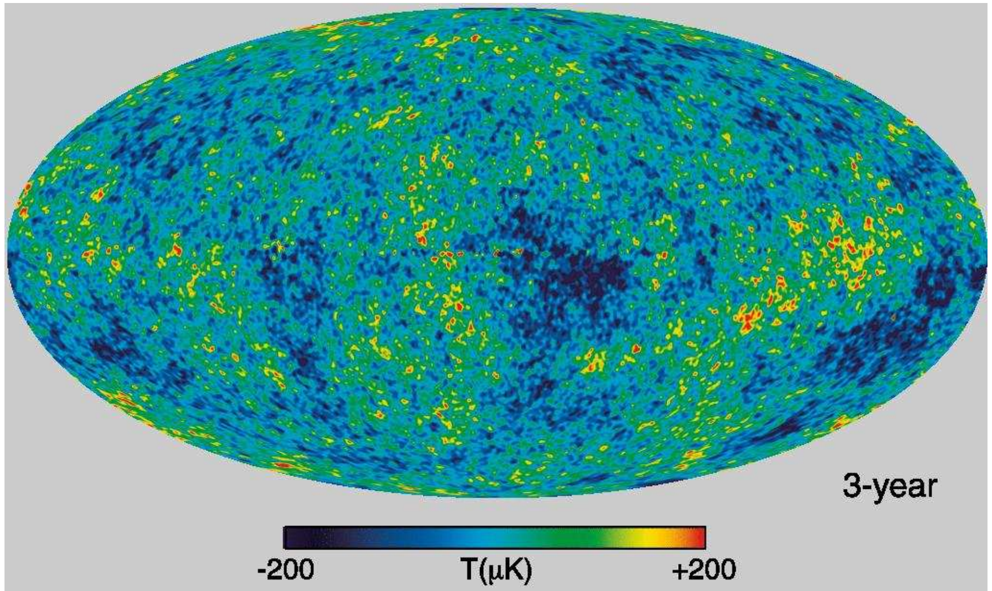
12–1

Inflation



(WMAP; Page et al., 2007)

12–2

Inflation

So far, have seen that BB works remarkably well in explaining the observed universe.

There are, however, many problems with the classical BB theories:

Horizon problem: CMB looks too isotropic $\Longrightarrow$ Why?

Flatness problem: Density close to BB was very close to $\Omega = 1$ (deviation $\sim 10^{-16}$ during nucleosynthesis) $\Longrightarrow$ Why?

Hidden relics problem: There are no observed magnetic monopoles, although predicted by GUT, neither gravitinos and other exotic particles $\Longrightarrow$ Why?

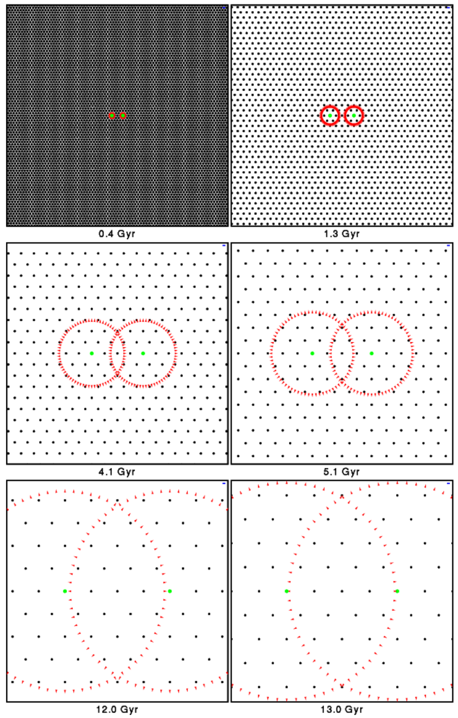
Vacuum energy problem: Energy density of vacuum is 10^{120} times smaller than predicted $\Longrightarrow$ Why?

Expansion problem: The universe expands $\Longrightarrow$ Why?

Baryogenesis: There is virtually no antimatter in the universe $\Longrightarrow$ Why?

