Elliptical Galaxies

Photometry

NGC 5846 and NGC 5850 – `http://www.krneki.ws`

Surface brightness: modified Sérsic's formula:

$$\log\left(\frac{I(R)}{I(R_e)}\right) = b\left[\left(\frac{R}{R_e}\right)^{1/n} - 1\right] \qquad (5.1)$$

where R_e: effective radius, i.e., radius containing half of the total luminosity.

In many respects, elliptical galaxies are similar to bulges of disk galaxies.

Photometry

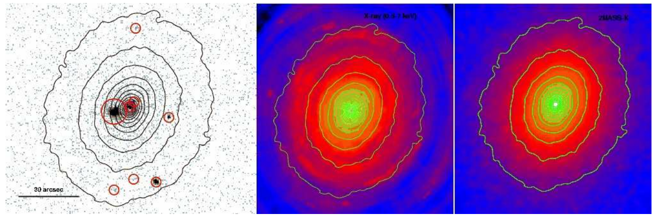
Three classes of elliptical galaxies:

- Luminous giant ellipticals: $L > 2 \times 10^{10} L_\odot$
- Midsized ellipticals: $3 \times 10^9 L_\odot < L < 2 \times 10^{10} L_\odot$
- Dwarf ellipticals: $L < 3 \times 10^9 L_\odot$.

Hubble type: En, where $n = 10(1-b/a)$ (a: major axis of isophotes, b: minor axis; $(1-b/a) = \epsilon$).

E0 galaxies: circular; E5 galaxies: axial ratio 1 : 2.

The Hubble type of an elliptical galaxy depends on our viewing direction! Characteristic parameters are largely determined by luminosity.

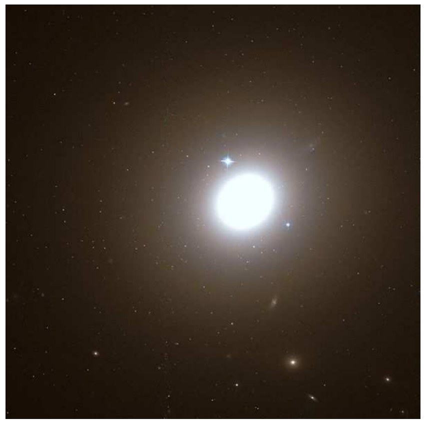


Near-IR isophotes vs. diffuse X-ray emission of M 32 (Revnivtsev et al., 2007).

Photometry

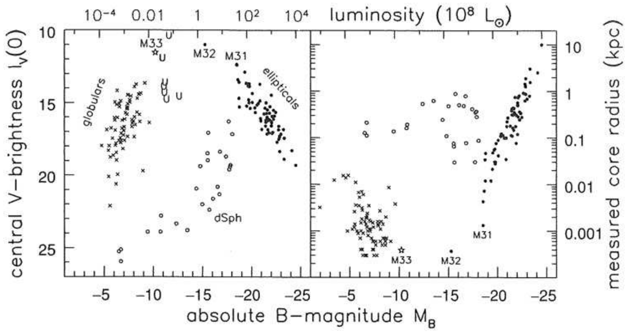
The most luminous of all galaxies are the cD galaxies, the central galaxies of groups or clusters.

$R^{1/4}$–law fulfilled out to $\sim\ 20R_e$. Beyond that, excess surface brightness from cluster stars or *shredded* debris of cannibalized dwarf galaxies.



HST image of NGC 1399 in Fornax cluster

Photometry



Sparke & Gallagher Fig 6.6

Central brightness and core radius are tightly linked to the luminosity for luminous and midsized ellipticals:

The more luminous the galaxy, the lower its central surface brightness and the larger its core. Opposite trend for dwarf ellipticals.

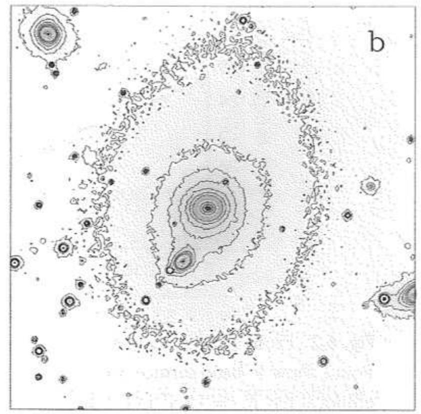
There is no such link for disk galaxies as a whole but disk bulges fit into the same trend as elliptical galaxies.

Photometry

A couple of complications:

- Luminosity/Central-Brightness correlation and Luminosity/Core-Radius correlation observationally similar to color-magnitude diagram relations for stars, but much harder to understand (in stars, the mass determines both luminosity and temperature; in galaxies, the processes in galaxy formation are most likely dominating)
- Surface brightness measurements are hampered by seeing and angular resolution. cD galaxies tend to have cores with approximately constant surface brightness but less-bright ellipticals have often central *cusps* (surface brightness keeps rising). ⇒ Core-radius measurements are uncertain.
- The isophotes of some (luminous) elliptical galaxies *twist* from the inner to the outer isophotes. ⇒ Evidence for *triaxiality*.

Photometry



Sparke & Gallagher Fig.6.1b

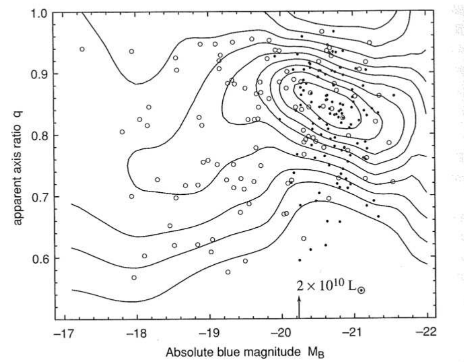
Twisted isophotes of a giant elliptical galaxy: the long axis of the inner isophotes is roughly horizontal, while the outer ones are near-vertical.

The 3D Shapes of Elliptical Galaxies

The appearance (Hubble type) of an elliptical galaxy depends on the direction from which it is observed. A near-circular does not guarantee that the ellipsoid has a true spherical three-dimensional structure.

⇒ Distribution of apparent shapes has to be studied.

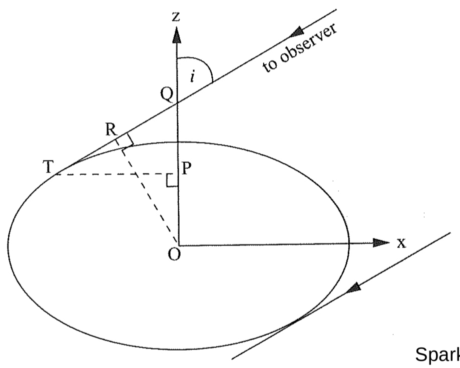
First clues: There are no ellipticals in the sky more flattened than E7 ($b \sim 0.3a$) and bright ellipticals on average appear rounder.



Sparke & Gallagher Fig.6.9

The 3D Shapes of Elliptical Galaxies

Try to understand apparent shapes with oblate spheroids:



Sparke & Gallagher Fig.6.9

Stellar density $\rho(\mathbf{x})$ is

$$\rho(\mathbf{x}) = \rho(m^2), \text{ where } \quad m^2 = \frac{x^2 + y^2}{A^2} + \frac{z^2}{B^2} \qquad (5.2)$$

with $A > B > 0$ (the true major and minor axes of the oblate spheroid).

The 3D Shapes of Elliptical Galaxies

Eq. 5.2 describes and ellipse for constant m^2, i.e., for constant stellar density, i.e., for isophotes.

$\Rightarrow$ the oblate spheroid appears elliptical under all viewing angles.

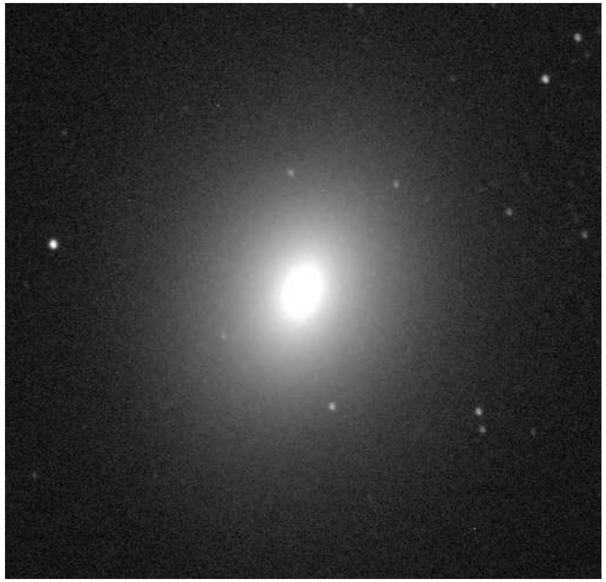
Following Sparke & Gallagher (Sect. 6.8), it can be shown easily that the observed axial ratio q_{obl} is given by

$$q_{\text{obl}}^2 = (b/a)^2 = (B/A)^2 \sin^2 i + \cos^2 i \quad . \qquad (5.3)$$

(For prolate galaxies, there is a fully analogous statement.)

A spheroidal galaxy never appears more flattened than its true axial ratio $A/B \Rightarrow$ E7 galaxies really are the most elongated systems.

Triaxial systems are possible. Particularly at high luminosity, some elliptical galaxies seem to be triaxial.



“Faber-Jackson” law for elliptical galaxies:

The luminosity L of an elliptical galaxy roughly scales with its intrinsic velocity dispersion, σ, as $L \propto \sigma^4$.

Note that ellipticals have virtually no neutral Hydrogen
$\Longrightarrow$ cannot use 21 cm.

Relation shows a broad scatter.

M32 (companion of Andromeda), courtesy W. Keel

Ellipticals: $M_{\text{B}} = -19.38 \pm 0.07 - (9.0 \pm 0.7)(\log\sigma - 2.3)$ (5.4)

Lenticulars (Type S0): $M_{\text{B}} = -19.65 \pm 0.08 - (8.4 \pm 0.8)(\log\sigma - 2.3)$ (5.5)

D_n-σ

The Faber-Jackson law is a specialized case of the more general D_n–σ-relation:

The intensity profile of an elliptical galaxy is given by de Vaucouleurs' $r^{1/4}$ law:

$$I(r) = I_0 \exp\left(-(r/r_0)^{1/4}\right) \quad \Longrightarrow \quad L = \int I \propto I_0 r_0^2 \qquad (5.6)$$

Because of the virial theorem ($E_{\text{kin}} = -E_{\text{pot}}/2$):

$$\frac{1}{2} m\sigma^2 = G\frac{mM}{r_0} \quad \Longleftrightarrow \quad \sigma^2 \propto \frac{M}{r_0} \qquad (5.7)$$

where σ: velocity dispersion.

Assume a mass-to-light ratio

$$M/L \propto M^\alpha \qquad (5.8)$$

($\alpha \sim 0.25$) and use r_0 from Eq. (5.6) to obtain

$$L^{1+\alpha} \propto \sigma^{4-4\alpha} I_0^{\alpha-1} \qquad (5.9)$$

This is called the “fundamental plane” relationship (Dressler et al., 1987).

D_n-σ

Observational version of the fundamental plane relationship: Instead of inserting r_0 and I_0, measure diameter D_n of aperture to reach some mean surface brightness (typically sky brightness, 20.75 mag arcsec^{-2} in B), and use calibration.

Note: Assumptions are

1. M/L same everywhere.
2. ellipticals have same stellar population everywhere

Calibration paper: Kelson et al. (2000).

Note: Bulges of spiral galaxies also follow the Faber-Jackson relation.

Kelson D.D., Illingworth G.D., Tonry J.L., et al., 2000, ApJ 529, 768

Revnivtsev M., Churazov E., Sazonov S., et al., 2007, A&A 473, 783

Cosmology – Basic Facts

Basic Facts

Cosmology deals with answering the questions about the universe as a whole.

The main question is:

How did the universe evolve into what it is now?

For this, *four major facts* need to be taken into account:

The universe is:
- expanding,
- isotropic,
- and homogeneous.

The isotropy and homogeneity of the universe is called the *cosmological principle*.

Perhaps (for us) the most important fact is:

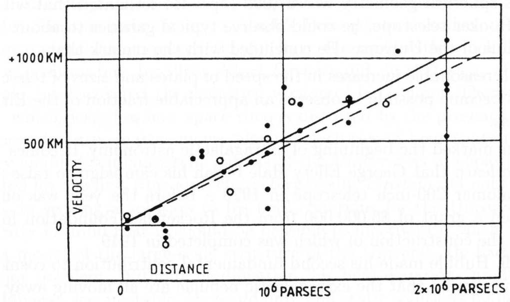
- The universe is habitable to humans.

i.e., the *anthropic principle*.

The one question cosmology does not attempt to answer is: How came the universe into being?

$\Longrightarrow$ Realm of theology!

Expansion



(Hubble, 1929, Fig. 1)

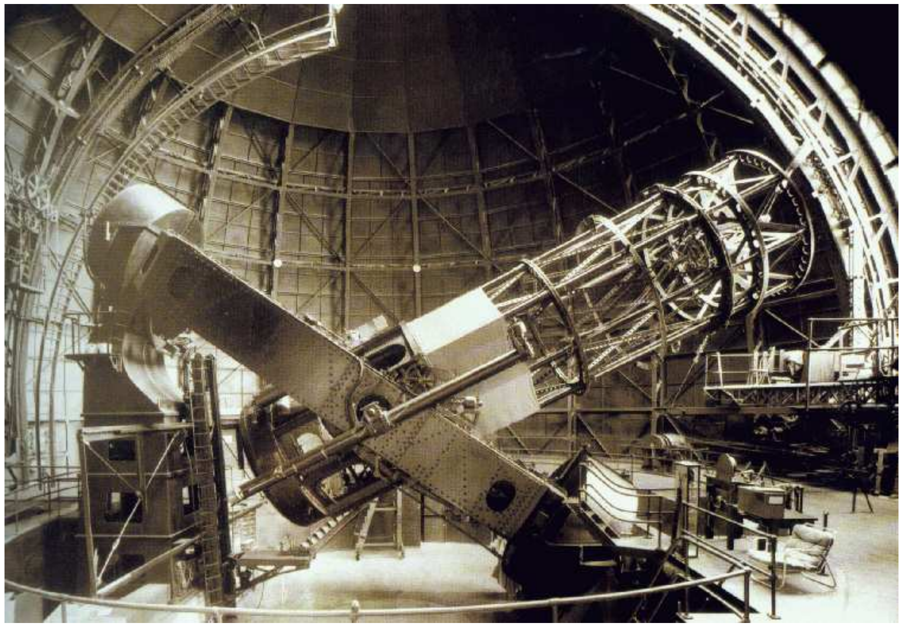
Hubble (1929): Velocity v (defined as $v/c := z = \Delta\lambda/\lambda$) for galaxy at distance r is

$$v(r) = H_0 r + v_X \cos\alpha\cos\delta + v_Y \sin\alpha\cos\delta + v_Z \sin\delta \quad (6.1)$$

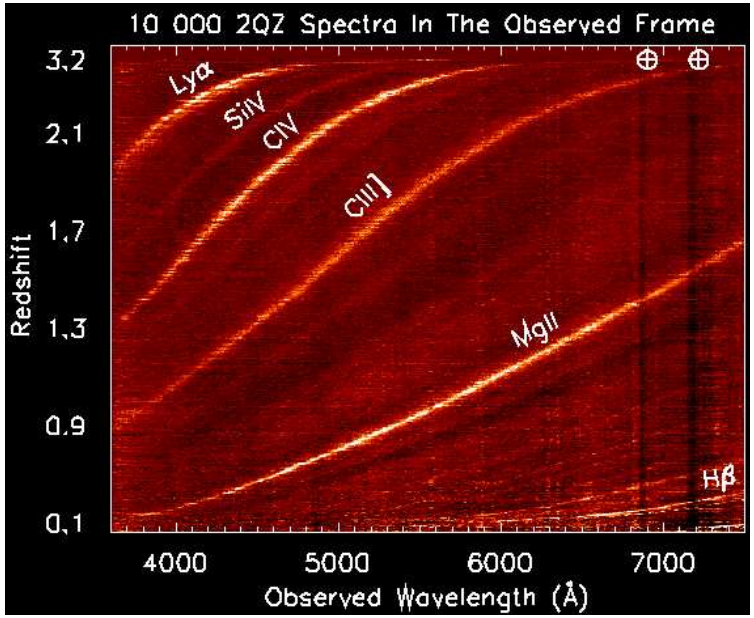
(v_X, v_Y, v_Z) velocity due to motion of solar system ($\sim$ 350 km s^{-1} towards $l = 264°$, $b = 48°$, Bennet et al., 1996)

H_0: "Hubble parameter"; *intrinsic* component of velocity due to *expansion* of the universe.

Old usage: "Hubble constant", but $H_0 \neq$ const. (cf. Eq. (7.49)).

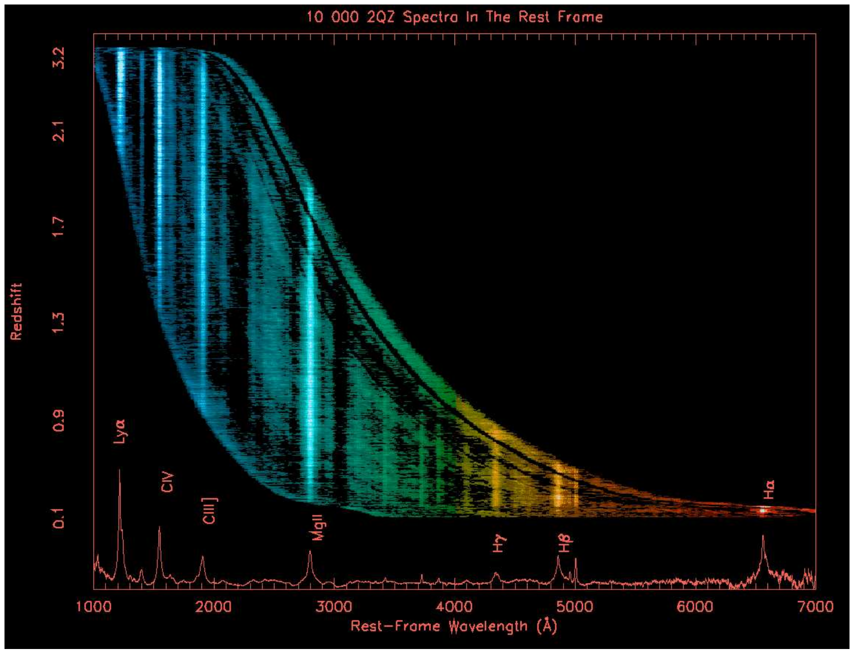


"Hubble's" 2.5 m (100-inch) telescope on Mt. Wilson (Image: `http://sckim.kasi.re.kr/Images/hooker2_5m.gif`)

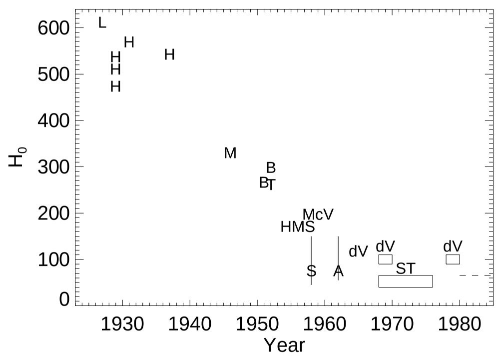


courtesy 2dF QSO Redshift survey

As a consequence of the cosmological redshift, for different z different parts of the spectrum of a distant source are visible.



Expansion



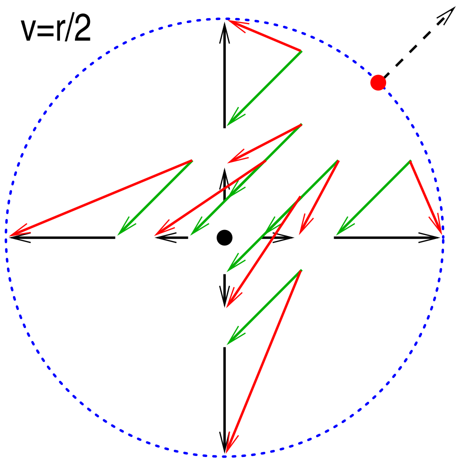
(after Trimble, 1997)

Currently accepted value:
$H_0 \sim 75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.
The systematic uncertainty of H_0 is $\sim 10\,\mathrm{km^{-1}\,s^{-1}\,Mpc^{-1}}$.
Parameterize uncertainty in formulae by defining

$$H_0 = 100\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \cdot h$$
$$H_0 = 75\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \cdot h_{75} \qquad (6.2)$$

Note: H_0^{-1} has units of time: $H_0^{-1} = 9.78\,\mathrm{Gyr}/h$: Hubble-Time;
for $h = 0.75$, the Hubble-Time is 13 Gyr.

Expansion



Expansion law $\mathbf{v} = H_0\mathbf{r}$ is unchanged under rotation and translation: isomorphism.

Proof:

Rotation: Trivial.

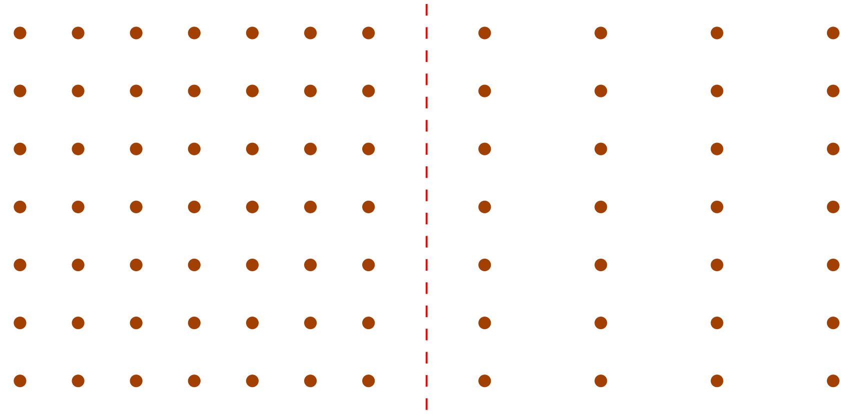
Translation: Observations from place with position $\mathbf{r}'$ and velocity $\mathbf{v}'$: Observed distance is $\mathbf{r}_\circ = \mathbf{r} - \mathbf{r}'$, observed velocity is $\mathbf{v}_\circ = \mathbf{v} - \mathbf{v}'$. Because of the Hubble law,

$$\mathbf{v}_\circ = H_0\mathbf{r} - H_0\mathbf{r}' = H_0\left(\mathbf{r} - \mathbf{r}'\right) = H_0\mathbf{r}_\circ$$

This isomorphism is a direct consequence of the homogeneity of the universe.

