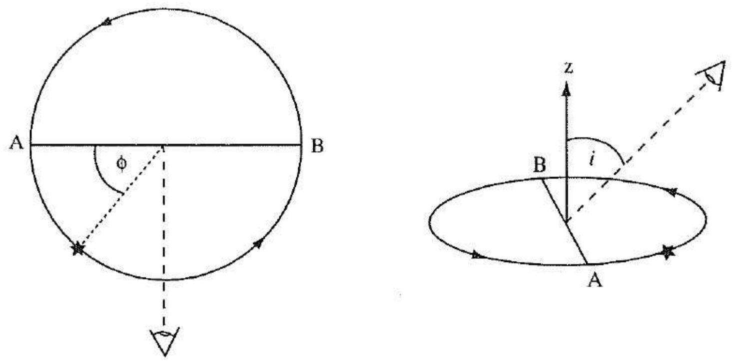
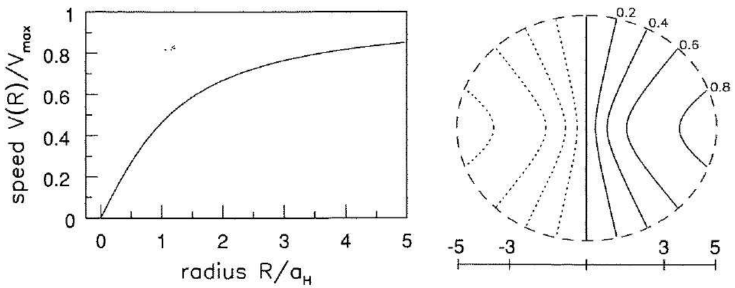
Spider Diagrams



Now consider motion of gas in spiral galaxies.

Good assumption for now: gas is moving on approximately circular orbits

Spider Diagrams

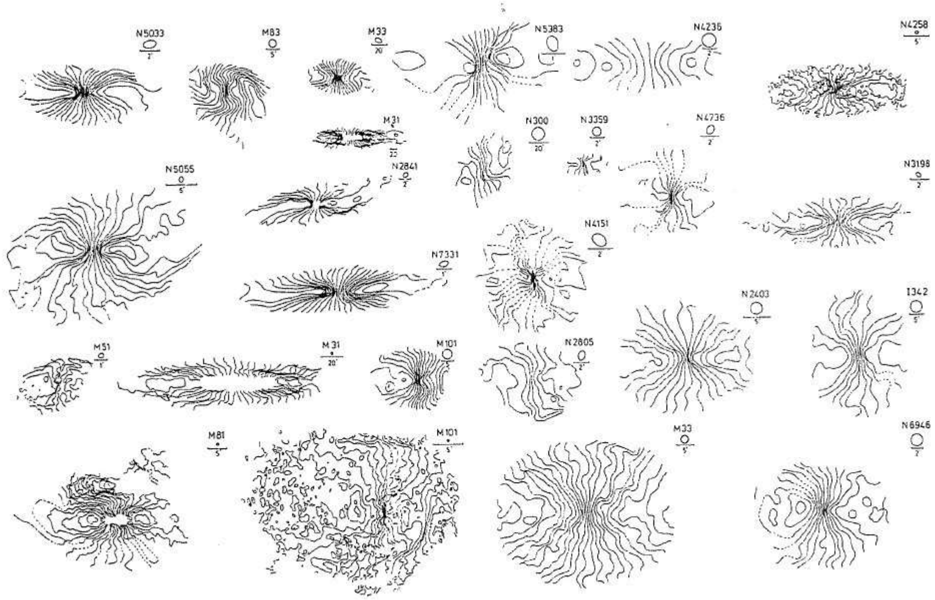


Because we look at inclined galaxy, observed radial velocity is given by

$$v_r(R,i) = v_{\mathrm{syst}} + v(R)\sin i \cos\phi \qquad (4.10)$$

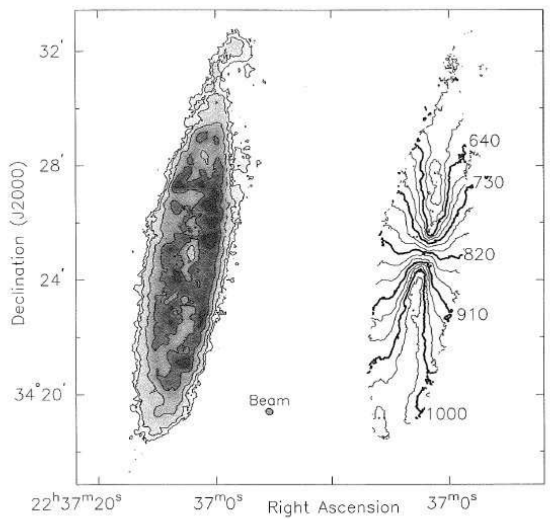
where v_{syst} is the systemic velocity (e.g., peculiar velocity plus Hubble flow), and where $v(R)$ is the rotational velocity.

