

Supernovae

Basic observations: easy:

- Detect SN in rise \Rightarrow CTIO 4 m
- Follow SN for \sim 2-3 months with 2-4 m class telescopes, HST, Keck. ...

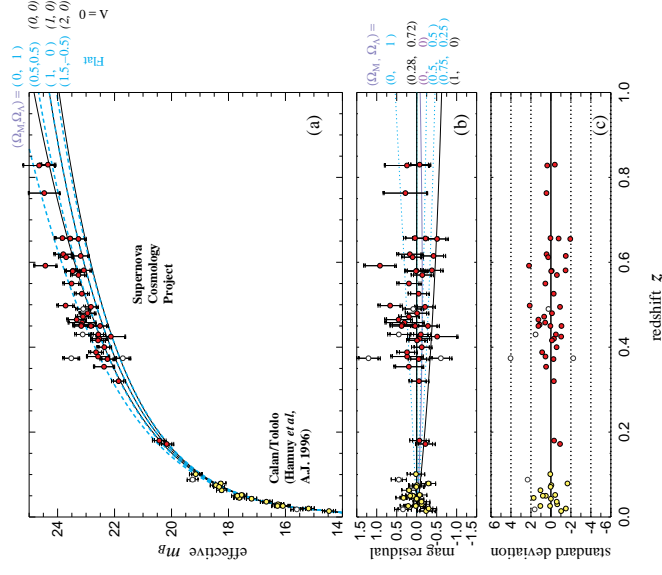
More technical problems in data analysis: Conversion into source frame:

- Correction of photometric flux for redshift: "K-correction"
- Correct for time dilatation in SN light curve

Further things to check

- SN internal extinction
- Galactic extinction
- Galactic reddening
- Photometric cross calibration
- Peculiar motion of SN

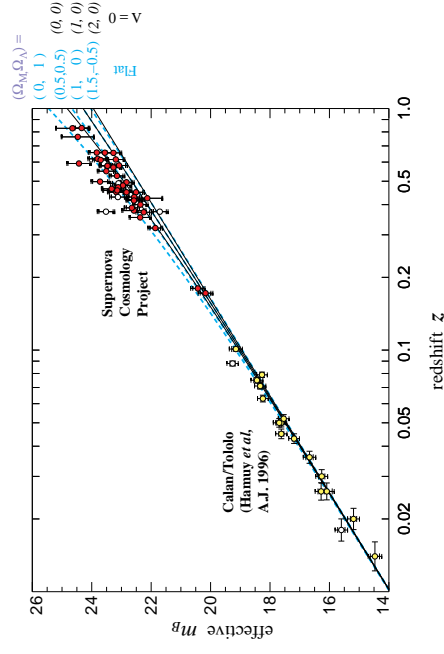
Determination of Ω_M



Best fit: $\Omega_M, \text{flat} = 0.28^{+0.09}_{-0.08}$,
 $\chi^2/\text{DOF} = 56/50$
 corresponding best free fit:
 $(\Omega_M, \Omega_\Lambda) = (0.73, 1.32)$.

(Perlmutter et al., 1999, Fig. 2)

Supernovae

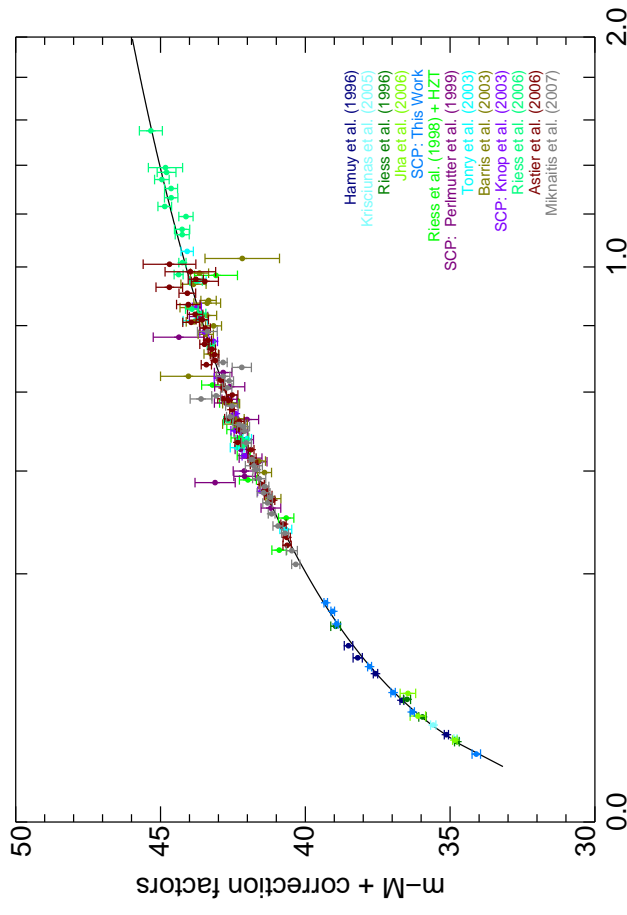


Vertical error bars:
 measurement uncertainty plus 0.17 mag intrinsic mag. dispersion
 Horizontal error bars:
 300 km s⁻¹ peculiar velocity uncertainty

(Perlmutter et al., 1999, Fig. 1)

42 SNe from SCP, 18 low redshift from Calán/Tololo SN Survey

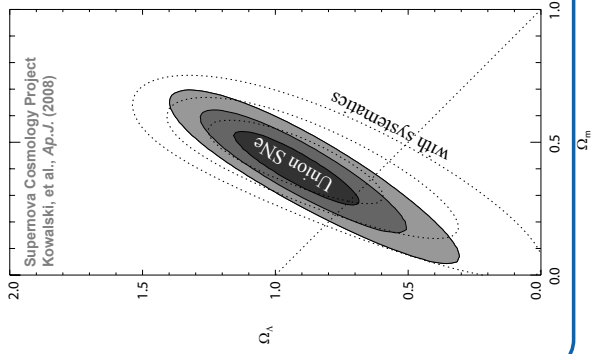
Determination of Ω_M



Kowalski et al. (2008): Union2 compilation of 414 SNe (307 usable)



Supernovae



Best fit to SN data to an $\Omega_c\text{-}\Lambda$ -universe *without* any prior assumptions gives results consistent with a flat universe.

Assuming a flat universe:

$$\Omega_\Lambda = 0.713^{+0.026}_{-0.029} (\text{stat})^{+0.036}_{-0.039} (\text{syst}) \quad (12.63)$$

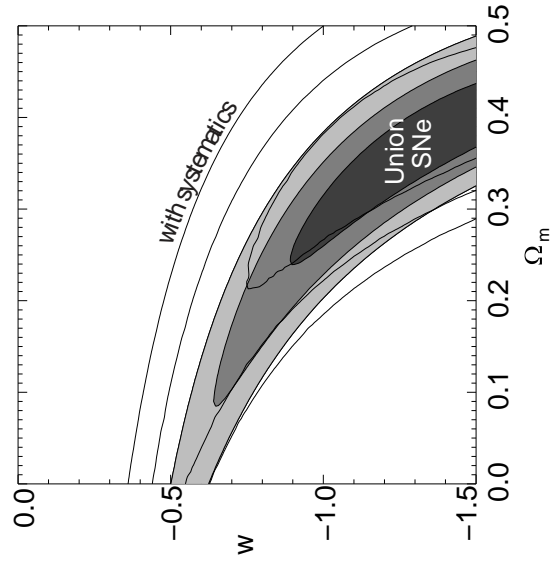
(Kowalski et al., 2008)

Determination of Ω_Λ

8



Supernovae



Data are now good enough to measure "equation of state" of whatever is Λ .

One possible model to solve the cosmological constant problem (=explain what Λ is) is quintessence or dark energy, with the equation of state

$$p_q = w \rho_q c^2 \quad (12.64)$$

where $w = -1$ for Einstein's Λ .

Current data are consistent with $w = -1$.

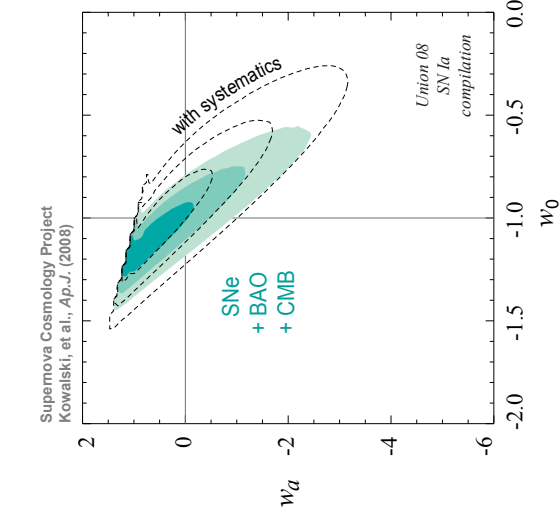
(Kowalski et al., 2008)

Determination of Ω_Λ

9



Supernovae



Physical models for quintessence are derived from string theory, M-theory, and other alternative gravities, which predict w to be time dependent (Linder & Miquel, 2004):

$$w(a) = w_0 + w_a(1 - a) \quad (12.65)$$

where $a = R/R_0$.

Note that since $a = (1 + z)^{-1}$ this implies

$$w(a) = w_0 + w_a \frac{z}{1 - z} \quad (12.66)$$

Current data are still consistent with $w_0 = -1$ and $w_a = 0$.

A better determination of w_a will be possible in the next years.

(Kowalski et al., 2008)

Determination of Ω_Λ

10



Summary

For all practical purposes, currently the best values of Ω_m and Ω_Λ are

$$\Omega_m \sim 0.3 \quad \text{and} \quad \Omega_\Lambda = 0.7$$

Even if $\Omega \neq 1$:

$$\Omega_\Lambda \neq 0$$

And therefore

Baryons are an energetically unimportant constituent of the universe.

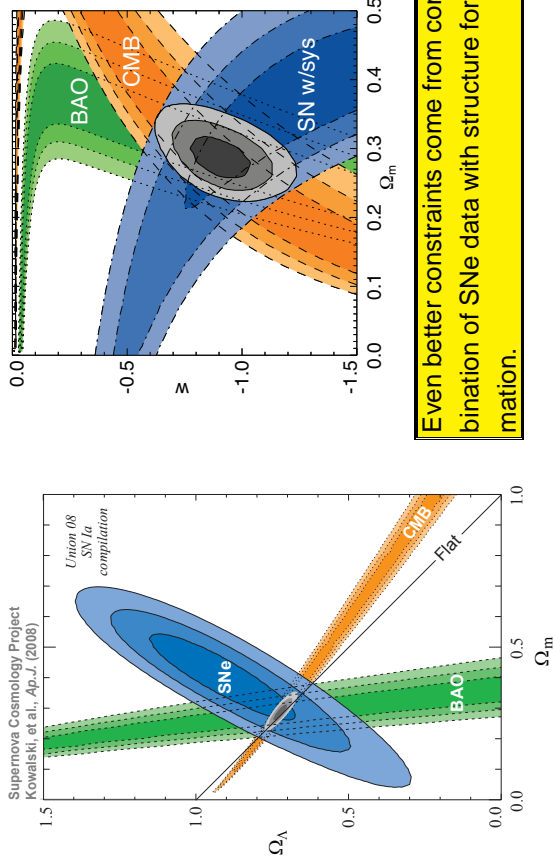
"The dark side of the force..." :-)

Summary

1



Outlook



Summary

12-54

- Alcock, C., et al., 2001, *ApJ*, 552, 259
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- Carroll, S. M., 2001, *Living Rev. Rel.*, 4, 2001
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- Peacock, J. A., 1999, *Cosmological Physics*, (Cambridge: Cambridge Univ. Press)
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- Riess, A. G., et al., 1998, *ApJ*, 116, 1009
- Stiell, M., 2009, in *Proc. Heavy Quarks and Leptons*, (arXiv:0906.0464)
- Turner, M. S., 1999, in *The Third Stromlo Symposium: The Galactic Halo*, ed. B. K. Gibson, T. S. Axelrod, M. E. P. Tumann, ASP, in press (astro-ph/9811454)
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- Wise, M. W., McNamara, B. R., & Murray, S. S., 2004, *ApJ*, 601, 184

2



Large Scale Structures and Structure Formation



The Lumpy Universe

So far: treated universe as smooth universe.

In reality:

Universe contains structures!

Last part of this class:

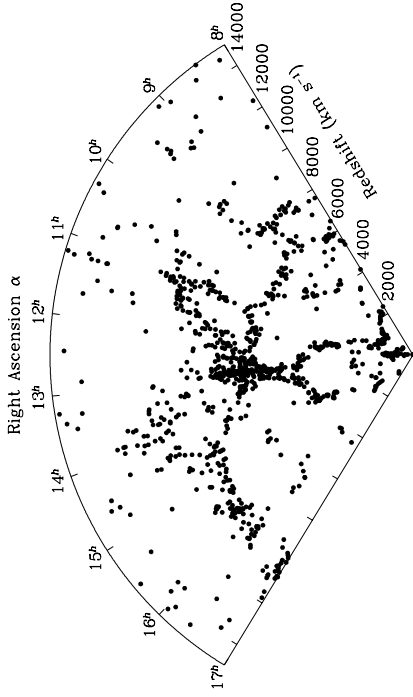
1. What are structures?
2. How can we quantify them?
3. How do structures form?
4. How do structures evolve?

Will see that all these questions are deeply connected with parameters of the universe seen so far:

1. H_0
2. $\Omega_0, \Omega_b, \Omega_m, \Omega_\Lambda, \dots$
3. Existence and Nature of Dark Matter

The Lumpy Universe

Introduction, I



$$26.5^\circ < \delta < 32.5^\circ$$

(de Lapparent, Geller & Huchra, 1986, limiting mag $m_B = 15.6$)

Lumpy universe: spatial distribution of galaxies and greater structures.

Redshift Surveys

Introduction, II

How do we study the structure of the Universe?

- ⇒ We need distance information for many ($10^4 \dots 10^7$) objects
 - ⇒ Large redshift surveys
- Review: Strauss & Willick (1995)

Redshift survey: Survey of (patch of) sky determining galaxy z and position to predefined magnitude or z .

First larger survey: de Lapparent, Geller & Huchra (1986)

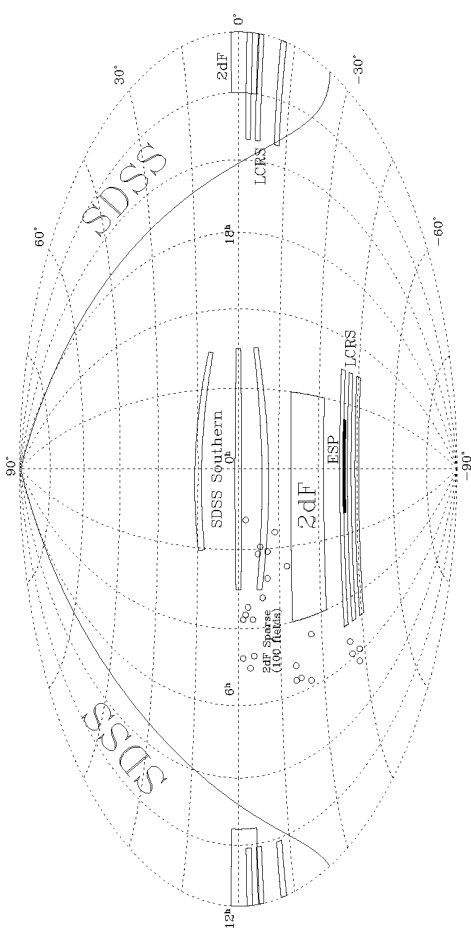
Classification:

- 1D-surveys:** very deep exposures of small patch of sky, e.g., HST Deep Field, Lockman Hole Survey, COSMOS Field, Marano Field.
 - 2D-surveys:** cover long strip of sky, e.g., CfA-Survey ($1.5 \times 100^\circ$), 2dF-Survey ("2 degree Field").
 - 3D-surveys:** cover part of the sky, e.g., Sloan Digital Sky Survey.
- These surveys attempt to go to certain limit in z or m .