Structure formation: Standard BB theory produces no explanation for lumpiness of universe.

Inflation attempts to answer all of these questions.



courtesy E. Wright.

Horizon problem

COBE and WMAP: There are temperature fluctuations in CMB on 10° scales:

$$\frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} \sim 2 \times 10^{-5} \tag{12.1}$$

Size of observable universe at given epoch ("particle horizon") is given by coordinate distance traveled by photons since the big bang (Eq. 7.56):

$$d_{\text{h}} = R_0 \cdot r_{\text{H}}(t) = \int_0^t \frac{c\,\mathrm{d}t}{a(t)} \tag{12.2}$$

For a matter dominated universe with $\Omega = 1$,

$$a(t) = \left(\frac{3H_0}{2}t\right)^{2/3} \tag{7.72}$$

such that for $t = t_0 = 2/(3H_0)$ (Eq. 7.73):

$$d_{\text{h}}(t_0) = \frac{3c}{(3H_0/2)^{2/3}} t_0^{1/3} = \frac{2c}{H_0} \tag{12.3}$$

Horizon problem

For matter dominated universes at redshift z, Eq. (12.3) works out to

$$d_{\text{h}} \approx \frac{6000\,\text{Mpc}}{h\sqrt{\Omega z}} \tag{12.4}$$

(Peacock, 1999, eq. 11.2)

Since CMB decoupled at $z \sim 1000$, at that time $d_{\text{h}} \sim 200\,\text{Mpc}$, while today $d_{\text{h}} \sim 6000\,\text{Mpc}$

$\Longrightarrow$ current observable volume $\sim 30000\times$ larger!

Note: we use $a \Longrightarrow$ all scales refer to what they are *now*, not what they *were* when the photons started!

Horizon problem: Why were causally disconnected areas on the sky so similar when CMB last interacted with matter?

Note that the horizon distance is larger than Hubble length:

$$d_{\text{h}} = \frac{2c}{H_0} > \frac{2c}{3H_0} = c \cdot t_0 = d_{\text{H}} \tag{12.5}$$

Reason for this is that universe expanded while photons traveled towards us

$\Longrightarrow$ The currently observable volume is larger than the volume expected in a non-expanding universe.

Flatness problem

Current observations of density of universe roughly imply

$$0.01 \lesssim \Omega \lesssim 2 \quad \text{i.e., } \Omega \sim 1 \tag{12.6}$$

(will be better constrained later)

$\Omega \sim 1$ imposes very strict conditions on initial conditions of universe:

The Friedmann equation (e.g., Eq. 7.41) can be written in terms of Ω:

$$\Omega - 1 = \frac{k}{a^2H^2} = \frac{ck}{\dot{a}^2} \tag{12.7}$$

For a nearly flat, matter dominated universe, $a(t) \propto t^{2/3}$, such that

$$\frac{\Omega(t) - 1}{\Omega(t_0) - 1} = \left(\frac{t}{t_0}\right)^{2/3} \tag{12.8}$$

while for the radiation dominated universe with $a(t) \propto t^{1/2}$,

$$\frac{\Omega(t) - 1}{\Omega(t_0) - 1} = \frac{t}{t_0} \tag{12.9}$$

Flatness problem

Today: $t_0 = 3.1 \times 10^{17}\,h^{-1}\,\text{s}$, i.e., observed flatness predicts for era of nucleosynthesis ($t = 1\,\text{s}$):

$$\frac{\Omega(1\,\text{s}) - 1}{\Omega(t_0) - 1} \sim 10^{-12} \ldots 10^{-16} \tag{12.10}$$

i.e., $\Omega(1\,\text{s})$ was very close to unity.

Flatness problem: It is very unlikely that Ω was so close to unity at the beginning without a physical reason.

Had Ω been different from 1, the universe would immediately have been collapsed or expanded too quickly

$\Longrightarrow$ Anthropocentric point of view *requires* $\Omega = 1$.