Observations: plot curves of constant v_r: Spider diagram



Combes et al. (Fig. 3.5)

Spider Diagrams

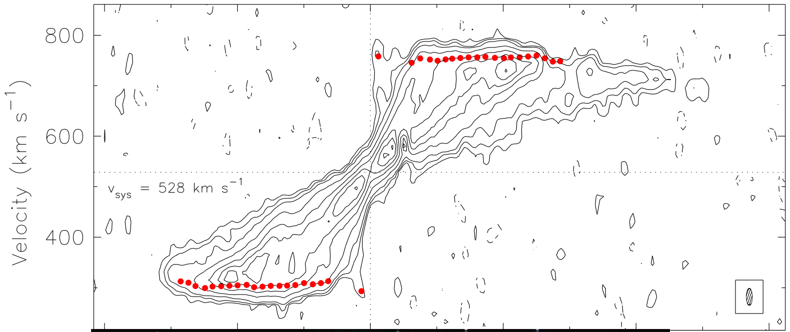


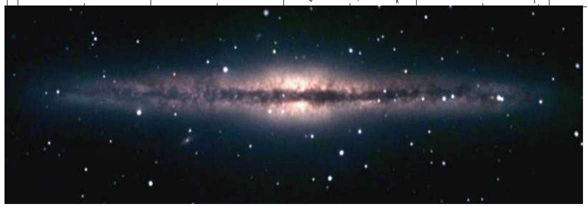
Major properties of spider diagrams:

- central regions: contours parallel to minor axis $\Longrightarrow V(R) \propto R$
- outer regions: contours radial $\Longrightarrow V(R) \sim$ const.

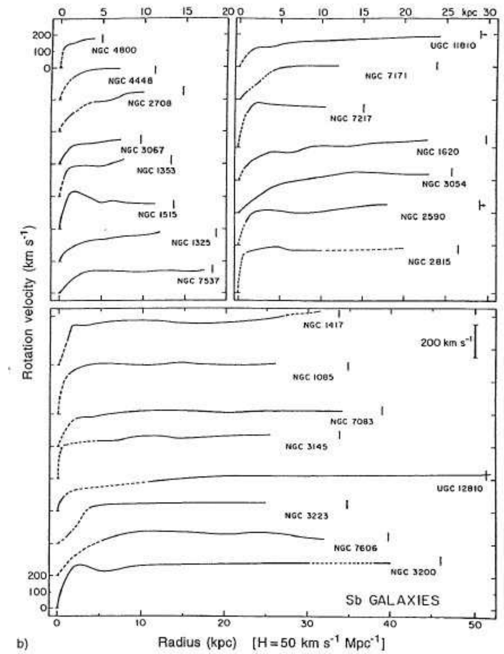
H I gas surface density and gas velocity in NGC 7331 (SG, Fig. 5.13)