Other approaches: use pre-existing galaxy catalogues (e.g., QDOT Survey [IRAS galaxies], APM survey, ...).

We will concentrate here on the larger surveys based on no other catalogue.
Redshift Surveys

Introduction, III

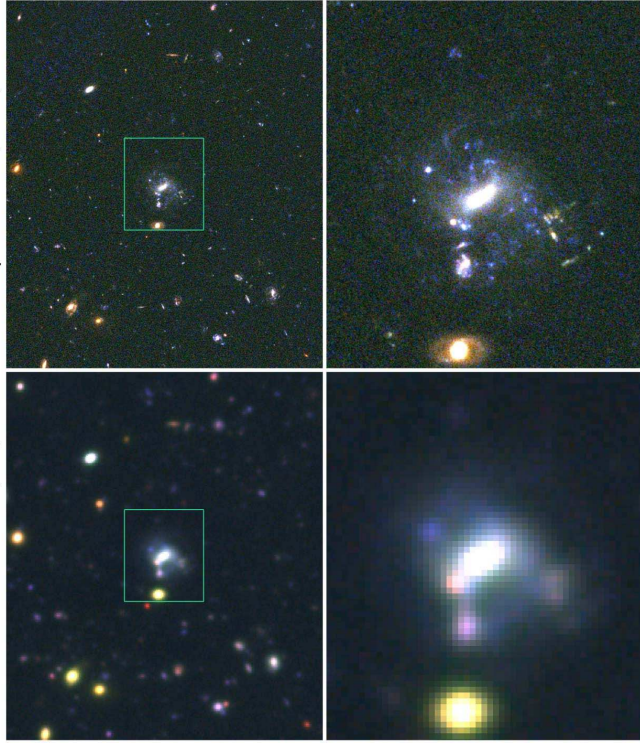


(Strauss, 1999)

Redshift Surveys

Ground: Subaru (8m)

Space: HST (2.4m)



To go deep one needs to go to space



STScl



13-8

Hubble Space Telescope, II

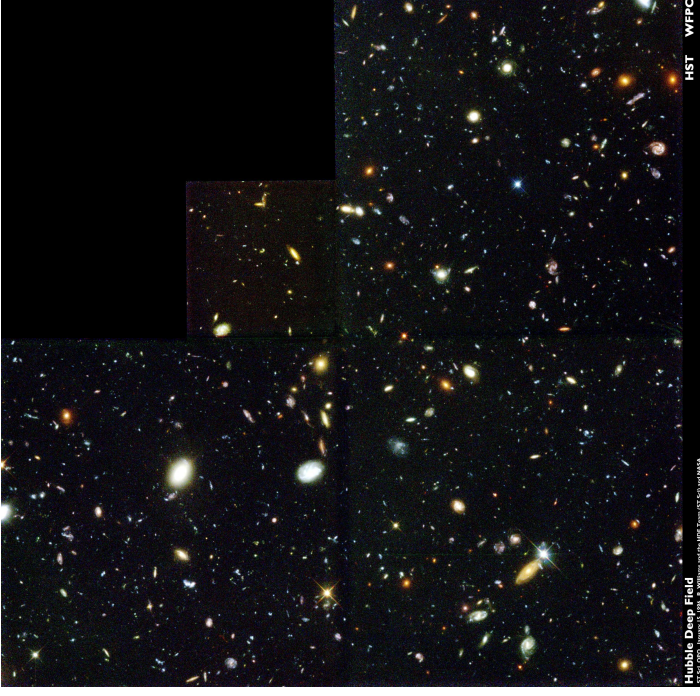
The Hubble Space Telescope has a large set of instruments well suited for cosmological observations:

Current HST Instruments :

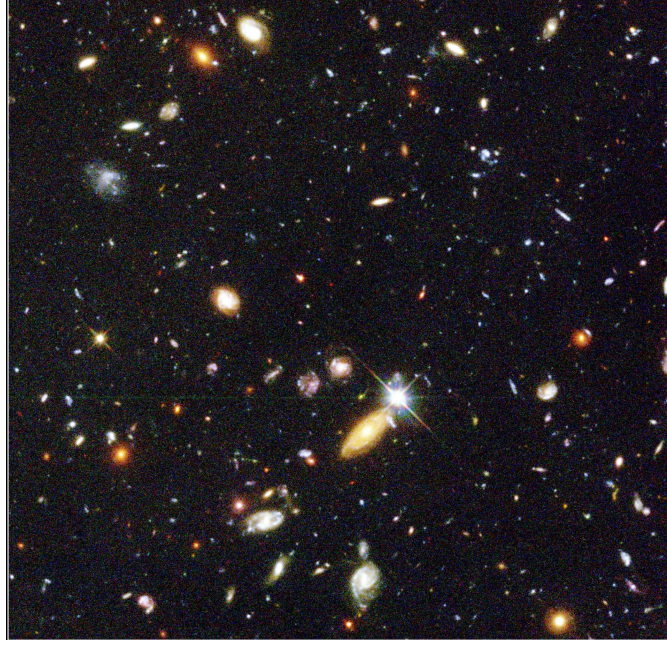
- WFC3: Wide Field Camera 3 (05.2009-)
- COS: Cosmic Origins Spectrograph (05.2009-)
- ACS: Advanced Camera for Surveys (03.2002-)
- STIS: Space Telescope Imaging Spectrograph (02.1997-)
- NICMOS: Near Infrared Camera and Multi Object Spectrometer (02.1997-)
- FGS: Fine Guidance Sensors

Former Generation Instruments :

- FOC: The Faint Object Camera (04.1990-03.2002)
- FOS: The Faint Object Spectrograph (04.1990-02.1997)
- GHRS: The Goddard High Resolution Spectrograph (04.1990-02.1997)
- HSP: The High Speed Photometer (04.1990-10.1993)
- WF/PC-1: Wide Field Planetary Camera 1 (04.1990-10.1993)
- WFPC2 The Wide Field Planetary Camera 2 (12.1993-05.2009)

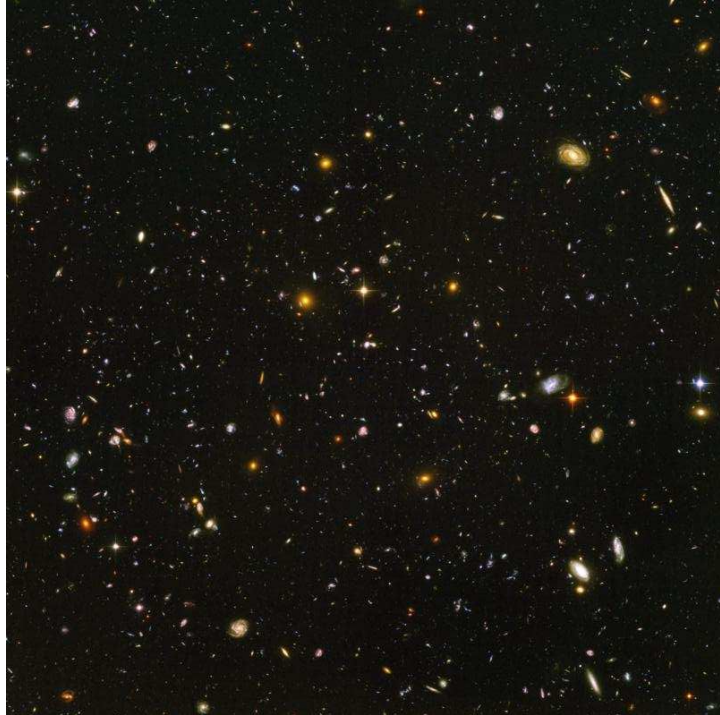
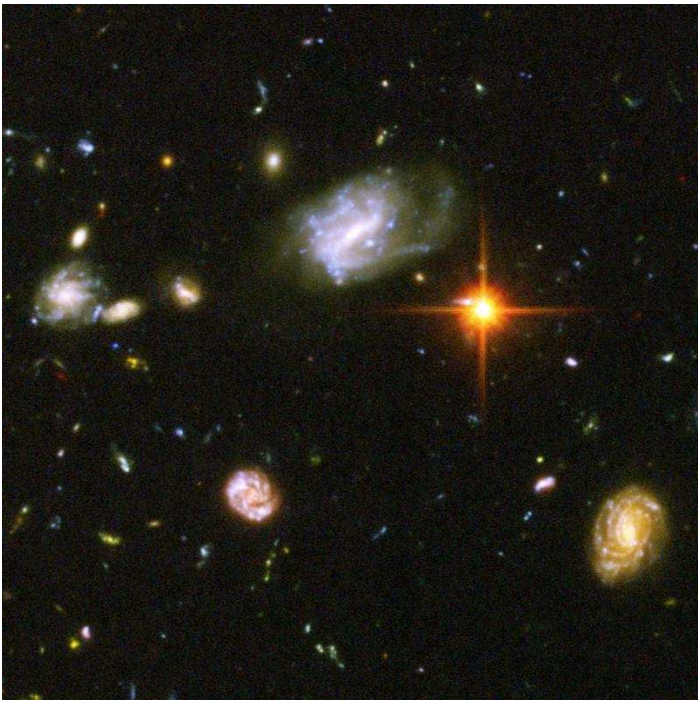


1995 December: Hubble Deep Field: ~150 ksec/Filter for four HST Filters
 Many galaxies with weird shapes \implies protogalaxies!
 Redshifts: $z \in [0.5, 5.3]$
 (Fernández-Soto et al., 1999)



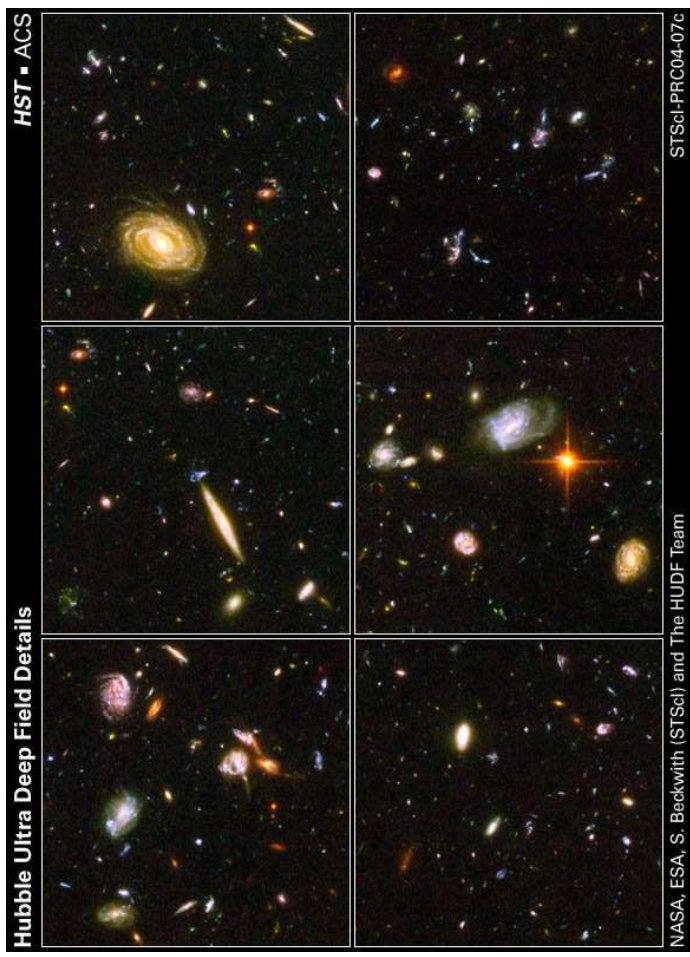


1998: Hubble Deep Field South, 10d of total observing time!



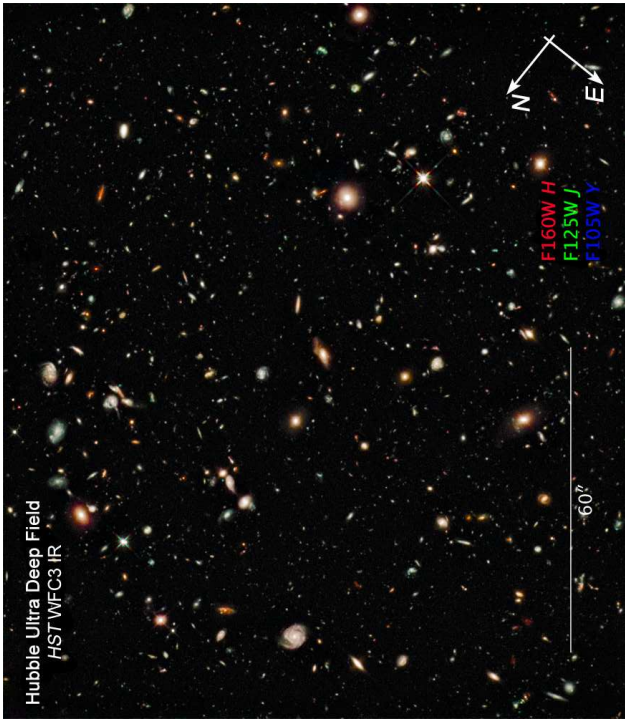
2004: Hubble Ultra Deep Field, 1 Msec long exposure of field in Fornax. Uses updated HST with Advanced Camera for Surveys (ACS) and Near Infrared Camera and Multi-Object Spectrometer (NICMOS); diameter: 3' ($2 \times$ HDF) Limiting magnitude: 30 mag, ~ 10000 galaxies visible, up to $z \gtrsim 7$ IR reveals many red- dened objects