Despite everything receding from us, we are not at the center of the universe $\Longrightarrow$ Copernicus principle still holds.

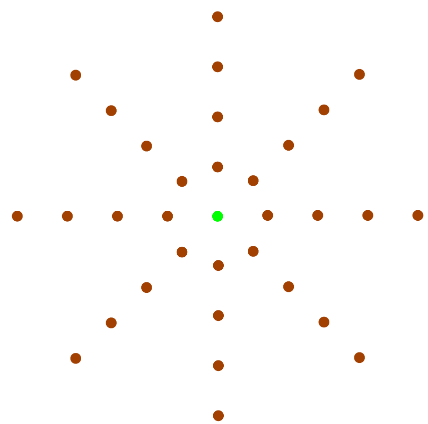
Homogeneity and Isotropy



after Silk (1997, p. 8).

Note that homogeneity does not imply isotropy!

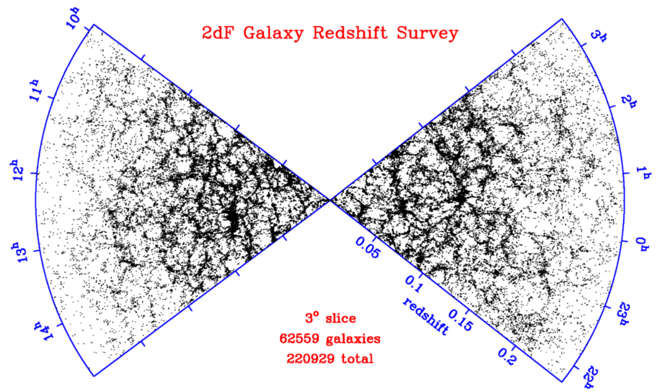
Homogeneity and Isotropy



Neither does isotropy *around one point* imply homogeneity!

$\Longrightarrow$ *Both* assumptions need to be tested.

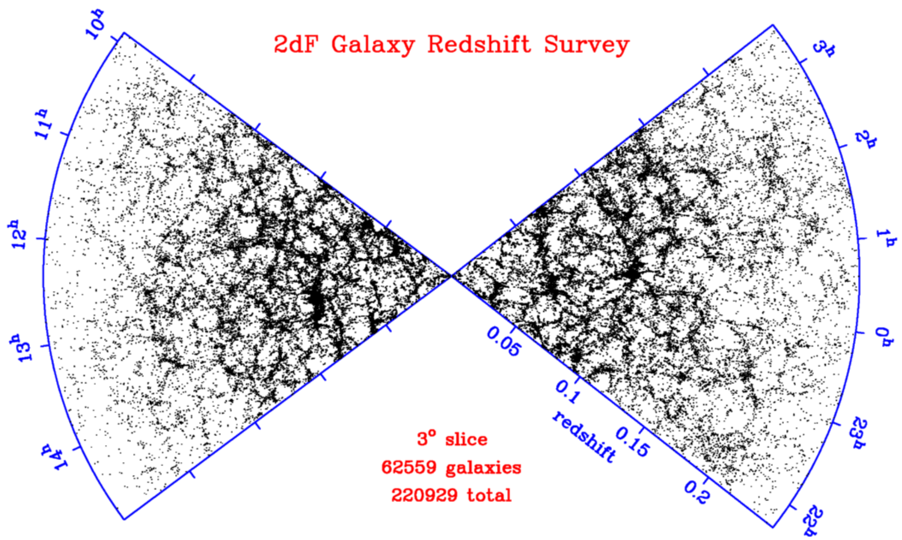
Homogeneity



2dF Survey, ~220000 galaxies total

The universe is homogeneous $\Longleftrightarrow$ The universe looks the same everywhere in space

Testable by observing spatial distribution of galaxies.

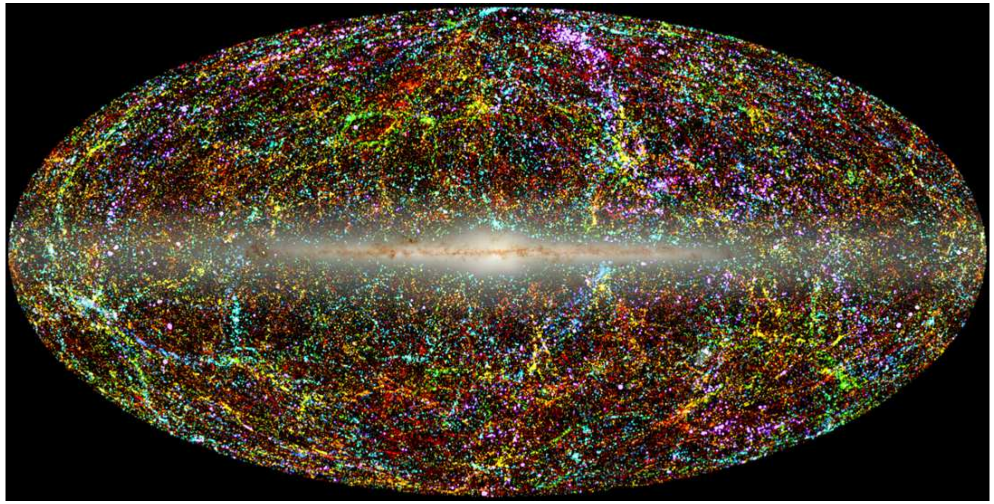


2dF Survey, ~220000 galaxies total

On scales $\gg$100 Mpc the universe looks indeed the same.

Below that: structure.

Structures seen are galaxy clusters (gravitationally bound) and superclusters (larger structures, not [yet] gravitationally bound).

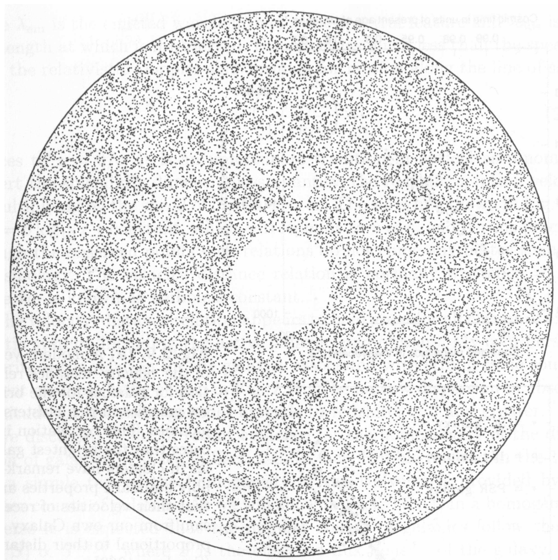


(Jarrett, 2004, Fig. 1)

Distribution of Galaxy redshifts in the 2MASS galaxy catalogue

(color code: blue - $z < 0.01$; green - $0.01 < z < 0.04$; red - $0.04 < z < 0.1$)

Isotropy



The universe is isotropic

$\Longleftrightarrow$ The universe looks the same in all directions

Radio galaxies are mainly quasars

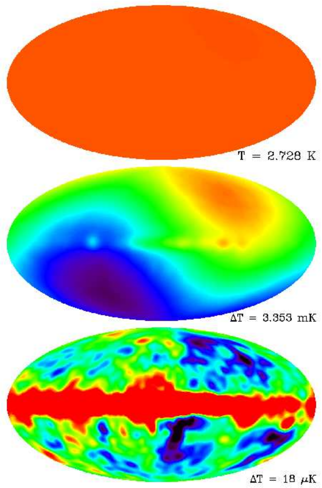
$\Longrightarrow$ Sample large space volume ($z \gtrsim 1$)

$\Longrightarrow$ Clear isotropy.

Peebles (1993): Distribution of 31000 objects at λ =6 cm from the Greenbank Catalogue.

Anisotropy in the image: galactic plane, exclusion region around Cyg A, Cas A, and the north celestial pole.

Isotropy



Best evidence for isotropy: Intensity of 3 K Cosmic Microwave Background (CMB) radiation.

First: dipole anisotropy due to motion of Sun (see slide 6–3), after subtraction: $\Delta T/T \lesssim 10^{-4}$ on scales from $10''$ to 180°.

At level of 10^{-5}: structure in CMB due to structure of surface of last scattering of the CMB photons, i.e., structure at the time when Hydrogen recombined.

6–15

Bennet, C. L., Banday, A. J., Gorski, K. M., et al. 1996, ApJ, 464, L1

Hubble, E. P., 1929, 15, 168

Jarrett, T., 2004, Proc. Astron. Soc. Aust., 21, 396

Peebles, P. J. E., 1993, Principles of Physical Cosmology, (Princeton: Princeton Univ. Press)

Silk, J., 1997, A Short History of the Universe, Scientific American Library 53, (New York: W. H. Freeman)

Trimble, V., 1997, Space Sci. Rev., 79, 793

World Models

Structure

Observations: cosmological principle holds: The universe is homogeneous and isotropic.

$\Longrightarrow$ Need theoretical framework obeying the cosmological principle.

Use combination of

- General Relativity
- Thermodynamics
- Quantum Mechanics

$\Longrightarrow$ Complicated!

For 99% of the work, the above points can be dealt with separately:

1. Define metric obeying cosmological principle.
2. Obtain equation for evolution of universe using Einstein field equations.
3. Use thermo/QM to obtain equation of state.
4. Solve equations.

GRT vs. Newton

Before we can start to think about universe: Brief introduction to assumptions of general relativity.

⟹ See theory lectures for the gory details, or check with the literature (Weinberg or MTW).

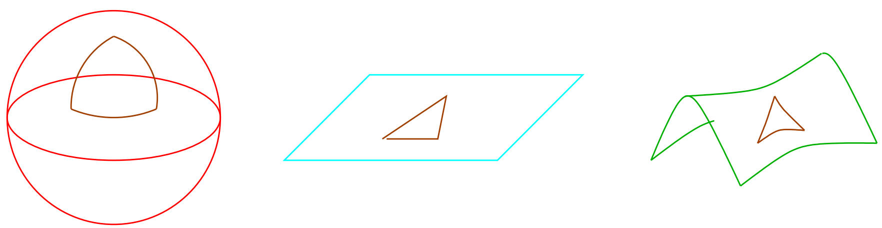
Assumptions of GRT:

- Space is 4-dimensional, might be curved
- Matter (=Energy) modifies space (Einstein field equation).
- Covariance: physical laws must be formulated in a coordinate-system independent way.
- Strong equivalence principle: There is no experiment by which one can distinguish between free falling coordinate systems and inertial systems.
- At each point, space is locally Minkowski (i.e., locally, SRT holds).

⟹ Understanding of geometry of space necessary to understand physics.

2D Metrics

Before describing the 4D geometry of the universe: first look at 2D spaces (easier to visualize).



After Silk (1997, p. 107)

There are three classes of isotropic and homogeneous two-dimensional spaces:

- 2-sphere ($\mathscr{S}^2$) positively curved
- x-y-plane ($\mathbb{R}^2$) zero curvature
- hyperbolic plane ($\mathscr{H}^2$) negatively curved

(curvature $\approx \sum$angles in triangle $>$, $=$, or $<180^\circ$)

We will now calculate what the metric for these spaces looks like.

2D Metrics

The metric describes the local geometry of a space.

Differential distance, $\mathrm{d}s$, in Euclidean space, $\mathbb{R}^2$:

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 \quad (7.1)$$

The metric tensor, $g_{\mu\nu}$, is defined through

$$\mathrm{d}s^2 = \sum_\mu \sum_\nu g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu =: g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \quad (7.2)$$

(Einstein's summation convention)

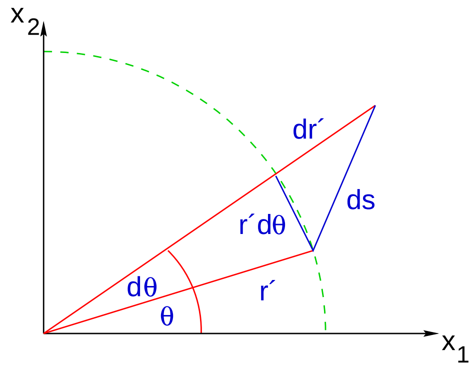
Thus, for the $\mathbb{R}^2$,

$$g_{11} = 1 \quad g_{12} = 0$$
$$g_{21} = 0 \quad g_{22} = 1 \quad (7.3)$$

2D Metrics

But: Other coordinate-systems are also possible in the plane!

Changing to polar coordinates r', θ, defined by



$$x_1 =: r'\cos\theta$$
$$x_2 =: r'\sin\theta \quad (7.4)$$

it is easy to see that

$$\mathrm{d}s^2 = \mathrm{d}r'^2 + r'^2\,\mathrm{d}\theta^2 \quad (7.5)$$

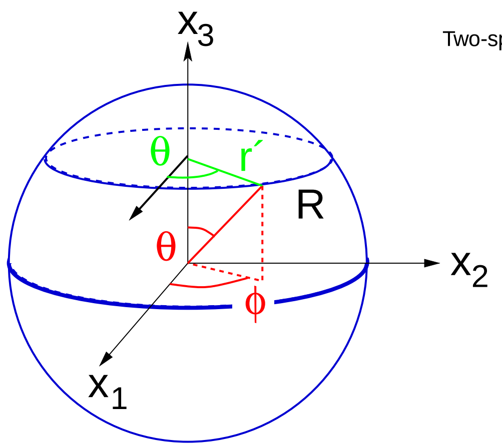
Performing a change of scale by substituting $r' = Rr$, then gives

$$\mathrm{d}s^2 = R^2\{\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2\} \quad (7.6)$$

2D Metrics

A more complicated case occurs if space is curved.

Easiest case: surface of three-dimensional sphere (a two-sphere).



After Kolb & Turner (1990, Fig. 2.1)

Two-sphere with radius R in $\mathbb{R}^3$:

$$x_1^2 + x_2^2 + x_3^2 = R^2 \tag{7.7}$$

Length element of $\mathbb{R}^3$:

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2$$

Eq. (7.7) gives

$$x_3 = \sqrt{R^2 - x_1^2 - x_2^2}$$

such that

$$\mathrm{d}x_3 = \frac{\partial x_3}{\partial x_1}\,\mathrm{d}x_1 + \frac{\partial x_3}{\partial x_2}\,\mathrm{d}x_2 = -\frac{x_1\,\mathrm{d}x_1 + x_2\,\mathrm{d}x_2}{\sqrt{R^2 - x_1^2 - x_2^2}} \tag{7.8}$$

2D Metrics

Introduce again polar coordinates r', θ in x_3-plane:

$$x_1 =: r'\cos\theta \quad x_2 =: r'\sin\theta \tag{7.4}$$

(note: r', θ are only unique in upper or lower half-sphere)

The differentials are given by

$$\mathrm{d}x_1 = \cos\theta\,\mathrm{d}r' - r'\sin\theta\,\mathrm{d}\theta \quad \text{and} \quad \mathrm{d}x_2 = \sin\theta\,\mathrm{d}r' + r'\cos\theta\,\mathrm{d}\theta \tag{7.9}$$

In cartesian coordinates, the length element on $\mathscr{S}^2$ is

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \frac{(x_1\,\mathrm{d}x_1 + x_2\,\mathrm{d}x_2)^2}{R^2 - x_1^2 - x_2^2} \tag{7.10}$$

inserting eq. (7.9) gives after some algebra

$$= r'^2\,\mathrm{d}\theta^2 + \frac{R^2}{R^2 - r'^2}\,\mathrm{d}r'^2 \tag{7.11}$$

finally, defining $r = r'/R$ (i.e., $0 \le r \le 1$) results in

$$\mathrm{d}s^2 = R^2\left\{\frac{\mathrm{d}r^2}{1-r^2} + r^2\,\mathrm{d}\theta^2\right\} \tag{7.12}$$

2D Metrics

Alternatively, we can work in spherical coordinates on $\mathscr{S}^2$

$$\begin{aligned} x_1 &= R\sin\theta\cos\phi \\ x_2 &= R\sin\theta\sin\phi \\ x_3 &= R\cos\theta \end{aligned} \tag{7.13}$$

($\theta \in [0,\pi]$, $\phi \in [0, 2\pi]$).

Going through the same steps as before, we obtain after some tedious algebra

$$\mathrm{d}s^2 = R^2\left\{\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right\} \tag{7.14}$$

2D Metrics

(Important) remarks:

1. The 2-sphere has no edges, has no boundaries, but has still a finite volume, $V = 4\pi R^2$.
2. Expansion or contraction of sphere caused by variation of $R \Longrightarrow R$ determines the *scale* of volumes and distances on $\mathscr{S}^2$.

   R is called the *scale factor*

3. Positions on $\mathscr{S}^2$ are defined, e.g., by r and θ, *independent* on the value of R

   r and θ are called *comoving coordinates*

4. Although the metrics Eq. (7.10), (7.12), and (7.14) look *very* different, they still describe the *same* space $\Longrightarrow$ that's why physics should be covariant, i.e., independent of the coordinate system!

2D Metrics

The hyperbolic plane, $\mathscr{H}^2$, is defined by

$$x_1^2 + x_2^2 - x_3^2 = -R^2 \tag{7.15}$$

If we work in Minkowski space, where

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 - \mathrm{d}x_3^2 \tag{7.16}$$

then

$$= \mathrm{d}x_1^2 + \mathrm{d}x_2^2 - \frac{(x_1\,\mathrm{d}x_1 + x_2\,\mathrm{d}x_2)^2}{R^2 + x_1^2 + x_2^2} \tag{7.17}$$

$\Longrightarrow$ substitute $R \to iR$ (where $i = \sqrt{-1}$) to obtain same form as for sphere (eq. 7.11)!

Therefore,

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1+r^2} + r^2\,\mathrm{d}\theta^2 \right\} \tag{7.18}$$

Transcript of Maple session to obtain Eq. (7.20):

```
> x1:=r*sinh(theta)*cos(phi);
```
$$x1 := r\sinh(\theta)\cos(\phi)$$
```
> x2:=r*sinh(theta)*sin(phi);
```
$$x2 := r\sinh(\theta)\sin(\phi)$$
```
> x3:=r*cosh(theta);
```
$$x3 := r\cosh(\theta)$$
```
> dx1:=diff(x1,theta)*dtheta+diff(x1,phi)*dphi;
```
$$dx1 := r\cosh(\theta)\cos(\phi)\,dtheta - r\sinh(\theta)\sin(\phi)\,dphi$$
```
> dx2:=diff(x2,theta)*dtheta+diff(x2,phi)*dphi;
```
$$dx2 := r\cosh(\theta)\sin(\phi)\,dtheta + r\sinh(\theta)\cos(\phi)\,dphi$$
```
> ds2:=dx1*dx1+dx2*dx2-(x1*dx1+x2*dx2)^2/(r^2+x 1^2+x2^2);
```
$$ds2 := (r\cosh(\theta)\cos(\phi)\,dtheta - r\sinh(\theta)\sin(\phi)\,dphi)^2 + (r\cosh(\theta)\sin(\phi)\,dtheta + r\sinh(\theta)\cos(\phi)\,dphi)^2 - (r\sinh(\theta)\cos(\phi)\,(r\cosh(\theta)\cos(\phi)\,dtheta - r\sinh(\theta)\sin(\phi)\,dphi) + r\sinh(\theta)\sin(\phi)\,(r\cosh(\theta)\sin(\phi)\,dtheta + r\sinh(\theta)\cos(\phi)\,dphi))^2 \Big/ (r^2 + r^2\sinh(\theta)^2\cos(\phi)^2 + r^2\sinh(\theta)^2\sin(\phi)^2)$$
```
> expand(ds2);
```
$$r^2\cosh(\theta)^2\cos(\phi)^2\,dtheta^2 + r^2\sinh(\theta)^2\sin(\phi)^2\,dphi^2 + r^2\cosh(\theta)^2\sin(\phi)^2\,dtheta^2 + r^2\sinh(\theta)^2\cos(\phi)^2\,dphi^2 - \frac{r^4\sinh(\theta)^2\cos(\phi)^4\cosh(\theta)^2\,dtheta^2}{\%1} - 2\,\frac{r^4\sinh(\theta)^2\cos(\phi)^2\cosh(\theta)^2\,dtheta^2\sin(\phi)^2}{\%1} - \frac{r^4\sinh(\theta)^2\sin(\phi)^4\cosh(\theta)^2\,dtheta^2}{\%1}$$
$$\%1 := r^2 + r^2\sinh(\theta)^2\cos(\phi)^2 + r^2\sinh(\theta)^2\sin(\phi)^2$$
```
> simplify(",{cosh(theta)^2-sinh(theta)^2=1}, [cosh(theta)]);
```
$$r^2\,dtheta^2 + r^2\sinh(\theta)^2\,dphi^2$$

2D Metrics

The analogy to spherical coordinates on the hyperbolic plane are given by

$$\begin{aligned} x_1 &= R\sinh\theta\cos\phi \\ x_2 &= R\sinh\theta\sin\phi \\ x_3 &= R\cosh\theta \end{aligned} \tag{7.19}$$

($\theta \in [-\infty, +\infty]$, $\phi \in [0, 2\pi]$).

A session with Maple (see handout) will convince you that these coordinates give

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \sinh^2\theta\,\mathrm{d}\phi^2 \right\} \tag{7.20}$$

Remark:

$\mathscr{H}^2$ is unbound and has an infinite volume.