Rotation Curves

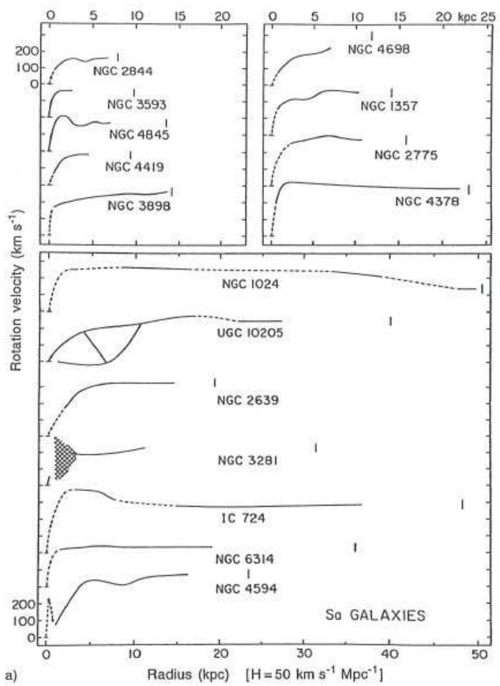




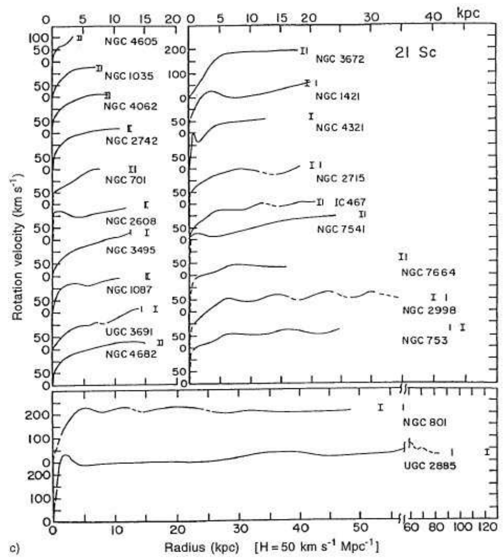
NGC 891 (Swaters et al., 1997, ApJ 491, 140 / Paul LeFevre, S&T Nov. 2002)



Ratation curves of Sb galaxies (Combes et al., Fig. 3.1)

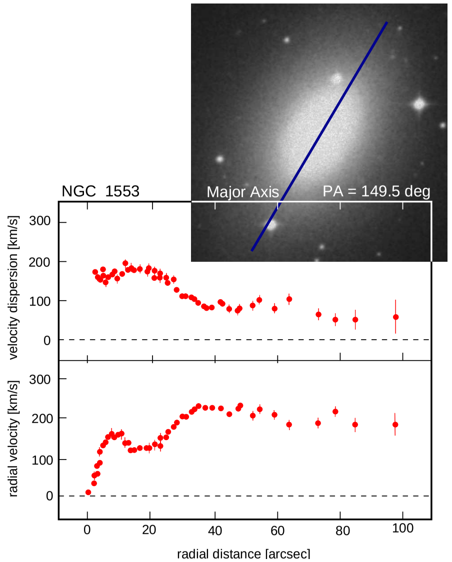


Ratation curves of Sa galaxies (Combes et al., Fig. 3.1)



Ratation curves of Sc galaxies (Combes et al., Fig. 3.1)

Rotation Curves



Spiral galaxy rotation curves are flat!

"Galaxy rotation problem", first discovered by Vera Rubin (1970)

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← NGC 1553 (S0) (after Kormendy, 1984, ApJ 286, 116)

Rotation Curves

To derive gas motion: assume that gas is in dynamic equilibrium and that the motion is solely determined by gravity.

The equation of motion

$$\ddot{\boldsymbol{r}} = -\nabla\Phi \tag{4.11}$$

where the potential is obtained from solving Poisson's equation

$$\Delta\Phi(R) = 4\pi G\rho(R) \tag{4.12}$$

The potential will generally be axisymmetric, i.e., $\Phi = \Phi(R, z)$ (no dependence on angular coordinate ϕ).

In this case, use cylindrical coordinates and the components of Eq. (4.11):

$$\frac{d}{dt}\left(R^2\dot{\phi}\right) = 0 \tag{4.13}$$

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial\Phi}{\partial R} \tag{4.14}$$

$$\ddot{z} = -\frac{\partial\Phi}{\partial z} \tag{4.15}$$

Rotation Curves

Because $\frac{d}{dt}\left(R^2\dot{\phi}\right) = 0$:

The z-component of the angular momentum (per unit mass) is conserved:

