $A$ is a constant used to define the zero point (defined by $m=0$ for Vega).

For a filter with transmission function $\phi_{\nu}$,

$$
\begin{equation*}
m_{i}=A_{i}-2.5 \log \int \phi_{\nu} F_{\nu} \mathrm{d} \nu \tag{5.9}
\end{equation*}
$$

where, e.g., $i=U, B, V$.

To enable comparison of luminosities: define absolute magnitude $M=$ magnitude at distance 10 pc

Thus, since $m=A-2.5 \log \left(L / 4 \pi d^{2}\right)$,

$$
\begin{equation*}
M=m-5 \log \left(\frac{d_{\mathrm{L}}}{10 \mathrm{pc}}\right) \tag{5.10}
\end{equation*}
$$

The difference $m-M$ is called the distance modulus, $\mu_{0}$ :

$$
\begin{equation*}
\mu_{0}=\mathrm{DM}=m-M=5 \log \left(\frac{d_{\mathrm{L}}}{10 \mathrm{pc}}\right) \tag{5.11}
\end{equation*}
$$

Often, distances are given in terms of $m-M$, and not in pc.

Main Sequence Fitting, I

after Rowan-Robinson (1985, Fig. 2.11)
All open clusters are comparably young
$\Longrightarrow$ Hertzsprung Russell Diagram (HRD) dominated by Zero Age Main Sequence (ZAMS).
$\Longrightarrow$ Measure HRD (or Color Magnitude Diagram; CMD), shift magnitude scale until main sequence aligns $\Longrightarrow$ distance modulus.

Distance Determination


## Caveats:

1. Location of ZAMS more age dependent than expected (van Leeuwen, 1999).
2. interstellar extinction $\Longrightarrow \mu_{0}=\mu_{\mathrm{V}}-A_{\mathrm{V}}$, where $\mu_{\mathrm{V}}, A_{\mathrm{V}}$ DM/extinction measured in V-band.
3. metals: line blanketing (change in stellar continuum due to metal absorption lines, see figure) $\Longrightarrow$ Changes color $\Longrightarrow$ horizontal shift in CMD.
van den Bergh (1977): $Z_{\text {Hyades }} \sim 1.6 Z_{\odot}$, while other open clusters have solar metallicity $\Longrightarrow$ Cepheid DM were overestimated by 0.15 mag .
4. identification of unevolved stars crucial (evolution to larger magnitudes on MS during stellar life).
Currently: distances to $\sim 200$ open clusters known (Fenkart \& Binggeli, 1979).
Distance limit $\sim 7 \mathrm{kpc}$.


(M68, Straniero, Chieffi \& Limongi, 1997, Fig. 11)
Globular clusters: HRD different from open clusters:

- population II $\Longrightarrow Z \ll Z_{\odot}$
- evolved

Use theoretical HRDs (isochrones) to obtain distance.
For distant clusters: MS unobservable $\Longrightarrow$ position of horizontal branch.

## Baade-Wesselink

Basic principle (Baade, 1926): Assume black body $\Longrightarrow$ Use color/spectrum to get $k T_{\text {eff }} \Longrightarrow$ Emitted intensity is Planckian $\Longrightarrow$ Observed Intensity is $I_{\nu} \propto \pi r_{*}^{2} B_{\nu}$.
Radius from integrating velocity profile of spectral lines:

$$
\begin{equation*}
R_{2}-R_{1}=p \int_{1}^{2} v \mathrm{~d} t \tag{5.12}
\end{equation*}
$$

( $p$ : projection factor between velocity vector and line of sight).
Wesselink (1947): Determine brightness for times of same color $\Longrightarrow$ rather independent of knowledge of stellar spectrum (deviations from $B_{\nu}$ ).
Stars: Calibration using interferometric diameters of nearby giants.

> Baade-Wesselink works for pulsating stars such as RR Lyr, Cepheids, Miras, and expanding supernova remnants.


M2: Lee \& Carney (1999, Fig. 2)
RR Lyrae variables: Stars crossing instability strip in HRD
$\Longrightarrow$ Variability ( $P \sim 0.2 \ldots 1 \mathrm{~d}$ )
$\Longrightarrow$ RR Lyr gap (change in color!).
Absolute magnitude of RR Lyr gap:
$M_{\mathrm{V}}=0.6, M_{\mathrm{B}}=0.8$, i.e., $\left.L_{\mathrm{RR}} \sim 50 L_{\odot}\right)$.
$M$ determined from ZAMS fitting, statistical parallax, and Baade-Wesselink method.


Lightcurve (here: Lee \&
Carney, 1999, Fig. 5) shows characteristic color variations over pulsation (temperature change!), and a fast rise, slow decay behavior.

RR Lyr in GCs show bimodal number distribution: RRab with $P>0.5 \mathrm{~d}$ and most probable period of $P_{\mathrm{ab}} \sim 0.7 \mathrm{~d}$, and RRc, with $P<0.5 \mathrm{~d}$ and $P_{\mathrm{c}} \sim 0.3 \mathrm{~d}$ (metallicity effect).