2D Metrics

To summarize:

Sphere: $$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1-r^2} + r^2\,\mathrm{d}\theta^2 \right\} \tag{7.12}$$

Plane: $$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 \right\} \tag{7.6}$$

Hyperbolic Plane: $$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1+r^2} + r^2\,\mathrm{d}\theta^2 \right\} \tag{7.18}$$

$\Longrightarrow$ All three metrics can be written as

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1-k\,r^2} + r^2\,\mathrm{d}\theta^2 \right\} \tag{7.21}$$

where k defines the geometry:

$$k = \begin{cases} +1 & \text{spherical} \\ 0 & \text{planar} \\ -1 & \text{hyperbolic} \end{cases} \tag{7.22}$$

2D Metrics

For "spherical coordinates" we found:

Sphere: $\mathrm{d}s^2 = R^2\left\{\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right\}$ (7.14)

Plane: $\mathrm{d}s^2 = R^2\left\{\mathrm{d}\theta^2 + \theta^2\mathrm{d}\phi^2\right\}$ (7.6)

Hyperbolic Plane: $\mathrm{d}s^2 = R^2\left\{\mathrm{d}\theta^2 + \sinh^2\theta\,\mathrm{d}\phi^2\right\}$ (7.20)

$\Longrightarrow$ All three metrics can be written as

$$\mathrm{d}s^2 = R^2\left\{\mathrm{d}\theta^2 + S_k^2(\theta)\,\mathrm{d}\phi^2\right\} \quad (7.23)$$

where

$$S_k(\theta) = \begin{cases} \sin\theta & \text{for } k = +1 \\ \theta & \text{for } k = 0 \\ \sinh\theta & \text{for } k = -1 \end{cases} \quad \text{and} \quad C_k(\theta) = \sqrt{1 - kS_k^2(\theta)} = \begin{cases} \cos\theta & \text{for } k = +1 \\ 1 & \text{for } k = 0 \\ \cosh\theta & \text{for } k = -1 \end{cases} \quad (7.24)$$

The cos-like analogue of S_k, C_k, will be needed later

Note that, compared to the earlier formulae, some coordinates have been renamed. This is confusing, but legal…

RW Metric

- Cosmological principle + expansion $\Longrightarrow$ $\exists$ freely expanding cosmical coordinate system.
  - Observers =: fundamental observers
  - Time =: cosmic time

  This is the coordinate system in which the 3K radiation is isotropic, clocks can be synchronized, e.g., by adjusting time to the local density of the universe.

  $\Longrightarrow$ Metric has temporal and spatial part.

  This also follows directly from the equivalence principle.
- *Homogeneity and isotropy* $\Longrightarrow$ spatial part is spherically symmetric:

$$\mathrm{d}\psi^2 := \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 \quad (7.25)$$

- *Expansion:* $\exists$ scale factor, $R(t)$ $\Longrightarrow$ measure distances using comoving coordinates.

$\Longrightarrow$ metric looks like

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\left[f^2(r)\,\mathrm{d}r^2 + g^2(r)\,\mathrm{d}\psi^2\right] \quad (7.26)$$

where $f(r)$ and $g(r)$ are arbitrary.

RW Metric

Metrics of the form of eq. (7.26) are called Robertson-Walker (RW) metrics (introduced in 1935).

Previously studied by Friedmann and Lemaître…

One common choice is

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\left[\mathrm{d}r^2 + S_k^2(r)\,\mathrm{d}\psi^2\right] \quad (7.27)$$

where

$R(t)$: scale factor, containing the physics

t: cosmic time

r, θ, ϕ: comoving coordinates (remember Eq. (7.25) ($\mathrm{d}\psi^2 := \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2$)!)

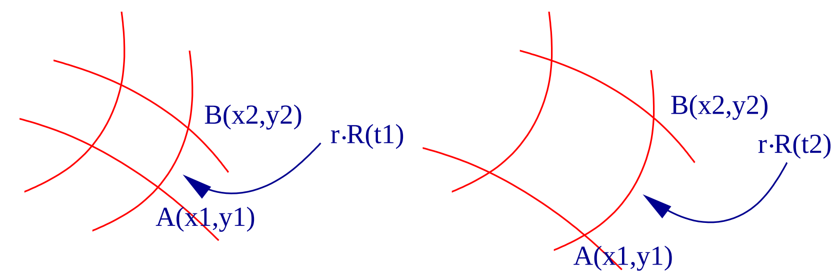
k: defines curvature, integer

$S_k(r)$ was defined in Eq. (7.24).

Remark: θ and ϕ describe directions on sky, as seen from the arbitrary center of the coordinate system (=us), r can be interpreted as a radial coordinate.

RW Metric

The RW metric defines an universal coordinate system tied to expansion of space:



Scale factor $R(t)$ describes evolution of universe.

- r is called the comoving distance.
- $D(t) := r \cdot R(t)$ is called the proper distance,

(e.g., $r \cdot R(t)$ is measured in Mpc)

RW Metric

Other forms of the RW metric are also used:

1. Substitution $S_k(r) \longrightarrow r$ gives

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\left\{\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\psi^2\right\} \quad (7.28)$$

(i.e., other definition of comoving radius r, which is still dimensionless).

2. A metric with a dimensionless scale factor,

$$a(t) := \frac{R(t)}{R(t_0)} = \frac{R(t)}{R_0} \quad (7.29)$$

(where t_0 = today, i.e., $a(t_0) = 1$), gives

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)\left\{\mathrm{d}r^2 + \frac{S_k^2(R_0r)}{R_0^2}\,\mathrm{d}\psi^2\right\} \quad (7.30)$$

RW Metric

3. Using $a(t)$ *and* the substitution $S_k(r) \longrightarrow r$ is also possible:

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)\left\{\frac{\mathrm{d}r^2}{1-k\cdot(R_0r)^2} + r^2\,\mathrm{d}\psi^2\right\} \quad (7.31)$$

The units of R_0r are Mpc $\Longrightarrow$ *Used for observations!*

4. Replace cosmic time, t, by conformal time, $\mathrm{d}\eta = \mathrm{d}t/R(t)$

$\Longrightarrow$ conformal metric,

$$\mathrm{d}s^2 = R^2(\eta)\left\{\mathrm{d}\eta^2 - \frac{\mathrm{d}r^2}{1-kr} - r^2\,\mathrm{d}\psi^2\right\} \quad (7.32)$$

Theoretical importance of this metric: For $k = 0$, i.e., a flat space, the RW metric = Minkowski line element $\times\, R^2(\eta) \Longrightarrow$ Equivalence principle!

RW Metric

5. Finally, the metric can also be written in the isotropic form,

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \frac{R(t)}{1+(k/4)r^2}\left\{\mathrm{d}r^2 + r^2\mathrm{d}\psi^2\right\} \quad (7.33)$$

Here, the term in $\{\dots\}$ is just the line element of a 3d-sphere $\Longrightarrow$ isotropy!

Note: There are as many notations as authors, e.g., some use $a(t)$ where we use $R(t)$, etc. $\Longrightarrow$ Be careful!

Note 2: Local homogeneity and isotropy (i.e., within a Hubble radius, $r = c/H_0$), do not imply *global* homogeneity and isotropy $\Longrightarrow$ Cosmologies with a non-trivial topology are possible (e.g., also with more dimensions...).

Friedmann Equations

General relativistic approach: Insert metric into Einstein equation to obtain differential equation for $R(t)$:

Einstein equation:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathscr{R}g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \quad (7.34)$$

where

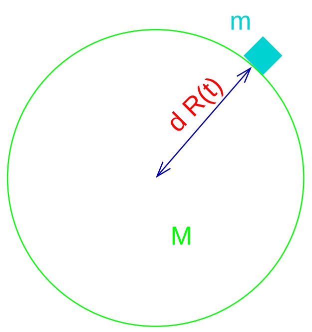
$g_{\mu\nu}$: Metric tensor ($\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$)
- $R_{\mu\nu}$: Ricci tensor (function of $g_{\mu\nu}$)
- $\mathscr{R}$: Ricci scalar (function of $g_{\mu\nu}$)
- $G_{\mu\nu}$: Einstein tensor (function of $g_{\mu\nu}$)

$T_{\mu\nu}$: Stress-energy tensor, describing curvature of space due to fields present (matter, radiation,...)

Λ: Cosmological constant

$\Longrightarrow$ Messy, but doable

Friedmann Equations



Here, Newtonian derivation of Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (McCrea, 1937)

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \tag{7.35}$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\big(d\,R(t)\big) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}m \tag{7.36}$$

Canceling $m \cdot d$ gives momentum equation:

$$\ddot{R}(t) = -\frac{4\pi G}{3}\frac{\rho_0}{R(t)^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{7.37}$$

Multiplying Eq. (7.37) with $\dot{R}$ and integrating yields the energy equation:

$$\frac{1}{2}\dot{R}(t)^2 = +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.} = +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.} \tag{7.38}$$

where the constant can only be obtained from GR.

Friedmann Equations

Problems with the Newtonian derivation:

1. Cloud is implicitly assumed to have $r_{\text{cloud}} < \infty$
   (for $r_{\text{cloud}} \to \infty$ the force is undefined)
   $\Longrightarrow$ violates cosmological principle.
2. Particles move *through* space
   $\Longrightarrow v > c$ possible
   $\Longrightarrow$ violates SRT.

Why do we get correct result?

GRT $\longrightarrow$ Newton for small scales and mass densities

Since universe is isotropic: scale invariance on Mpc scales

$\Longrightarrow$ Newton sufficient (classical limit of GR).

(In fact, point 1 above *does* hold in GR: Birkhoff's theorem).

Friedmann Equations

The exact GR derivation of Friedmanns equation gives:

$$\begin{aligned}\ddot{R} &= -\frac{4\pi G}{3}R\left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right] \\ \dot{R}^2 &= +\frac{8\pi G\rho}{3}R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right]\end{aligned} \tag{7.39}$$

Notes:

1. For $\Lambda = 0$: Eq. (7.39) $\longrightarrow$ Eq. (7.38).
2. k determines the curvature of space (and is *not* an integer here!).
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. There is energy associated with the vacuum, parameterized by the parameter Λ.

In Eq. 7.49, it will be shown that the Hubble parameter can be expressed as $H(t) = \frac{\dot{R}}{R}$.

For $\Lambda = 0$, it evolves as:

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2(t) = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} \tag{7.40}$$

The Critical Density

Solving Eq. (7.40) for k:

$$\frac{R^2}{c}\left(\frac{8\pi G}{3}\rho - H^2\right) = k \tag{7.41}$$

$\Longrightarrow$ Sign of curvature parameter k only depends on density, ρ. With

$$\rho_c = \frac{3H^2}{8\pi G} \quad \text{and} \quad \Omega = \frac{\rho}{\rho_c} \tag{7.42}$$

it is easy to see that:

$$\begin{aligned}\Omega > 1 &\Longrightarrow k > 0 \Longrightarrow \text{closed universe} \\ \Omega = 1 &\Longrightarrow k = 0 \Longrightarrow \text{flat universe} \\ \Omega < 1 &\Longrightarrow k < 0 \Longrightarrow \text{open universe}\end{aligned}$$

ρ_c is called the critical density.

For $\Omega \leq 1$ the universe will expand until ∞,

For $\Omega > 1$ we will see the "big crunch".

Current value of ρ_c: $\sim 1.67 \times 10^{-24}\,\text{g cm}^{-3}$ (3…10 H-atoms m^{-3}).

The Critical Density

Ω has a second order effect on the expansion:

Taylor series of $R(t)$ around $t = t_0$:

$$\frac{R(t)}{R(t_0)} = \frac{R(t_0)}{R(t_0)} + \frac{\dot{R}(t_0)}{R(t_0)}(t - t_0) + \frac{1}{2}\frac{\ddot{R}(t_0)}{R(t_0)}(t - t_0)^2 \qquad (7.43)$$

The Friedmann equation Eq. (7.37) can be written

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\rho = -\frac{4\pi G}{3}\Omega\frac{3H^2}{8\pi G} = -\frac{\Omega H^2}{2} \qquad (7.44)$$

Since $H(t) = \dot{R}/R$ (Eq. 7.49), Eq. (7.43) is

$$\frac{R(t)}{R(t_0)} = 1 + H_0(t - t_0) - \frac{1}{2}\frac{\Omega_0}{2}H_0^2(t - t_0)^2 \qquad (7.45)$$

where $H_0 = H(t_0)$ and $\Omega_0 = \Omega(t_0)$.

The subscript 0 is often omitted in the case of Ω.

Often, Eq. (7.45) is written using the deceleration parameter:

$$q := \frac{\Omega}{2} = -\frac{\ddot{R}(t_0)R(t_0)}{\dot{R}^2(t_0)} \qquad (7.46)$$

Hubble's Law

Hubble's Law follows from the variation of $R(t)$:

Small scales $\Longrightarrow$ Euclidean geometry. Then the proper distance between two observers is:

$$D(t) = d \cdot R(t) \qquad (7.47)$$

where d: comoving distance.

Expansion $\Longrightarrow$ proper separation changes:

$$\frac{\Delta D}{\Delta t} = \frac{R(t+\Delta t)d - R(t)d}{\Delta t} \Longrightarrow \lim_{\Delta t \to 0} \Longrightarrow \quad v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}\,d = \frac{\dot{R}}{R}D =: H\,D \qquad (7.48)$$

$\Longrightarrow$ Identify local Hubble "constant" as

$$H = \frac{\dot{R}}{R} = \dot{a}(t) \quad (a(t) \text{ from Eq. 7.29, } a(\text{today}) = 1) \qquad (7.49)$$

Since $R = R(t) \Longrightarrow H$ is time-dependent!

Redshift

The cosmological redshift is a consequence of the expansion of the universe:

The comoving distance is constant, thus in terms of the proper distance:

$$d = \frac{D(t = \text{today})}{R(t = \text{today})} = \frac{D(t)}{R(t)} = \text{const.} \qquad (7.50)$$

Set $a(t) = R(t)/R(t = \text{today})$, then eq. (7.50) implies

$$\lambda_{\text{obs}} = \frac{\lambda_{\text{emit}}}{a_{\text{emit}}} \qquad (7.51)$$

(λ_{obs}: observed wavelength, λ_{emit}: emitted wavelength)

Thus the observed redshift is

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1 = \frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} - 1 \qquad (7.52)$$

$$\Longrightarrow \quad 1 + z = \frac{1}{a_{\text{emit}}} = \frac{R(t = \text{today})}{R(t)} = \frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} \qquad (7.53)$$

Light emitted at $z = 1$ was emitted when the universe was half as big as today!

z: measure for *relative size* of universe at time the observed light was emitted.

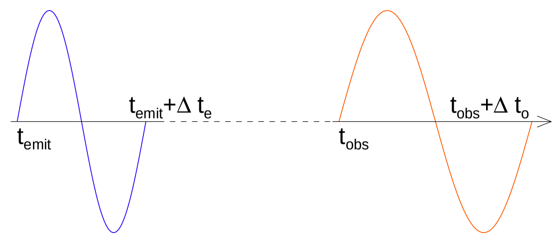
Note that the definition of H allows us to derive Hubble's relation for the case of small v, i.e., $v \ll c$. In this case, the red-shift is

$$z = \frac{v}{c} \Longrightarrow z = \frac{Hd}{c} \qquad (7.54)$$

An alternative derivation of the cosmological redshift follows directly from general relativity, using the basic GR fact that for photons $\mathrm{d}s^2 = 0$. Inserting this into the metric, and assuming without loss of generality that $\mathrm{d}\psi^2 = 0$, one finds

$$0 = c^2\,\mathrm{d}t^2 - R^2(t)\,\mathrm{d}r^2 \Longrightarrow \mathrm{d}r = \pm\frac{c\,\mathrm{d}t}{R(t)} \qquad (7.55)$$

Since photons travel forward, we choose the $+$-sign.



The *comoving* distance traveled by photons emitted at cosmic times t_{emit} and $t_{\text{emit}} + \Delta t_e$ is

$$r_1 = \int_{t_{\text{emit}}}^{t_{\text{obs}}} \frac{c\,\mathrm{d}t}{R(t)} \quad \text{and} \quad r_2 = \int_{t_{\text{emit}}+\Delta t_e}^{t_{\text{obs}}+\Delta t_o} \frac{c\,\mathrm{d}t}{R(t)} \qquad (7.56)$$

But the comoving distances are equal, $r_1 = r_2$! Therefore

$$0 = \int_{t_\mathrm{emit}}^{t_\mathrm{obs}} \frac{c\,\mathrm{d}t}{R(t)} - \int_{t_\mathrm{emit}+\Delta t_\mathrm{e}}^{t_\mathrm{obs}+\Delta t_\mathrm{o}} \frac{c\,\mathrm{d}t}{R(t)} \tag{7.57}$$

$$= \int_{t_\mathrm{emit}}^{t_\mathrm{emit}+\Delta t_\mathrm{e}} \frac{c\,\mathrm{d}t}{R(t)} - \int_{t_\mathrm{obs}}^{t_\mathrm{obs}+\Delta t_\mathrm{o}} \frac{c\,\mathrm{d}t}{R(t)} \tag{7.58}$$

If Δt small $\Longrightarrow R(t) \approx$ const.:

$$= \frac{c\,\Delta t_\mathrm{e}}{R(t_\mathrm{emit})} - \frac{c\,\Delta t_\mathrm{o}}{R(t_\mathrm{obs})} \tag{7.59}$$

For a wave: $c\Delta t = \lambda$, such that

$$\frac{\lambda_\mathrm{emit}}{R(t_\mathrm{emit})} = \frac{\lambda_\mathrm{obs}}{R(t_\mathrm{obs})} \iff \frac{\lambda_\mathrm{emit}}{\lambda_\mathrm{obs}} = \frac{R(t_\mathrm{emit})}{R(t_\mathrm{obs})} \tag{7.60}$$

From this equation it is straightforward to derive Eq. (7.52).

Redshift

Outside of the local universe: Eq. (7.53) only valid interpretation of z.

$\Longrightarrow$ It is common to interpret z as in special relativity:

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{7.61}$$

This is WRONG

Redshift is due to expansion of space, not due to motion of galaxy.

What *is* true is that z is accumulation of many infinitesimal red-shifts à la Eq. (7.54), see, e.g., Peacock (1999).

Time Dilatation

For light, $D = c\,\Delta t$. Then a consequence of Eq. (7.50) is

$$\frac{c\,\Delta t_\mathrm{emit}}{R(t_\mathrm{emit})} = \frac{c\,\Delta t_\mathrm{obs}}{R(t_\mathrm{obs})} \quad\Longrightarrow\quad \frac{\mathrm{d}t}{R} = \text{const.} \tag{7.59}$$

In other words:

$$\frac{\mathrm{d}t_\mathrm{obs}}{\mathrm{d}t_\mathrm{emit}} = \frac{R(t_\mathrm{obs})}{R(t_\mathrm{emit})} = 1 + z \tag{7.62}$$

$\Longrightarrow$ Time dilatation of events at large z.

This cosmological time dilatation has been observed in the light curves of supernova outbursts.

All other observables apart from z (e.g., number density $N(z)$, luminosity distance d_L, etc.) require explicit knowledge of $R(t)$

$\Longrightarrow$ Need to look at the dynamics of the universe.

Equation of state

Evolution of the universe determined by three different kinds of equation of state:

1. Matter: Normal (nonrelativistic) particles get diluted by expansion of the universe:

$$\rho_\mathrm{m} \propto R^{-3} \tag{7.63}$$

Matter is also often called dust by cosmologists.

2. Radiation: The energy density of radiation decreases because of volume expansion and because of the cosmological redshift (Eq. 7.60: $\lambda_\mathrm{obs}/\lambda_\mathrm{emit} = \nu_\mathrm{emit}/\nu_\mathrm{obs} = R(t_\mathrm{obs})/R(t_\mathrm{emit})$) such that

$$\rho_\mathrm{r} \propto R^{-4} \tag{7.64}$$

3. Vacuum: The vacuum energy density (=Λ) is independent of R:

$$\rho_\mathrm{v} = \text{const.} \tag{7.65}$$

Inserting these equations of state into the Friedmann equation and solving with the boundary condition $R(t=0) = 0$ then gives a specific world model.

Equation of state

Current scale factor is determined by H_0 and Ω_0:

Friedmann for $t = t_0$:

$$\dot{R}_0^2 - \frac{8\pi G}{3}\rho R_0^2 = -kc^2 \qquad (7.66)$$

Insert Ω and note $H_0 = \dot{R}_0/R_0$

$$\Longleftrightarrow \quad H_0^2 R_0^2 - H_0^2 \Omega_0 R_0^2 = -kc^2 \qquad (7.67)$$

And therefore

$$R_0 = \frac{c}{H_0}\sqrt{\frac{k}{\Omega - 1}} \qquad (7.68)$$

For $\Omega \longrightarrow 0$, $R_0 \longrightarrow c/H_0$, the Hubble length.

For $\Omega = 1$, R_0 is arbitrary.

We now have everything we need to solve the Friedmann equation and determine the evolution of the universe for $k = 0$, $+1$, and -1.

$k = 0$, Matter dominated

For the matter dominated, flat case (the Einstein-de Sitter case), the Friedmann equation is

$$\dot{R}^2 - \frac{8\pi G}{3}\frac{\rho_0 R_0^3}{R^3} R^2 = 0 \qquad (7.69)$$

For $k = 0$: $\Omega = 1$ and

$$\frac{8\pi G \rho_0}{3} = \Omega_0 H_0^2 R_0^3 = H_0^2 R_0^3 \qquad (7.70)$$

Therefore, the Friedmann eq. is

$$\dot{R}^2 - \frac{H_0^2 R_0^3}{R} = 0 \quad \Longrightarrow \quad \frac{\mathrm{d}R}{\mathrm{d}t} = H_0 R_0^{3/2} R^{-1/2} \qquad (7.71)$$

Separation of variables and setting $R(0) = 0$,

$$\int_0^{R(t)} R^{1/2}\,\mathrm{d}R = H_0 R_0^{3/2} t \quad \Longrightarrow \quad \frac{2}{3}R^{3/2}(t) = H_0 R_0^{3/2} t \quad \Longrightarrow \quad R(t) = R_0 \left(\frac{3H_0}{2}\, t\right)^{2/3} \qquad (7.72)$$

Therefore, for $k = 0$, the universe expands until ∞, its current age ($R(t_0) = R_0$) is given by

$$t_0 = \frac{2}{3H_0} \qquad (7.73)$$

Reminder: The Hubble-Time is $H_0^{-1} = 9.78\,\mathrm{Gyr}/h$.