$$L_z = R^2\dot{\phi} = \text{const.} \tag{4.16}$$

Introduce the effective potential

$$\Phi_{\text{eff}} = \Phi(R, z) + \frac{L_z^2}{2R^2} \tag{4.17}$$

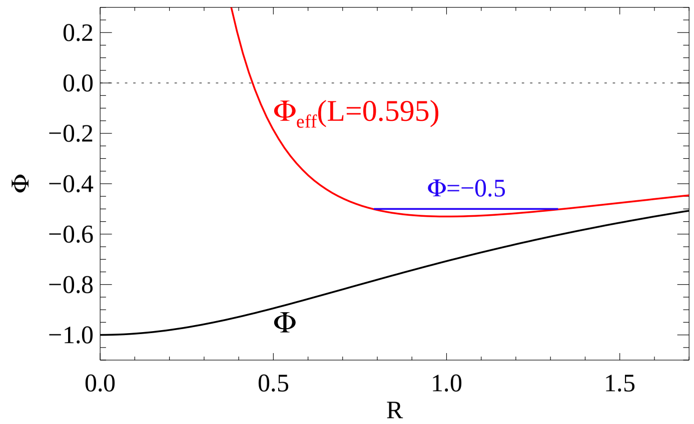
to simplify Eq. (4.13)ff. further to obtain

$$\ddot{R} = -\frac{\partial\Phi_{\text{eff}}}{\partial R} \quad \text{and} \quad \ddot{z} = -\frac{\partial\Phi_{\text{eff}}}{\partial z} \tag{4.18}$$

Multiply by $\dot{R}$ and integrate to obtain the energy equation

$$\frac{1}{2}\dot{R}^2 + \Phi_{\text{eff}}(R, z) = \text{const.} \tag{4.19}$$

Rotation Curves



Energy equation defines the orbits of stars, yields (in general complicated) orbits between minimum and maximum radius.

Note "angular momentum barrier" for small R.

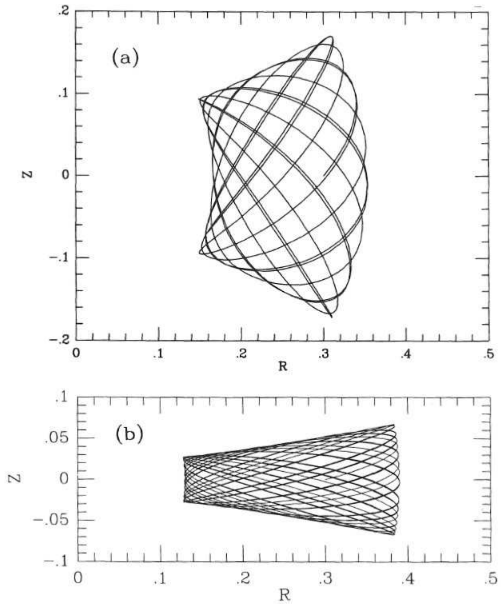


Figure 3-3. Two orbits in the potential of equation (3-50) with $q = 0.9$. Both orbits are at energy $E = -0.8$ and angular momentum $L_z = 0.2$, and we assume $v_0 = 1$.

Binney & Tremaine (Fig. 3.3)

Rotation Curves

To derive mass of galaxy from observed motion, a possible Ansatz is to assume circular orbits and to write

$$v^2(R) = R\frac{\partial\Phi}{\partial R} \qquad (4.20)$$

where Φ is some suitable potential. An ansatz originally due to Alar Toomre is to notice that

$$\Phi_\pm(R,z) = \exp(-k|z|)J_0(kR) \qquad (4.21)$$

is a solution of Laplace equation (J_0: Bessel function).

The surface density distribution causing this potential is (use Gauss' theorem; assume $z = 0$)

$$\Sigma_k(R) = -\frac{k}{2\pi G}J_0(kR) \qquad (4.22)$$

Real surface densities can be generated by superposition:

$$\Sigma(R) = \int_0^\infty S(k)\Sigma_k(R)dk = -\frac{1}{2\pi G}\int_0^\infty S(k)J_0(kR)kdk \qquad (4.23)$$

and the corresponding potential is

$$\Phi(R,z) = \int_0^\infty S(k)\Phi_k(R,z)dk = \int_0^\infty S(k)J_0(kR)e^{-k|z|}dk \qquad (4.24)$$

Exponential Disks

It is possible to show that

$$S(k) = -2\pi G\int_0^\infty J_0(kR)\Sigma(R)RdR \qquad (4.25)$$

Remember similar quantities for Fourier series!