Caveat: $M$ dependent on metallicity: larger for higher $Z$ (i.e., metal-rich RR Lyr are fainter, i.e., difference in RR Lyr from population I and II).
Works out to LMC and other dwarf galaxies of local group, however, used mainly for globular clusters.

Previous methods: Selection of methods for distances within Milky Way (and Magellanic Clouds): Basis for extragalactic distance scale.

Primary extragalactic distance indicators: Distance can be calibrated from observations within milky way or from theoretical grounds.

Primary indicators usually work within our neighborhood (i.e., out to Virgo cluster at $15-20 \mathrm{Mpc}$ ).
Examples: Cepheids, light echos,...

## Secondary extragalactic distance indicators: <br> Distance calibrated from primary distance indicators.

Examples: Type la SNe , methods based on integral galaxy properties.

## UWarwick

Distance Determination


To get a feel for the distances in our "neighborhood":

50 kpc : LMC, SMC, some other dwarf galaxies


700 kpc : M31 (Andromeda)


Palomar Schmidt

2-3 Mpc: Sculptor, M81 group (groups similar to local group: a few large spirals, plus smaller stuff).

NGC 300 (Sculptor; Laustsen, Madsen, West, 1991)
5-7 Mpc: M101 group ("pinwheel galaxy"). Important because of high $L$.


Cygnus
Void

source: http://anzwers.org/free/universe/200mill.html

15-20 Mpc: Virgo cluster.


Distance Determination

(Gieren et al., 2000, Fig. 3)
Cepheids: Luminous stars ( $L \sim 1000 L_{\odot}$ ) in instability strip (He II-He III ionization) with large amplitude variation, $P \sim 2 .$. . 150 d (easily measurable). Recent review: Feast (1999).


STScl PR94-49


PL relation for the LMC
Cepheids (after Mould, Kennicutt, Jr. \& Freedman, 2000, Fig. 2).

Henrietta Leavitt (1907): Period-Luminosity (PL) relation: $M_{\mathrm{V}} \propto-2.76 \log P$.

Low luminosity Cepheids have lower periods. Good indications that also influence of color $\Longrightarrow$ Period-Luminosity-Color (PLC) relation

## Cepheids, IV

Physics of Period-Luminosity-Color relation:
Star pulsates such that outer parts remain bound:

$$
\begin{equation*}
\frac{1}{2}\left(\frac{R}{P}\right)^{2} \lesssim \frac{G M}{R} \Longrightarrow \frac{M}{R^{3}} \propto P^{-2} \tag{5.13}
\end{equation*}
$$

where $P$ period. Therefore:

$$
\begin{equation*}
P \propto \rho^{-1 / 2} \quad \Longleftrightarrow \quad P \rho^{1 / 2}=Q \tag{5.14}
\end{equation*}
$$

( $Q$ : pulsational constant, $\rho \propto M R^{-3}$ mean density). But Radius $R$ related to luminosity $L$ :

$$
\begin{equation*}
L=4 \pi R^{2} \sigma T^{4} \quad \Longrightarrow \quad R \propto L^{1 / 2} T^{-2} \tag{5.15}
\end{equation*}
$$

Inserting everything into Eq. (5.14) gives:

$$
\begin{gather*}
P L^{-3} T^{3}=\text { const. }  \tag{5.16}\\
 \tag{5.17}\\
\Longleftrightarrow \quad \log P-3 \log L+3 \log T=\text { const. }
\end{gather*}
$$

## But:

bolometric magnitude: $M_{\mathrm{bol}} \propto-\log L$;
colors: $\mathrm{B}-\mathrm{V} \propto \log T$
such that

$$
\begin{equation*}
c_{1} \log P+c_{2} M_{\mathrm{bol}}+c_{3}(\mathrm{~B}-\mathrm{V})=\mathrm{const} . \tag{5.18}
\end{equation*}
$$

where $c_{1,2,3}$ calibration constants.

## Cepheids, V

Calibration: Need slope and zero point of PLC. Slope is easy: Observations of nearby galaxies (e.g., open clusters in LMC, see previous slide). Zero point is difficult:

- Cepheids in galactic clusters, distance to these via ZAMS fitting $\Longrightarrow$ problematic due to age dependency of ZAMS.
- Hipparcos: geometrical distances $\Longrightarrow$ problematic due to low SNR (resulting in 9\% systematic error.
- Baade-Wesselink using IR info (low metallicity dependence).
Typical relations (Mould et al., 2000, 32 Cepheids):

$$
\begin{align*}
M_{\mathrm{V}} & =-2.76 \log P-1.40+C(Z) \\
M_{\mathrm{I}} & =-3.06 \log P-1.81+C(Z) \tag{5.1}
\end{align*}
$$

The metallicity (color) dependence is roughly

$$
\begin{equation*}
(m-M)_{\text {true }}=(m-M)_{\mathrm{PL}}-\gamma \log Z / Z_{\mathrm{LMC}} \tag{5.2}
\end{equation*}
$$

where $\gamma=-0.11 \pm 0.03 \mathrm{mag} /$ dex ( $Z$ : metallicity) (=Cepheids with larger $Z$ are fainter).