Hubble Ultra Deep Field Details

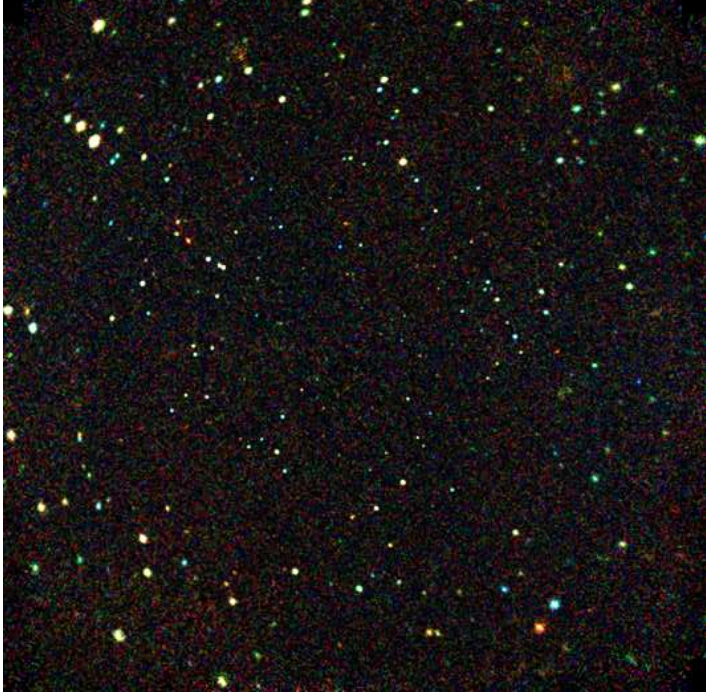


NASA, ESA, S. Beckwith (STScI) and The HUDF Team

STScI-PRC04-07C

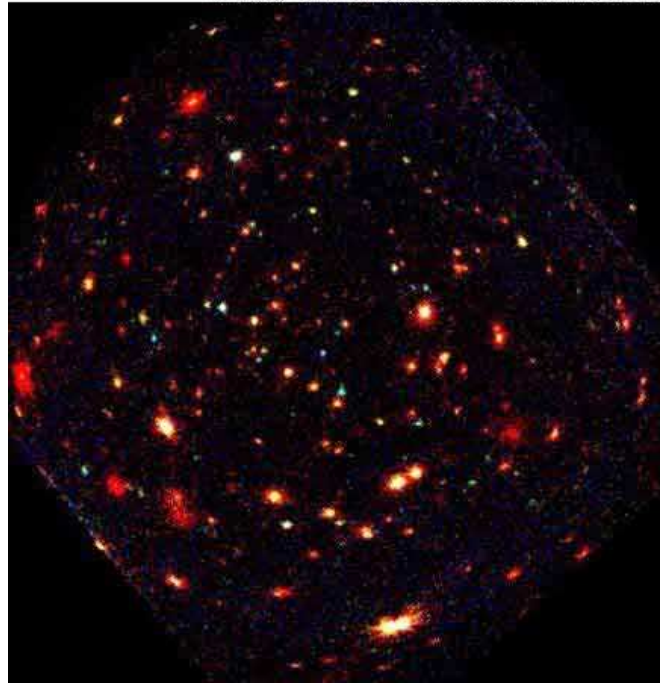


2009 August 26 – 2009 September 2009: WFC3 pushes HUDF even deeper (data taken in same region as HUDF, "Hubble Ultra Deep Field Infrared"). Exposure: 48 h



Chandra Deep Field South: 1 Msec (10.8 days) on one region in Fornax ($\alpha_{J2000.0} = 3^{\text{h}}32^{\text{m}}28.0^{\text{s}}$, $\delta_{J2000.0} = -27^{\circ}48'30''$, coaligned with HDF-S
Deepest X-ray field ever
color code: spectral hardness

scale: $15' \times 15'$; courtesy NASA/JHU/AUI/R.Giacconi et al.

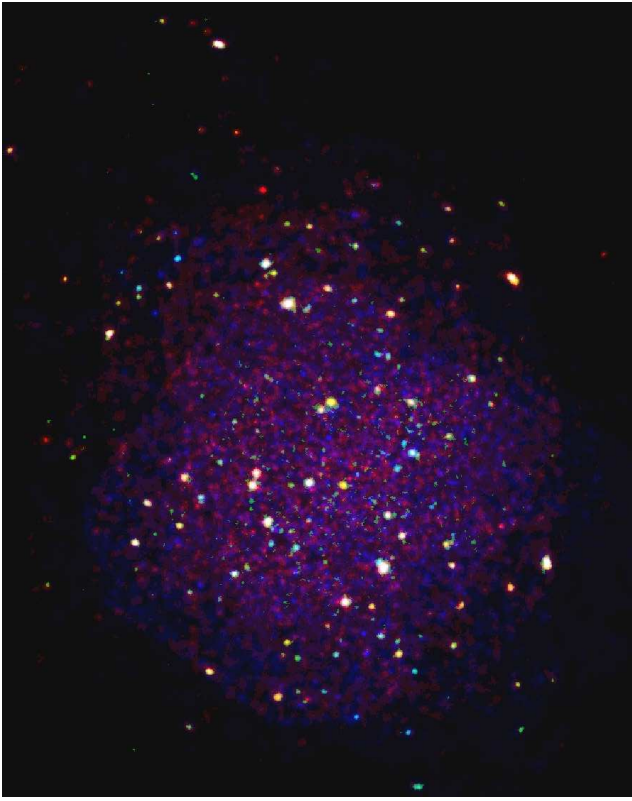


COSMOS-field: large X-ray survey for Galaxy Clusters

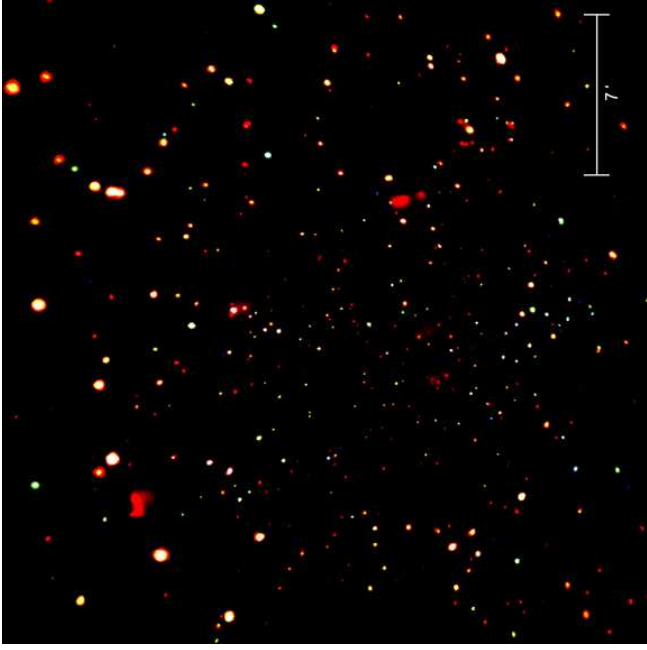
Lockman Hole: Northern Sky region with very low N_{H} \implies low interstellar absorption \implies "Window in the sky" \implies X-rays: evolution of active galaxies with z !

XMM-Newton, Hasinger et al., 2001,
blue: hard X-ray spectrum,
red: soft X-ray spectrum

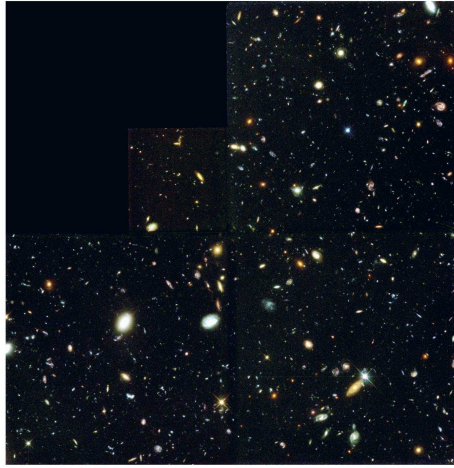
GUNTHER HASINGER/ASTROPHYSIS INSTITUT FÜR POTSDAM



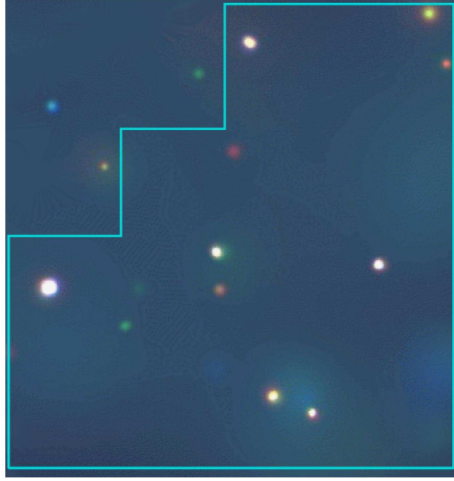
Deep XMM-Newton image of the Marano Field (IAAT/AIP/MPE)



⇒ GOODS-Survey (Great Observatories Origins Deep Survey), centered on CDF-S (same image as before, this time smoothed)



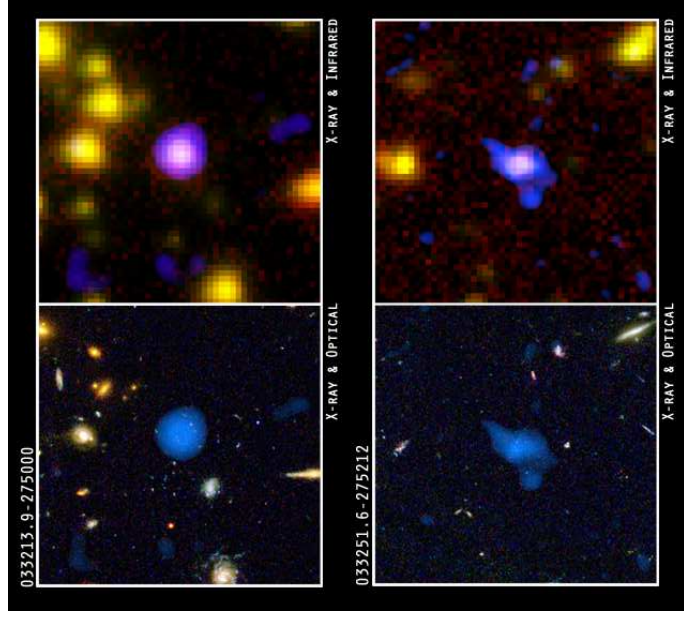
HST



Chandra

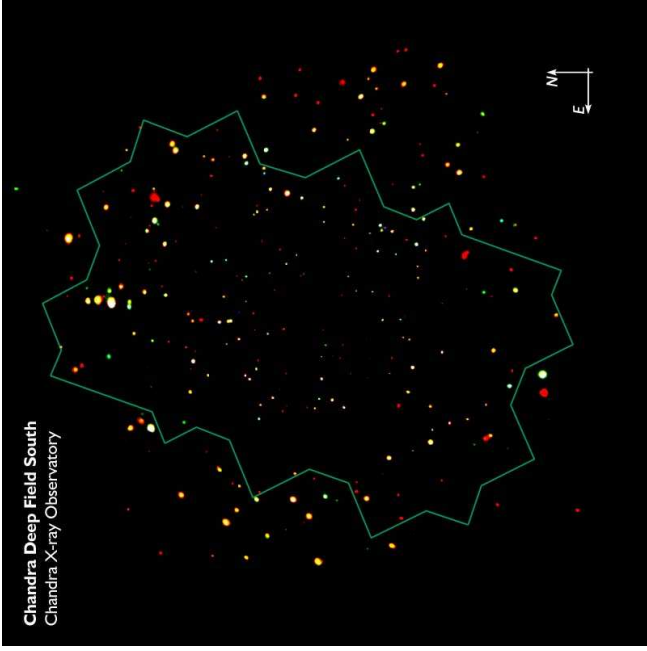
Chandra/HST Image of Hubble Deep Field North; 500 ksec

Joint multi-wavelength campaigns allow the measurement of broad-band spectra of sources in the early universe!

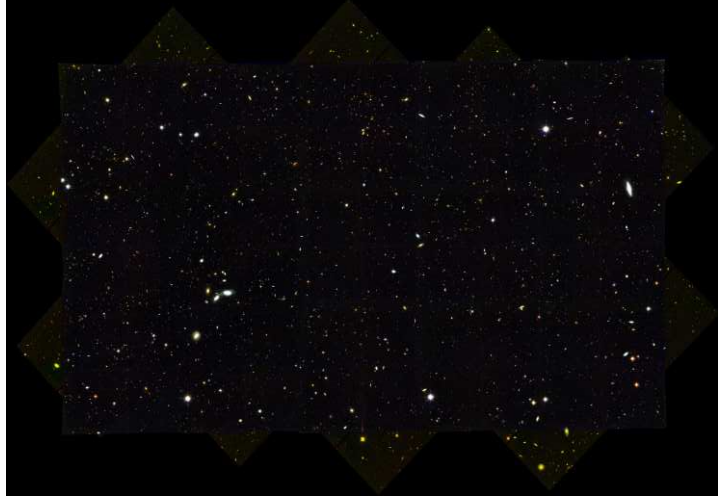


IR, optical, and X-ray image of small fraction of GOODS

CXC/NASA

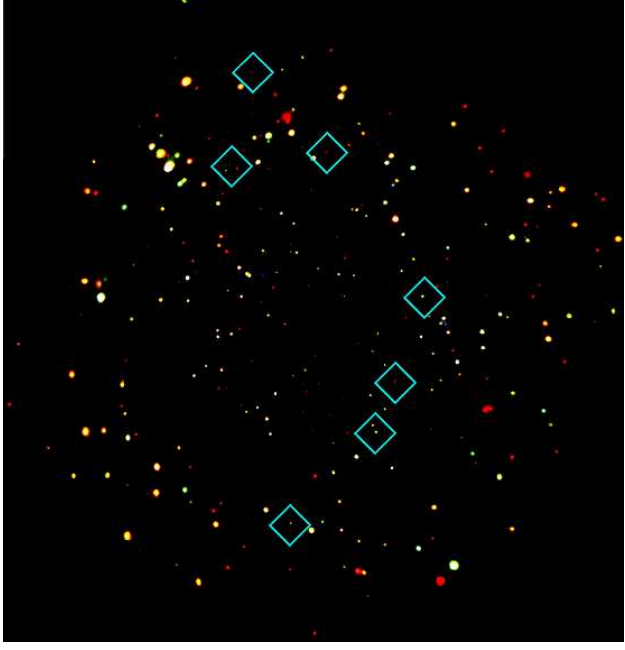


Chandra and HST fields aligned

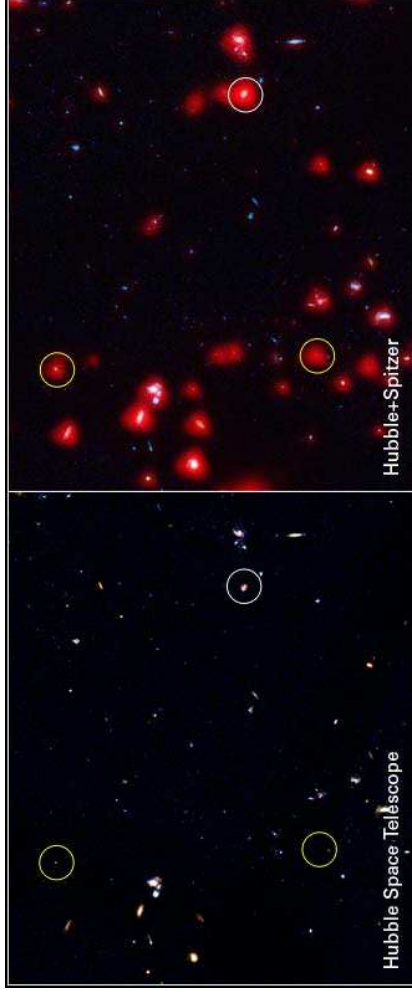


HST ACS observations of whole area of CDF-S

CXC



CDFS: blue boxes contain objects not visible in HST
⇒ farthest black holes known



1/200th of the whole GOODS field in optical and IR

2D/3D Surveys: Technology

Future for Large Scale Structure: 2D and 3D Surveys observing large part of sky with dedicated instruments.

Currently largest surveys:

Las Campanas Redshift Survey (LCRS): 26418 redshifts in six $1.5 \times 80^\circ$ slices around NGP and SGP, out to $z = 0.2$.

CfA Redshift Survey: 30000 galaxies

APM: (Oxford University) $2 \sim 10^6$ galaxies, 10^7 stars around SGP, 10% of sky, through $B = 21$ mag.