Hidden relics problem

Modern theories of particle physics predict the following particles to exist:

Gravitinos: Particle with $mc^2 \sim 100\,\text{GeV}$ and spin $3/2$ predicted by supergravity, if it exists, then nucleosynthesis would not work if BB started at $kT > 10^9\,\text{GeV}$.

Moduli: Spin-0 particles predicted by superstring theory, contents of vacuum at high energies.

Magnetic Monopoles: Predicted by grand unifying theories, but not observed.

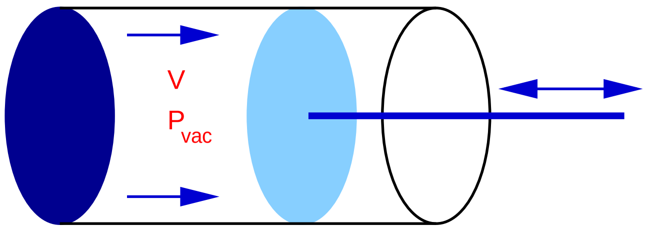
Hidden relics problem: If there was a normal big bang, then strange particles should exist, which are not observed today.

Vacuum, Λ

What is vacuum? *Not empty space* but rather ground state of some physical theory

(Reviews: Carroll et al. 1992, Carroll 2001)

Since ground state should be same in all coordinate systems $\Longrightarrow$ Vacuum is Lorentz invariant.



(after Peacock, 1999, Fig. 1.3)

Equation of state (Zeldovich, 1967):

$$P_{\text{vac}} = -\rho_{\text{vac}} c^2 \qquad (12.11)$$

This follows directly from 1st law of thermodynamics: ρ_{vac} should be constant if compressed or expanded, which is true only for this type of equation of state:

$$\mathrm{d}E = \mathrm{d}U + P\,\mathrm{d}V = \rho_{\text{vac}} c^2\,\mathrm{d}V - \rho_{\text{vac}} c^2\,\mathrm{d}V = 0 \qquad (12.12)$$

Vacuum, Λ

Remember the Friedmann equation:

$$\dot{R}^2 = +\frac{8\pi G\rho}{3}R^2 - kc^2 + \frac{1}{3}\Lambda c^2 R^2 \qquad (7.39)$$

From this we can identify the relationship between Einstein's cosmological constant, Λ, and ρ_{vac}:

$$\frac{8\pi G\rho_{\text{vac}}}{3} = \frac{1}{3}\Lambda c^2 \qquad (12.13)$$

and therefore

$$\Lambda = \frac{8\pi G\rho_{\text{vac}}}{c^2} \quad \text{and} \quad \rho_{\text{vac}} = \frac{\Lambda c^2}{8\pi G} \qquad (12.14)$$

This then allows us to define

$$\Omega_\Lambda = \frac{\rho_{\text{vac}}}{\rho_{\text{crit}}} = \frac{\rho_{\text{vac}}}{3H^2/8\pi G} = \frac{\Lambda c^2}{3H^2} \qquad (12.15)$$

Vacuum, Λ

How do we generate a vacuum energy?

Classical physics: Particles have energy

$$E = T + V \qquad (12.16)$$

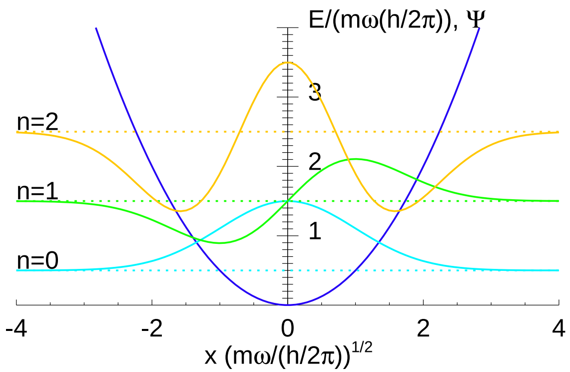
and force is $F = -\nabla V$, i.e., can add constant without changing equation of motion

$\Longrightarrow$ In classical physics, we are able to define $\rho_{\text{vac}} = 0$!

Quantum mechanics is (as usual) more difficult.