## UWarwick

Distance Determination

## Cepheids, VI

Notes:

1. Pulsational constant $Q=Q(\rho, P)$ ? $\qquad$ possible deviation from PLC, especially at high luminosity $\Longrightarrow$ adds uncertainty at large distances.
2. $M_{\mathrm{V}}$ depends on metallicity (LMC Cepheids are bluer [ $Z_{\mathrm{LMC}}<Z_{\odot}$ ]), but $\gamma$ very uncertain.
For V and I magnitudes, most probably
$\delta(m-M)_{0} / \delta[\mathrm{O} / \mathrm{H}] \lesssim-0.4$ mag dex $^{-1}$, however, others find $+0.75 \mathrm{mag} \mathrm{dex}^{-1}$, see Ferrarese et al. (2000) for details...
3. Stellar evolution unclear (multiple crossings of instability strip possible).

W Vir stars, also called type ॥ Cepheids = "little brother of Cepheids" (present in globular clusters).
Less luminous than normal Cepheids, similar PLC relation, first confused with Cepheids $\Longrightarrow$ Cause for early thoughts of much smaller universe.
Cause for early confusion with Cepheids by Hubble (realization vastly increased assumed size of universe).

## Light echo: specialized way to determine distance to LMC using Supernova 1987A.



STScl PR94-22
February 1987: Supernova in Large Magellanic Cloud. 87 d after explosion: Ring of ionized C and N around SN
$\Longrightarrow$ Excitation of $\mathrm{C}, \mathrm{N}$ in ring-like shell (ejecta from stars equator during red giant phase?).
Observed size: $1.66^{\prime \prime} \times 1.21^{\prime \prime}$

Assuming ring-geometry: direct geometrical determination of distance to LMC possible:


Time delay SN - close side of ring:

$$
\begin{align*}
c t_{1} & =r(1-\sin i)  \tag{5.21}\\
& =86 \pm 6 \mathrm{~d}
\end{align*}
$$

Time delay SN - far side of ring:

$$
\begin{align*}
c t_{2} & =r(1+\sin i)  \tag{5.22}\\
& =413 \pm 24 \mathrm{~d}
\end{align*}
$$

The radius is (Eq. $5.21+$ Eq. 5.22):

$$
\begin{equation*}
r=c \frac{t_{1}+t_{2}}{2}=250 \pm 12 \text { It d } \tag{5.23}
\end{equation*}
$$

and the inclination is (Eq. 5.21+Eq. 5.22):

$$
\begin{equation*}
\sin i=\frac{t_{2}-t_{1}}{t_{1}+t_{2}} \quad \Longrightarrow \quad i \sim 41^{\circ} \tag{5.24}
\end{equation*}
$$

From ring-geometry: $\cos i=1.21^{\prime \prime} / 1.66^{\prime \prime} \Longrightarrow i \sim 43^{\circ}$ ). Thus from angular size of ring:

$$
\begin{equation*}
1.66^{\prime \prime}=\frac{r \cos i}{d} \Longrightarrow d=52 \pm 3 \mathrm{kpc} \tag{5.25}
\end{equation*}
$$

Large Magellanic Cloud (LMC) distance: "anchor point" of extragalactic distance scale.


After Gaia Science Workgroup
Problems that are still not understood:

- Strong dependence on Hipparcos calibration. Values between $18.7 \pm 0.1$ (Feast \& Catchpole) and $18.57 \pm 0.11$ (Madore \& Freedman) obtained.
- Eclipsing binaries and red clump stars: $\mu_{\mathrm{LMC}} \sim 18.23$ (Mould, Kennicutt, Jr. \& Freedman, 2000) $\Longrightarrow$ Inconsistent with other methods!?!

Currently, the distance to the LMC is less well known than desirable.

## PN Luminosity Function, I


(Ciardullo et al., 1989, Fig. 4)
Planetary Nebulae have empirical universal
luminosity function:

$$
\begin{equation*}
N(M) \propto \mathrm{e}^{0.307 M}\left(1-\mathrm{e}^{3\left(M_{\mathrm{PN}}-M\right)}\right) \tag{5.26}
\end{equation*}
$$

Measurement of "cutoff magnitude" $M_{\mathrm{PN}} \Longrightarrow$ Standard candle!
PN detection with narrow band filter of O[III] $\lambda 5007$ Å.

(Ferrarese et al., 2000, Fig. 3), left to right: LMC, M31, NGC 300, M81, M101, NGC 3368, and several galaxy groups.
Result of calibration using Cepheid distances
(Ferrarese et al., 2000):
Cutoff of luminosity function:

$$
\begin{equation*}
M_{\mathrm{PN}}=-4.58 \pm 0.13 \mathrm{mag} \tag{5.27}
\end{equation*}
$$

Out to $\sim 40 \mathrm{Mpc}$ with 8 m class telescope.