2MASS: IR Survey of complete sky (Mt. Hopkins/CTIO) completed 2000 October 25), 3 bands, $\sim 2 \times 10^6$ galaxies, accompanying redshift survey (8dF, CfA)

Sloan Digital Sky Survey (SDSS): dedicated 2000 October 5, Apache Point Obs., NM, 25% of whole sky, $\sim 10^8$ objects, now in Google Earth

And many more (e.g., Keck, ESO, LSST, ...).

Redshift Surveys

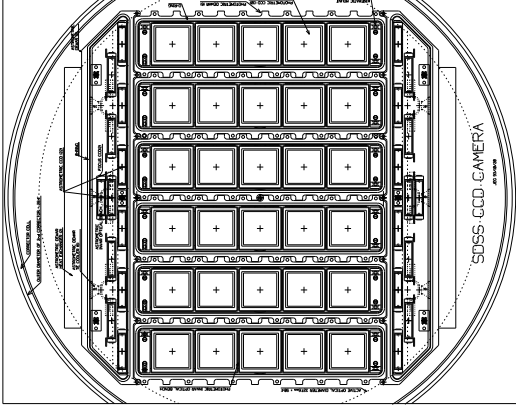
25



SDSS 2.5 m telescope at Apache Point Observatory

courtesy SDSS

2D/3D Surveys: Technology



CCD alignment of SDSS:

- focal plane: 2.5° ,
- 5 rows of 2048×2048 CCDs with $r, i,$
 u, z, g filters, saturation at $r = 14$
- 22 2048×400 CCD, saturation at $r = 6.6$ for astrometry

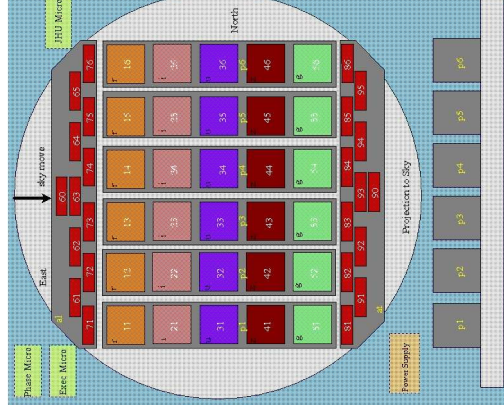
Imaging by slewing over CCD Array

(Strauss, 1999, Fig. 5)

Redshift Surveys

27

2D/3D Surveys: Technology



CCD alignment of SDSS:

- focal plane: 2.5° ,
- 5 rows of 2048×2048 CCDs with $r, i,$
 u, z, g filters, saturation at $r = 14$
- 22 2048×400 CCD, saturation at $r = 6.6$ for astrometry

Imaging by slewing over CCD Array

SDSS

Redshift Surveys

28



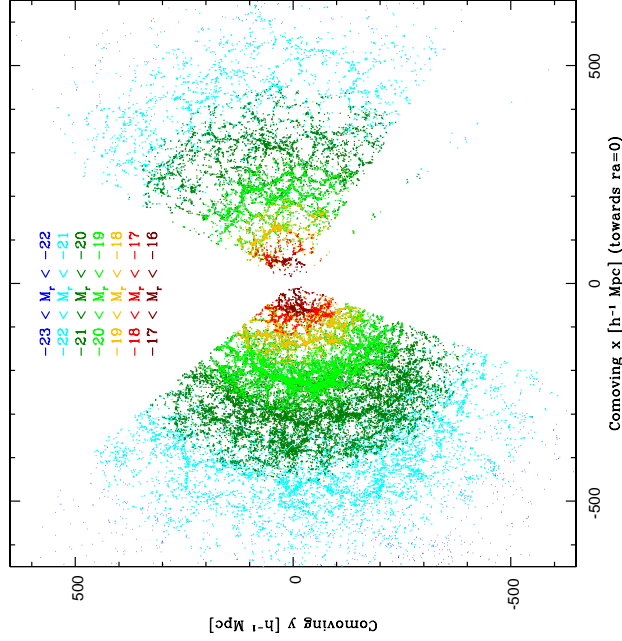
2D/3D Surveys: Technology



courtesy SDSS

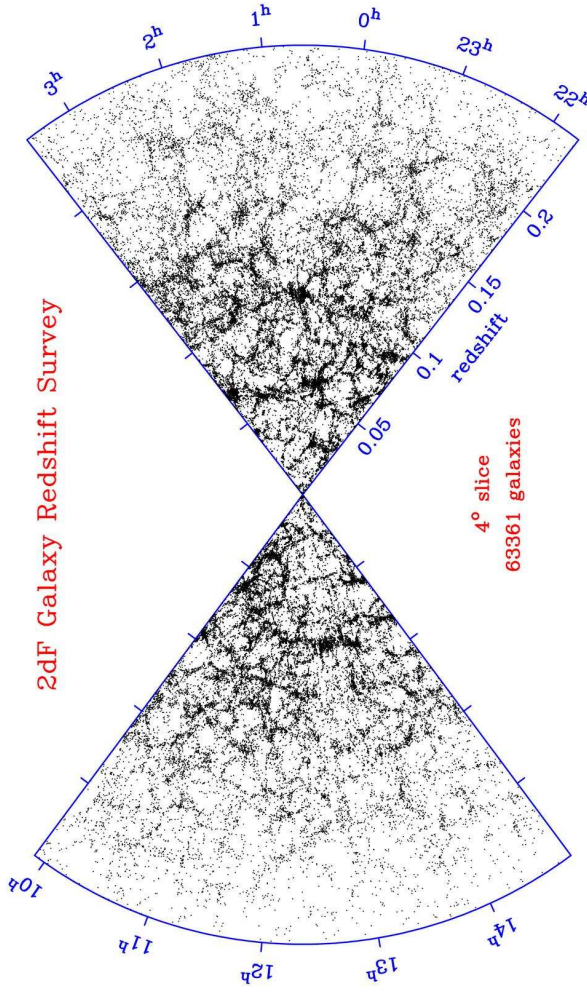
Spectroscopy with grism (combination of prism and grating), light from objects via optical fibers and plug plate.

Redshift Surveys

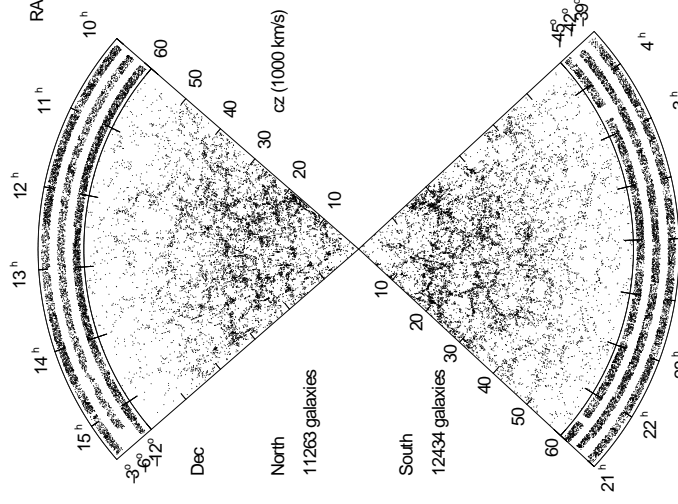


Galaxy distribution from the SDSS

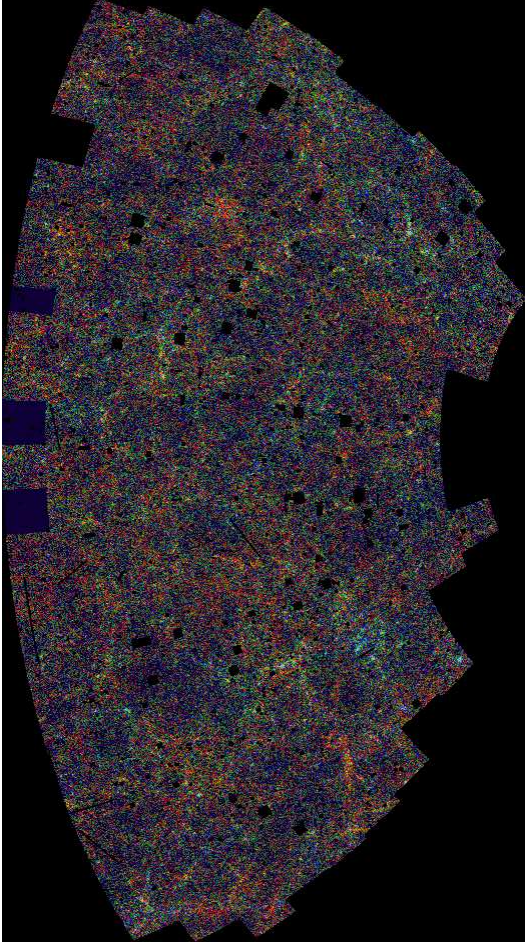
(Tegmark et al., 2004, Fig. 4)



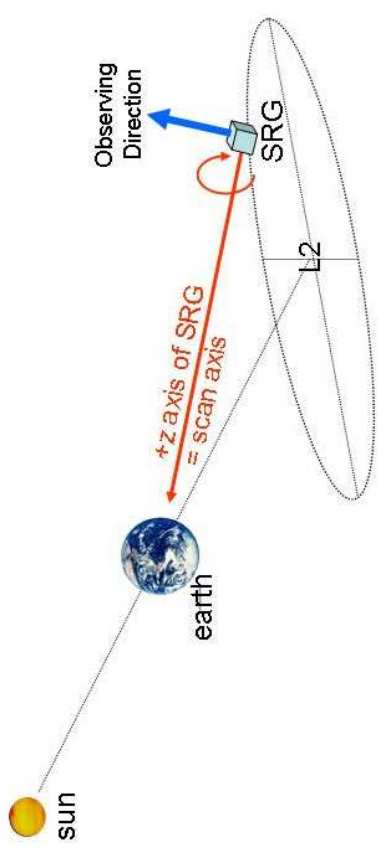
courtesy 2dF collaboration



The complete LCRS survey (at cz large: reach mag. limit)

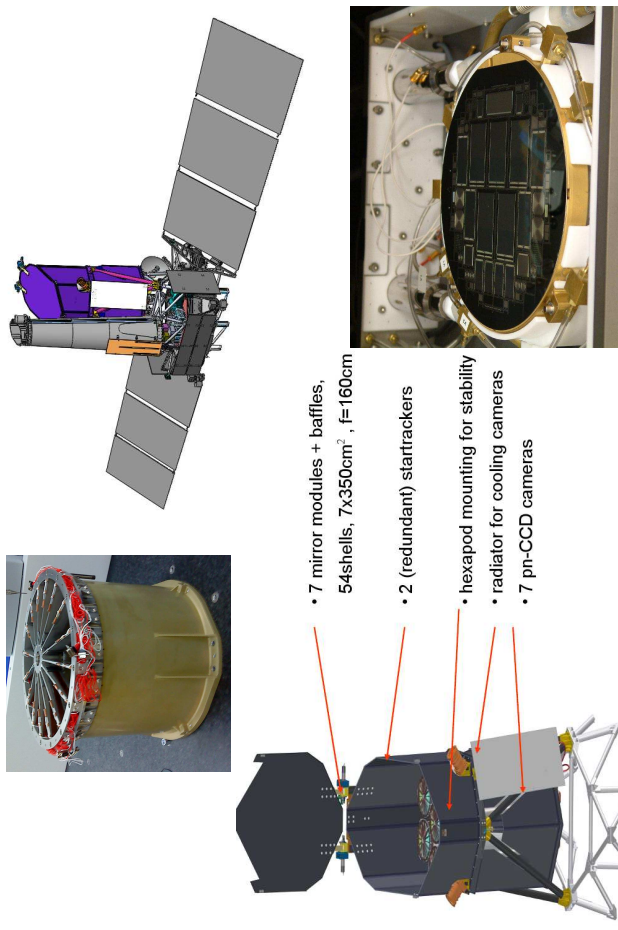
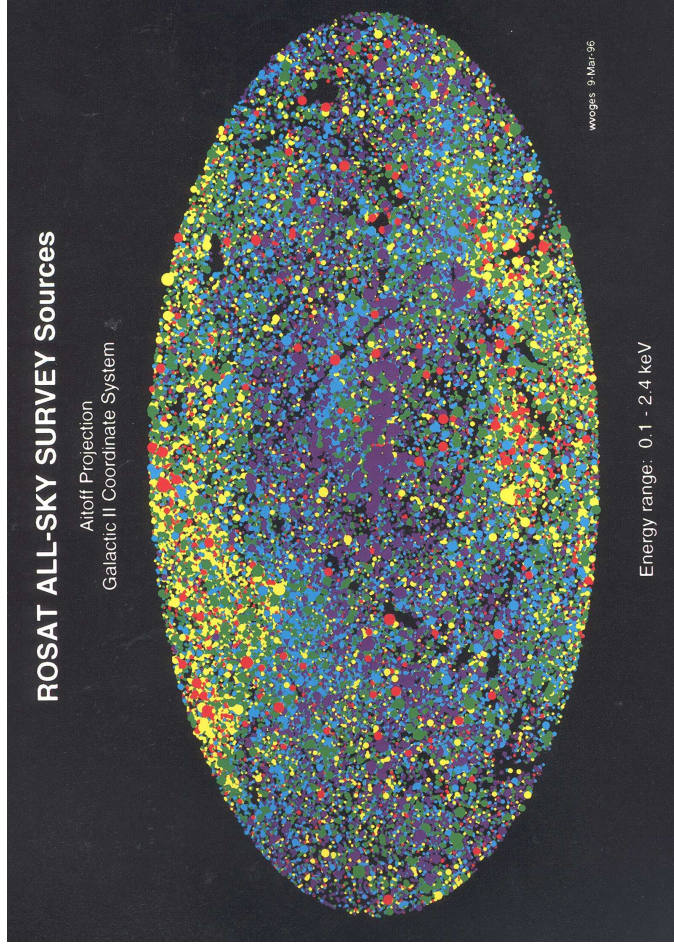


Galaxies in APM catalogue, color: avg. B in pixel: blue (18) – green (19) – red (20)



eROSITA (extended ROentgen Survey with an Imaging Telescope Array) on Spectrum-XG

- Launch: Fall 2012, 4 year survey + pointed phase.
- Collaboration: Germany (MPE, IAT, AIP, FAU, Hamburg)+Russia
- Extends ROSAT survey to 10 keV, 30× deeper
- 3000 000 supermassive black holes
- 100 000 galaxy clusters $\Rightarrow \Delta, w$



Correlation Function, I

Sky surveys show:

Galaxies are *not* evenly distributed: "cosmic web"!

- Structures at scales up to several 10 Mpc
 - But: Over-density even in clusters not too dramatic ($\sim 100\times$ denser than average)
 - Voids on scales $50 h^{-1}$ Mpc
- \implies Need quantitative description of structures.
- \implies Need physical explanation of structures.
- \implies Need to understand what we see (do galaxies trace matter distribution??).

Structures: Quantitative Description

1

Correlation Function, II

Mathematical description of clustering: Correlation function!

Assume *uniform* distribution of galaxies with galaxy density n galaxies Mpc^{-3} .