$\Longrightarrow$ Obtain $\Sigma(R)$ from observations, then get $S(k)$

The predicted velocity profile is then:

$$v^2(R) = R\left.\frac{\partial\Phi}{\partial R}\right|_{z=0} = -R\int_0^\infty S(k)J_1(kR)kdk \qquad (4.26)$$

For exponential disks, $\Sigma(R) = \Sigma_0\exp(-R/R_\mathrm{d})$, one finds

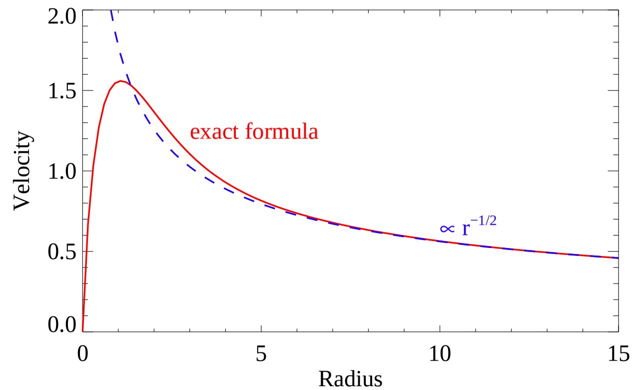
$$S(k) = -\frac{2\pi G\Sigma_0 R_\mathrm{d}^2}{[1+(kR_\mathrm{d})^2]^{3/2}} \qquad (4.27)$$

and after a little bit tedious calculation

$$v^2(R) = 4\pi G\Sigma_0 R_\mathrm{d} y^2\left[I_0(y)K_0(y) - I_1(y)K_1(y)\right] \qquad (4.28)$$

where I_j, K_j: modified Bessel functions and $y = R/(2R_\mathrm{d})$.

Exponential Disks



For a disk, the velocity is predicted to fall at sufficiently large radii, $v \propto r^{-1/2}$

As expected:

$$\frac{GM(\le r)}{r^2} = \frac{v_\mathrm{rot}^2(r)}{r} \quad\Longrightarrow\quad v = \sqrt{\frac{GM}{r}} \qquad (4.29)$$

Exponential Disks

What mass distribution do we expect?

Intensity profile of disk in spiral galaxies can be well described by

$$I(r) = I_0 \exp(-r/h) \quad (4.1)$$

Therefore the luminosity emitted within the radial distance r_0 is:

$$L(r < r_0) = I_0 \int_0^{r_0} \exp(-r/h)\, 2\pi r\, \mathrm{d}r = 2\pi I_0 \left(h^2 - \exp(-r_0/h) h(h + r_0)\right) \quad (4.30)$$

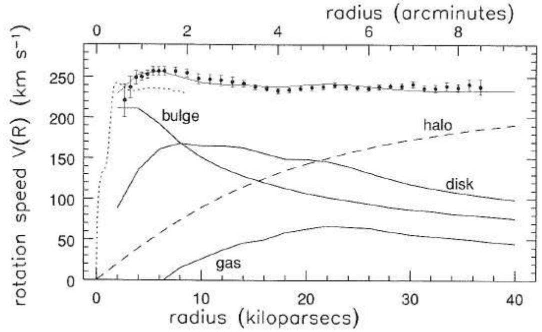
i.e., for $r_0 \longrightarrow \infty$: $L(r < r_0) \to$ const..

$\Longrightarrow$ If all light comes from stars, i.e., light traces mass, and the population of stars does not change with position then $M/L \sim$ const..

This implies that outside of a certain radius $M(< r) \sim$ const. and thus $v \propto r^{-1/2}$

This is not what is observed!

Dark Matter Halos



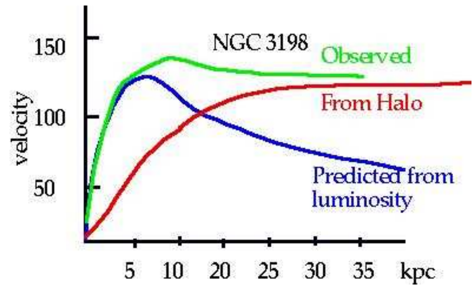
Radial Velocity Curve of NGC 7331 (SG, Fig. 5.20)

To obtain the flat rotation curves, disk profiles are not sufficient.