## UWarwick

Distance Determination

Caveats: Effects of metallicity, population age, parent galaxy most probably small, but

- Contamination by H II regions (but distinguish using $\mathrm{H} \alpha /[\mathrm{O} I I I]$ ratio.
- Background emission-line galaxies at $z=3.1$
- intracluster PNe (i.e., PNe outside galaxies)


The VLT Looks Deep into a Spiral Galaxy

ESO PR Photo 20/98 ( 23 June 1998)
(C) ESO European Southern Observatory

M83

## Brightest Stars, II

Brightest Stars= O, B, A supergiants, absolute magnitudes usable in local group, large scatter. Brightest stars possible: upper limit to stellar luminosity due to mass loss in supergiants

Possible Improvement: Strength of Balmer series lines. $\mathrm{H} \alpha$ and $\mathrm{H} \beta$ appear biased (class of supergiants with anomalously strong Balmer lines?).

## Problems:

- Contamination by foreground halo stars $\Longrightarrow$ Choose stars with unusual color (rare, i.e. less foreground contamination): $\mathrm{B}-\mathrm{V}<0.4$ or $\mathrm{B}-\mathrm{V}>2.0 \Longrightarrow$ Tip of Red Giant Branch
- Internal extinction.
- Scatter in max. $L \Longrightarrow$ Average over brightest $N$ stars (Sandage, Tammann: $N=3$ ).
- Metallicity dependence.

(Ferrarese et al., 2000, Fig. 1)
Tip of Red Giant Branch: Usable within local group, possibly out to Virgo.
Calibration:

$$
\begin{equation*}
M_{\mathrm{I}}=-4.06 \pm 0.13 \mathrm{mag} \tag{5.28}
\end{equation*}
$$

## Globular Cluster



Globular Cluster Luminosity Function very stable $\approx$ Gaussian $\Longrightarrow$ Use maximum of distribution ("turnover magnitude", $M_{\mathrm{T}}$ ) as standard candle.
(MW GCs, Abraham \& van den Bergh, 1995, Fig. 1)
From Virgo and Fornax Cepheid distances
(Ferrarese et al., 2000):

$$
\begin{equation*}
M_{\mathrm{T}, \mathrm{~V}}=-7.60 \pm 0.25 \mathrm{mag} \tag{5.29}
\end{equation*}
$$

## Caveats:

1. $M_{\mathrm{T}}$ depends on luminosity and type of host galaxy (GC of dwarf galaxies weaker by $\sim 0.3$ in V ).
2. Metallicity of galaxy cluster influences $M_{\mathrm{T}}$.
3. Measurement difficult (need the weak GCs!).
4. Large scatter in data $\Longrightarrow$ Method rather unreliable.

## UWarwick



For early type galaxies:
Assume $N$ stars in picture element (pixel), with average flux $f$.
$\Longrightarrow$ Mean pixel intensity:

$$
\begin{equation*}
\mu=N f \tag{5.30}
\end{equation*}
$$

$\mu$ independent of distance, since $N \propto r^{2}$ and $f \propto r^{-2}$.
(Ajhar et al., 1997, Fig. 3d)
Standard Deviation (Poisson):

$$
\begin{equation*}
\sigma=\sqrt{N} f \propto r^{-1} \tag{5.31}
\end{equation*}
$$

Therefore:

$$
\begin{equation*}
f=\frac{\sigma^{2}}{\mu}=\frac{L}{4 \pi r^{2}} \tag{5.32}
\end{equation*}
$$

which gives the distance $r$.
Review: Blakeslee, Ajhar \& Tonry (1999).
Complication: Adjacent pixels not independent (point spread function of telescope!)
$\Longrightarrow$ Use radial power spectrum to obtain $\sigma^{2}$ and $\mu$.

## UWarwick

Distance Determination

(Ferrarese et al., 2000, Fig. 5)
Luminosity of galaxy dominated by Red Giant Branch stars
$\Longrightarrow$ Strong wavelength and color dependence $\Longrightarrow$ Primary calibration: I-band plus broad-band color dependency to give standard candle.
Often also used: HST WFPC2 plus F814W filter (close to I-band),

$$
\begin{align*}
M_{\mathrm{F} 814 \mathrm{~W}}=(-1.70 \pm & 0.16) \\
& +(4.5 \pm 0.3)\left[(\mathrm{V}-\mathrm{I})_{0}-1.15\right] \tag{5.33}
\end{align*}
$$

Works out to $\sim 70 \mathrm{Mpc}$ with HST.

(Nova in M31, Arp, 1956, p. 18)
"classical nova"= explosion on surface of white dwarf
Novae only in binary systems $\Longrightarrow$ slow accretion of material onto WD $\Longrightarrow$ outer skin reaches $M_{\text {crit }}$ for fusion $\Longrightarrow$ explosion $\Longrightarrow$ ejection of $10^{-6} \ldots 10^{-4} M_{\odot}$ with $v \sim 500 \mathrm{~km} / \mathrm{s}$
Explosion produces characteristic lightcurve.