Vacuum, Λ

Vacuum in quantum mechanics:



Simplest case: harmonic oscillator:

$$V(x) = \frac{1}{2}m\omega^2x^2 \quad \text{i.e.,} \quad V(0) = 0 \tag{12.17}$$

However, particles can only have energies

$$E_n = \frac{1}{2}\hbar\omega + n\hbar\omega \quad \text{where } n \in \mathbb{N} \tag{12.18}$$

$\Longrightarrow$ Vacuum state has zero point energy

$$E_0 = \frac{1}{2}\hbar\omega \tag{12.19}$$

Simple consequence of uncertainty principle!

In QM, we could normalize $V(x)$ such that $E_0 = 0$, important here is that vacuum state energy *differs* from classical expectation!

Vacuum, Λ

Quantum field theory: Field as collection of harmonic oscillators of all frequencies. Simplest case: spinless boson ("scalar field", ϕ).

$\Longrightarrow$ Vacuum energy is the sum of all contributing ground state modes:

$$E_0 = \sum_j \frac{1}{2}\hbar\omega_j \tag{12.20}$$

Calculate sum by putting system in box with volume L^3, and then $L \longrightarrow \infty$.

Box $\Longrightarrow$ periodic boundary conditions:

$$\lambda_i = L/n_i \quad \Longleftrightarrow \quad k_i = 2\pi/\lambda_i = 2\pi n_i/L \tag{12.21}$$

for $n_i \in \mathbb{N} \Longrightarrow$ there are $\mathrm{d}k_i L/2\pi$ discrete wavenumbers in $[k_i, k_i + \mathrm{d}k_i]$, such that

$$E_0 = \frac{1}{2}\hbar L^3 \int \frac{\omega_{\mathbf{k}}}{(2\pi)^3}\mathrm{d}^3\mathbf{k} \quad \text{where} \quad \omega_k^2 = k^2 + m^2/\hbar^2 \tag{12.22}$$

Imposing cutoff k_{max}:

$$\rho_{\mathrm{vac}}c^2 = \lim_{L\to\infty}\frac{E_0}{L^3} = \hbar\frac{k_{\mathrm{max}}^4}{16\pi^3} \tag{12.23}$$

Divergent for $k_{\mathrm{max}} \longrightarrow \infty$ ("ultraviolet divergence").

Not worrisome since simple QM will break down at large energies anyway (ignored collective effects, etc.).

Vacuum, Λ

When does classical quantum mechanics break down?

Estimate: Formation of "Quantum black holes":

$$\lambda_{\text{de Broglie}} = \frac{2\pi\hbar}{mc} < \frac{2Gm}{c^2} = r_{\text{Schwarzschild}} \tag{12.24}$$

$\Longrightarrow$ Defines Planck mass:

$$m_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}} \mathrel{\widehat{=}} 1.22 \times 10^{19}\,\mathrm{GeV} \tag{12.25}$$

Corresponding length scale: Planck length:

$$l_{\mathrm{P}} = \frac{\hbar}{m_{\mathrm{P}}} = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-37}\,\mathrm{cm} \tag{12.26}$$

…and time scale (Planck time):

$$t_{\mathrm{P}} = \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-47}\,\mathrm{s} \tag{12.27}$$

$\Longrightarrow$ Limits of current physics until successful theory of quantum gravity.

The system of units based on l_{P}, m_{P}, t_{P} is called the system of Planck units.

Vacuum, Λ

To calculate the QFT vacuum energy density, choose

$$k_{\mathrm{max}} = m_{\mathrm{P}}c^2/\hbar \tag{12.28}$$

Inserting into Eq. (12.23) gives

$$\rho_{\mathrm{vac}}c^2 = 10^{74}\,\mathrm{GeV}\,\hbar^{-3} \quad \text{or} \quad \rho_{\mathrm{vac}} \sim 10^{92}\,\mathrm{g\,cm^{-3}} \tag{12.29}$$

…a tad bit on the high side ($\sim 10^{120}$ higher than observed).

Inserting ρ_{vac} in Friedmann equation: $T < 3\,\mathrm{K}$ at $t = 10^{-41}\,\mathrm{s}$ after Big Bang.