For the matter dominated, closed case, Friedmann's equation is

$$\dot{R}^2 - \frac{8\pi G}{3}\frac{\rho_0 R_0^3}{R} = -c^2 \quad \Longleftrightarrow \quad \dot{R}^2 - \frac{H_0^2 R_0^3 \Omega_0}{R} = -c^2 \qquad (7.74)$$

Inserting R_0 from Eq. (7.68) gives

$$\dot{R}^2 - \frac{H_0^2 c^3 \Omega_0}{H_0^3(\Omega - 1)^{3/2}}\frac{1}{R} = -c^2 \qquad (7.75)$$

which is equivalent to

$$\frac{\mathrm{d}R}{\mathrm{d}t} = c\left(\frac{\xi}{R} - 1\right)^{1/2} \quad \text{with} \quad \xi = \frac{c}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \qquad (7.76)$$

With the boundary condition $R(0) = 0$, separation of variables gives

$$ct = \int_0^{R(t)} \frac{\mathrm{d}R}{(\xi/R - 1)^{1/2}} = \int_0^{R(t)} \frac{\sqrt{R}\,\mathrm{d}R}{(\xi - R)^{1/2}} \qquad (7.77)$$

Integration by substitution gives the "cycloid solution"

$$R = \xi \sin^2\frac{\theta}{2} = \frac{\xi}{2}(1 - \cos\theta) \quad \text{and} \quad ct = \frac{\xi}{2}(\theta - \sin\theta) \qquad (7.78)$$

where θ is an implicit parameter.

The age of the universe, t_0, is obtained by solving

$$R_0 = \frac{c}{H_0(\Omega_0 - 1)^{1/2}} = \frac{\xi}{2}(1 - \cos\theta_0) = \frac{1}{2}\frac{c}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}(1 - \cos\theta_0) \qquad (7.79)$$

(remember Eq. 7.68!). Therefore

$$\cos\theta_0 = \frac{2 - \Omega_0}{\Omega_0} \quad \Longleftrightarrow \quad \sin\theta_0 = \frac{2}{\Omega_0}\sqrt{\Omega_0 - 1} \qquad (7.80)$$

Inserting this into Eq. (7.78) gives

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}\left[\arccos\left(\frac{2 - \Omega_0}{\Omega_0}\right) - \frac{2}{\Omega_0}\sqrt{\Omega_0 - 1}\right] \qquad (7.81)$$

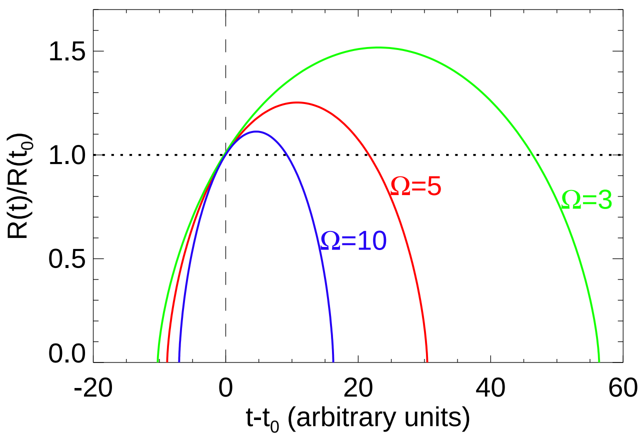
The cycloid solution shows that for $\Omega > 1$, the universe has a finite lifetime, i.e., it expands to a maximum and then becomes smaller and dies in a "big crunch". The max. expansion occurs at $\theta = \pi$, with a maximum scale factor of

$$R_{\text{max}} = \xi = \frac{c}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \qquad (7.82)$$

The big crunch will happen at $\theta = 2\pi$, such that the lifetime of the closed universe is

$$t_{\text{life}} = \frac{\pi}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \qquad (7.83)$$

$k = +1$, Matter dominated



For the closed universe, one finds

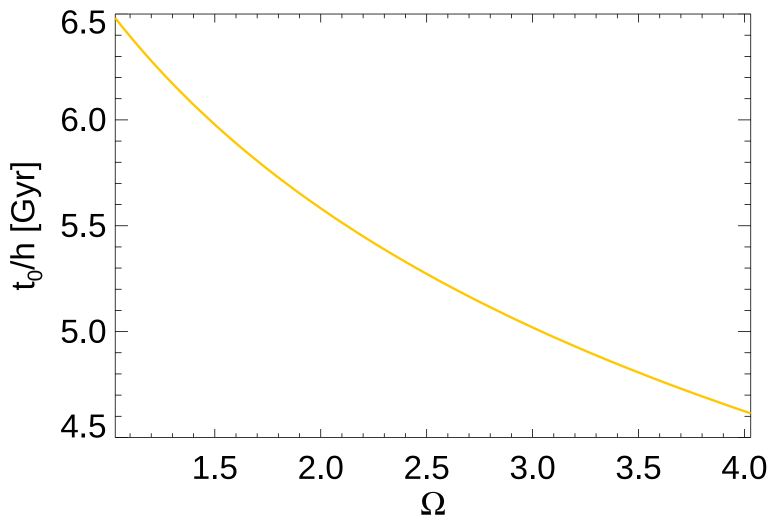
$$R = \frac{\xi}{2}(1-\cos\theta) \qquad ct = \frac{\xi}{2}(\theta - \sin\theta) \tag{7.78}$$

Note that R is a cyclic function

$\Longrightarrow$ The closed universe has a finite lifetime, given by

$$t_{\text{life}} = \frac{\pi}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \tag{7.83}$$

$k = +1$, Matter dominated



Age of a closed and matter dominated universe.

$k = -1$, Matter dominated

Finally, the matter dominated, open case. This case is very similar to the case of $k = +1$:

For $k = -1$, the Friedmann equation becomes

$$\frac{dR}{dt} = c\left(\frac{\zeta}{R} + 1\right)^{1/2} \tag{7.84}$$

where

$$\zeta = \frac{c}{H_0}\frac{\Omega_0}{(1-\Omega_0)^{3/2}} \tag{7.85}$$

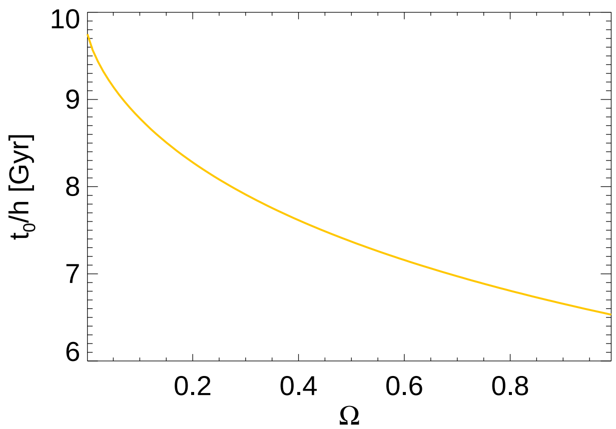
Separation of variables gives after a little bit of algebra

$$R = \frac{\zeta}{2}(\cosh\theta - 1) \qquad ct = \frac{\zeta}{2}(\sinh\theta - 1) \tag{7.86}$$

where the integration was again performed by substitution.

Note: θ here has *nothing* to do with the coordinate angle θ!

$k = -1$, Matter dominated



To obtain the age of the universe, note that at the present time,

$$\cosh\theta_0 = \frac{2-\Omega_0}{\Omega_0} \qquad \sinh\theta_0 = \frac{2}{\Omega_0}\sqrt{1-\Omega_0} \tag{7.87}$$

(identical derivation as that leading to Eq. 7.79)

therefore,

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(1-\Omega_0)^{3/2}} \cdot \left\{\frac{2}{\Omega_0}\sqrt{1-\Omega_0} - \ln\left(\frac{2-\Omega_0+2\sqrt{1-\Omega_0}}{\Omega_0}\right)\right\} \tag{7.88}$$

Summary

For the matter dominated case, our results from Eqs. (7.78), and (7.86) can be written with the functions S_k and C_k (Eq. 7.24) in form of the cycloid solution:

$$R = k\mathscr{R}\left(1 - C_k(\theta)\right) \qquad ct = k\mathscr{R}\left(\theta - S_k(\theta)\right) \tag{7.89}$$

with

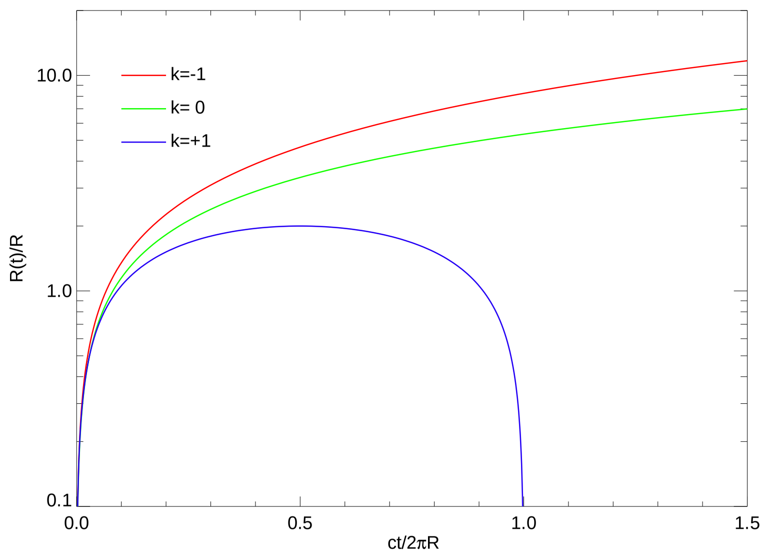
$$S_k(\theta) = \begin{cases} \sin\theta \\ \theta \\ \sinh\theta \end{cases} \text{and} \quad C_k(\theta) = \begin{cases} \cos\theta & \text{for } k = +1 \\ 1 & \text{for } k = 0 \\ \cosh\theta & \text{for } k = -1 \end{cases} \tag{7.24}$$

and where the characteristic radius, $\mathscr{R}$, is given by

$$\mathscr{R} = \frac{c}{H_0} \frac{\Omega_0/2}{\left(k\left(\Omega_0 - 1\right)\right)^{3/2}} \tag{7.90}$$

Notes:

1. Eq. (7.89) can also be derived as the result of the Newtonian collapse/expansion of a spherical mass distribution.
2. θ is called the development angle, it is equal to the *conformal time* (Eq. (7.32)).



7–39

Silk, J., 1997, A Short History of the Universe, Scientific American Library 53, (New York: W. H. Freeman)

Distance Ladder and H_0

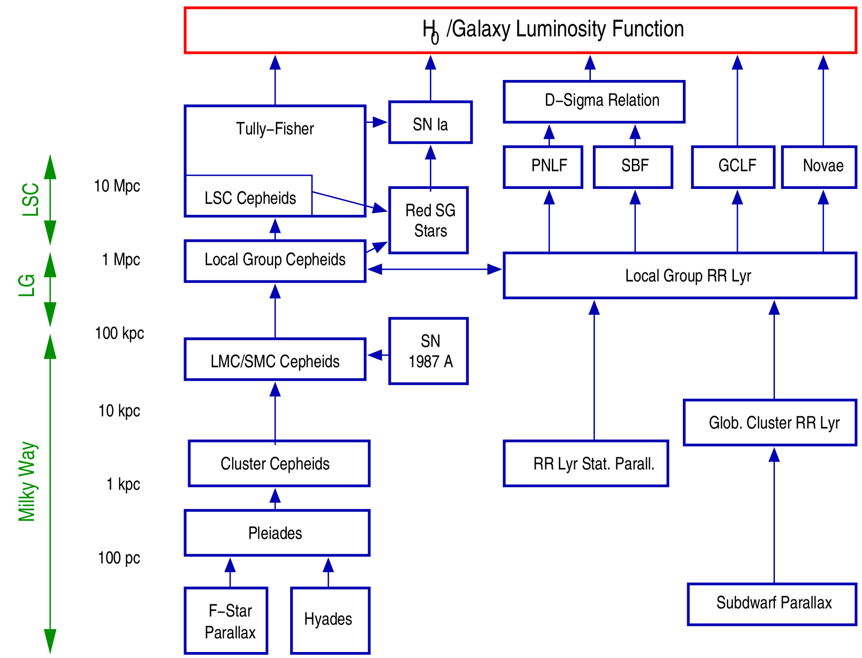
Classical Cosmology

To understand what universe we live in, we need to determine observationally the following numbers:

1. The Hubble constant, H_0
   $\Longrightarrow$ Requires distance measurements.
2. The current density parameter, Ω_0
   $\Longrightarrow$ Requires measurement of the mass density.
3. The cosmological constant, Λ
   $\Longrightarrow$ Requires acceleration measurements.
4. The age of the universe, t_0, for consistency checks
   $\Longrightarrow$ Requires age measurements.

The determination of these numbers is the realm of classical cosmology.

First part: Distance determination and H_0!



(after Jacoby et al., 1992, Fig. 1)

Introduction

Distances are required for determination of H_0.

$\Longrightarrow$ Need to measure distances out to $\sim$200 Mpc to obtain reliable values.

To get this far: cosmological distance ladder.

1. Trigonometric Parallax and Moving Cluster
2. Main Sequence Fitting
3. RR Lyr
4. Baade-Wesselink
5. Cepheids
6. (Light echos)
7. Brightest Stars
8. Type Ia Supernovae
9. Tully-Fisher
10. D_n-σ for ellipticals
11. Brightest Cluster Galaxies
12. Gravitational Lenses

Still the best reference on this subject is ROWAN-ROBINSON, M., 1985, The Cosmological Distance Ladder, New York: Freeman. Newer: Freedman & Madore (2010).

Units

Basic unit of length in astronomy: Astronomical Unit (AU).

Colloquial Definition: 1 AU $=$ mean distance Earth–Sun.

Measurement: (Venus) radar ranging, interplanetary satellite positions, χ^2 minimization of N-body simulations of solar system

1 AU $\sim 149.6 \times 10^6$ km

In the astronomical system of units (IAU 1976), the AU is defined via Gaussian gravitational constant (k), where the acceleration

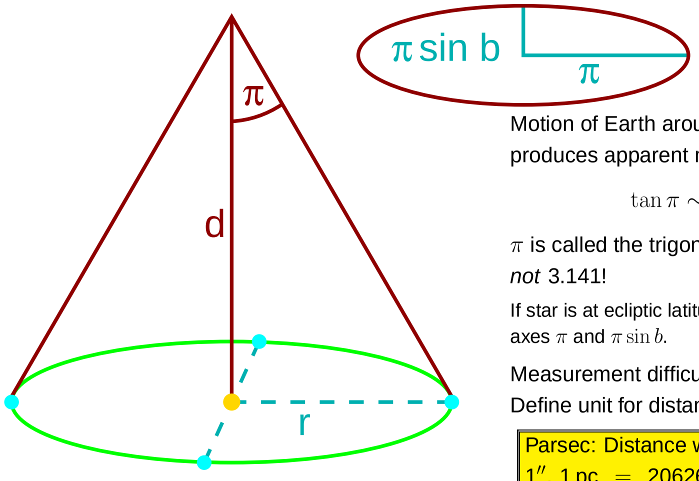
$$\ddot{\mathbf{r}} = -\frac{k^2\mathbf{r}}{r^3}$$

where $k := 0.01720209895$, leading to $a_\oplus = 1.00000105726665$, and 1 AU=1.4959787066 $\times 10^{11}$ m (Seidelmann, 1992).

Reason for this definition: k much better known than G.

(2006 CODATA: $G = 6.67428(67) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, so only known to 4 significant digits)

Trigonometric Parallax



Motion of Earth around Sun $\Longrightarrow$ Parallax produces apparent motion by amount

$$\tan\pi \sim \pi = r_\oplus/d \qquad (8.1)$$

π is called the trigonometric parallax, and *not* 3.141!

If star is at ecliptic latitude b, then ellipse with axes π and $\pi \sin b$.

Measurement difficult: $\pi \lesssim 0.76''$ (α Cen).

Define unit for distance:

Parsec: Distance where 1 AU has $\pi = 1''$. 1 pc $=$ 206265 AU $=$ 3.08×10^{18} cm $= 3.26$ ly

after Rowan-Robinson (1985, Fig. 2.1)

Trigonometric Parallax

Best measurements to date: Hipparcos satellite (1989–1993)

- systematic error of position: ~0.5 mas for stars brighter 9 mag
- effective distance limit: 1 kpc
- standard error of proper motion: ~1 mas yr^{-1}
- broad band photometry
- narrow band: B − V, V − J
- magnitude limit: 12 mag
- complete to mag: 7.3–9.0

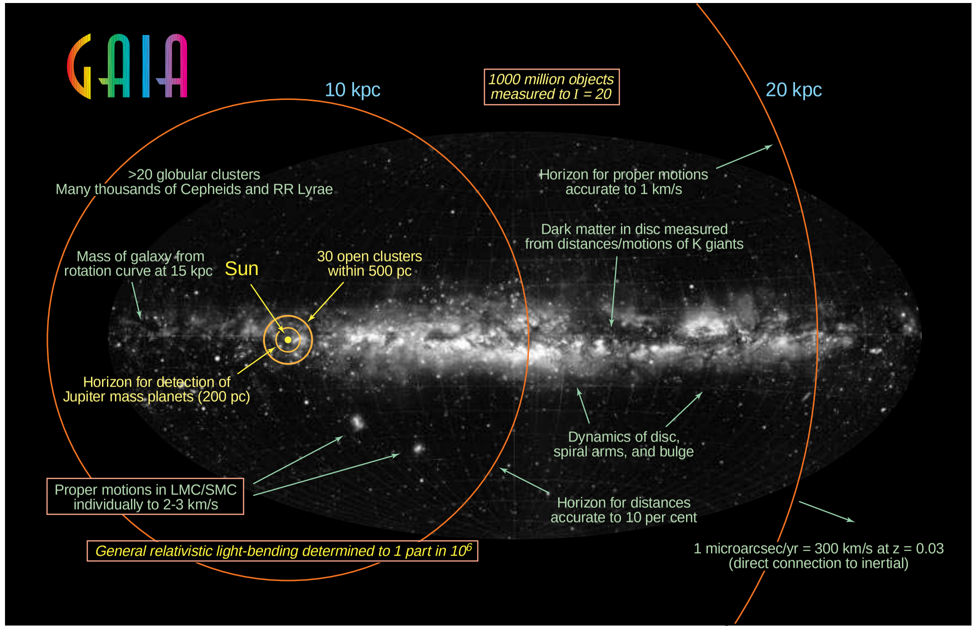
Results available at `http://www.rssd.esa.int/index.php?project=HIPPARCOS`

Hipparcos catalogue: 118 218 objects with milliarcsecond precision.

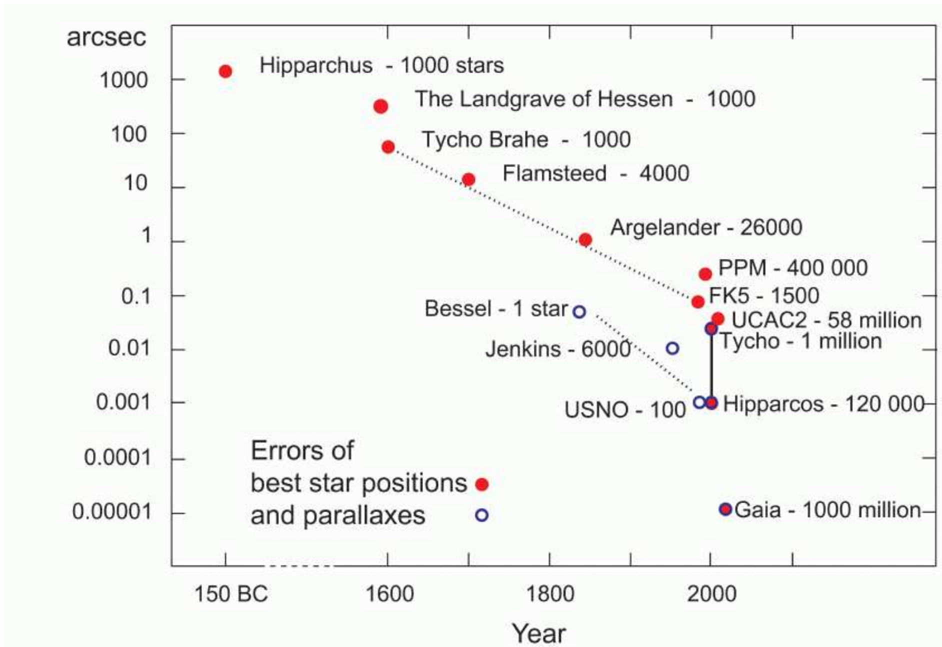
Tycho catalogue: 2 539 913 stars with 20–30 mas precision, two-band photometry (99% complete down to 11 mag)

Revised Hipparcos calibration: see van Leeuwen (2007).

GAIA (ESA mission, to be launched 2012 Nov on Soyuz from Kourou):



GAIA: $\sim 4\mu$arcsec precision, 4 color to $V = 20$ mag, 10^9 objects.



ESA/M. Perryman

Development of the precision of astronomical position measurements

Interlude

Parallax (and Moving Cluster [only of historical interest] and Megamaser [see later]): geometrical methods.

All other methods (exception: light echoes): standard candles.