(van den Bergh \& Pritchet, 1986, Fig. 1).
Strong scatter in lightcurves (higher $L_{\max } \Longrightarrow$ faster decline, but typically $\sim 3 \times$ brighter than Cepheids), but good Correlation luminosity vs. decline timescale ( $t_{i}$, time to reach $\left.m\left(t_{i}\right)=m_{\text {max }}+i\right)$.
Calibration: galactic novae.

## SN1994d (HST WFPC)

Supernovae have luminosities comparable to whole galaxies: $\sim 10^{51} \mathrm{erg} / \mathrm{s}$ in light, $100 \times$ more in neutrinos.

## Type la Supernovae, II


(Filippenko, 1997, Fig. 1); $t$ : time after maximum light; $\tau$ : time after core collapse; P Cyg profiles give $v \sim 10000 \mathrm{~km} \mathrm{~s}^{-1}$

Rough classification (Minkowski, 1941):
Type I: no hydrogen in spectra; subtypes la, lb, lc
Type II: hydrogen present, subtypes II-L, II-P
Note: pre 1985 subtypes la, lb had different definition than today $\Longrightarrow$ beware when reading older texts.


## Type la Supernovae, IV


(Filippenko, 1997, Fig. 3)

Light curves of SNe I all very similar, SNe II have much more scatter.

SNe II-L ("linear") resemble SNe I
SNe II-P ("plateau") have const. brightness to within 1 mag for extended period of time.


(SN 1998bu in M96, Jha et al., 1999, Figs. 2 and 4)

(SN 1998bu, Jha et al., 1999, Fig. 6)


90 cm CTIO, N. Suntzeff

Clue on origin from supernova statistics:

- SNe II, Ib, Ic: never seen in ellipticals; rarely in SO; generally associated with spiral arms and H II regions.
$\Longrightarrow$ progenitor of SNe II, Ib, Ic: massive stars ( $\lesssim 8 M_{\odot}$ ) $\Longrightarrow$ core collapse
- SNe la: all types of galaxies, no preference for arms.
$\Longrightarrow$ progenitor of SNe la: accreting carbon-oxygen white dwarfs, undergoing thermonuclear runaway

after P. Höflich


## UWarwick

## Type la Supernovae, VII

SN Ia = Explosion of CO white dwarf when pushed over Chandrasekhar limit ( $1.4 M_{\odot}$ ) (via accretion?).

## $\Longrightarrow$ Always similar process

$\Longrightarrow$ Very characteristic light curve: fast rise, rapid fall, exponential decay with half-time of 60 d .

60 d time scale from radioactive decay $\mathrm{Ni}^{56} \rightarrow \mathrm{Co}^{56} \rightarrow \mathrm{Fe}^{56}$ ("self calibration" of lightcurve if same amount of $\mathrm{Ni}^{56}$ produced everywhere).

Calibration: SNe la in nearby galaxies where Cepheid distances known. At maximum light:

$$
\begin{equation*}
M_{\mathrm{B}}=-18.33 \pm 0.11+5 \log h_{100} \tag{5.34}
\end{equation*}
$$

( $L \sim 10^{9 \ldots 10} L_{\odot}$ ).
Intrinsic dispersion: $\lesssim 0.25 \mathrm{mag}$ (possibly due to size of clusters analyzed?!?)
Observable out to 1000 Mpc

(Phillips et al., 1999, Fig. 8)
Caveats:

1. Are they really identical? $\Longrightarrow$ history of pre-WD star?
2. Correction for extinction in parent galaxy difficult.
3. Baade-Wesselink for calibration Eq. (5.34) depends crucially on assumed ( $\mathrm{B}-\mathrm{V}$ ) $-T_{\text {eff }}$ relation.
4. Some SN Iae spectroscopically peculiar $\Longrightarrow$ Do not use these!
5. Decline rate and color vary, but max. brightness and decline rate correlate (see figure).


Lightcurves of Hamuy et al. SN la sample ( 18 SNe discovered within 5 d past maximum, with
$3.6<\log c z<4.5$, i.e., $z<0.1$, after correction of systematic effects and time dilatation (Kim et al., 1997).

## Type la Supernovae, X

Recalibration of SN Ia distances with Cepheids gives (Gibson et al., 2000):

$$
\begin{gather*}
\log H_{0}=0.2\left\{M_{\mathrm{B}}^{\max }-0.720( \pm 0.459)\right. \\
\cdot\left[\Delta m_{\mathrm{B}, 15, t}-1.1\right]-1.010( \pm 0.934) \\
\left.\cdot\left[\Delta m_{\mathrm{B}, 15, t}-1.1\right]^{2}+28.653( \pm 0.042)\right\} \tag{5.35}
\end{gather*}
$$

where

$$
\begin{equation*}
\Delta m_{\mathrm{B}, 15, t}=\Delta m_{\mathrm{B}, 15}+0.1 E(\mathrm{~B}-\mathrm{V}) \tag{5.36}
\end{equation*}
$$

where
$\Delta m_{\mathrm{B}, 15}$ : observed 15 d decline rate, $E(\mathrm{~B}-\mathrm{V})$ : total extinction (galactic+intrinsic).