To obtain current universe we require $k_{\mathrm{max}} = 10^{-2}\,\mathrm{eV} \Longrightarrow$ Less than binding energy of Hydrogen, where QM definitively works!

Vacuum energy problem: Contributions from virtual fluctuations of all particles must cancel to very high precision to produce observable universe.

Casimir effect: force between conducting plates of area A and distance a in vacuum is $F_{\text{Casimir}} = \hbar c A\pi^2/(240a^4) \Longrightarrow$ caused by incomplete cancellation of quantum fluctuations. Confirmed by Lamoreaux in 1996 at 5% level.

Expansion problem

Cosmological Expansion:

GR predicts expansion of the universe, but initial conditions for expansion are not set!

Classical cosmology: "The unverse expands since it has expanded in the past"

$\Longrightarrow$ Hardly satisfying...

Cosmological Expansion Problem: What is the physical mechanism responsible for the expansion of the universe?

To put it more bluntly:

"The Big Bang model explains nothing about the origin of the universe as we now perceive it, because all the most important features are 'predestined' by virtue of being built into the assumed initial conditions near to $t = 0$." (Peacock, 1999, p. 324)

Baryogenesis

Quantitatively: Today:

$$\frac{N_{\mathrm{p}}}{N_\gamma} \sim 10^{-9} \quad \text{but} \quad \frac{N_{\bar{\mathrm{p}}}}{N_\gamma} \sim 0 \tag{12.30}$$

Assuming isotropy and homogeneity, this is puzzling: Violation of Copernican principle!

Antimatter problem: There are more particles than antiparticles in the observable universe.

Sakharov (1968): Asymmetry implies three fundamental properties for theories of particle physics:

1. CP violation (particles and antiparticles must behave differently in reactions, observed, e.g., in the K^0 meson),
2. Baryon number violating processes (more baryons than antibaryons $\Longrightarrow$ Prediction by GUT),
3. Deviation from thermal equilibrium in early universe (CPT theorem: $m_{\mathrm{X}} = m_{\bar{\mathrm{X}}} \Longrightarrow$ *same* number of particles and antiparticles in thermal equilibrium).

Structure formation

Final problem: structure formation

In the classical BB picture, the initial conditions for structure formation observed are not explained. Furthermore, assuming the observed Ω_{baryons}, the observed structures (=us) cannot be explained.

The theory of inflation attempts to explain all of the problems mentioned by invoking phase of exponential expansion in the very early universe ($t \lesssim 10^{-16}\,\mathrm{s}$).

Basic Idea

Use the Friedmann equation with a cosmological constant:

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{12.31}$$

Basic assumption of inflationary cosmology:

During the big bang there was a phase where Λ dominated the Friedmann equation.

$$H(t) = \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}} = \text{const.} \tag{12.32}$$

since $\Lambda = \text{const.}$ (probably...). Solution of Eq. (12.32):

$$a \propto e^{Ht} \tag{12.33}$$

and inserting into Eq. (12.7) shows that

$$\Omega - 1 = \frac{k}{a^2H^2} \propto e^{-2Ht} \tag{12.34}$$

Basic Idea

When did inflation happen?

Typical assumption: Inflation = phase transition of a scalar field ("inflaton") associated with Grand Unifying Theories.

Therefore the assumptions:

- temperature $kT_{\text{GUT}} = 10^{15}$ GeV, when $1/H \sim 10^{-34}\,\text{sec}$ ($t_{\text{start}} \sim 10^{-34}$ s).
- inflation lasted for 100 Hubble times, i.e., for $\Delta T = 10^{-32}$ s.

With Eq. (12.33): Inflation: Expansion by factor $e^{100} \sim 10^{43}$.

...corresponding to a volume expansion by factor $\sim 10^{130}$

$\Longrightarrow$ solves hidden relics problem!

Furthermore, Eq. (12.34) shows

$$\Omega - 1 = 10^{-86} \qquad (12.35)$$

$\Longrightarrow$ solves flatness problem!