Probability to find galaxy in volume ΔV :

$$P \propto n \Delta V \quad (13.1)$$

Probability to find galaxies in *two* volumes 1 and 2:

$$P = P_1 \cdot P_2 \propto n^2 \Delta V_1 \Delta V_2 \quad (13.2)$$

Universe inhomogeneous: measure (distance dependent) deviation from mean:

$$P \propto n^2 (1 + \xi(r_{12})) \Delta V_1 \Delta V_2 \quad (13.3)$$

$\xi(r_{12})$ is called the two-point spatial correlation function.

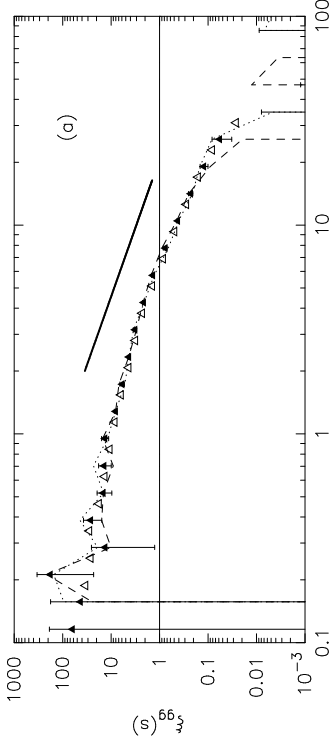
For *small* r :

$$\xi(r) > 0 \implies \text{clustering}$$

Structures: Quantitative Description

2

Correlation Function, III



s [h^{-1} Mpc]

(galaxy-galaxy correlation function from the Las Campanas Redshift Survey; Tucker et al., 1997, Fig. 1)

Rough description: power law

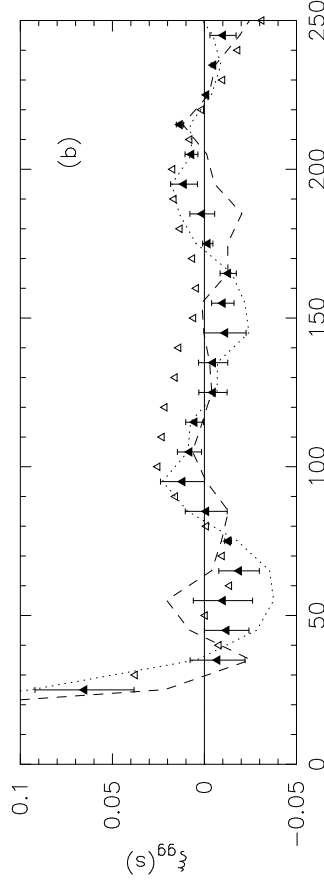
$$\xi(r) = (r/r_0)^{-\gamma} \quad (13.4)$$

where $r_0 \sim 6 h^{-1}$ Mpc (correlation length), and $\gamma \sim 1.5 \dots 1.8$.

Structures: Quantitative Description

3

Correlation Function, IV



s [h^{-1} Mpc]

(galaxy-galaxy correlation function from the Las Campanas Redshift Survey; Tucker et al., 1997, Fig. 1)

Rough description: power law

$$\xi(r) = (r/r_0)^{-\gamma} \quad (13.5)$$

where $r_0 \sim 6 h^{-1}$ Mpc (correlation length), and $\gamma \sim 1.5 \dots 1.8$.

Above $r = 30 h^{-1}$ Mpc: oscillation due to voids.

Structures: Quantitative Description

4



Correlation Function, V

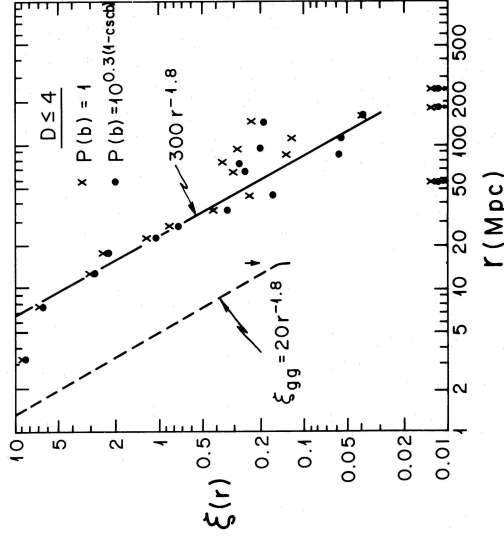
The spatial correlation function for rich (Abell) galaxy clusters is similar to the one for galaxies,

$$\xi_{\text{clusters}}(r) = 360(rh)^{-1.8} \quad (13.6)$$

for $r \leq 150h^{-1}$ Mpc.

For $r > 150h^{-1}$ Mpc no correlations are observed.

Note that richer clusters show stronger correlations.



(Bahcall & Soneira, 1983, Fig. 9a)

Structures: Quantitative Description

5



Correlation Function, VI

ξ is related to the density contrast $\Delta(x)$:

Write the density n as

$$n(\mathbf{x}) = n_0(1 + \Delta(\mathbf{x})) \iff \Delta(\mathbf{x}) = \delta n/n \quad (13.7)$$

Average joint probability to have galaxies at \mathbf{x} and $\mathbf{x} + \mathbf{r}$:

$$P = \langle n(\mathbf{x})dV_1 \cdot n(\mathbf{x} + \mathbf{r})dV_2 \rangle \quad (13.8)$$

$$= \langle n_0^2(1 + \Delta(\mathbf{x}))(1 + \Delta(\mathbf{x} + \mathbf{r})) \rangle dV_1dV_2 \quad (13.9)$$

Since $\langle \Delta \rangle = 0$, only the cross product survives the averaging:

$$= n_0^2 \langle 1 + \langle \Delta(\mathbf{x})\Delta(\mathbf{x} + \mathbf{r}) \rangle \rangle dV_1dV_2 \quad (13.10)$$

where $\langle \dots \rangle$ denotes averaging over an appropriate volume, i.e.,

$$\langle f(\mathbf{r}) \rangle = \frac{1}{V} \int_V f(\mathbf{r}) d^3r \quad (13.11)$$

Structures: Quantitative Description

6



Correlation Function, VII

The two-point correlation function was defined via

$$P \propto n^2(1 + \xi(r_{12}))\Delta V_1\Delta V_2 \quad (13.3)$$

while we just found

$$P = n_0^2(1 + \langle \Delta(\mathbf{x})\Delta(\mathbf{x} + \mathbf{r}) \rangle) dV_1dV_2 \quad (13.10)$$

Comparing these two equations shows:

$$\xi(r) = \langle \Delta(\mathbf{x})\Delta(\mathbf{x} + \mathbf{r}) \rangle \quad (13.12)$$

$\xi(r)$ is a measure for the average density contrast at places separated by distance r .

Structures: Quantitative Description

7



Power Spectrum, I

To describe the strength of fluctuations, a good measure is the variance of the density fluctuations.

Let's calculate this around an arbitrary location, $\mathbf{x} = 0$.

$$\text{Var}(n) = \frac{1}{V} \int (n(\mathbf{r}) - \langle n \rangle)^2 dV \quad (13.13)$$

$$= \frac{1}{V} \int (n_0(1 + \Delta(\mathbf{r})) - n_0)^2 dV \quad (13.14)$$

$$= \frac{1}{V} \int \Delta(\mathbf{r})^2 dV \quad (13.15)$$

To evaluate the integral, it is more convenient for most calculations to work in Fourier space than in "normal" space.

Define the Fourier transform in spatial coordinates through:

$$\Delta_r(\mathbf{r}) = \frac{V}{(2\pi)^3} \int \Delta_k(\mathbf{k}) \exp(-i\mathbf{k} \cdot \mathbf{r}) d^3k \quad (13.16)$$

$$\Delta_k(\mathbf{k}) = \frac{1}{V} \int \Delta(\mathbf{r}) \exp(+i\mathbf{k} \cdot \mathbf{r}) d^3r \quad (13.17)$$

Structures: Quantitative Description

8



Power Spectrum, II

Therefore

$$\text{Var}(n) = \frac{1}{V} \int \Delta_r(\mathbf{r})^2 dV \quad (13.18)$$

$$= \frac{1}{V} \int \left(\frac{V}{(2\pi)^3} \int \Delta_k \exp(-i\mathbf{k} \cdot \mathbf{r}) d^3k \right)^2 dV \quad (13.19)$$

$$= \frac{V}{(2\pi)^6} \int \left(\int \Delta_k \exp(-i\mathbf{k} \cdot \mathbf{r}) d^3k \right) \left(\int \Delta_{k'} \exp(-i\mathbf{k}' \cdot \mathbf{r}) d^3k' \right) dV \quad (13.20)$$

$$= \frac{V}{(2\pi)^6} \int \int \Delta_k \Delta_{k'} \exp(-i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{r}) d^3k d^3k' d^3V \quad (13.21)$$

$$= \frac{V}{(2\pi)^6} \int \int \Delta_k \Delta_{k'} (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') d^3k d^3k' \quad (13.22)$$

$$= \frac{V}{(2\pi)^3} \int \Delta_k^2 d^3k \quad (13.23)$$

This result is called Parseval's theorem

$$\frac{1}{V} \int \Delta_r^2(\mathbf{r}) d^3x = \frac{V}{(2\pi)^3} \int \Delta_k^2(\mathbf{k}) d^3k \quad (13.24)$$

(from signal theory: the power in a time series is the same as the power in the associated Fourier transform)

Structures: Quantitative Description

9



Power Spectrum, III

Therefore

$$\text{Var}(n) = \langle \Delta^2 \rangle = \frac{V}{(2\pi)^3} \int \Delta_k^2 d^3k = \frac{V}{(2\pi)^3} \int P(k) d^3k \quad (13.25)$$

where the power spectrum is defined by

$$P(k) = \Delta_k^2 = \left(\frac{1}{V} \int \Delta(\mathbf{r}) \exp(+i\mathbf{k} \cdot \mathbf{r}) d^3r \right)^2 \quad (13.26)$$

The power spectrum is a measure for the strength of density fluctuations at wave number k .

Structures: Quantitative Description

10



Power Spectrum, IV

How are $\langle \Delta^2 \rangle$ and ξ related?

⇒ Use brute force computation or make use of the correlation theorem.

For functions g, h , the correlation theorem states that the Fourier transform of the correlation,

$$\text{Corr}(g, h) = \int g(x+r)h(r) dx \quad (13.27)$$

is given by

$$\text{FT}(\text{Corr}(g, h)) = GH^* \quad (13.28)$$

where $G = \text{FT}(g)$, etc.

Therefore, setting $g = \Delta(\mathbf{r})$ and $h = \Delta(\mathbf{r})$,

$$\xi(r) = \langle \Delta(\mathbf{x})\Delta(\mathbf{x} + \mathbf{r}) \rangle = \frac{V}{(2\pi)^3} \int |\Delta_k|^2 \exp(i\mathbf{k} \cdot \mathbf{r}) d^3k \quad (13.29)$$

The power spectrum and ξ are Fourier transform pairs.

(remember Eq. 13.26, $P(k) = \Delta_k^2$)

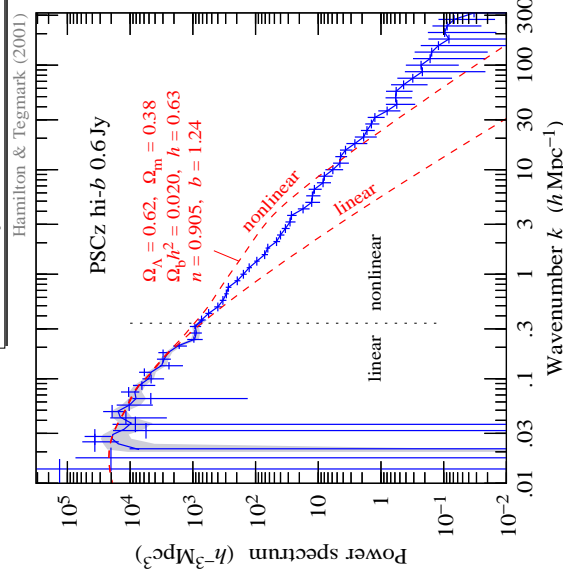
See Peebles (1980, sect. 3f) for 100s of pages of the properties of ξ, P , etc.

Structures: Quantitative Description

11



Power Spectrum: Measurements, I



The power spectrum of high galactic latitude IR galaxies is a power-law, roughly described by

$$P(k) \sim 150 k^{-1.46} h^{-3} \text{Mpc}^3 \quad (13.30)$$

As we will see later, this result gives problems with results of theory of structure formation, see dashed lines in Figure.

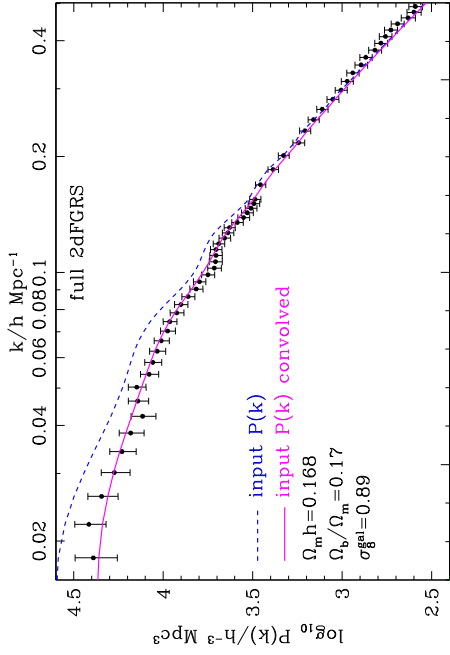
(Hamilton & Tegmark, 2002, Fig. 6)

Structures: Quantitative Description

12



Power Spectrum: Measurements, II



(Cole et al., 2005, Fig. 12)

The galaxy-galaxy-power spectrum flattens towards small k .

Structures: Quantitative Description



Power Spectrum: Interpretation

To understand ξ and P better, let's assume an isotropic universe...

Since

$$\xi(r) \propto \int P(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{r}) d^3k \quad (13.29)$$

using spherical coordinates in k space, one finds:

$$\mathbf{k} \cdot \mathbf{r} = kr \cos \theta \quad (13.31)$$

$$dV = k^2 \sin \theta d\theta d\phi dk \quad (13.32)$$

such that

$$\xi(r) \propto \int_0^\infty \int_0^\pi \int_0^{2\pi} P(k) \exp(ikr \cos \theta) k^2 \sin \theta d\theta d\phi dk \quad (13.33)$$

$$= 2\pi \int_0^\infty \int_0^\pi \xi(r) \exp(ikr \cos \theta) r^2 d(\cos \theta) dr \quad (13.34)$$

$$= \frac{V}{2\pi^2} \int_0^\infty P(k) \frac{\sin kr}{kr} dr \quad (13.35)$$

(the last Eq. is exact).

For $kr < \pi$: $\sin kr / kr > 0$, while oscillation for $kr > \pi$

\implies only wavenumbers $k \lesssim r^{-1}$ contribute to amplitude on scale r .

Structures: Quantitative Description



Power Spectrum: Interpretation

For a power law spectrum,

$$P(k) \propto k^n \quad (13.36)$$

the correlation function is

$$\xi(r) \propto \int_0^\infty \frac{\sin kr}{kr} k^{n+2} dk \sim \int_0^{1/r} k^{n+2} dk \propto r^{-(n+3)} \quad (13.37)$$

But the mass within a fluctuation is $M \sim \rho r^3$, i.e., the mass fluctuation spectrum is

$$\xi(M) \propto M^{-(n+3)/3} \quad (13.38)$$

and therefore the rms density fluctuation at mass scale M is

$$\frac{\delta \rho}{\rho} = \xi(M)^{1/2} \propto M^{-(n+3)/6} \quad (13.39)$$

For $n > -3$, the rms mass fluctuations decrease with $M \implies$ isotropic universe on largest scales

Structures: Quantitative Description



Power Spectrum: Interpretation

What spectra do we expect?