Eq. (5.35) valid for B-band, equivalent formulae exist for V and I .
Overall, the calibration is good to better than 0.2 mag in B.

## 5-58

## Tully-Fisher, I






(Sakai et al., 2000, Fig. 1)
Tully-Fisher relation for spiral galaxies: Width of 21 cm line of H correlated with galaxy luminosity:

$$
\begin{equation*}
M=-a \log \left(\frac{W_{20}}{\sin i}\right)-b \tag{5.3}
\end{equation*}
$$

where $W_{20}$ : $20 \%$ line width ( $\mathrm{km} / \mathrm{s}$; typically
$W_{20} \sim 300 \mathrm{~km} / \mathrm{s}$ ), $i$ inclination angle.
For the B- and I-Bands (Sakai et al., 2000):

|  | $B$ | I |
| :---: | :---: | :---: |
| a | $7.97 \pm 0.72$ | $9.24 \pm 0.75$ |
| b | $19.80 \pm 0.11$ | $21.12 \pm 0.12$ |

Qualitative Physics: Line width related to mass of galaxy: $W / 2 \sim V_{\max }$, where $V_{\max }$ max. velocity of rotation curve
$\Longrightarrow$ Assume $M / L=$ const. (good assumption)
$\Longrightarrow$ width related to luminosity.
Detailed physical basis unknown. Might be related to galaxy formation in CDM models ("hierarchical clustering", see later).

I-band is better (less internal extinction).
Caveats:

1. Determination of inclination $i$.
2. Influence of turbulent motion within galaxy.
3. Constants dependent on galaxy type (Sa and Sb similar, Sc more luminous by factor of $\sim 2$ ).
4. Optical extinction.
5. Intrinsic dispersion $\sim 0.2$ mag.
6. Barred Galaxies problematic.


M32 (companion of Andromeda), courtesy W. Keel
"Faber-Jackson" law for elliptical galaxies:
The luminosity $L$ of an elliptical galaxy scales with its intrinsic velocity dispersion, $\sigma$, as $L \propto \sigma^{4}$.

Note that ellipticals have virtually no Hydrogen

$$
\Longrightarrow \text { cannot use } 21 \mathrm{~cm} .
$$

Ellipticals:

$$
\begin{equation*}
M_{\mathrm{B}}=-19.38 \pm 0.07-(9.0 \pm 0.7)(\log \sigma-2.3) \tag{5.38}
\end{equation*}
$$

Lenticulars:

$$
\begin{equation*}
M_{\mathrm{B}}=-19.65 \pm 0.08-(8.4 \pm 0.8)(\log \sigma-2.3) \tag{5.39}
\end{equation*}
$$

## UWarwick

Distance Determination

The Faber-Jackson law is a specialized case of the more general $D_{n}-\sigma$-relation:
The intensity profile of an elliptical galaxy is given by de Vaucouleurs' $r^{1 / 4}$ law:

$$
\begin{equation*}
I(r)=I_{0} \exp \left(-\left(r / r_{0}\right)^{1 / 4}\right) \quad \Longrightarrow \quad L=\int I \propto I_{0} r_{0}^{2} \tag{5.40}
\end{equation*}
$$

Because of the virial theorem ( $E_{\text {kin }}=-E_{\text {pot }} / 2$ ):

$$
\begin{equation*}
\frac{1}{2} m \sigma^{2}=G \frac{m M}{r_{0}} \Longleftrightarrow \sigma^{2} \propto \frac{M}{r_{0}} \tag{5.41}
\end{equation*}
$$

where $\sigma$ : velocity dispersion.
Assume mass-to-light ratio

$$
\begin{equation*}
M / L \propto M^{\alpha} \tag{5.42}
\end{equation*}
$$

( $\alpha \sim 0.25$ ). and use $r_{0}$ from Eq. (5.40) to obtain

$$
\begin{equation*}
L^{1+\alpha} \propto \sigma^{4-4 \alpha} I_{0}^{\alpha-1} \tag{5.43}
\end{equation*}
$$

This is called the "fundamental plane" relationship (Dressler et al., 1987).

Observationally easier: Instead of inserting $r_{0}, I_{0}$, measure diameter $D_{n}$ of aperture to reach some mean surface brightness (typically sky brightness, $20.75 \mathrm{mag} \mathrm{arcsec}^{-2}$ in B ), and use calibration.
Note: Assumptions are

1. $M / L$ same everywhere.
2. ellipticals have same stellar population everywhere
Calibration paper: Kelson et al. (2000).

For very large distances: use brightest cluster galaxies as indicators.
Assumption: Galaxy clusters are similar, brightest galaxy has similar brightness.
Calibration: Close clusters.
10 close galaxy clusters: brightest galaxy has

$$
\begin{equation*}
M_{\mathrm{V}}=-22.82 \pm 0.61 \tag{5.44}
\end{equation*}
$$

Problems:

- Cosmological evolution (e.g., galaxy cannibalism)
- Scatter in brightest galaxy large $\Longrightarrow$ Use 2nd, 3rd brightest, or average brightest $N$ galaxies.
$\Longrightarrow$ The method of brightest cluster galaxies should not be used anymore.