Requirements for standard candles (Mould et al., 2000a):

1. Physical basis should be understood.
2. Parameters should be measurable objectively.
3. No corrections ("fudges") required.
4. Small intrinsic scatter ($\Longrightarrow$ requiring small number of measurements!).
5. Wide dynamic range in distance.

Magnitudes

Assuming isotropic emission, distance and luminosity are related ("inverse square law") $\Longrightarrow$ luminosity distance:

$$F = \frac{L}{4\pi d_\mathrm{L}^2} \qquad (8.2)$$

where F is the measured flux (erg cm^{-2} s^{-1}) and L the luminosity (erg s^{-1}).

Definition also true for flux densities, I_ν (erg cm^{-2} s^{-1} Å^{-1}).

The magnitude is defined by

$$m = A - 2.5 \log_{10} F \qquad (8.3)$$

where A is a constant used to define the zero point (defined by $m = 0$ mag for Vega).

For a filter with transmission function ϕ_ν,

$$m_i = A_i - 2.5 \log \int \phi_\nu F_\nu \, \mathrm{d}\nu \qquad (8.4)$$

where, e.g., $i =$ U, B, V.

Magnitudes

To enable comparison of luminosities: define

absolute magnitude M = magnitude at distance 10 pc

Thus, since $m = A - 2.5 \log(L/4\pi d^2)$,

$$M = m - 5 \log \left(\frac{d_\mathrm{L}}{10\,\mathrm{pc}} \right) \qquad (8.5)$$

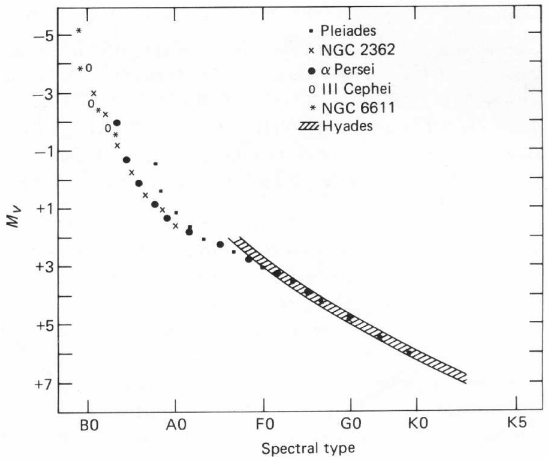
The difference $m - M$ is called the distance modulus, μ_0:

$$\mu_0 = \mathrm{DM} = m - M = 5 \log \left(\frac{d_\mathrm{L}}{10\,\mathrm{pc}} \right) \qquad (8.6)$$

Often, distances are given in terms of $m - M$, and not in pc.

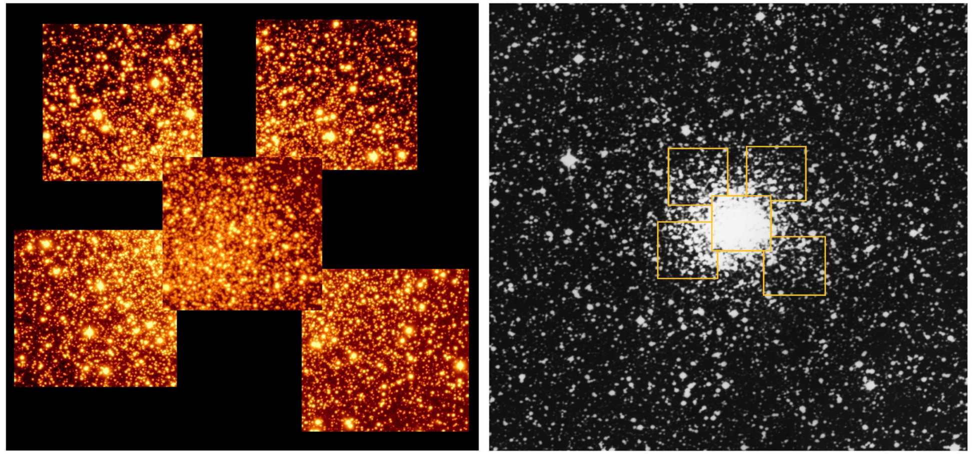
DM [mag]	3	5	10	15	20	25	30
d	40 pc	100 pc	1 kpc	10 kpc	100 kpc	1 Mpc	10 Mpc

Main Sequence Fitting



after Rowan-Robinson (1985, Fig. 2.11)

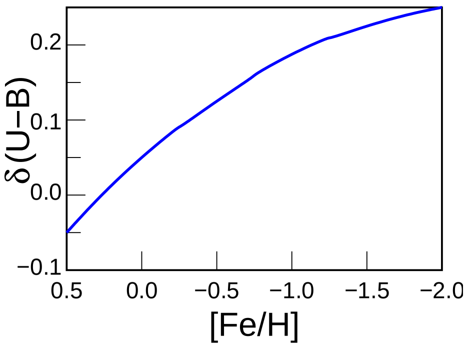
All open clusters are comparably young
$\Longrightarrow$ Hertzsprung Russell Diagram (HRD) dominated by Zero Age Main Sequence (ZAMS).
$\Longrightarrow$ Measure HRD (or Color Magnitude Diagram; CMD), shift magnitude scale until main sequence aligns
$\Longrightarrow$ distance modulus.



Globular Cluster NGC 6712

ESO PR Photo 06a/99 (18 February 1999)

Main Sequence Fitting



(after Rowan-Robinson, 1985, Fig. 2.12)

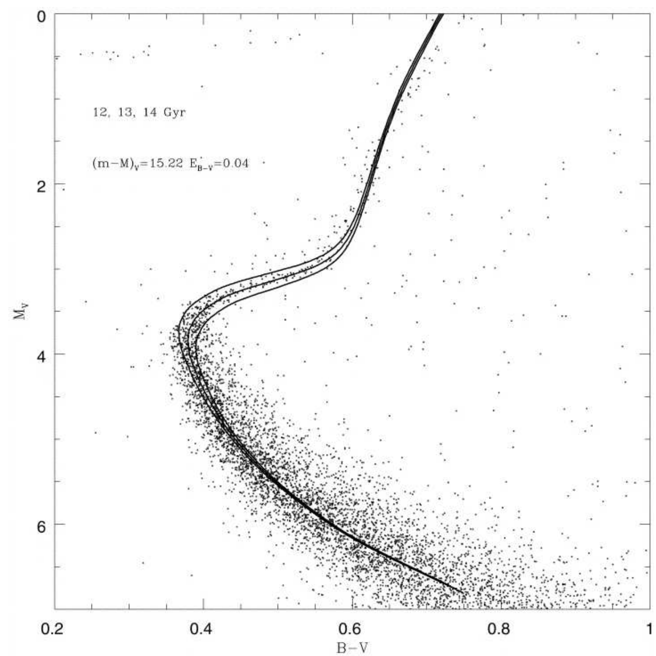
Caveats:

1. Location of ZAMS more age dependent than expected (van Leeuwen, 1999).
2. interstellar extinction
   $\Longrightarrow \mu_0 = \mu_V - A_V$, where μ_V, A_V DM/extinction measured in V-band.
3. metals: line blanketing (change in stellar continuum due to metal absorption lines, see figure)
   $\Longrightarrow$ Changes color
   $\Longrightarrow$ horizontal shift in CMD.

van den Bergh (1977): $Z_{\text{Hyades}} \sim 1.6 Z_\odot$, while other open clusters have solar metallicity $\Longrightarrow$ Cepheid DM were overestimated by 0.15 mag.

4. identification of unevolved stars crucial (evolution to larger magnitudes on MS during stellar life).

Currently: distances to ~200 open clusters known (Fenkart & Binggeli, 1979), limit ~7 kpc.



(M68, Straniero et al., 1997, Fig. 11)

Globular clusters: HRD different from open clusters:

- population II
  $\Longrightarrow Z \ll Z_\odot$
- evolved

Use theoretical HRDs (isochrones) to obtain distance.

For distant clusters: MS unobservable
$\Longrightarrow$ position of horizontal branch.

Baade-Wesselink

For pulsating stars: Basic principle (Baade, 1926) – Assume black body

$\Longrightarrow$ Use color/spectrum to get kT_{eff}

$\Longrightarrow$ Emitted intensity is Planckian, B_ν

$\Longrightarrow$ Observed Intensity is $I_\nu \propto \pi R_*^2 \cdot B_\nu$.

Radius from integrating velocity profile of spectral lines:

$$R_2 - R_1 = p \int_1^2 v \, \mathrm{d}t \qquad (8.7)$$

(p: projection factor between velocity vector and line of sight).

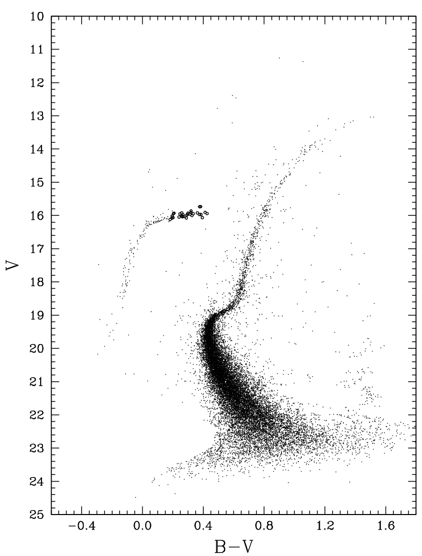
Wesselink (1947): Determine brightness for times of same color

$\Longrightarrow$ rather independent of knowledge of stellar spectrum (deviations from B_ν).

Stars: Calibration using interferometric diameters of nearby giants.

Baade-Wesselink works for pulsating stars such as RR Lyr, Cepheids, Miras, and expanding supernova remnants.

RR Lyr



RR Lyrae variables: Stars crossing instability strip in HRD

$\Longrightarrow$ Variability ($P \sim 0.2 \ldots 1$ d)

$\Longrightarrow$ RR Lyr gap (change in color!).

Absolute magnitude of RR Lyr gap:

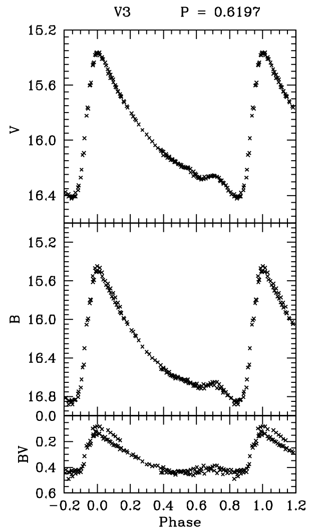
$M_{\text{V}} = 0.6$, $M_{\text{B}} = 0.8$ mag, i.e.,

$L_{\text{RR}} \sim 50 \, L_\odot$.

M determined, e.g., from ZAMS fitting or Baade-Wesselink method.

M2: Lee & Carney (1999, Fig. 2)

RR Lyr



Lightcurve shows characteristic color variations over pulsation (temperature change!), and a fast rise, slow decay behavior.

RR Lyr in GCs show bimodal number distribution due to a metallicity effect:

- RRab with $P > 0.5$ d and most probable period of $P_{\text{ab}} \sim 0.7$ d, and
- RRc, with $P < 0.5$ d and $P_{\text{c}} \sim 0.3$ d.

M is larger for higher Z, i.e., metal-rich RR Lyr are *fainter*

$\Longrightarrow$ difference in RR Lyr from population I and II.

RR Lyr work out to LMC and other dwarf galaxies of local group, however, used mainly for globular clusters.

(Lee & Carney, 1999, Fig. 5)

Interlude

Previous methods: Selection of methods for distances within Milky Way (and Magellanic Clouds): Basis for extragalactic distance scale.

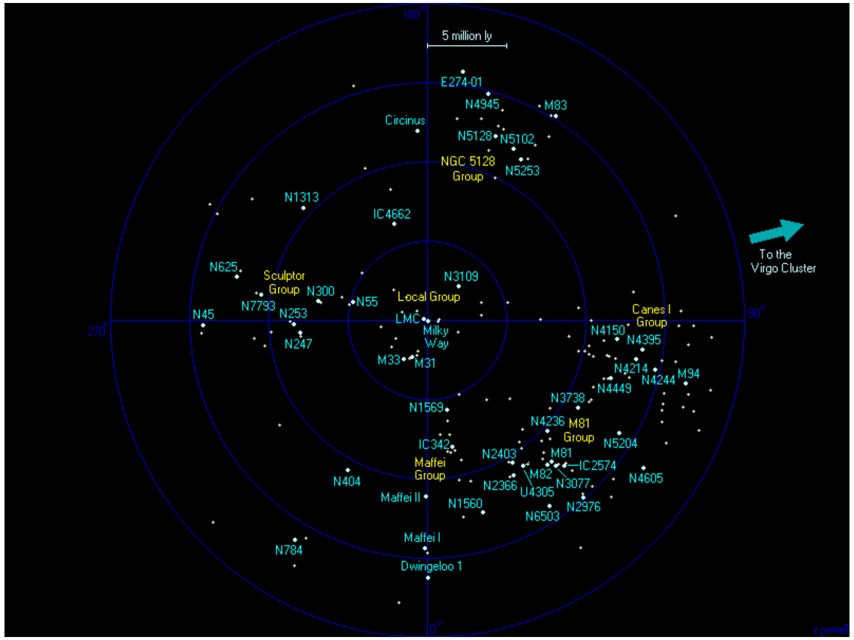
Primary extragalactic distance indicators: Distance can be calibrated from observations *within* milky way or from theoretical grounds.

Primary indicators usually work within our neighborhood (i.e., out to Virgo cluster at 15–20 Mpc).

Best example: Cepheids

Secondary extragalactic distance indicators: Distance calibrated from primary distance indicators.

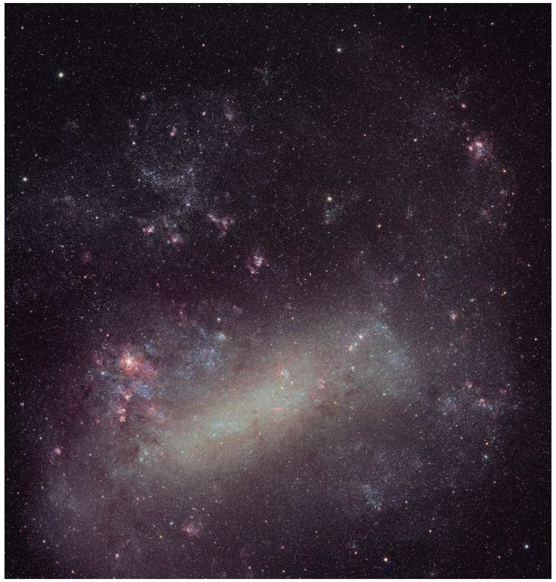
Examples: Type Ia SNe, methods based on integral galaxy properties.



Our $\sim$ 12 Mpc-Backyard (source: `http://www.atlasoftheuniverse.com/galgrps.html`)

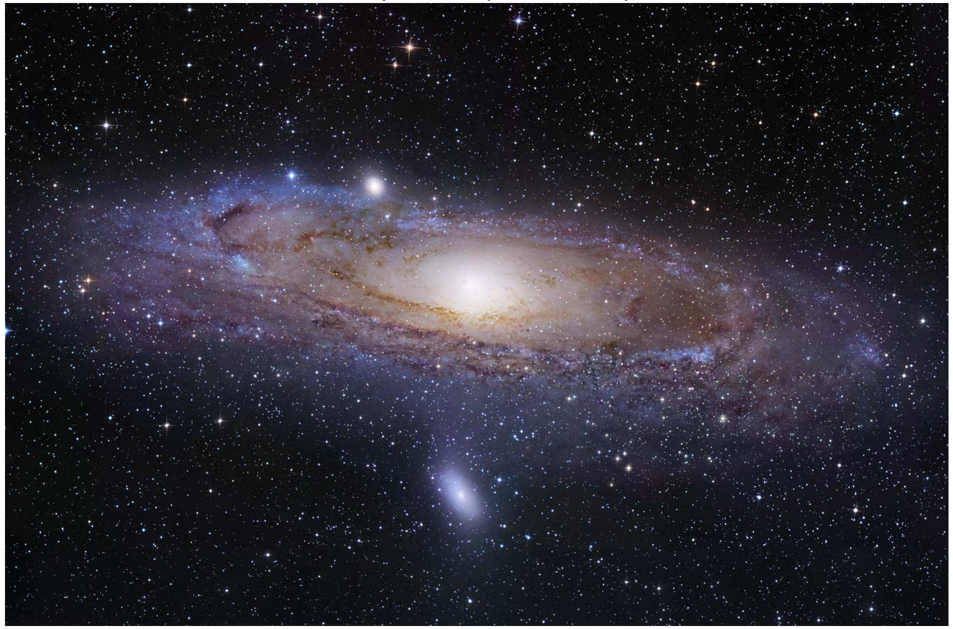
To get a feel for the distances in our "neighborhood":

50 kpc: LMC, SMC, some other dwarf galaxies



Loke Kun Tan

700 kpc: M31 (Andromeda)



Robert Gendler

the largest astronomical picture ever taken, 21904 × 14454 pixels

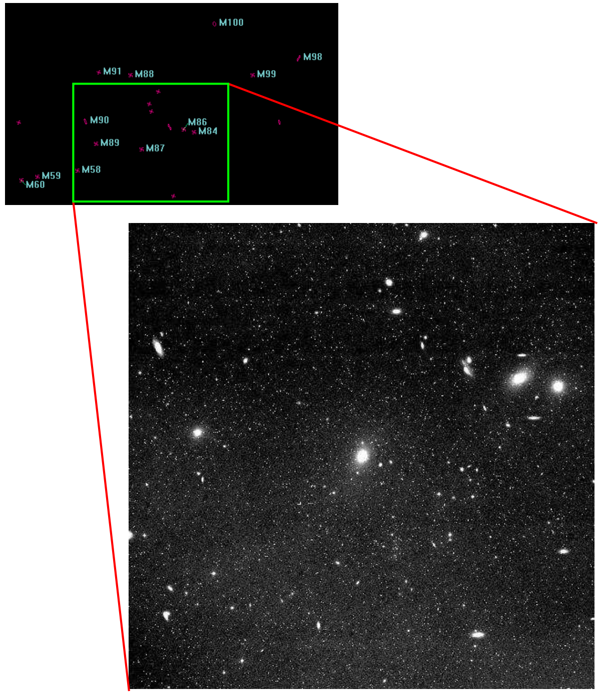
2–3 Mpc: Sculptor and M81 group

(groups similar to local group: a few large spirals, plus smaller stuff).

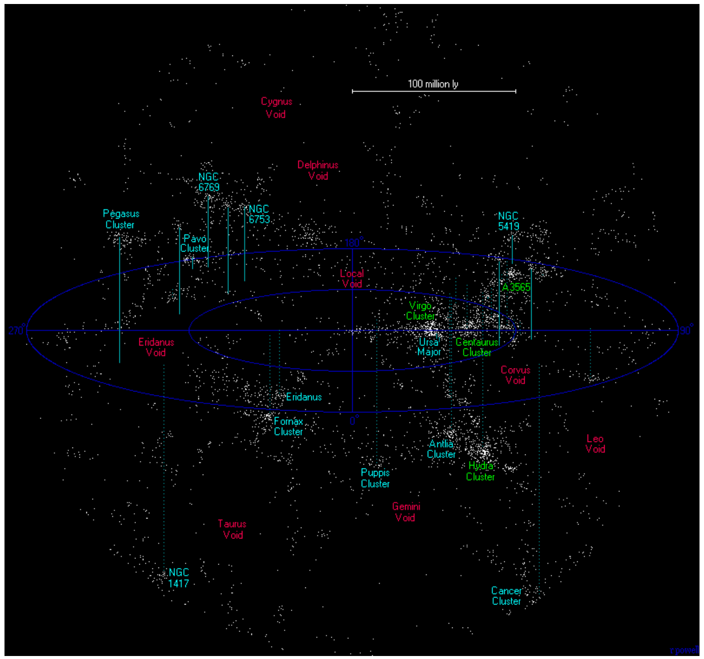
NGC 300 (Sculptor; Laustsen, Madsen, West, 1991)

5–7 Mpc: M101 group (“pinwheel galaxy”). Important because of high L.