Two simple cases:

Poisson noise: Random statistical fluctuations in number of particles on

scale r :

$$\frac{\delta N}{N} = \frac{1}{N} \implies \frac{\delta M}{M} = \frac{1}{M} \quad (13.40)$$

and therefore $n = 0$ ($\rho \propto M!$) ("white noise").

Zeldovich spectrum: defined by $n = 1$. Thus

$$\frac{\delta \rho}{\rho} \propto M^{-2/3} \quad (13.41)$$

... will be important later

The Zeldovich spectrum is the spectrum expected for the case when initial density fluctuations coming through the horizon had the same amplitude.

Structures: Quantitative Description



Linear Theory, II

Onset of structure formation: $\delta(t), \epsilon(t) \ll 1$ ("linear regime") \implies Ignore all higher orders of δ and ϵ .

Left hand side of Friedmann:

$$\dot{a}^2 = (\dot{a} - \dot{a}\epsilon - \dot{a}\delta)^2 = \dot{a}^2 - 2\dot{a}^2\epsilon - 2\dot{a}^2\delta + \dots \implies \dot{a}^2 - 2\dot{a} \frac{d}{dt}(\dot{a}\epsilon) \quad (13.45)$$

Right hand side of Friedmann:

$$\frac{8\pi G}{3} \bar{\rho}(1 + \delta)\bar{a}^2(1 - \epsilon)^2 + H_0^2(1 - \Omega_0) = \frac{8\pi G}{3} \bar{\rho}\bar{a}^2(1 + \delta)(1 - 2\epsilon) + H_0^2(1 - \Omega_0) \quad (13.46)$$
$$= \frac{8\pi G}{3} \bar{\rho}\bar{a}^2(1 + \delta - 2\epsilon) + H_0^2(1 - \Omega_0)$$

Therefore Eq. (13.45)=Eq. (13.46):

$$\dot{a}^2 - 2\dot{a} \frac{d}{dt}(\dot{a}\epsilon) = \frac{8\pi G}{3} \bar{\rho}\bar{a}^2(1 + \delta - 2\epsilon) + H_0^2(1 - \Omega_0) \quad (13.47)$$

Use the Friedmann Equation for \dot{a} (Eq. 13.44) to simplify this to

$$2\dot{a} \cdot \frac{d}{dt}(\dot{a}\epsilon) = \frac{8\pi G}{3} \bar{\rho}\bar{a}^2(\delta - 2\epsilon) \quad (13.48)$$

Structure Formation



Linear Theory, III

To solve Eq. (13.48): Assume for simplicity $\Omega = 1$, matter-dominated universe.

Matter domination $\implies \rho a^3 = \text{const.} \implies$

$$\bar{\rho}(1 + \delta)\bar{a}^3(1 - \epsilon)^3 \sim \bar{\rho}\bar{a}^3(1 - 3\epsilon + \delta) \stackrel{!}{=} \text{const.} \quad (13.49)$$

and therefore

$$\epsilon = \delta/3 \quad (13.50)$$

\implies Eq. (13.48) becomes

$$2\dot{a} \cdot \frac{d}{dt}(\dot{a}\delta) = \frac{8\pi G}{3} \bar{\rho}\bar{a}^2\delta \quad (13.51)$$

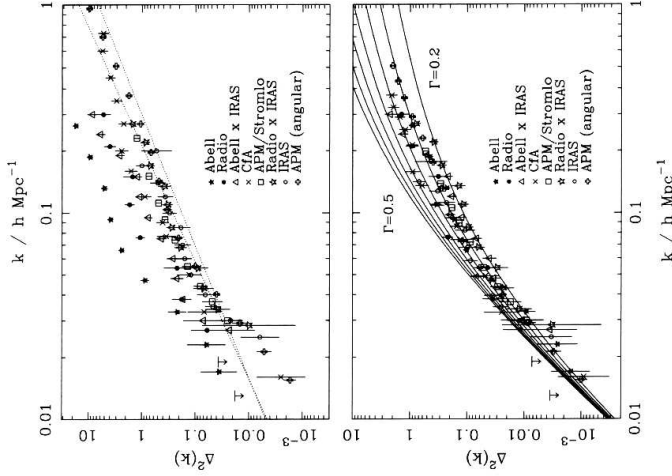
In a $k = 0$ universe,

$$\dot{a}(t) = \left(\frac{3H_0}{2} t\right)^{2/3} =: a_0 t^{2/3} \quad (8.72)$$

and because of $\rho a^3 = \text{const.}$,

$$\bar{\rho}(t) \propto t^{-2} =: \rho_0 t^{-2} \quad (13.52)$$

Structure Formation



(Peacock, 1999, Fig. 16.4)

The measured power spectrum is more complicated \implies Structure formation to understand details!

Structure formation = evolution of overdensity in universe with time.

Describe density and scale factor with respect to normal expansion:

$$\rho(t) = \bar{\rho}(t) \cdot (1 + \delta(t)) \quad (13.42)$$

$$a(t) = \bar{a}(t) \cdot (1 - \epsilon(t)) \quad (13.43)$$

Sign:

$\delta > 0 \implies$ Overdensity

$\epsilon > 0 \implies$ collapse

Seek mathematical model for collapse of gravitating material in expanding universe

Question is similar to that used when deriving Friedmann equations.

\implies Equation describing structure formation:

$$\dot{a}(t) = \frac{8\pi G}{3} \rho(t)a^2(t) + H_0^2(1 - \Omega_0) \quad (13.44)$$

We will drop the explicit t dependency in the following

Structure Formation





Linear Theory, IV

Insert \bar{a} , $\bar{\rho}$ into Eq. (13.51):

$$\frac{4a_0}{3}t^{-1/3} \left(\frac{2a_0}{3}t^{-1/3}\delta + a_0t^{2/3}\dot{\delta} \right) = \frac{8\pi G}{3}\rho_0t^{-2}a_0^4/3\delta \quad (13.53)$$

and simplify

$$t^{-2/3}\delta + t^{1/3}\dot{\delta} = 2\pi G\rho_0t^{-2/3}\delta \quad (13.54)$$

which gives

$$t\dot{\delta} + (1 - 2\pi G\rho_0)\delta = 0 \quad (13.55)$$

The general solution of Eq. (13.55) is a power-law

⇒ Growth of structure!

Since also *negative* PL indexes possible

⇒ Some initial perturbations can be damped out!

We need a better theory to do that in detail...

Structure Formation

4



Linear Theory, V

Better linear theory: Use linearized equations of motion from hydrodynamics.

Detailed theory very difficult, see handout for a few ideas of what is going on...

Classical approach, considering a collapsing sphere:

Potential energy and kinetic energy content of sphere:

$$U = -\frac{1}{2} \int \rho(x)\Phi(x) d^3x \sim -\frac{16\pi^2}{15}G\rho^2r^5 \quad \text{and} \quad T \sim \frac{c_s^2}{2} \frac{4\pi r^3\rho}{3} \quad (13.56)$$

c_s : speed of sound; for neutral Hydrogen, $c_s = \sqrt{5T/3m_p}$.

Sphere collapses for $|U| > T$, i.e., when

$$2r \gtrsim \sqrt{\frac{5}{2\pi}} \sqrt{\frac{c_s^2}{G\rho}} \sim c_s \sqrt{\frac{\pi}{G\rho}} =: \lambda_J \quad (13.57)$$

λ_J is called the Jeans length, the corresponding mass is the Jeans mass,

$$M_J = \frac{\pi}{6}\rho\lambda_J^3 \quad (13.58)$$

Structures with $m < M_J$ cannot grow.

Note that c_s is time dependent ⇒ M_J can change with time!

Structure Formation

5

A better derivation of the Jeans length comes from considering the evolution of a fluid in an expanding universe. Assuming that the initial density perturbations were small, we can use perturbation theory for obtaining deviations from homogeneity (structures).

In a Friedmann universe, for length scales $< 1/H$, dynamical equations are Newtonian to first order, but we need to still use the scale factor, $a(t)$ in the fluid equations.

Continuity equation:

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (13.59)$$

Euler's equation:

$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \left(\Phi + \frac{\dot{a}}{a} \right) \quad (13.60)$$

$$(13.61)$$

$$\nabla^2 \Phi = 4\pi G \rho$$

$$(13.62)$$

Without perturbations (i.e., the zeroth order solution) is given by the normal Friedmann solutions: (dilation by expansion)

$$(13.63)$$

$$\rho_0(t, \mathbf{r}) = \frac{\dot{a}(t)}{a(t)} \mathbf{r} \quad (\text{Hubble law})$$

$$(13.64)$$

$$\Phi_0(t, \mathbf{r}) = \frac{2\pi G \rho_0 r^2}{3} \quad (\text{soln. of Poisson with } \rho = \text{const.})$$

Convert into comoving coordinates ($\mathbf{x} = \mathbf{r}/a(t)$) to get rid of the $a(t)$'s and write down perturbation equations:

$$(13.65)$$

$$\rho(t, \mathbf{x}) = \rho_0(t) + \rho_1(t) =: \rho_0(t) (1 + \delta(t, \mathbf{x}))$$

$$(13.66)$$

$$\mathbf{v}(t, \mathbf{x}) = \mathbf{v}_0(t, \mathbf{x}) + \mathbf{v}_1(t, \mathbf{x})$$

$$(13.67)$$

$$\Phi(t, \mathbf{x}) = \Phi_0(t, \mathbf{x}) + \Phi_1(t, \mathbf{x})$$

where $|\mathbf{v}_1|, |\Phi_1|$ small (δ is called density perturbation field).

Since the equations are spatially homogeneous, we can Fourier transform them to search for plane wave solutions. The general perturbation solution can then later be found by performing linear combinations of these plane waves.

$$\delta(t, \mathbf{x}) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{k} \cdot \mathbf{x}} \delta(t, \mathbf{k}) d^3k \iff \delta(t, \mathbf{k}) = \int e^{-i\mathbf{k} \cdot \mathbf{x}} \delta(t, \mathbf{x}) d^3x \quad (13.68)$$

Inserting into hydro equations gives

$$\dot{\delta}(t, \mathbf{k}) + 2 \frac{\dot{a}(t)}{a(t)} \delta(t, \mathbf{k}) + \left(\frac{k^2 c_s^2}{a^2(t)} - 4\pi G \rho_0 \right) \delta(t, \mathbf{k}) = 0 \quad (13.69)$$

13-58

where the sound speed is $c_s^2 = (\partial p / \partial \rho)_{\text{adiabatic}}$.

Solutions to Eq. 13.69 grow or decrease depending on sign of

$$k_J = \left(\frac{k^2 c_s^2}{a^2(t)} - 4\pi G \rho_0 \right) \quad (13.70)$$

Thus, growth is only possible for $k > k_J$ where

$$(13.71)$$

$$k_J = \sqrt{\frac{4\pi G \rho_0 a^2(t)}{c_s^2}}$$

or, in terms of physical wavelengths,

$$(13.72)$$

$$\lambda_J = \frac{2\pi a(t)}{k_J} = c_s \sqrt{\frac{\pi}{G \rho_0}}$$

the Jeans length.



Classical Structure Formation, I

Early universe: radiation dominates:

$$c_s = c/\sqrt{3} \quad \text{and} \quad \rho_M c^2 = \sigma T^4 \quad (13.73)$$

and therefore

$$\lambda_{J,\text{rad}} = c^2 \sqrt{\pi} 3 G \sigma T^4 \propto a^2 \quad \text{and} \quad M_J \propto \rho_m \lambda_{J,\text{rad}}^3 \propto a^3 \quad (13.74)$$

In the early universe, the Jeans mass grows quickly.

At time of radiation – matter equilibrium,

$$\rho_m = \rho_{\text{rad}} = \sigma T_{\text{eq}}^4 / c^2 \quad (13.75)$$

and

$$M_J(t_{\text{eq}}) = \frac{\pi^{5/2}}{18\sqrt{3}} \frac{c^4}{G^{3/2} \sigma^{1/2} T_{\text{eq}}} \sim \frac{3.6 \times 10^{16} (\Omega_0 h^2)^{-2} M_\odot}{(T/T_{\text{eq}})^3} \quad (13.76)$$

assuming $1 + z_{\text{eq}} = 24000 \Omega_0 h^2$.

\Rightarrow much larger than mass in galaxy cluster (\sim mass of (50 Mpc)³-cube)

Overdense regions with $m < M_{J,\text{rad}}$ are smoothed out by the radiation coupling to matter.

Much larger structures also cannot grow since λ is larger than horizon radius \Rightarrow Mass spectrum of possible structures.

Structure Formation

6



Classical Structure Formation, II

After t_{eq} not much happens until $T_{\text{rec}} \sim 3000$ K

\Rightarrow recombination

\Rightarrow Sound speed drops dramatically (radiation and matter decouple):

$$c_s \sim \frac{kT}{m_p} \sim 5 \text{ km s}^{-1} \quad (13.77)$$

$\Rightarrow M_J$ drops by 10^{11} ;

$$M_{J,\text{eq}} = \frac{\pi \bar{\rho}}{6} \left(\frac{\pi k T_{\text{rec}}}{G \bar{\rho} m_p} \right)^{1/2} \sim 5 \times 10^5 (\Omega_0 h^2)^{-1/2} M_\odot \quad (13.78)$$

after that, M_J drops because of expansion.

So, in a pure matter universe: huge structures (Zeldovich pancakes) form early, and then fragment at recombination. \Rightarrow “top-down model”

Problem: This is not really what has been observed (i.e., galaxy clusters are not yet fully formed, but galaxies are)

Solution: Dark matter

Structure Formation

7



Structure Formation and DM

Structure formation with dark matter:

DM unaffected by radiation pressure \Rightarrow collapse of smaller structures possible
 \Rightarrow bottom-up model

As long as DM is relativistic:

$$M_{J,\text{HDM}} = \frac{\pi \rho_{\text{DM}}}{6} \left(\frac{\pi c_{\text{DM}}}{G \rho_{\text{DM}}} \right)^{3/2} \quad (13.79)$$

Hot Dark Matter: $c_{\text{HDM}} \sim c/\sqrt{3}$

Cold Dark Matter: $c_{\text{CDM}} \ll c/\sqrt{3}$

Standard CDM Scenario:

- DM cools long before t_{rec}
- CDM structures form, M_J about galaxy mass, while baryons coupled to radiation \Rightarrow stays smooth
- t_{rec} : matter decouples, falls in DM gravity wells

CDM “seeds” structures!

Gives not exactly observed power spectrum

\Rightarrow Currently preferred: combination of CDM and Λ DM

Structure Formation

8



Structure Formation and DM

Finally, the real linear theory has to be done in linearized or even full general relativity

\Rightarrow very, very complicated.

Full fledged, detailed structure formation is mainly done numerically.

N -body codes: describe particles (=galaxies) as points, compute mutual interactions in expanding universe
Requires massive computing power.