To obtain $H_{0}$ : need two things:

1. distances, and
2. redshifts

Distances:
Hubble Space Telescope Key Project on
Extragalactic Distance Scale.
Summary paper: Freedman et al. (2001), there are a total of 29 papers on the HST key project!

Strategy:

1. Use high-quality standard candle: Cepheid variables as primary distance calibrator.
2. Calibrate secondary calibrators that work out to $c z=10000 \mathrm{~km} \mathrm{~s}^{-1}$ :

- Tully-Fisher,
- Type la Supernovae,
- Surface Brightness Fluctuations,
- Fundamental-plane for Ellipticals.

3. Combine uncertainties from these methods.

Redshift determination is obviously trivial compared to distance determination. . .

## Velocity Field, I

Before determining $H_{0}$ : correct for influence of velocity field (cluster motion wrt. comoving coordinates).
The observed redshift is given by

$$
\begin{equation*}
1+z=\left(1+z_{\mathrm{R}}\right)\left(1-\frac{v_{0}}{c}+\frac{v_{\mathrm{G}}}{c}\right) \tag{5.45}
\end{equation*}
$$

where
$v_{0}$ : observer's radial velocity in direction of galaxy $v_{\mathrm{G}}$ : radial velocity of the galaxy, difficult to find $z_{\mathrm{R}}$ : cosmological redshift

Older galaxy catalogues often attempt to correct the measured values of $z$ to produce "corrected redshifts", e.g., by setting $v_{\mathrm{G}}=0$ and

$$
\begin{equation*}
1+z=\left(1+z_{\mathrm{R}}\right)\left(1+\frac{v_{0}}{c}\right) \sim 1+z_{\mathrm{R}}-\frac{v_{0}}{c} \tag{5.46}
\end{equation*}
$$

and thus

$$
\begin{equation*}
z_{\mathrm{R}} \sim z+\frac{v_{0}}{c} \tag{5.4}
\end{equation*}
$$

since $v_{0}$ was up to COBE not well known $\Longrightarrow$ introduces unnecessary problems $\Longrightarrow$ correction not used anymore in recent redshift surveys!
see Harrison \& Noonan (1979) for details

(Bennett et al., 1996, COBE DMR;)
$v_{0}$ is easy to find $\Longrightarrow$ Measure velocity of Earth with respect to 3 K radiation. COBE finds speed of $(369.1 \pm 2.6) \mathrm{km} / \mathrm{s}$, such that

$$
\begin{equation*}
v_{0}=370 \mathrm{~km} \mathrm{~s}^{-1} \cdot \cos \theta_{\mathrm{CMB}} \tag{5.48}
\end{equation*}
$$

where $\theta_{\mathrm{CMB}}=\angle\left(\mathbf{v}, \mathbf{v}_{\mathrm{CMB}}\right)$, and $\mathbf{v}_{\mathrm{CMR}}$ points towards

$$
\begin{aligned}
(l, b) & =\left(264.26^{\circ} \pm 0.33^{\circ}, 48.22^{\circ} \pm 0.13^{\circ}\right) \\
(\alpha, \delta)_{\mathrm{J} 2000.0} & =\left(11^{\mathrm{h}} 12.2^{\mathrm{m}} \pm 0.8^{\mathrm{m}},-7.06^{\circ} \pm 0.16^{\circ}\right)
\end{aligned}
$$

in constellation Crater.
Velocity comes from measured Dipole temperature anisotropy of $\Delta T=3.353 \pm 0.024 \mathrm{mK}$ of 3K black-body spectrum of $T=2.725 \pm 0.020 \mathrm{~K}$, using $\Delta T / T=v / c$.

## UWarwick



The constellation Crater ("Becher") in Johan Elert Bode's Sternatlas (after Slawik/Reichert, Atlas der Sternbilder, Spektrum, 2004)

# To get feeling for $v_{G}$ 

 out to Virgo, need to study local velocity field surrounding local group and beyond.Two major velocity components:

1. Virgocentric infall (known since mid-1970s)
2. Motion towards great attractor ("Seven

## Samurai", 1980)

plus virialized galaxy motions within clusters.
General analysis: build maximum likelihood model of velocity field including above components plus Hubble flow. See Tonry et al. (2000) for details.

## 5-69

## Velocity Field, V


(Tonry et al., 2000, Fig. 20)
Decomposition of velocity field: (Mould et al., 2000, Tab. A1, note that Tonry et al. 2000 find slightly different values)

|  | $\alpha_{1950.0}$ | $\delta_{1950.0}$ | $v\left(\mathrm{~km} \mathrm{~s}^{-1}\right)$ |
| :--- | :--- | :--- | ---: |
| Virgo | $12^{\mathrm{h}} 28^{\mathrm{m}}$ | $+12^{\circ} 40^{\prime}$ | 957 |
| GA | $13^{\mathrm{h}} 20^{\mathrm{m}}$ | $+44^{\circ} 00^{\prime}$ | 4380 |
| Shapley | $13^{\mathrm{h}} 30^{\mathrm{m}}+31^{\circ} 00^{\prime}$ | 13600 |  |

( $v$ wrt. center of local group; not taking Hubble flow into account!).