One therefore postulates a massive halo, e.g., with a density distribution of the form

$$\rho_{\text{halo}}(R) = \frac{\rho_0}{1 + (R/a)^\gamma} \quad \Longrightarrow \quad v^2_{\text{halo}} = \frac{4\pi G}{r} \int_0^r x^2 \rho_{\text{halo}}(x)\, dx \quad (4.31)$$

Dark Matter Halos



Distribution of dark matter:

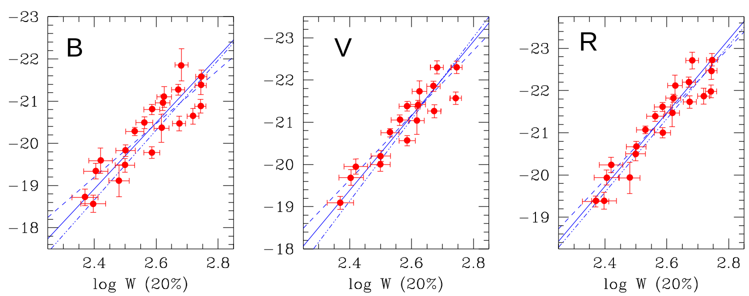
- luminosity to mass ratio: $L/M = 4$ (solar neighbourhood)
- convert luminosity to mass
- compute expected rotation curve form the mass distribution $v_{\text{lum}}(R)$
- distribution of dark matter:

$$M_{\text{dark}}(R) = \frac{M}{G}\left[v^2(R) - v^2_{\text{lum}}(R)\right] \quad (4.32)$$

Canonical interpretation: a large fraction of gravitating material does not emit light $\Longrightarrow$ spiral galaxies have large and massive halos made of dark matter

In general, the mass to light ratio of spiral galaxies is $5 \lesssim M/L \lesssim 25$.

Tully-Fisher



(after Sakai et al., 2000, Fig. 1)

The constant M/L for spirals leads to the Tully-Fisher relation for spiral galaxies: The width of 21 cm line of H is correlated with the galaxy luminosity, can be used as a distance indicator:

$$M = -a \log\left(\frac{W_{20}}{\sin i}\right) - b \quad (4.33)$$

where W_{20}: 20% line width (km s^{-1}; typically $W_{20} \sim 300$ km s^{-1}), i inclination angle.

For the B- and I-Bands (Sakai et al., 2000):

	B	I
a	7.97± 0.72	9.24± 0.75
b	19.80± 0.11	21.12± 0.12

Tully-Fisher

Qualitative Physics: Line width related to mass of galaxy: $W/2 \sim V_{\max}$, where $V_{\max}$ max. velocity of rotation curve

$\Longrightarrow$ Assume $M/L =$ const. (good assumption)

$\Longrightarrow$ width related to luminosity.

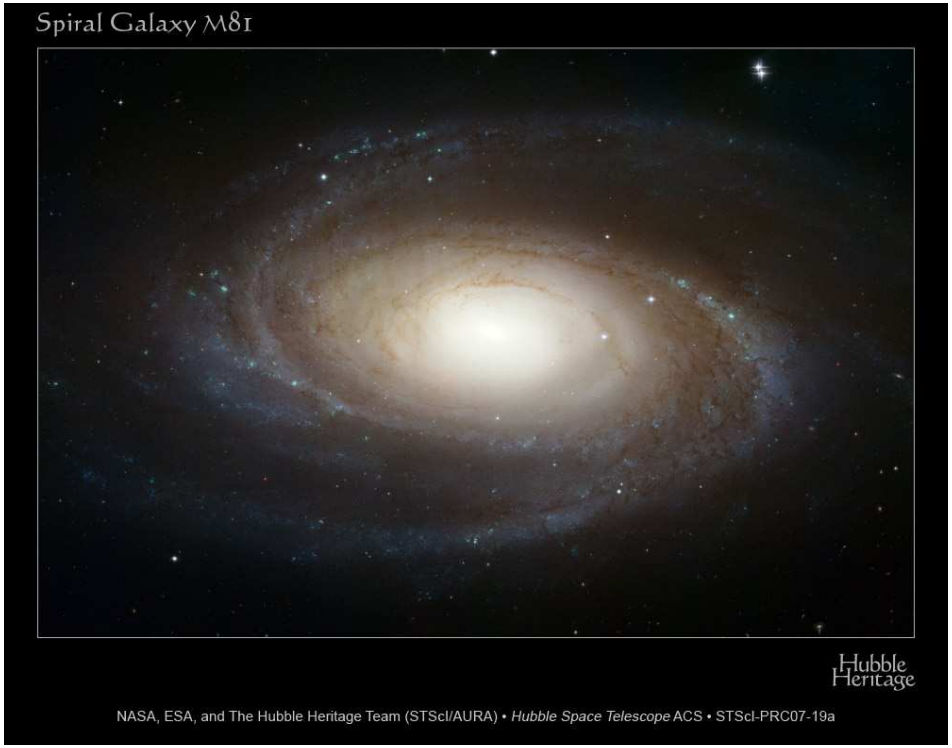
The detailed physical basis is still unknown. Might be related to galaxy formation ("hierarchical clustering", see later).