VIRGO consortium: USA, Canada, Germany, UK

Hubble Volume Simulation: Garching T3E (512 processors), 70 h CPU time followed by the Millenium Simulation (30 d CPU time)

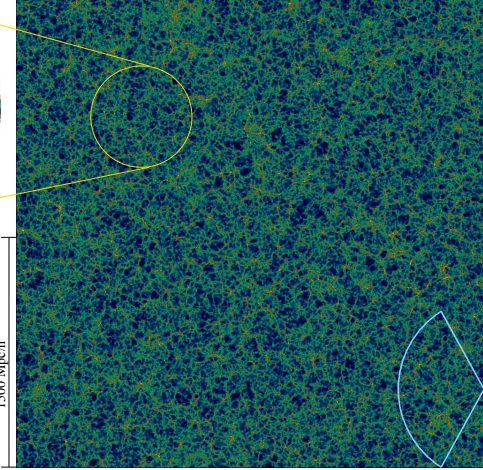
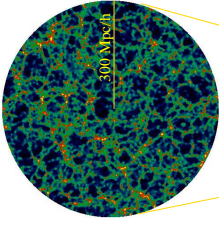
see Springel et al. (2005), Springel, Frenk & White (2006) and <http://www.mpa-garching.mpg.de/~virgo/virgo/>

Structure Formation

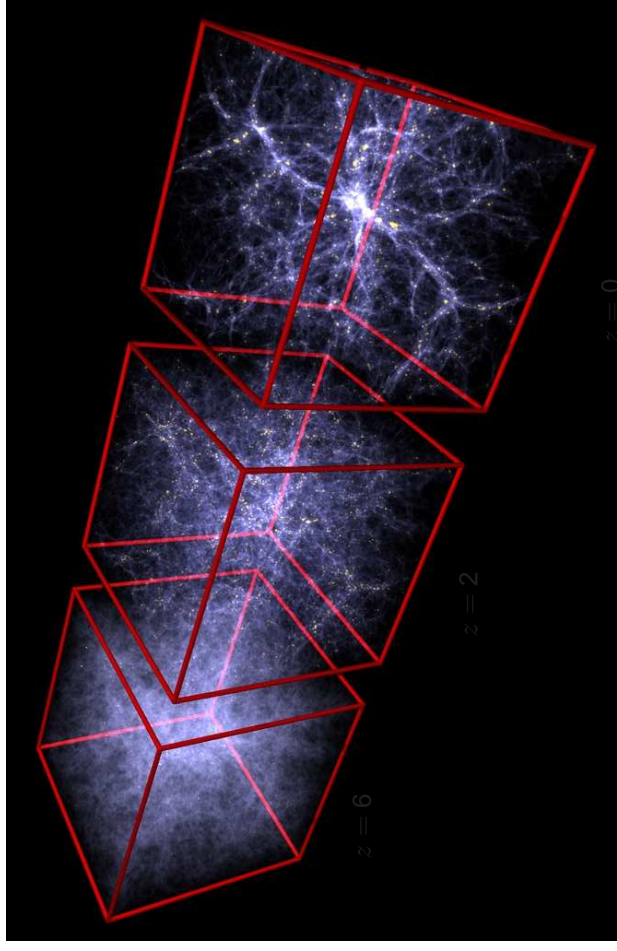
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The Hubble Volume Simulation

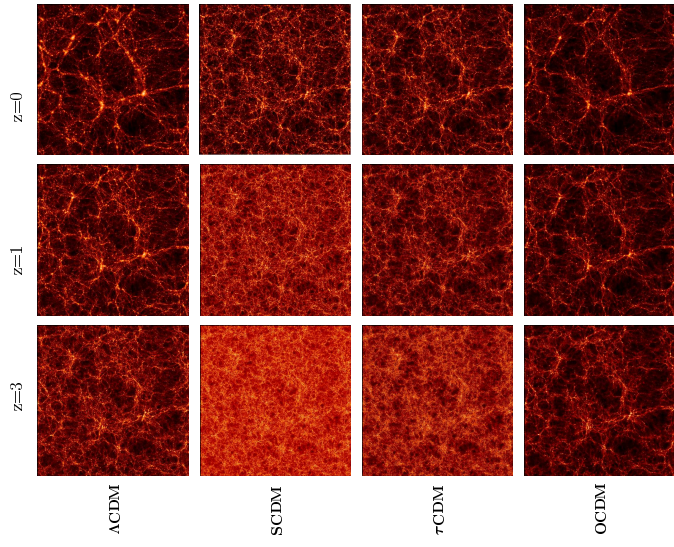
$\Omega = 0.3, \Lambda = 0.7, h = 0.7,$
 $\sigma_8 = 0.9$ (Λ CDM)
 $3000 \times 3000 \times 30 \text{ } h^3 \text{ Mpc}^3$
 P.M: $z_i = 35, \quad s = 100 \text{ } h^{-1} \text{ kpc}$
 1000^3 particles, 1024^3 mesh
 T3E(Garching) - 512cpu
 $M_{\text{particle}} = 2.2 \times 10^{12} \text{ } h^{-1} M_{\odot}$



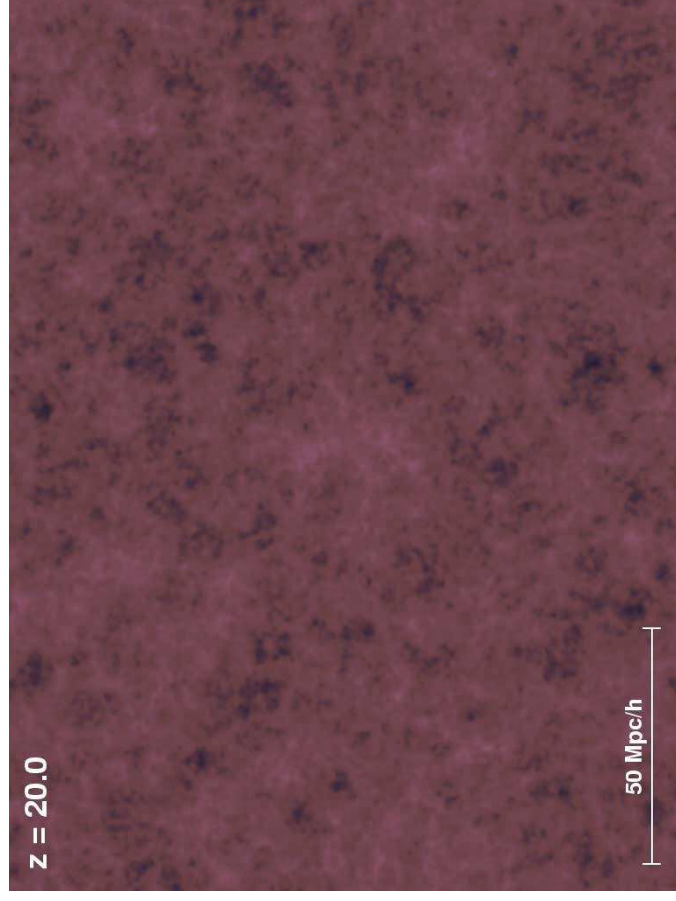
Λ DM, pie shows SDSS size



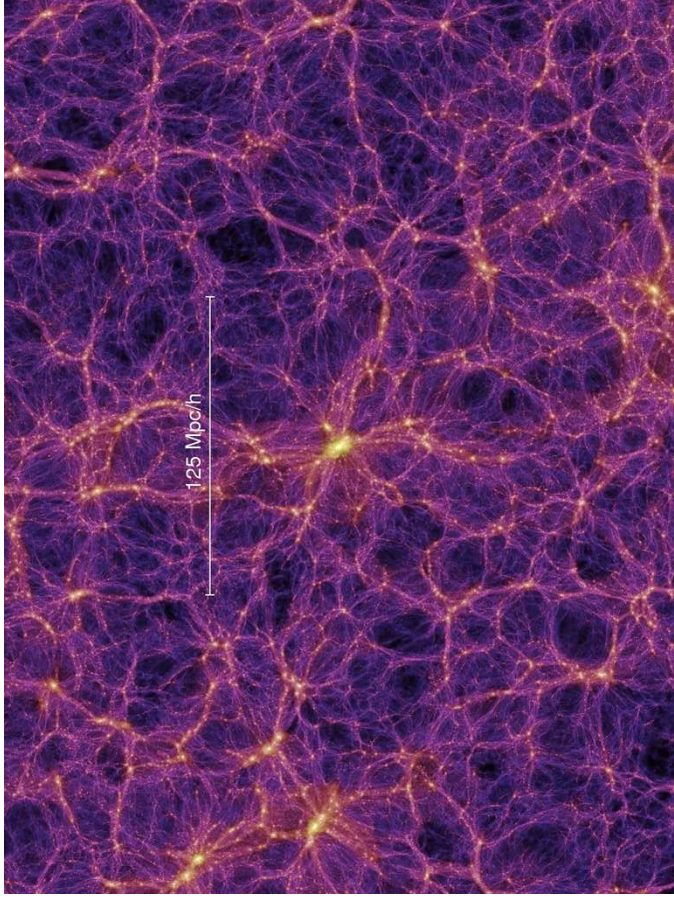
(V. Springe/MPA)



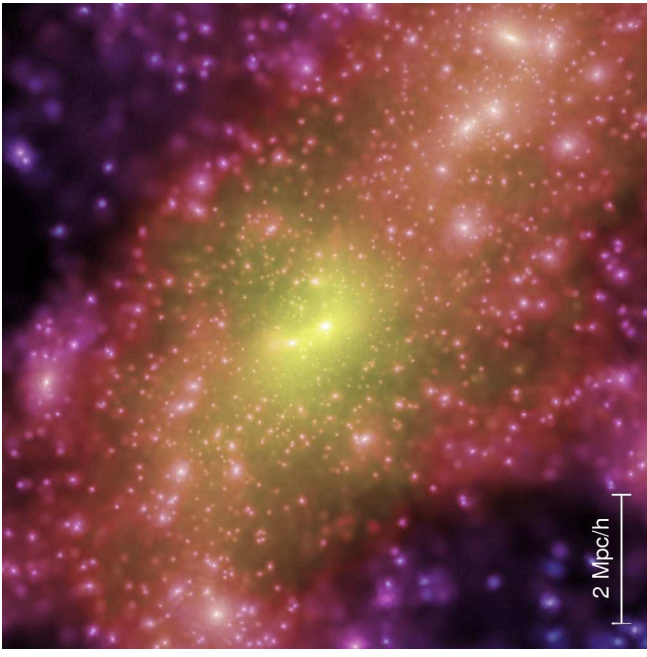
The VIRGO Collaboration 1996



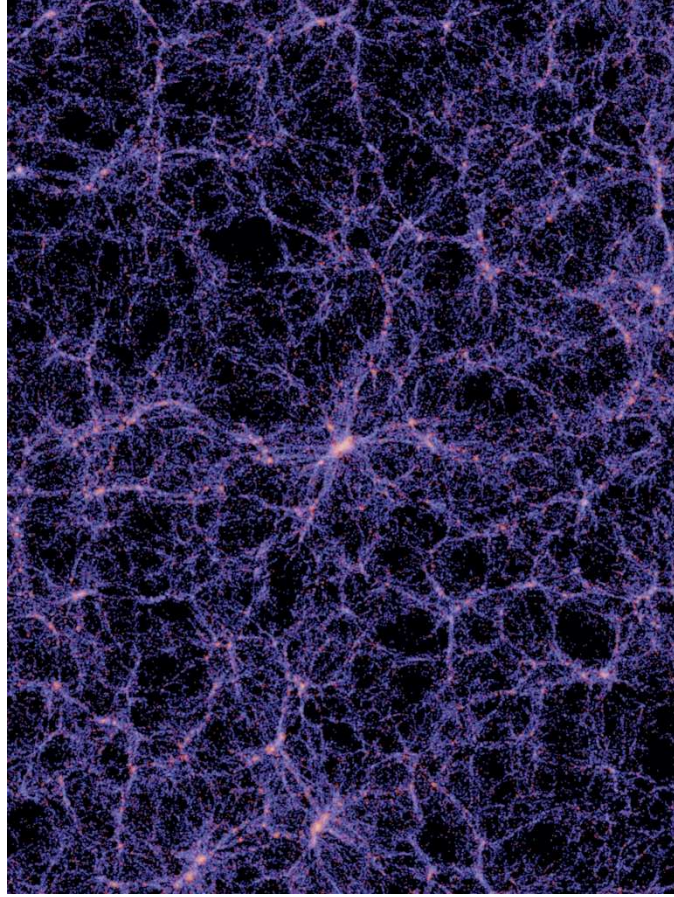
Evolution of structure in a Λ CDM Universe (MPAV, Springel)



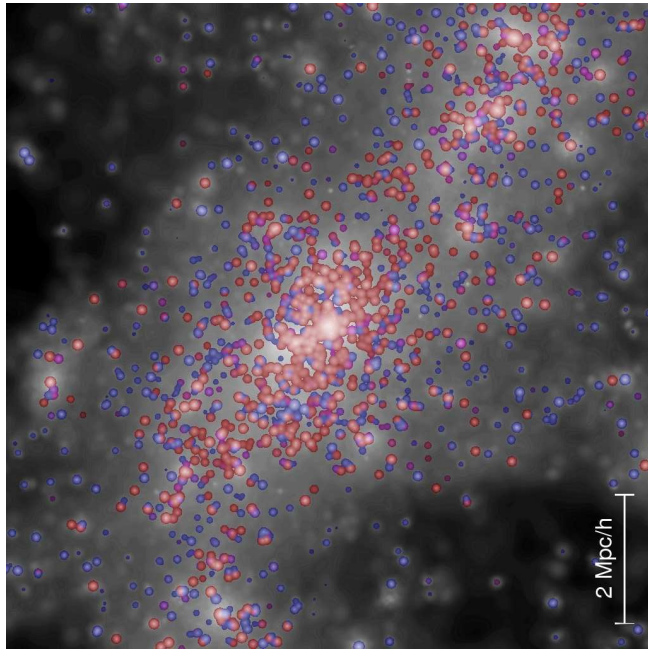
Today's dark matter distribution... (V. Springel/MPA)



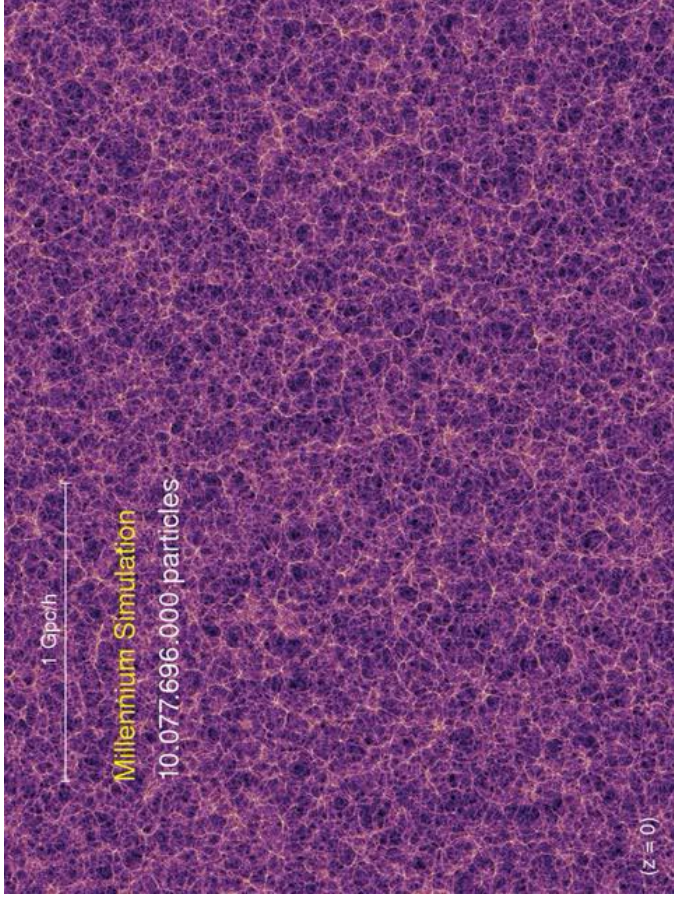
Today's dark matter distribution in a cluster... (V. Springel/MPA)