Basic Idea

Temperature behavior: *During* inflation universe supercools:

Remember: entropy density

$$s = \frac{\rho c^2 + P}{T} \qquad (11.71)$$

But for Λ:

$$p = -\rho c^2 \qquad (12.11)$$

so that the entropy density of vacuum

$$s_{\text{vac}} = 0 \qquad (12.36)$$

Trivial result since vacuum is just one quantum state $\Longrightarrow$ *very* low entropy.

Inflation produces no entropy $\Longrightarrow$ S existing *before* inflation gets diluted, since entropy *density* $s \propto a^{-3}$.

But for relativistic particles $s \propto T^3$ (Eq. 11.73), such that

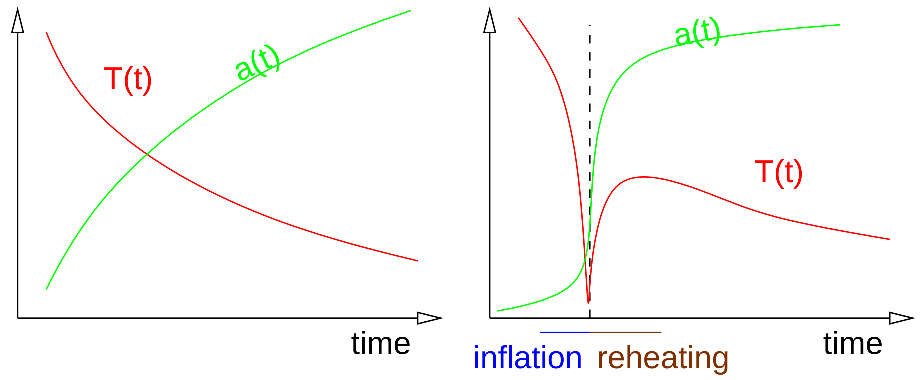
$$aT = \text{const.} \quad \Longrightarrow \quad T_{\text{after}} = 10^{-43} T_{\text{before}} \qquad (12.37)$$

When inflation stops: vacuum energy of inflaton field transferred to normal matter

$\Longrightarrow$ "Reheating" to temperature

$$T_{\text{reheating}} \sim 10^{15}\,\text{GeV} \qquad (12.38)$$

Summary



(after Bergström & Goobar, 1999, Fig. 9.1, and Kolb & Turner, Fig. 8.2)

Scalar Fields

For inflation to work: need short-term cosmological constant, i.e., need particles with negative pressure.

Basic idea (Guth, 1981): phase transition where suddenly a large Λ happens.

How? $\Longrightarrow$ Quantum Field Theory!

Describe hypothetical particle with a time-dependent quantum field, $\phi(t)$, and potential, $V(\phi)$.

Simplest example from QFT ($\hbar = c = 1$):

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \qquad (12.39)$$

where m: "mass of field". Particle described by ϕ: "inflaton".

For all scalar fields, particle physics shows:

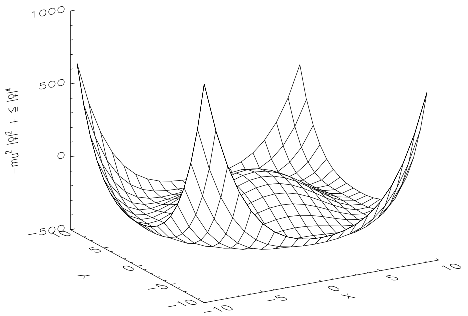
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad (12.40)$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad (12.41)$$

i.e., obeys vacuum equation of state!

"Vacuum": particle "sits" at minimum of V.

Scalar Fields



Typically: potential looks more complicated.
Due to symmetry, after harmonic oscillator, 2nd simplest potential: Mexican hat potential ("Higgs potential"),

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4 \qquad (12.42)$$

$\Longrightarrow$ Minimum of V still determines vacuum value.