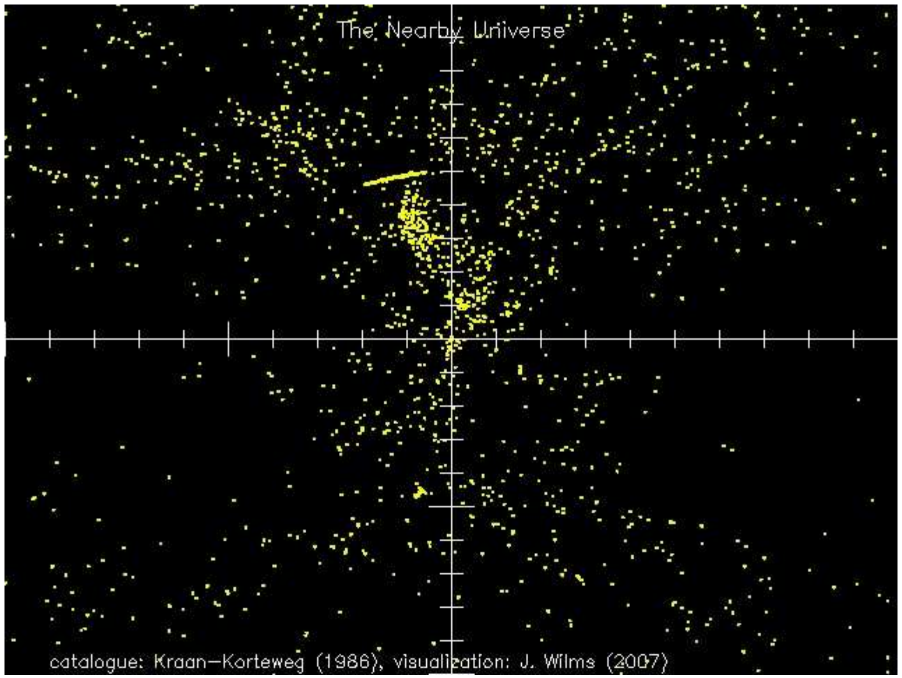
Adam Block/NOAO/AURA/NSF

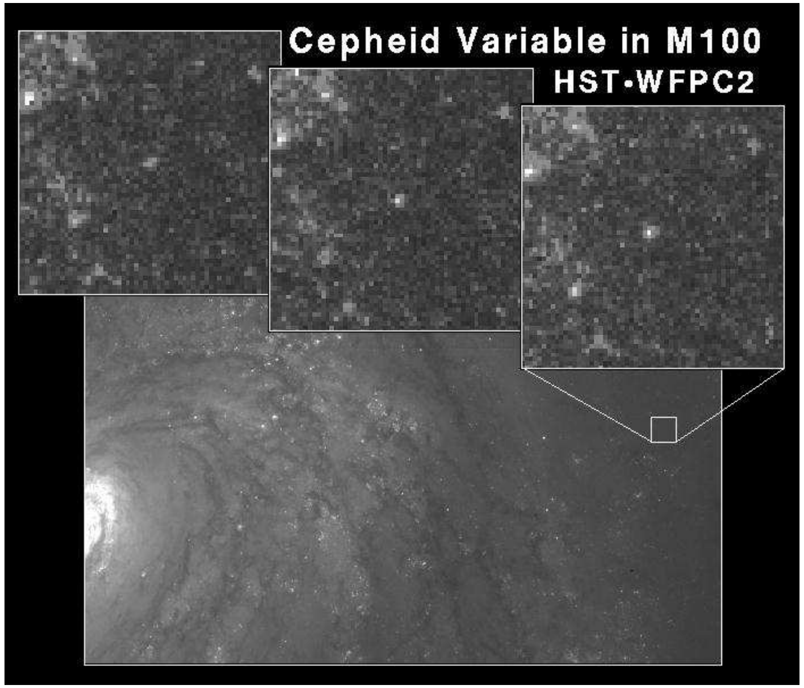


15–20 Mpc: Virgo cluster.



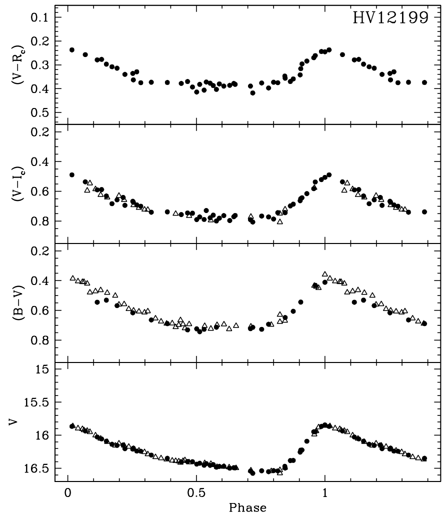
source: `http://www.atlasoftheuniverse.com/200mill.html`





STScI PR94-49

Cepheids

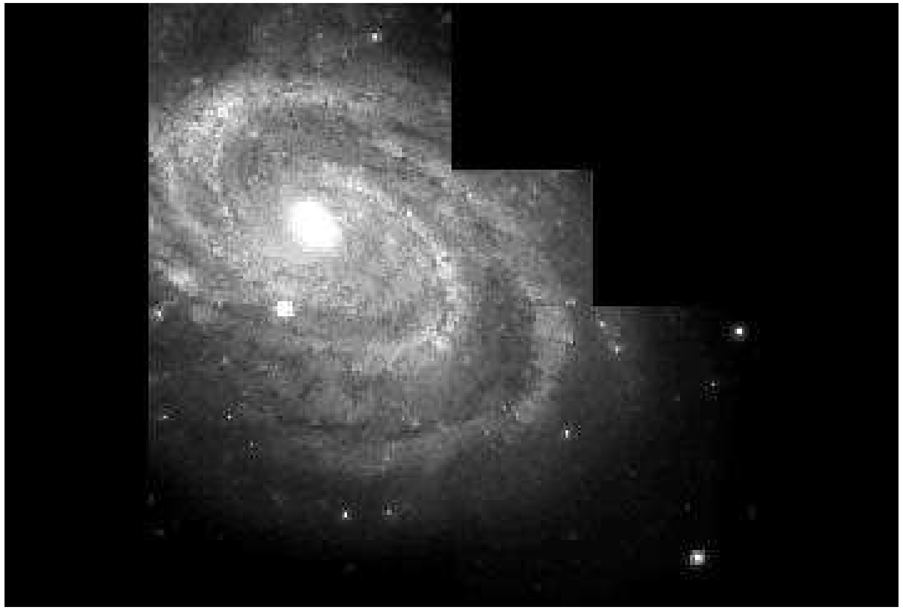


Cepheids:

- Luminous stars ($L \sim 1000\,L_\odot$) in instability strip (He II–He III ionization)
- large intensity amplitude variation,
- $P \sim 2\ldots150\,$d (easily measurable).

Review: Feast (1999).

(Gieren et al., 2000, Fig. 3)



STScI

Cepheids

© ASP

Henrietta Leavitt (1868–1921):

- Graduated from Radcliffe College
- from 1895: volunteer at Harvard Observatory
- was ill, and partially deaf as a result
- 1902: back at Harvard Obs
- discovered 1777 variable stars in LMC
- 1912: discovered Period-Luminosity relation of Cepheids in SMC, but was not allowed to follow up on this; Since 2009, called the Leavitt Law
- later: defined Harvard photographic magnitude system
- died of cancer in 1921

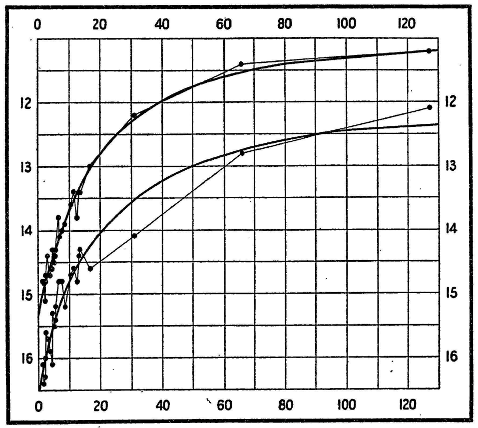


Fig. 1.

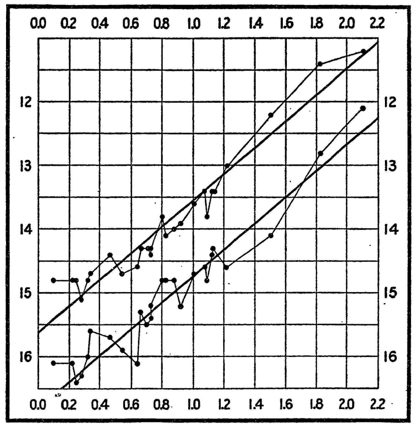


Fig. 2.

X-axis: period in days, Y-axis: magnitude

Leavitt & Pickering, 1912, Periods of 25 Variable Stars in the Small Magellanic Cloud, Harvard College Observatory Circular, vol. 173, pp. 1–3

Cepheids

Physics of Period-Luminosity relation:

Star pulsates such that outer parts remain bound:

$$\frac{1}{2}\left(\frac{R}{P}\right)^2 \lesssim \frac{GM}{R} \implies \frac{M}{R^3} \propto P^{-2} \tag{8.8}$$

where P period. Therefore:

$$P \propto \rho^{-1/2} \iff P\rho^{1/2} = Q \tag{8.9}$$

(Q: pulsational constant, $\rho \propto MR^{-3}$ mean density). But Radius R related to luminosity L:

$$L = 4\pi R^2 \sigma T^4 \implies R \propto L^{1/2} T^{-2} \tag{8.10}$$

Inserting everything into Eq. (8.9) gives:

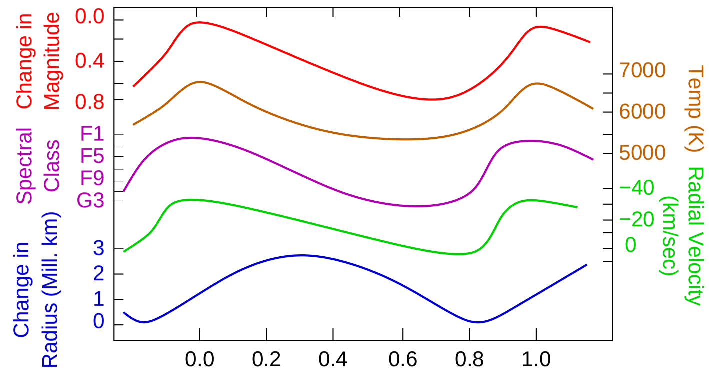
$$PL^{-3}T^3 = \text{const.} \iff \log P - 3\log L + 3\log T = \text{const.} \tag{8.11}$$

But: *bolometric magnitude:* $M_{\text{bol}} \propto -\log L$, and *colors:* B − V $\propto \log T$ such that

$$c_1 \log P + c_2 M_{\text{bol}} + c_3(\text{B} - \text{V}) = \text{const.} \tag{8.12}$$

where $c_{1,2,3}$ calibration constants. $\Longrightarrow$ Relation should also depend on the color.

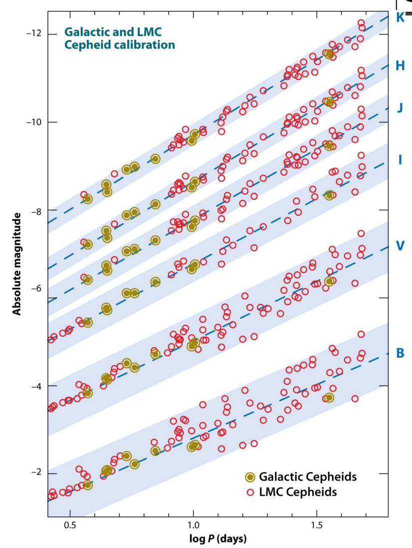
Cepheids



after `http://csep10.phys.utk.edu/astr162/lect/index.html`

Typical variation of measurable parameters over one pulsation.

Cepheids



Freedman WL, Madore BF. 2010. Annu. Rev. Astron. Astrophys. 48:673–710

The Leavitt Law (Period-Luminosity) relation: $M_{\text{V}} \propto -2.76 \log P$.

Low-luminosity Cepheids have smaller periods.

The Leavitt Law needs to consider color $\Longrightarrow$ Period-Luminosity-Color (PLC) relation

Note: W Vir stars, also called type II Cepheids = "little brother of Cepheids" (present in globular clusters). Less luminous than normal Cepheids, similar PLC relation, first confused with Cepheids $\Longrightarrow$ Cause for early thoughts of much smaller universe.

Cepheids

Calibration: Need slope and zero point of PLC.

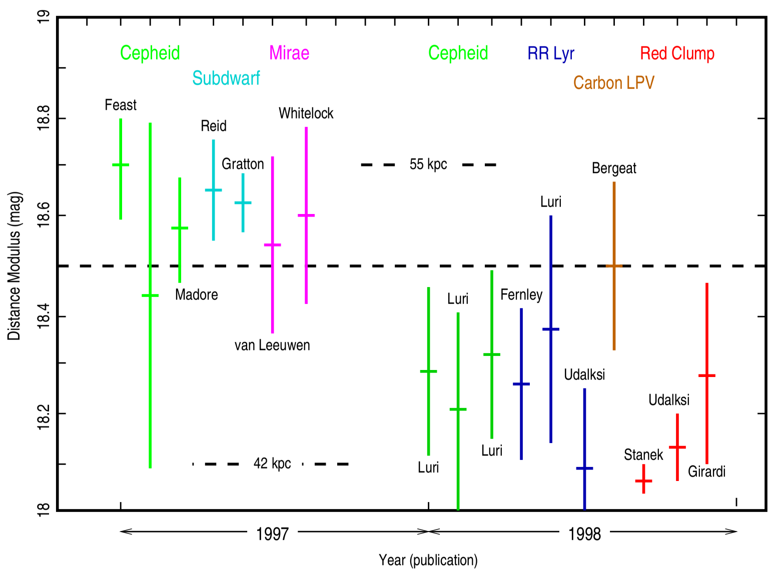
Slope: Observations of nearby galaxies (e.g., open clusters in LMC)

Zero point is difficult:

- All known Cepheids are more than 250 pc away $\Longrightarrow$ small geometrical parallaxes
- Hipparcos: geometrical distances
  $\Longrightarrow$ problematic due to low SNR (resulting in 9% systematic error).
- Parallaxes of 10 Cepheids (spanning a period range from 3.7 to 35.6 days) measured with HST (Benedict et al., 2007)

Error of zero-point calibration (and the Cepheid distance scale): $\pm 3\,\%$ (or ± 0.06 mag)

Improvement of the calibration awaits a larger sample of long-period Cepheids from the GAIA mission.



LMC distance: "anchor point" of extragalactic distance scale (e.g., several thousand known Cepheids in LMC).

After Gaia Science Workgroup

Strong dependence on Hipparcos calibration. DM ranges between 18.0 mag and 18.9 mag

For many years, the distance to the LMC was less well known than desirable. Now best value: 18.39 ± 0.06 mag (Freedman & Madore, 2010)

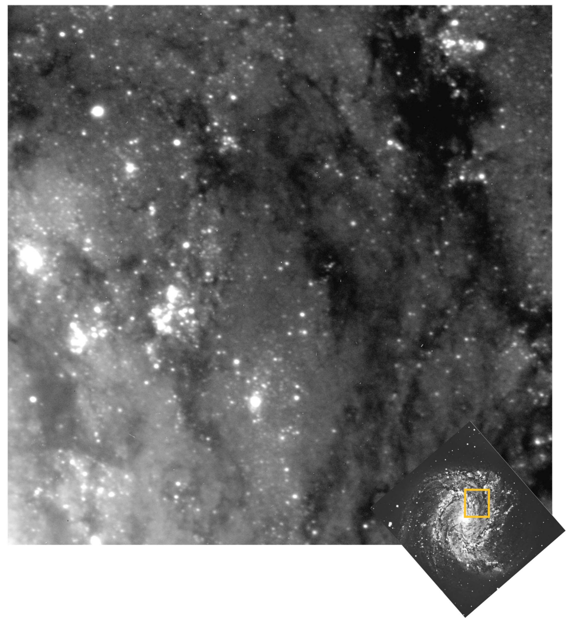
Cepheids

Notes:

1. Is the pulsational constant a constant? (or is $Q = Q(\rho, P)$?):
   $\Longrightarrow$ possible deviation from PLC, especially at high luminosity
   $\Longrightarrow$ adds uncertainty at large distances.
2. M_V depends on metallicity:

$$(m - M)_\mathrm{true} = (m - M)_\mathrm{PL} - \gamma \log Z/Z_\mathrm{LMC} \qquad (8.13)$$

   where $\gamma = -0.11 \pm 0.03$ mag/dex (Z: metallicity)
   (=Cepheids with larger Z are fainter).
   $\Longrightarrow$ LMC Cepheids *are* bluer [$Z_\mathrm{LMC} < Z_\odot$]), but the exact value of γ in Eq. (8.13) is very uncertain.
   For V and I magnitudes, most probably $\delta(m-M)_0/\delta[\mathrm{O/H}] \lesssim -0.4\,\mathrm{mag\,dex}^{-1}$, however, others find $+0.75\,\mathrm{mag\,dex}^{-1}$, see Ferrarese et al. (2000) for details...
3. Stellar evolution unclear (multiple crossings of instability strip are possible).



The VLT Looks Deep into a Spiral Galaxy

ESO PR Photo 20/98 (23 June 1998)

Brightest Stars

Brightest Stars= O, B, A supergiants, absolute magnitudes usable in local group, although there is a large scatter.

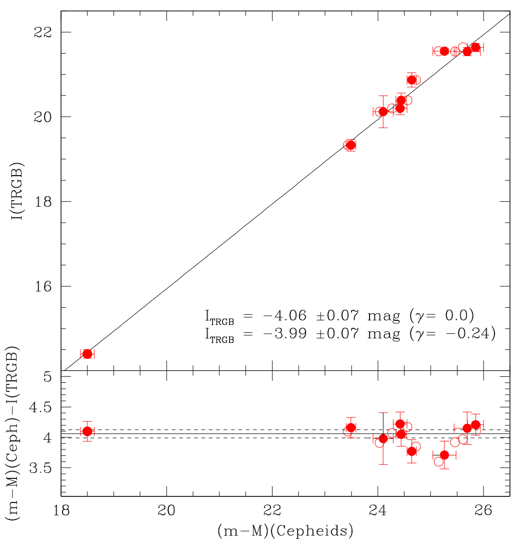
Reason: there is an upper limit to stellar luminosity due to mass loss in supergiants.

Possible Improvement: Strength of Balmer series lines. Hα and Hβ appear biased (class of supergiants with anomalously strong Balmer lines?).

Problems:

- Contamination by foreground halo stars
  $\Longrightarrow$ Choose stars with unusual color (rare, i.e. less foreground contamination): B − V $<$ 0.4 or B − V $>$ 2.0 $\Longrightarrow$ Tip of Red Giant Branch
- Internal extinction.
- Scatter in max. L
  $\Longrightarrow$ Average over brightest N stars (Sandage, Tammann: $N = 3$).
- Metallicity dependence.

Brightest Stars



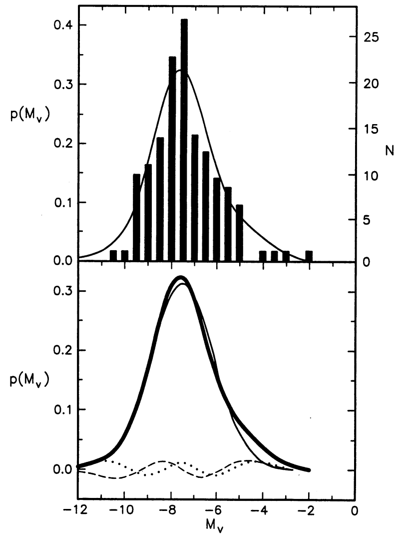
Tip of Red Giant Branch: Usable within local group, possibly out to Virgo.

Calibration:

$$M_{\rm I} = -4.06 \pm 0.13\,\text{mag} \quad (8.14)$$

(Ferrarese et al., 2000, Fig. 1)

Globular Clusters



Globular Cluster Luminosity Function is ~Gaussian
$\Longrightarrow$ Use maximum of distribution ("turnover magnitude", $M_{\rm T}$) as standard candle.
From Virgo and Fornax Cepheid distances (Ferrarese et al., 2000):

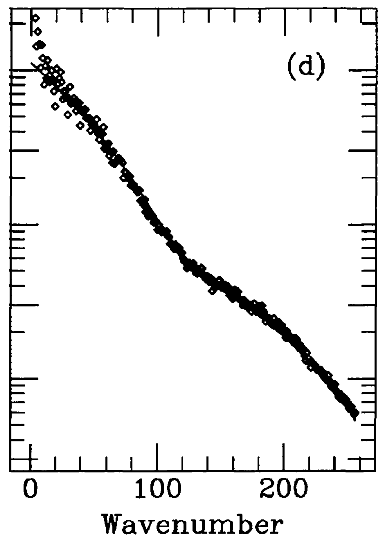
$$M_{\rm T,\,V} = -7.60 \pm 0.25\,\text{mag} \quad (8.15)$$

Caveats:

1. $M_{\rm T}$ depends on luminosity and type of host galaxy (GC of dwarf galaxies weaker by ~ 0.3 in V).
2. Metallicity of galaxy cluster influences $M_{\rm T}$.
3. Measurement difficult (need the weak GCs!).
4. Large scatter in data $\Longrightarrow$ Method rather unreliable.

(MW GCs, Abraham & van den Bergh, 1995, Fig. 1)

Surface Brightness Fluctuations



For early type galaxies: Assume N stars in picture element (pixel), with average flux f each.

$$\Longrightarrow \text{Mean pixel intensity:} \quad \mu = Nf \quad (8.16)$$

independent of distance, since $N \propto r^2$ and $f \propto r^{-2}$.
Standard deviation between pixels (Poisson!):

$$\sigma = \sqrt{N} f \propto r^{-1} \quad (8.17)$$

and therefore

$$f = \frac{\sigma^2}{\mu} = \frac{L}{4\pi r^2} \quad (8.18)$$

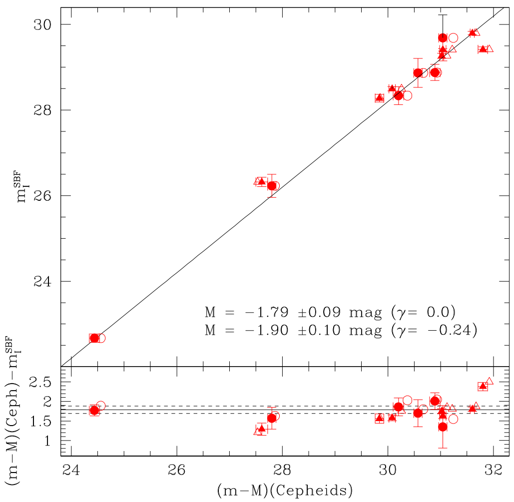
which gives the distance r.

Review: Blakeslee et al. (1999).

Complication: Adjacent pixels not independent (point spread function of telescope!)
$\Longrightarrow$ Use radial power spectrum to obtain σ^2 and μ.

(Ajhar et al., 1997, Fig. 3d)

Surface Brightness Fluctuations



Luminosity of galaxy dominated by Red Giant Branch stars

⟹ Strong wavelength and color dependence

⟹ Primary calibration: I-band plus broad-band color dependency to give standard candle.