I-band is better (less internal extinction).

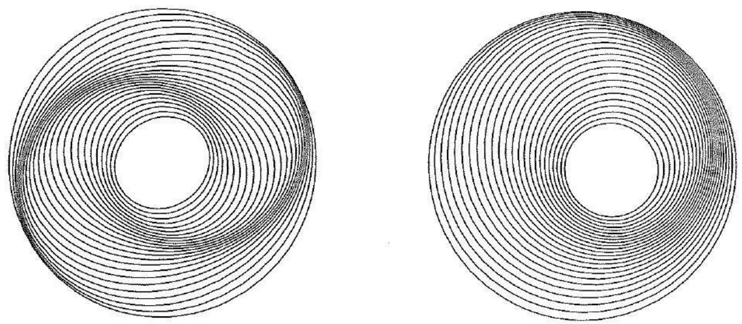
Caveats:

1. Determination of inclination i.
2. Influence of turbulent motion within galaxy.
3. Constants dependent on galaxy type (Sa and Sb similar, Sc more luminous by factor of ∼2).
4. Optical extinction.
5. Intrinsic dispersion ∼0.2 mag.
6. Barred Galaxies problematic.

M100, "grand design spiral galaxy" (NOAO)



Spiral Patterns



$R = R_g(1 + 0.075\cos(2(5 - 5R_g + \phi)))^{-1}$ and $R = R_g(1 + 0.15\cos((5 - 5R_g + \phi)))^{-1}$ (SG, Fig. 5.28)

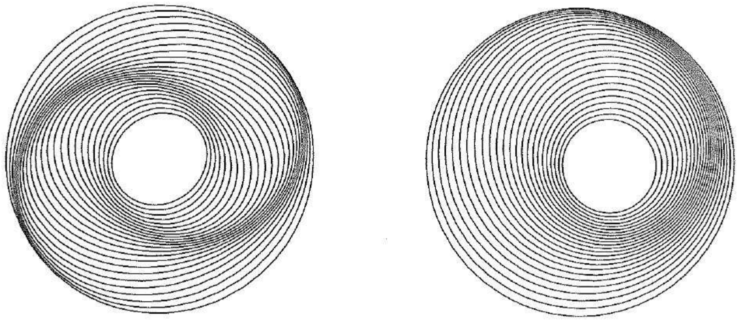
Spiral arms can be generally described via

$$\cos\left(m\left(\phi + f(R,t)\right)\right) = 1 \qquad (4.34)$$

where R, Φ: galactocentric coordinates, m: number of arms

$f(R,t)$ describes winding: $|\partial f/\partial R|$ large $\longrightarrow$ tightly wound arms.

Spiral Patterns



$R = R_g(1 + 0.075\cos(2(5 - 5R_g + \phi)))^{-1}$ and $R = R_g(1 + 0.15\cos((5 - 5R_g + \phi)))^{-1}$ (SG, Fig. 5.28)

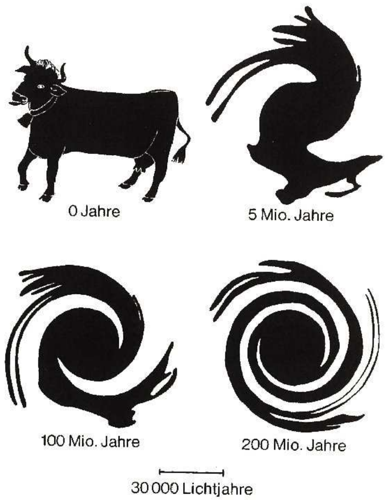
Opening angle of spiral ("pitch angle"):

$$\cot i = \frac{1}{\tan i} = \left|R\frac{\partial\phi}{\partial R}\right| = \left|R\frac{\partial f}{\partial R}\right| \qquad (4.35)$$

for Sa-galaxies: $i \sim 5^\circ$, for Sc $10^\circ < i < 30^\circ$.

Constant pitch angles lead to logarithmic spirals, $f(R,t) = \ln R + \text{const.}$.