For $T \neq 0$, we need to take interaction with thermal bath into account

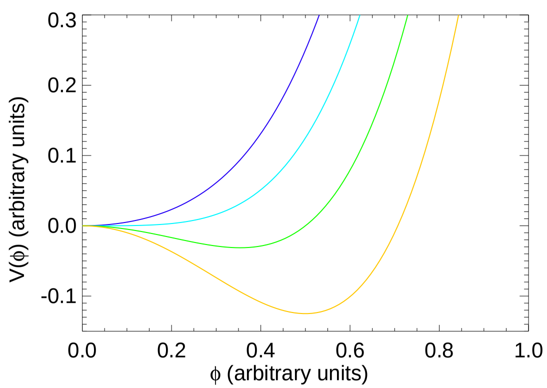
$\Longrightarrow$ Temperature dependent potential!

$$V_{\text{eff}}(\phi) = -(\mu^2 - aT^2)\phi^2 + \lambda\phi^4 \qquad (12.43)$$

where a is some constant.

(minimization of Helmholtz free energy, see Peacock, 1999, p. 329ff., for details)

Scalar Fields



The minimum of V is at

$$\phi = \begin{cases} 0 & \text{for } T > T_c \\ \sqrt{\frac{\mu^2 - aT^2}{2\lambda}} & \text{for } T < T_c \end{cases} \qquad (12.44)$$

where the critical temperature

$$T_c = \mu/\sqrt{a} \qquad (12.45)$$

and

$$V_{\text{min}} = \begin{cases} 0 & \text{for } T > T_c \\ -\frac{(\mu^2 - aT^2)^2}{4\lambda} & \text{for } T < T_c \end{cases} \qquad (12.46)$$

Since switch happens suddenly: phase transition

Scalar Fields

Minimum V_{min} for $T > T_c$ smaller than "vacuum minimum"

$\Longrightarrow$ Behaves like a cosmological constant!

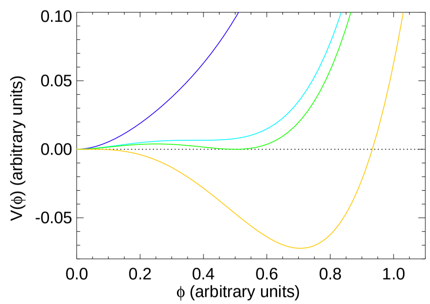
Since $T_c \propto \mu$,

Inflation sets in at mass scale of whatever scalar field produces inflation.

Grand Unifying Theories: $m \sim 10^{15}$ GeV.

The problem is, what $V(\phi)$ to use...

First-Order Inflation



(after Peacock, 1999, Fig. 11.2)

Original idea (Guth, 1981):

$$V(\phi, T) = \lambda|\phi|^4 - b|\phi|^3 + aT^2|\phi|^2 \qquad (12.47)$$

has two minima for T greater than a critical temperature:
$V_{\text{min}}(\phi = 0)$: false vacuum
$V_{\text{min}}(\phi > 0)$: true vacuum iff < 0.
Particle can tunnel between both vacua: first order phase transition $\Longrightarrow$ first order inflation.

Problem: vacuum tunnels between false and true vacua $\Longrightarrow$ formation of bubbles.

Outside of bubbles: inflation goes infinitely ("graceful exit problem").

First order inflation is not feasible.

Summary

First order inflation does not work

$\Longrightarrow$ Potentials derived from GUTs do not work.

$\Longrightarrow$ However, many empirical potentials do not suffer from these problems.

$\Longrightarrow$ inflation is *still* theory of choice for early universe.

Catchphrases (Liddle & Lyth, 2000, Ch. 8):

- chaotic inflation,
- supersymmetry/-gravitation $\Longrightarrow$ tree-level potentials,
- renormalizable global susy,
- power-law inflation,
- hybrid inflation (combination of *two* scalar fields) $\Longrightarrow$ spontaneous or dynamical susy breaking,
- scalar-tensor gravity

and many more. . .

All are somewhat *ad hoc*, and have more or less foundations in modern theories of QM and gravitation.

Information on what model is correct comes from

1. predicted seed to structure formation, and
2. values of Ω and Λ.

$\Longrightarrow$ Determine Ω and Λ!

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