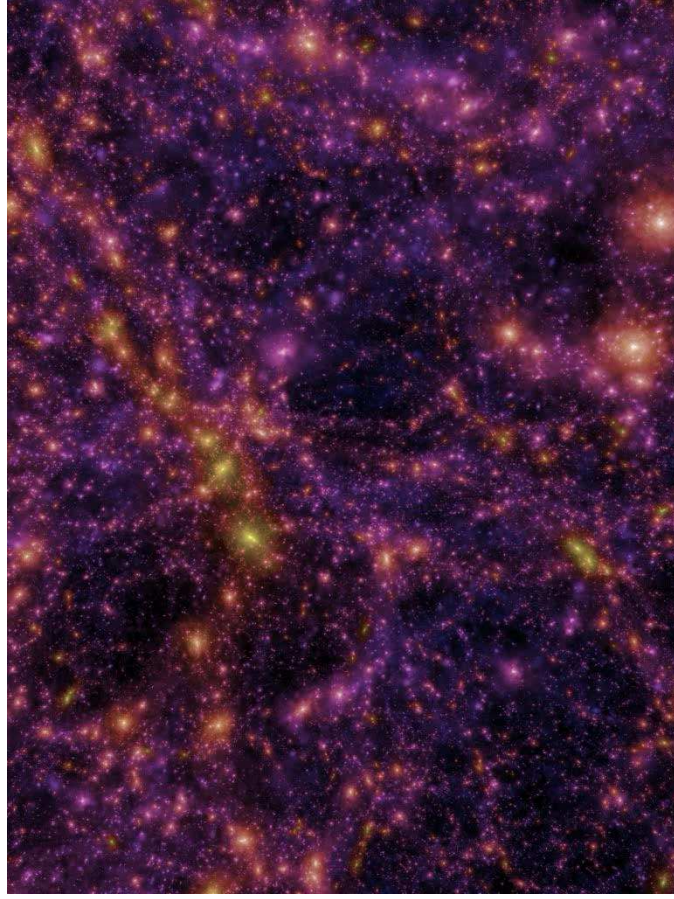
...and corresponding galaxy distribution (V. Springel/MPA)



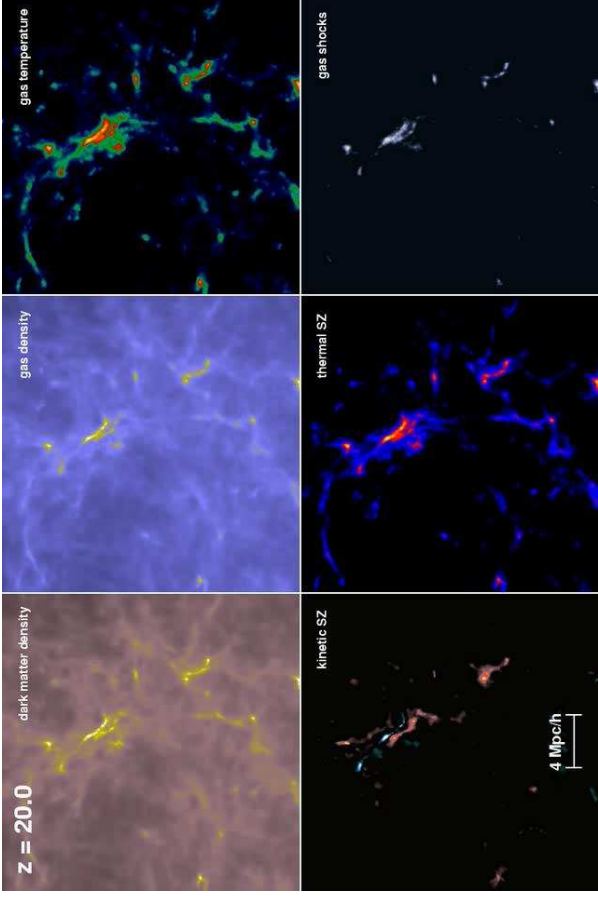
...and corresponding galaxy distribution (V. Springel/MPA)



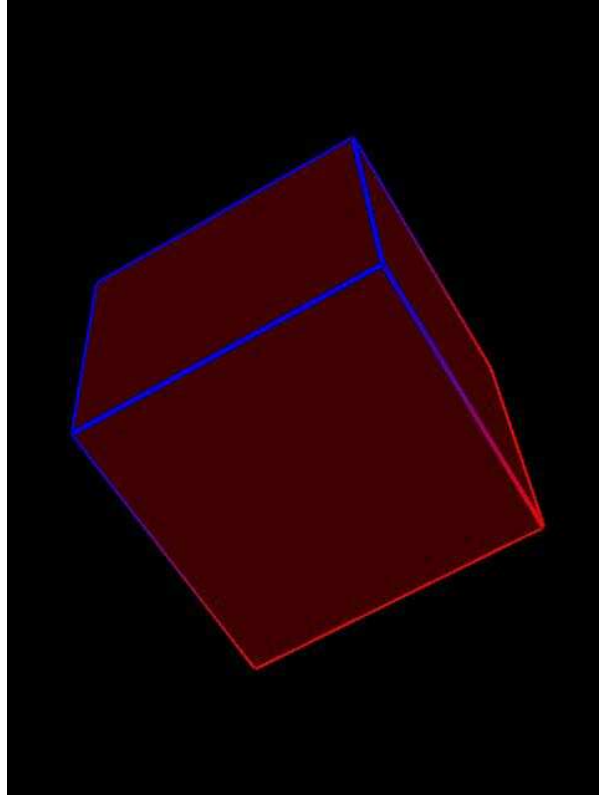
Zoom into the DM structure of the Millennium Simulation



Flight through the DM structure of the Millennium Simulation (V. Springel)

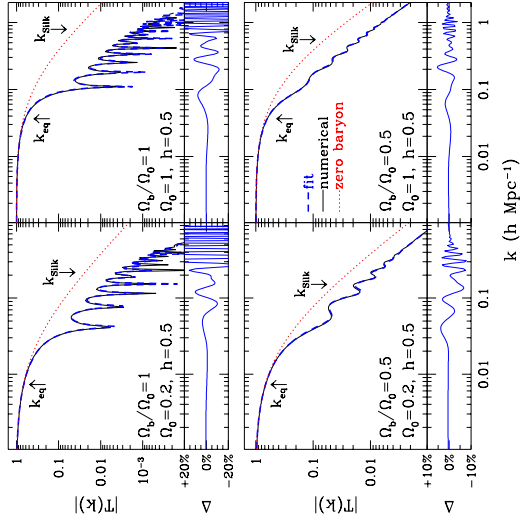


Formation of a galaxy cluster (V. Springel)



Reionization: ionization of intergalactic medium through UV radiation from the first stars
(movie courtesy N. Gnedin)

Formal Structure Formation



Calculation of real power spectra difficult: growth under self-gravitation pressure effects dissipation. To predict observations from today: define transfer function

$$\delta_b(z=0) = D(z)T_k \delta_k(z) \quad (13.80)$$

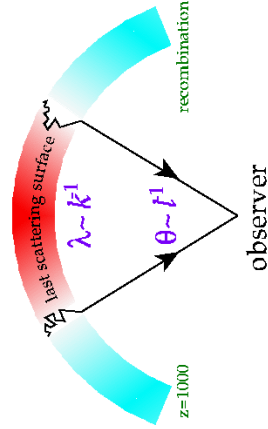
But: need initial conditions, $\delta_k(z)$!

(Eisenstein & Hu, 1999)

Initial conditions

1

CMBR



Matter and Radiation are coupled, i.e., large mass density = high photon density.

Photons from overdense regions: gravitational redshift ⇒ observable! (Sachs Wolfe Effect)

CMBR: Radiation from surface of last scattering ⇒ Gravitational redshift is observable as temperature fluctuation:

$$\frac{\Delta T}{T} \sim \frac{\Delta \Phi_g}{c^2} \quad (13.81)$$

CMBR Fluctuations trace gravitational potential at $z \sim 1100$!

Initial conditions

2

CMBR

Temperature fluctuations:

$$\frac{\Delta T}{T} \sim \frac{\Delta \Phi_g}{c^2} \quad (13.81)$$

where

$$\Delta \Phi_g \sim -\frac{2G\Delta M}{R} = \frac{8\pi G}{3} \rho R^2 \delta = -\delta(t) (H(t)R)^2 \quad (13.82)$$

Current angle of region on sky:

$$\alpha \sim R/d_A \quad (13.83)$$

where the angular diameter distance

$$d_A = d_L / (1+z)^2 \quad (13.84)$$

Therefore:

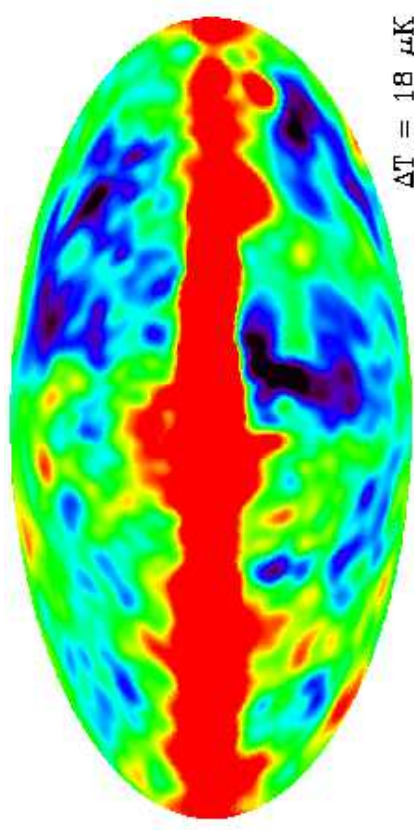
$$\frac{\Delta T}{T} \sim \frac{\Delta \Phi_g}{c^2} \propto \frac{\delta \alpha^2}{3} \quad (13.85)$$

Quotient 3 from more detailed theory, "Integrated Sachs Wolfe effect"

Initial conditions

3

CMBR



COBE: Resolution $\alpha \sim 7^\circ$ (corresponds to $\sim 10^{20} M_\odot$ at recombination). Temperature fluctuations imply $\delta \sim 10^{-3}$ at recombination.

This is small for pure matter dominated universe ⇒ **implies existence of dark matter!**

Initial conditions

4

CMBR

Detailed theory of fluctuations: Expand CMB fluctuations on sky in spherical harmonics:

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell, m} a_{\ell, m} Y_{\ell, m}(\theta, \phi) \tag{13.86}$$

Since rotationally symmetric, can express variation in terms of multipole coefficients, C_ℓ :

$$C(\theta) = \frac{1}{4\pi} \sum_{\ell} \sum_{m=-\ell}^{+\ell} |a_{\ell, m}|^2 P_\ell(\cos \theta) \tag{13.87}$$

$$=: \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_\ell P_\ell(\cos \theta) \tag{13.88}$$

where $C(\theta) = \langle \Delta T / T \rangle$ and where the P_ℓ are the Legendre polynomials.

Initial conditions

CMBR

Expect following features:

Large angle anisotropy: (small ℓ , scales \gtrsim horizon at decoupling): Flat part due to Sachs-Wolfe effect

Smaller angular scales: (larger ℓ): Influenced by photon-baryon interactions:

Matter falls in potential well

⇒ Pressure resists

⇒ acoustic oscillations

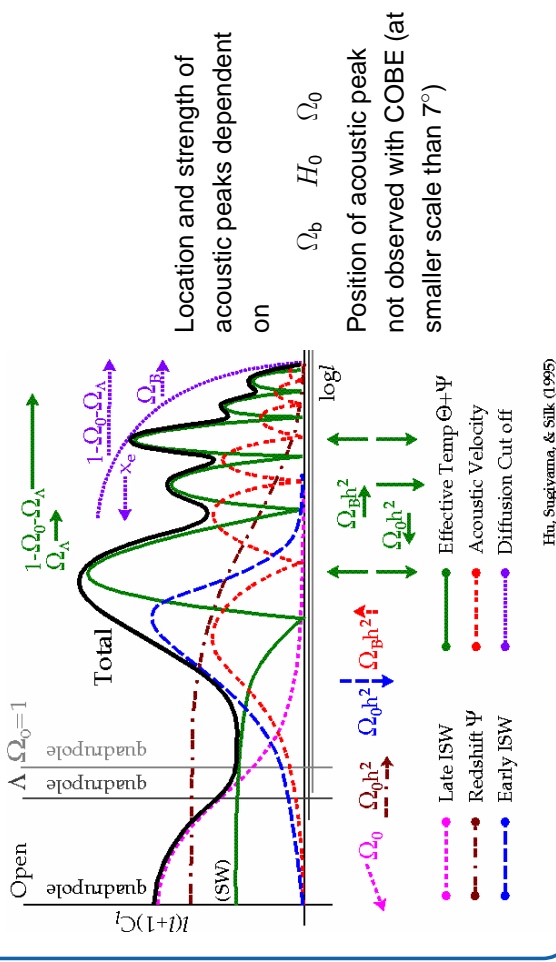
⇒ Power at selected scales!

Power from those density fluctuations which had their maximum amplitude at time of last scattering dominates ⇒ acoustic peak

Also damping from photon diffusion (Compton scattering; Silk damping [after Joseph Silk])

Initial conditions

CMBR



Initial conditions



courtesy BOOMERANG team

Enter: BOOMERANG (Balloon Observations of Millimetric Extragalactic Radiation and Geophysics), Flight in Antarctica 1998 December 29 – 1999 January 9



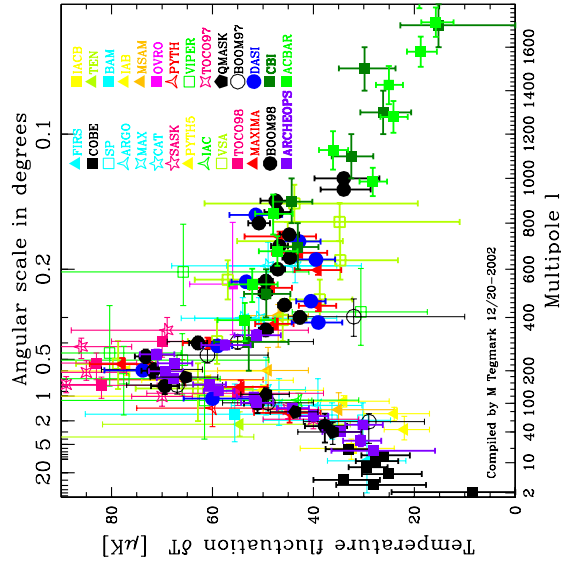
BOOMERANG before Mt. Erebus; courtesy BOOMERANG team

Other balloon missions: MAXIMA-1, ...



13-84

Summary: Pre-WMAP



1st acoustic peak found by BOOMERANG in 1999

(Jaffe et al., 2000)

Courtesy M. Tegmark