Winding Problem



Winding Problem: differential rotation in the Galaxy would smear out any structures (=spiral arms) in a short time:

- Assume stars on straight line, i.e., $\phi = \phi_0$
- Stars move on circular orbits with $\Omega(R) = V(R)/R$, i.e. at time t: $\phi(t) = \phi_0 + \Omega(R)t$, i.e., $f(R,t) = -\phi_0 - \Omega(R)t$.
- Since $\Omega(R)$ drops: trailing spiral develops

Example for solar vicinity ($R = 8$ kpc, $V(R) \sim 200\,\text{km}\,\text{s}^{-1}$ and roughly constant):

$$\cot i = \frac{200}{8}\left(\frac{t}{1\,\text{Gyr}}\right) \Longleftrightarrow i \sim 2^\circ\left(\frac{t}{1\,\text{Gyr}}\right) \qquad (4.36)$$

$\Longrightarrow$ very fast winding up

Kippenhahn

Epicyclic Orbits

To explain spiral pattern, we need to look at motions of stars in greater detail.

Of special interest: orbits around the minimum of the effective potential,

$$\nabla\Phi_{\text{eff}} = 0 \qquad (4.37)$$

This means:

z-direction: gradient is 0 at $z = 0$

R-direction:

$$\frac{\partial\Phi_{\text{eff}}}{\partial R} = 0 \quad\Longrightarrow\quad \left.\frac{\partial\Phi}{\partial R}\right|_{R_g,z=0} = \frac{L_z^2}{R_g^3} = R_g\dot\phi^2 \qquad (4.38)$$

$\Longrightarrow$ centrifugal force!

$\Longrightarrow$ circular orbit with angular speed $\dot\phi$!

The effective potential has a minimum at the radius of the circular orbit which has the angular momentum corresponding to L_z.

Epicyclic Orbits

Now look at small deviations in motion around $R = R_g$. Introduce

$$x = R - R_g \qquad (4.39)$$

(which is a small quantity). In this case (Taylor!):

$$\Phi_{\text{eff}} \sim \frac{1}{2}\left(\kappa^2x^2 + \nu^2z^2\right) + \mathcal{O}(xz) + \dots \qquad (4.40)$$

where

$$\kappa^2 = \left.\frac{\partial^2\Phi_{\text{eff}}}{\partial R^2}\right|_{R_g,0} = \left.\frac{\partial^2\Phi}{\partial R^2}\right|_{(R_g,0)} + \frac{3L_z^2}{R_g^4} \quad\text{and}\quad \nu^2 = \left.\frac{\partial^2\Phi}{\partial z^2}\right|_{(R_g,0)} \qquad (4.41)$$

But for circular motion

$$\left.\frac{\partial\Phi}{\partial R}\right|_{(R_g,0)} = \frac{L_z^2}{R_g^3} = R_g\Omega^2(R_g) \qquad (4.42)$$

where Ω is angular speed of circular orbit at $R = R_g$, such that

$$\kappa^2 = \left(R\frac{d(\Omega^2)}{dR} + 4\Omega^2\right)_{R=R_g} \qquad (4.43)$$

Epicyclic Orbits

Now insert Φ_{Eff} into equations of motion and solve them:

- z-direction:

$$\ddot{z} = -\frac{\partial \Phi_{\text{eff}}}{\partial z} = -\nu^2 z \quad \Longrightarrow \quad z(t) = Z\cos(\nu t + \zeta) \tag{4.44}$$

- x-direction:

$$\ddot{x} = -\kappa^2 x \quad \Longrightarrow \quad x(t) = X\cos(\kappa t + \psi) \tag{4.45}$$

- ϕ-direction:

$$\dot{\phi} = \frac{L_z}{R^2} = \frac{L_z}{R_g^2}\left(1 + \frac{x}{R_g}\right)^{-1} \sim \Omega_g\left(1 - \frac{2x}{R_g}\right)$$

$$\Longrightarrow \quad \phi(t) = \Omega_g t + \phi_0 - \frac{2\Omega_g X}{\kappa R_g}\sin(\kappa t + \psi) \tag{4.46}$$

The motion of stars close to the circular orbit can be described as a circular motion plus an elliptical motion around this guiding center.

Similar to pre-Keplerian theories of planetary motion $\Longrightarrow$ "epicyclic approximation"

4–52

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