## UWarwick

Hubble Diagram for Cepheids (flow-corrected)


Freedman et al. (2001, Fig. 1)
To obtain $H_{0}$ :

1. Determine $d$ with Cepheids and HST
2. Determine " $v$ ", corrected for local velocity field
3. Draw Hubble-diagram
4. Regression Analysis $\Longrightarrow H_{0}$

Value from HST Key Project:

$$
\begin{equation*}
H_{0}=75 \pm 10 \mathrm{~km} / \mathrm{s} / \mathrm{Mpc} \tag{5.49}
\end{equation*}
$$

## H from HST


(SN la Hubble relations; left: full sample, middle: excluding strongly reddened SN lae, right: same as middle, correcting for light-curve shape Freedman et al., 2001, Fig. 2)

## Cepheids alone: nearby $\Longrightarrow$ systematic

 uncertainty due to local flow correction and small overall $v \Longrightarrow$ use secondary candles to get to larger distances.Example above: magnitude-redshift diagram, analoguous to Hubble diagram ( $m \propto-5 \log I$, and $I \propto 1 / r^{2} \propto 1 / z^{2}$ because of Hubble $\Longrightarrow m \propto \log c z$ ).


Freedman et al. (2001, Fig. 4)
Combining all secondary methods, best value found:

$$
\begin{equation*}
H_{0}=72 \pm 8 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1} \tag{5.50}
\end{equation*}
$$

H from HST

(Mould et al., 2000, Fig. 5)
Major systematic uncertainty in current $H_{0}$ value: zero-point of Cepheid scale, i.e., distance to Large Magellanic Cloud.

Despite these problems:
$\Longrightarrow$ All current values approach
$\sim 70 \mathrm{~km} \mathrm{~s}^{-1} \mathrm{Mpc}^{-1}$, with uncertainty $\sim 10 \%$

## $H_{0}$ controversy is over

H from HST


For larger distances: Deviations from Hubble-Relation!
Before we understand why: Understand Big-Bang itself!

Bibliography

Abraham, R. G., \& van den Bergh, S., 1995, ApJ, 438, 218
Ajhar, E. A., Lauer, T. R., Tonry, J. L., Blakeslee, J. P., Dressler, A., Holtzman, J. A., \& Postman, M., 1997, Astron. J., 114, 626

Arp, H. C., 1956, Astron. J., 61, 15
Bennett, C. L., et al., 1996, ApJ, 464, L1
Blakeslee, J., Ajhar, E. A., \& Tonry, J. L., 1999, in Post-Hipparcos Cosmic Candles, ed. A. H. . F. Caputo, (Dordrecht: Kluwer), 181, astro-ph/9807124

Ciardullo, R., Jacoby, G. H., Ford, H. C., \& Neill, J. D., 1989, ApJ, 339, 53
Feast, M., 1999, PASP, 111, 775
Fenkart, R. F., \& Binggeli, B., 1979, ApJS, 35, 271
Ferrarese, L., et al., 2000, ApJ, 529, 745
Filippenko, A. V., 1997, ARA\&A, 35, 309
Freedman, W. L., et al., 2001, ApJ, 553, 47
Gibson, B. K., et al., 2000, ApJ, 529, 723
Gieren, W. P., Gómez, M., Storm, J., Moffett, T. J., Infante, L., Barnes, III, T. G., Geisler, D., \& Fouqué, P., 2000, Astrophys. J., Suppl. Ser., 129, 111

Harrison, E. R., \& Noonan, T. W., 1979, ApJ, 232, 18
Jacoby, G. H., et al., 1992, PASP, 104, 599
Jha, S., et al., 1999, Astrophys. J., Suppl. Ser., 125, 73
Kelson, D. D., et al., 2000, ApJ, 529, 768
Kim, A. G., et al., 1997, ApJ, 476, L63
Lee, J.-W., \& Carney, B. W., 1999, ApJ, 117, 2868
Mould, J., Kennicutt, Jr., R. C., \& Freedman, W., 2000, Rep. Prog. Phys., 63, 763
Mould, J. R., et al., 2000, ApJ, 529, 786
Phillips, M. M., Lira, P., Suntzeff, N. B., Schommer, R. A., Hamuy, M., \& Maza, J., 1999, Astron. J., 118, 1766

Rowan-Robinson, M., 1985, The Cosmological Distance Ladder, (New York: Freeman)
Sakai, S., et al., 2000, ApJ, 529, 698
Seidelmann, P. K., (eds.) 1992, Explanatory Supplement to the Astronomical Almanac, (Mill Valley, CA: University Science Books)

Straniero, O., Chieffi, A., \& Limongi, M., 1997, ApJ, 490, 425
Tonry, J. L., Blakeslee, J. P., Ajhar, E. A., \& Dressler, A., 2000, ApJ, 530, 625
van den Bergh, S., \& Pritchet, C. J., 1986, PASP, 98, 110