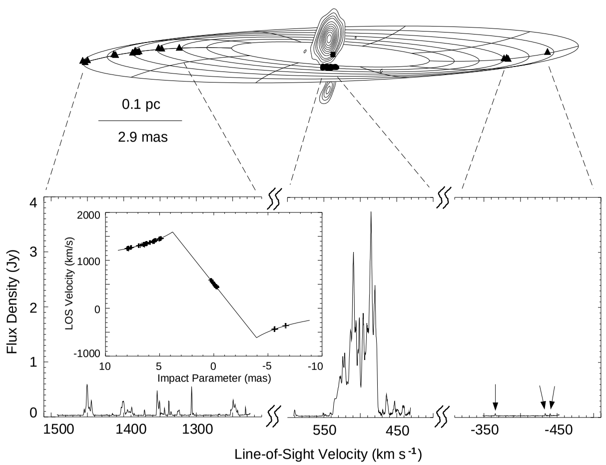
Often also used: HST WFPC2 plus F814W filter (close to I-band),

$$M_{\mathrm{F814W}} = (-1.70 \pm 0.16) + (4.5 \pm 0.3)\,[(\mathrm{V}-\mathrm{I})_0 - 1.15] \quad (8.19)$$

Works out to ~ 70 Mpc with HST.

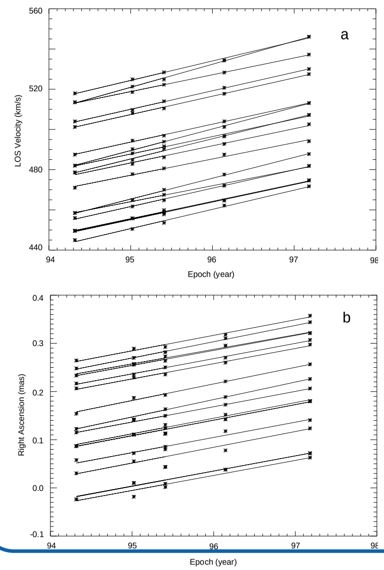
(Ferrarese et al., 2000, Fig. 5)

Megamasers



Miyoshi et al. (1995) and Herrnstein et al. (1999) found a Keplerian rotating disk at the center of NGC 4258 ⟹ Mass determination of central object ⟹ Black hole with $M = 3.6 \times 10^7 M_\odot$

Megamasers



(Herrnstein et al., 1999)

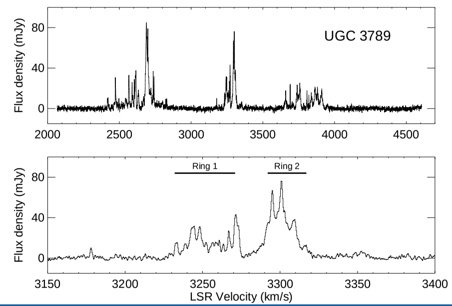
Observational principle: VLBI imaging of 22.3 GHz Megamaser emission

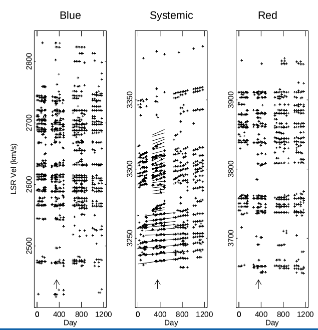
- 3 components: two high-velocity components offset by $\pm 1000\,\mathrm{km\,s^{-1}}$ and one systemic component
- High-velocity masers yield disk-rotation velocity and BH mass (from Kepler rotation curve)
- Velocities and acceleration of systemic masers yield distance: 7.2 ± 0.2 Mpc and 7.1 ± 0.2 Mpc
- NGC 4258 can serve as an independent anchor point for extragalactic distance determinations (next to the more widely used LMC, see later)

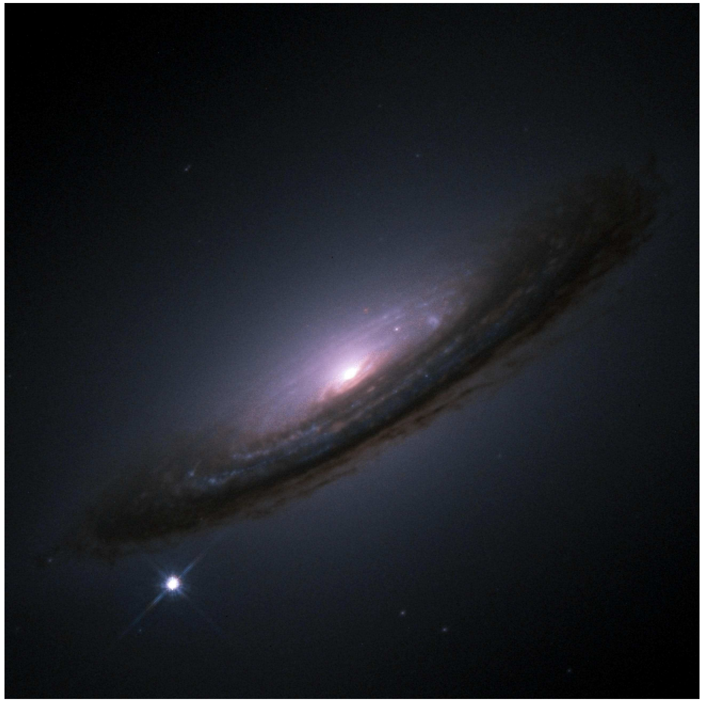
Megamasers

"The Megamaser Cosmology Project aims to determine H_0 by measuring angular-diameter distances to galaxies in the Hubble flow, $\sim 50 - 200$ Mpc distant, using the maser technique." (Braatz et al., 2010)

- Conducted with the High-Sensitivity Array: VLBA+Effelsberg+GBT+VLA
- Goal: measure ~ 10 galaxies ⟹ determine H_0 with ~ 3 % uncertainty
- So far: One measurement – UGC 3789 at 49.9 ± 7.0 Mpc (Braatz et al., 2010)

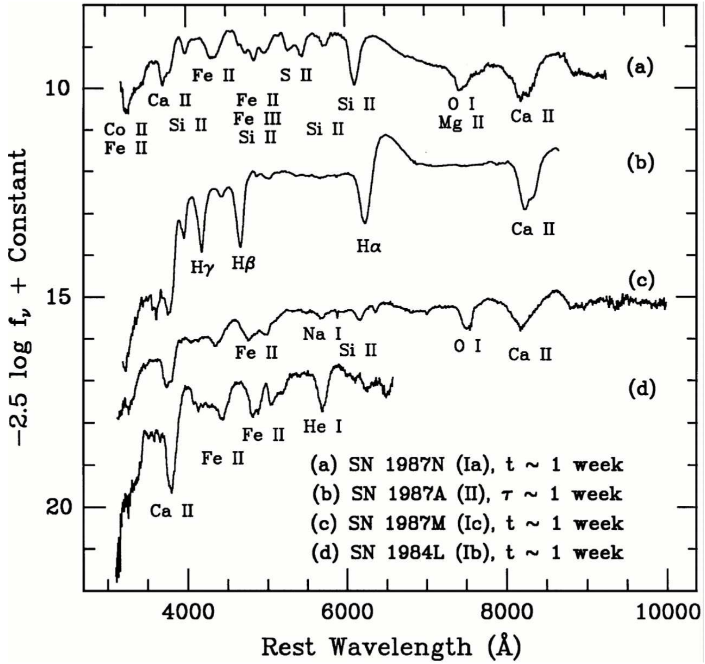






Supernovae have luminosities comparable to whole galaxies:
$\sim 10^{51}\,\mathrm{erg\,s^{-1}}$ in light, $100\times$ more in neutrinos.

SN1994d (HST WFPC)



(Filippenko, 1997, Fig. 1); t: time after maximum light; τ: time after explosion;
P Cyg profiles give $v \sim 10000\,\mathrm{km\,s^{-1}}$

Rough classification (Minkowski, 1941):

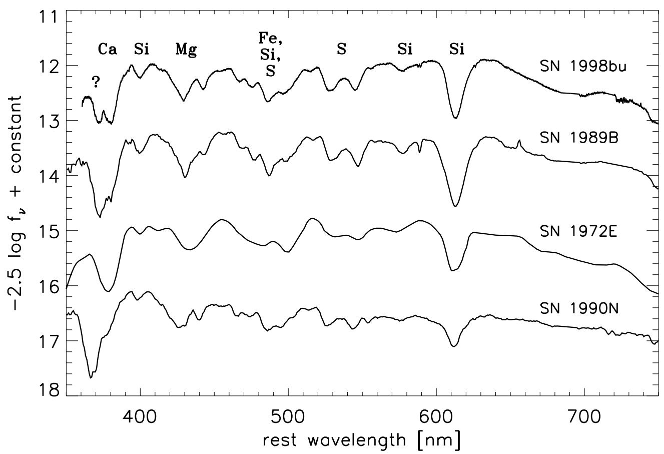
Type I: no hydrogen in spectra; subtypes Ia, Ib, Ic

Type II: hydrogen present, subtypes II-L, II-P

Note: pre 1985 subtypes Ia, Ib had different definition than today $\Longrightarrow$ beware when reading older texts.

8–51

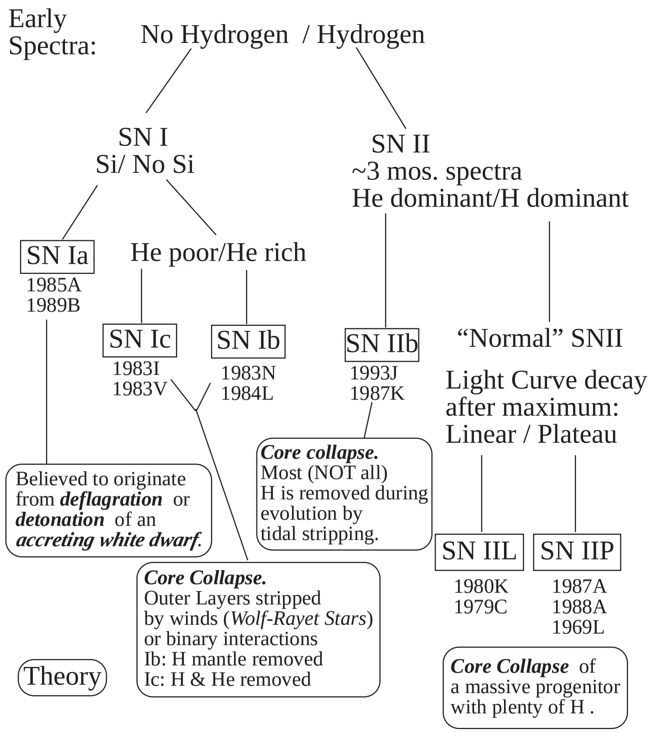
Type Ia Supernovae



(Spectra of several SNe at maximum light Jha et al., 1999, Fig. 6)

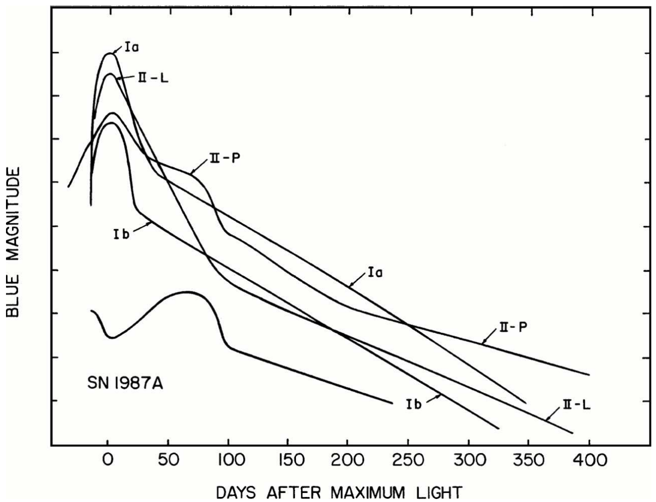
Different supernovae can have very similar spectra.

$\Longrightarrow$ Allows their classification.



courtesy M.J. Montes

Type Ia Supernovae

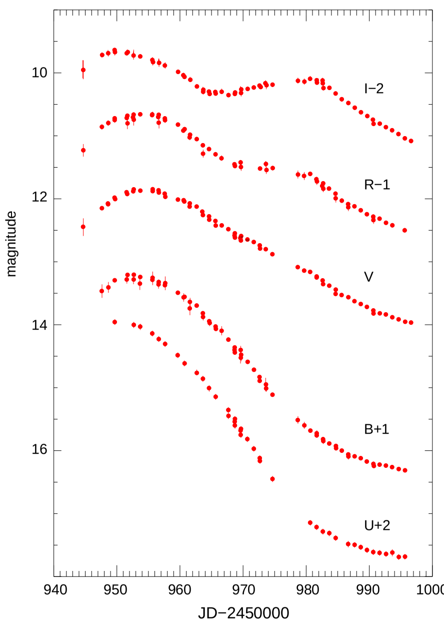


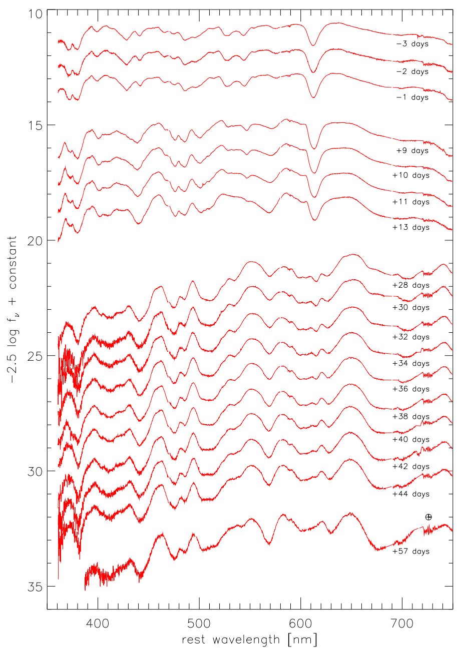
(Filippenko, 1997, Fig. 3)

Light curves of SNe I all very similar, SNe II have much more scatter.

SNe II-L ("linear") resemble SNe I

SNe II-P ("plateau") have const. brightness to within 1 mag for extended period of time.





(SN 1998bu in M96, Jha et al., 1999, Figs. 2 and 4)

Type Ia Supernovae

Clue on origin from supernova statistics:

- SNe II, Ib, Ic: never seen in ellipticals; rarely in S0; generally associated with spiral arms and H II regions.

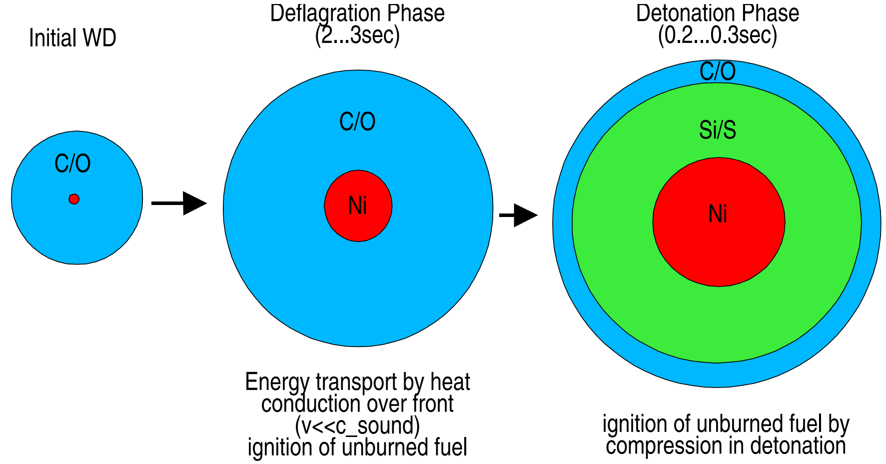
$\Longrightarrow$ *progenitor of SNe II, Ib, Ic:* massive stars ($\gtrsim 8\,M_\odot$) $\Longrightarrow$ core collapse

- SNe Ia: all types of galaxies, no preference for arms, almost no scatter in lightcurves

$\Longrightarrow$ *progenitor of SNe Ia:* accreting carbon-oxygen white dwarfs, undergoing thermonuclear runaway

Rule of thumb: 1. . . 3 SNe per galaxy and per century

Type Ia Supernovae



after P. Höflich

Type Ia Supernovae

SN Ia = Explosion of CO white dwarf when pushed over Chandrasekhar limit (1.4 $M_\odot$) (via accretion?).

$\Longrightarrow$ Always similar process

$\Longrightarrow$ Very characteristic light curve: fast rise, rapid fall, exponential decay ("FRED") with half-time of 60 d.

60 d time scale from radioactive decay $Ni^{56} \rightarrow Co^{56} \rightarrow Fe^{56}$ ("self calibration" of lightcurve if same amount of Ni^{56} produced everywhere).

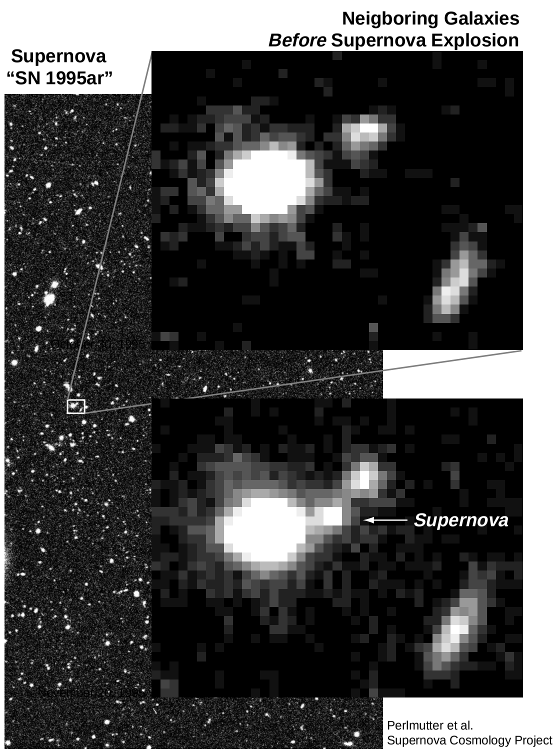
Calibration: SNe Ia in nearby galaxies where Cepheid distances known.

At maximum light:

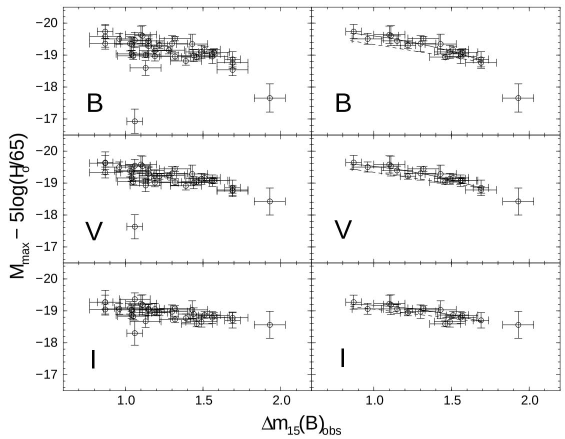
$$M_B = -18.33 \pm 0.11 + 5 \log h_{100} \quad (L \sim 10^{9\ldots10} L_\odot) \qquad (8.20)$$

Intrinsic dispersion: $\lesssim$0.25 mag (possibly due to size of clusters analyzed?!?)

Observable out to 1000 Mpc



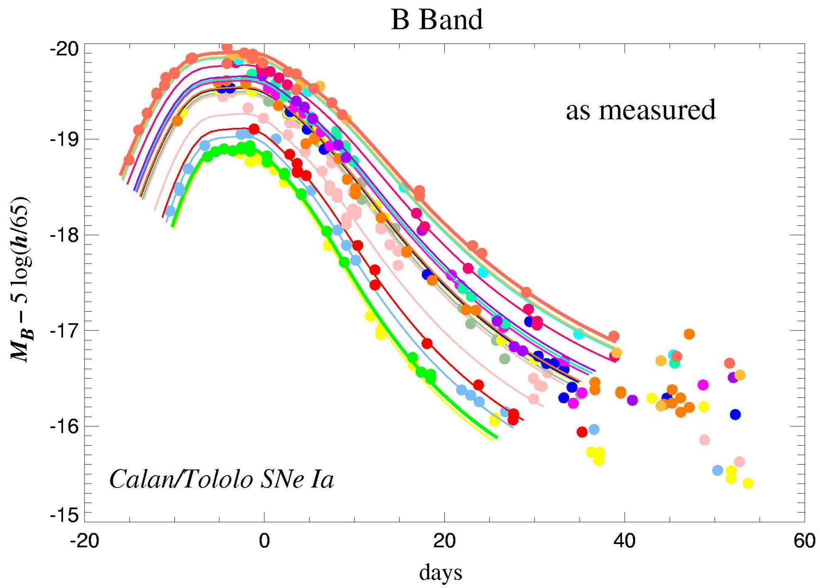
Type Ia Supernovae



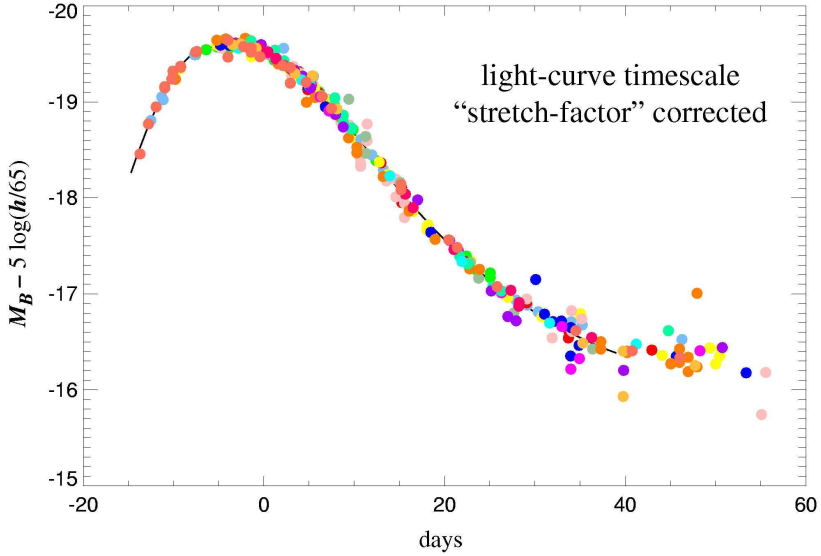
(Phillips et al., 1999, Fig. 8)

Caveats:

1. Are they *really* identical? $\Longrightarrow$ history of pre-WD star?
2. Correction for extinction in parent galaxy difficult.
3. Baade-Wesselink for calibration Eq. (8.20) depends crucially on assumed $(B - V)$-T_{eff} relation.
4. Some SN Iae spectroscopically peculiar $\Longrightarrow$ Do not use these!
5. Decline rate and color vary, but max. brightness and decline rate correlate (see figure).



Lightcurves of Hamuy et al. SN Ia sample (18 SNe discovered within 5 d past maximum, with 3.6 $< \log cz <$ 4.5, i.e., $z <$ 0.1)



Kim, *et al.* (1997)

Lightcurves of Hamuy et al. SN Ia sample (18 SNe discovered within 5 d past maximum, with $3.6 < \log cz < 4.5$, i.e., $z < 0.1$), after correction of systematic effects and time dilatation (Kim et al., 1997).

Type Ia Supernovae

Recalibration of SN Ia distances with Cepheids gives (Gibson et al., 2000):

$$\log H_0 = 0.2\{M_B^{\max} - 0.720(\pm 0.459) \cdot [\Delta m_{B,15,t} - 1.1] - 1.010(\pm 0.934) \cdot [\Delta m_{B,15,t} - 1.1]^2 + 28.653(\pm 0.042)\} \quad (8.21)$$

where

$$\Delta m_{B,15,t} = \Delta m_{B,15} + 0.1E(B-V) \quad (8.22)$$

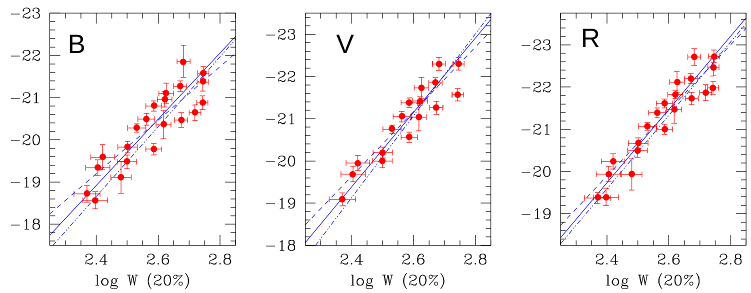
where

$\Delta m_{B,15}$: observed 15 d decline rate,
$E(B-V)$: total extinction (galactic+intrinsic).

Eq. (8.21) valid for B-band, equivalent formulae exist for V and I.

Overall, the calibration is good to better than 0.2 mag in B.

Tully-Fisher



(after Sakai et al., 2000, Fig. 1)

Tully-Fisher relation for spiral galaxies: Width of 21 cm line of H correlated with galaxy luminosity:

$$M = -a \log\left(\frac{W_{20}}{\sin i}\right) - b \quad (8.23)$$

where W_{20}: 20% line width (km s^{-1}; typically $W_{20} \sim 300$ km s^{-1}), i inclination angle.

For the B- and I-Bands (Sakai et al., 2000):

	B	I
a	7.97± 0.72	9.24± 0.75
b	19.80± 0.11	21.12± 0.12

Tully-Fisher

Qualitative Physics: Line width related to mass of galaxy: $W/2 \sim V_{\max}$, where $V_{\max}$ max. velocity of rotation curve

$\Longrightarrow$ Assume $M/L =$ const. (good assumption)

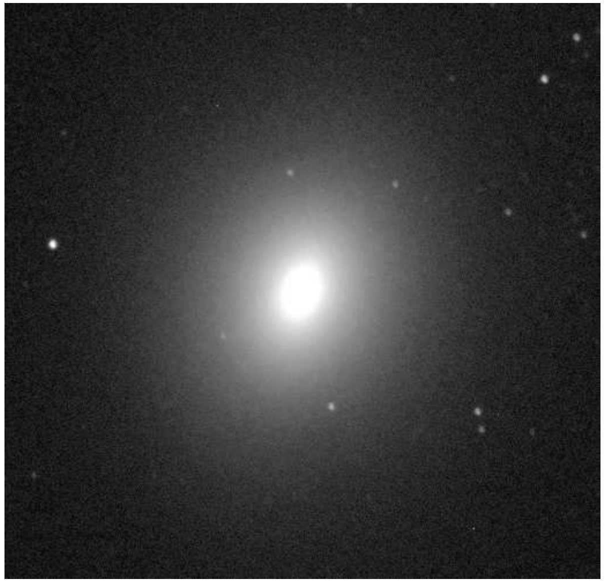
$\Longrightarrow$ width related to luminosity.

Detailed physical basis unknown. Might be related to galaxy formation (“hierarchical clustering”, see later).

I-band is better (less internal extinction).

Caveats:

1. Determination of inclination i.
2. Influence of turbulent motion within galaxy.
3. Constants dependent on galaxy type (Sa and Sb similar, Sc more luminous by factor of ∼2).
4. Optical extinction.
5. Intrinsic dispersion ∼0.2 mag.
6. Barred Galaxies problematic.



“Faber-Jackson” law for elliptical galaxies:

The luminosity L of an elliptical galaxy scales with its intrinsic velocity dispersion, σ, as $L \propto \sigma^4$.

Note that ellipticals have virtually no Hydrogen

$\Longrightarrow$ cannot use 21 cm.

M32 (companion of Andromeda), courtesy W. Keel

$$\text{Ellipticals:} \quad M_\mathrm{B} = -19.38 \pm 0.07 - (9.0 \pm 0.7)(\log\sigma - 2.3) \qquad (8.24)$$

$$\text{Lenticulars (Type S0):} \quad M_\mathrm{B} = -19.65 \pm 0.08 - (8.4 \pm 0.8)(\log\sigma - 2.3) \qquad (8.25)$$

D_n-σ

The Faber-Jackson law is a specialized case of the more general D_n–σ-relation:

The intensity profile of an elliptical galaxy is given by de Vaucouleurs’ $r^{1/4}$ law:

$$I(r) = I_0 \exp\left(-(r/r_0)^{1/4}\right) \quad \Longrightarrow \quad L = \int I \propto I_0 r_0^2 \qquad (5.6)$$

Because of the virial theorem ($E_\mathrm{kin} = -E_\mathrm{pot}/2$):

$$\frac{1}{2} m\sigma^2 = G\frac{mM}{r_0} \quad \Longleftrightarrow \quad \sigma^2 \propto \frac{M}{r_0} \qquad (8.26)$$

where σ: velocity dispersion.

Assume a mass-to-light ratio

$$M/L \propto M^\alpha \qquad (8.27)$$

($\alpha \sim 0.25$). and use r_0 from Eq. (5.6) to obtain

$$L^{1+\alpha} \propto \sigma^{4-4\alpha} I_0^{\alpha-1} \qquad (8.28)$$

This is called the “fundamental plane” relationship (Dressler et al., 1987).

D_n-σ

Observational version of the fundamental plane relationship: Instead of inserting r_0 and I_0, measure diameter D_n of aperture to reach some mean surface brightness (typically sky brightness, 20.75 mag arcsec^{-2} in B), and use calibration.

Note: Assumptions are

1. M/L same everywhere.
2. ellipticals have same stellar population everywhere

Calibration paper: Kelson et al. (2000).

Path to H_0

To obtain H_0, we need distances, and redshifts.

Redshifts: Trivial

Distances: Hubble Space Telescope Key Project on Extragalactic Distance Scale, Megamaser Cosmology Project.

Summary papers: Freedman et al. (2001), (Freedman & Madore, 2010)

Strategy:

1. Use high-quality candles: Cepheid variables as primary distance calibrator.
2. Calibrate secondary calibrators that work out to $cz = 10000\,\mathrm{km\,s^{-1}}$:
   - Tully-Fisher,
   - Type Ia Supernovae,
   - Surface Brightness Fluctuations,
   - Fundamental-plane for Ellipticals.
3. Combine uncertainties from these methods.

Velocity Field

Before determining H_0: correct for influence of velocity field (cluster motion with respect to comoving coordinates).

The observed redshift is given by

$$1 + z = (1 + z_R)\left(1 - \frac{v_0}{c} + \frac{v_G}{c}\right) \quad (8.29)$$

where

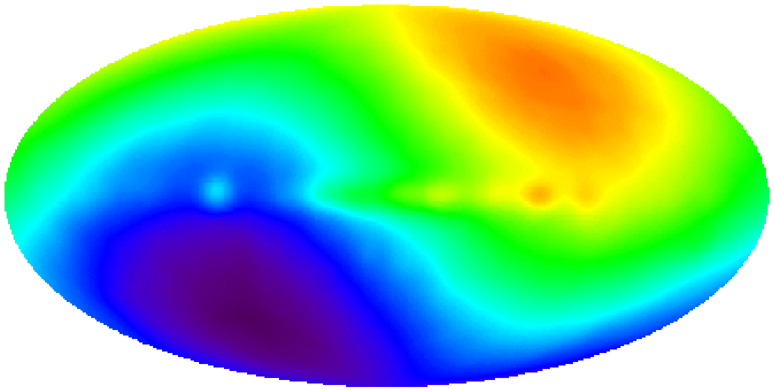
v_0: observer's radial velocity in direction of galaxy
v_G: radial velocity of the galaxy, difficult to find
z_R: cosmological redshift

Older galaxy catalogues often attempt to correct the measured values of z to produce "corrected redshifts", e.g., by setting $v_G = 0$ and

$$1 + z = (1 + z_R)\left(1 + \frac{v_0}{c}\right) \sim 1 + z_R - \frac{v_0}{c} \implies z_R \sim z + \frac{v_0}{c} \quad (8.30)$$

since v_0 was not well known before COBE $\Longrightarrow$ introduces unnecessary problems

$\Longrightarrow$ correction not used in recent redshift surveys! (see Harrison & Noonan, 1979, for details)



(COBE DMR; Bennett et al., 1996)

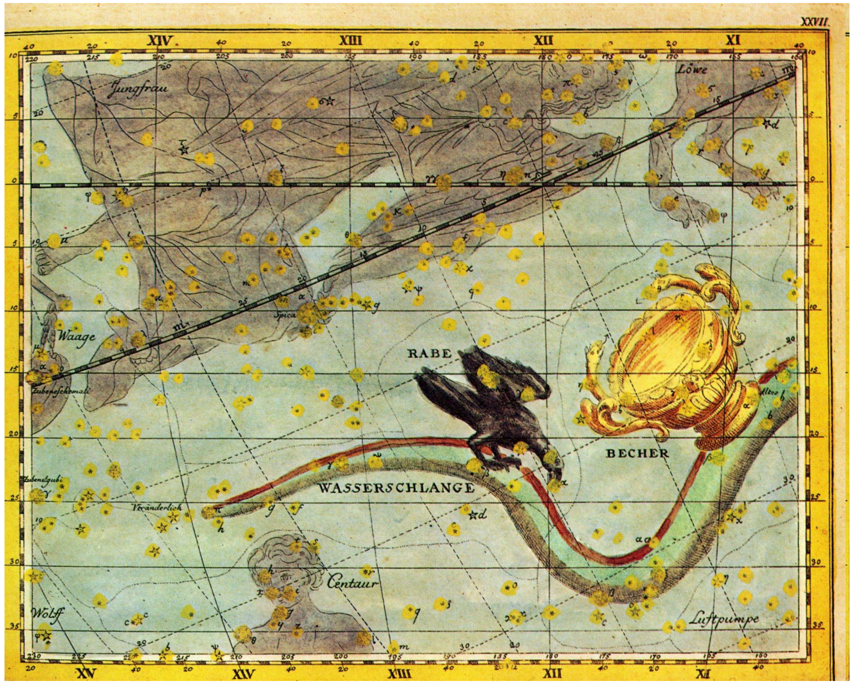
v_0 is easy to find $\Longrightarrow$ Measure velocity of Earth with respect to 3 K radiation. COBE finds $\Delta T = 3.353 \pm 0.024$ mK of 3K black-body spectrum of $T = 2.725 \pm 0.020$ K, using $\Delta T/T = v/c$.

$$v_0 = (369.1 \pm 2.6)\,\mathrm{km\,s^{-1}} \cdot \cos\theta_{\mathrm{CMB}} \quad (8.31)$$

where $\theta_{\mathrm{CMB}} = \angle(\mathbf{v}, \mathbf{v}_{\mathrm{CMB}})$, and $\mathbf{v}_{\mathrm{CMR}}$ points towards

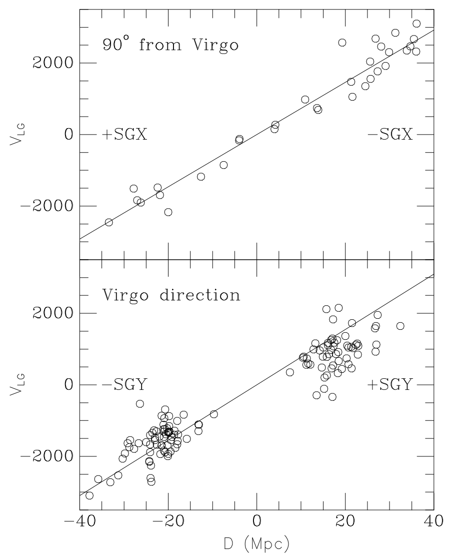
$$(l, b) = (264.^\circ 26 \pm 0.^\circ 33, 48.^\circ 22 \pm 0.^\circ 13)$$
$$(\alpha, \delta)_{\mathrm{J2000.0}} = (11^{\mathrm{h}}12.^{\mathrm{m}}2 \pm 0.^{\mathrm{m}}8, -7.^\circ 06 \pm 0.^\circ 16)$$

in constellation Crater.



The constellation Crater ("Becher") in Johan Elert Bode's Sternatlas (after Slawik/Reichert, Atlas der Sternbilder, Spektrum, 2004)

Velocity Field



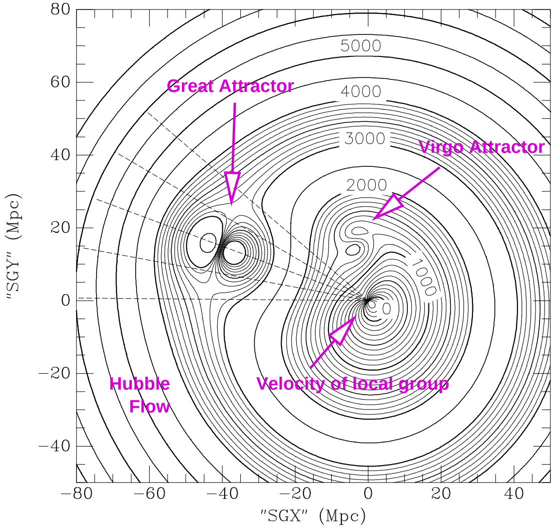
To get feeling for v_G out to Virgo, need to study local velocity field surrounding local group and beyond.

Two major velocity components:

1. Virgocentric infall (known since mid-1970s)
2. Motion towards great attractor ("Seven Samurai", 1980)

plus virialized galaxy motions within clusters.
General analysis: build maximum likelihood model of velocity field including above components *plus* Hubble flow. See Tonry et al. (2000) for details.

Galaxy moves within local group with $v \sim 630\,\mathrm{km\,s^{-1}}$



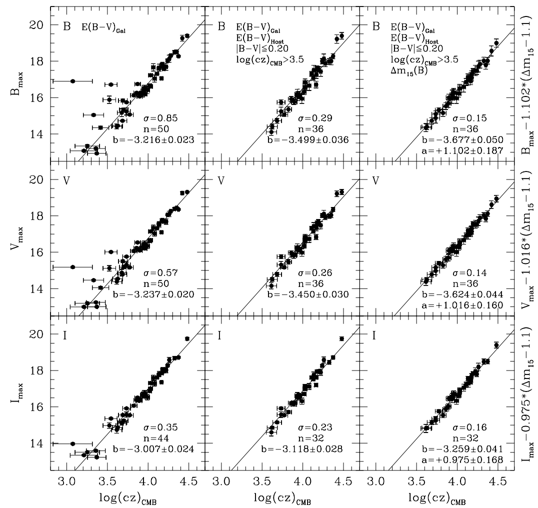
Decomposition of velocity field: (Mould et al., 2000b, Tab. A1, note that Tonry et al. 2000 find slightly different values):

	$\alpha_{1950.0}$	$\delta_{1950.0}$	v (km s^{-1})
Virgo	12^h28^m	$+12°40'$	200
GA	13^h20^m	$+44°00'$	400
Shapley	13^h30^m	$+31°00'$	85

(v wrt. center of local group; *not* taking Hubble flow into account!).

(Tonry et al., 2000, Fig. 20)

H from HST



Cepheids alone: nearby

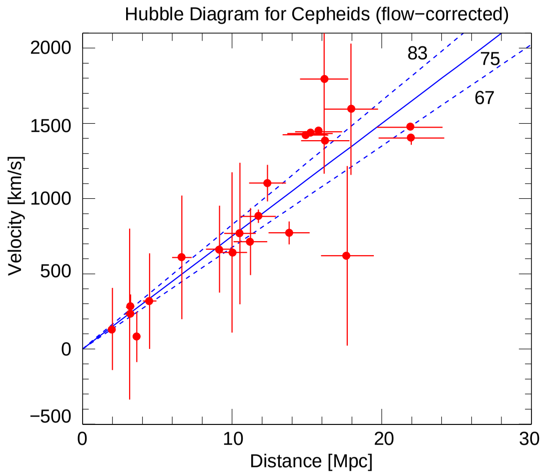
$\Longrightarrow$ systematic uncertainties due to local flow correction and small overall v

$\Longrightarrow$ use secondary candles to get to larger distances.

Example: magnitude-redshift diagram, analoguous to Hubble diagram ($m \propto -5\log I$, and $I \propto 1/r^2 \propto 1/z^2$ because of Hubble $\Longrightarrow m \propto \log cz$).

(SN Ia Hubble relations; left: full sample, middle: excluding strongly reddened SN Iae, right: same as middle, correcting for light-curve shape Freedman et al., 2001, Fig. 2)

H from HST



Freedman et al. (2001, Fig. 1)

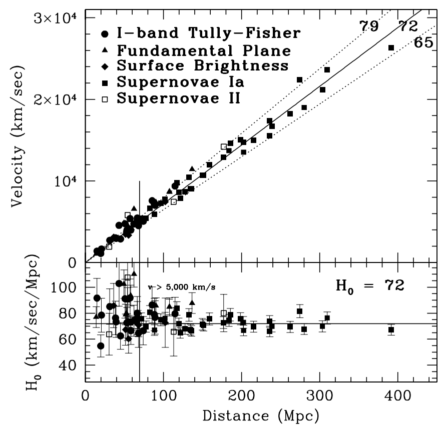
To obtain H_0:

1. Determine d with Cepheids and HST
2. Determine "v", corrected for local velocity field
3. Draw Hubble-diagram
4. Regression Analysis $\Longrightarrow H_0$

Value from HST Key Project:

$$H_0 = 75 \pm 10\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{8.32}$$

H from HST

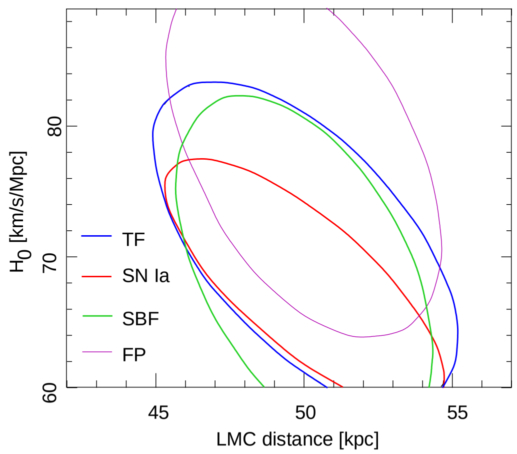


Freedman et al. (2001, Fig. 4)

Combining all secondary methods, best value found (Freedman & Madore, 2010):

$$H_0 = 73 \pm 2_{\text{(stat)}} \pm 4_{\text{(sys)}}\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{8.33}$$

H from HST



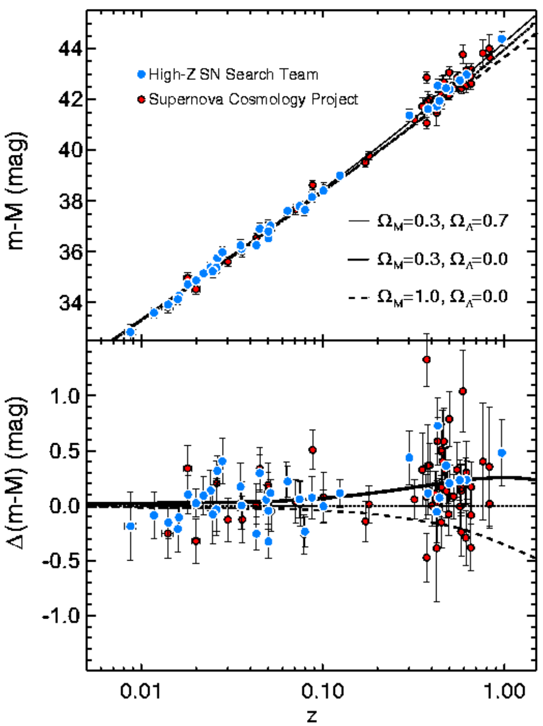
(after Mould et al., 2000b, Fig. 5)

Major systematic uncertainty in current H_0 value: zero-point of Cepheid scale.

Before 2007, this was dominated by the distance to Large Magellanic Cloud. Now, LMC used for slope but Galactic Cepheids used for zero-point calibration.

$\Longrightarrow$ All current values approach $\sim$70 km s^{-1} Mpc^{-1}, with uncertainty $\sim$10%

H_0 controversy is over



For larger distances: There are deviations from Hubble-Relation! Before we understand why: Need to understand the Big-Bang itself!

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