



Determination of Ω and Λ

Inflation

Remember that

$$\Omega_m = \frac{\rho_m}{\rho_{\text{crit}}} = \frac{8\pi G \rho}{3H^2} \quad (7.42)$$

and

$$\Omega_\Lambda = \frac{\rho_{\text{vac}}}{\rho_{\text{crit}}} = \frac{\rho_{\text{vac}}}{3H^2/8\pi G} = \frac{\Lambda c^2}{3H^2} \quad (12.15)$$

As for a typical ensemble of stars,

$$\frac{M}{L} \approx \text{const.} \quad (13.2)$$

we often express Ω in terms of a mass to luminosity ratio:

Using canonical luminosity density of universe, one can show (Peacock, 1999, p. 368, for the B-band):

$$\left. \frac{M}{L} \right|_{\text{crit}} = 1390 h \frac{M_\odot}{L_\odot} \quad (13.3)$$

... which means that there *must* be lots of dark matter.

Motivation



Inflation

Previous lectures: Inflation requires

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} = \Omega_m + \Omega_\Lambda = 1 \quad (13.1)$$

Here,

Ω_m : Ω due to gravitating stuff,

Ω_Λ : Ω due to vacuum energy or other exotic stuff.

To decide whether that is true:

- need inventory of gravitating material in the universe,
- need to search for evidence of non-zero Λ

Also search for evidence in structure formation \Rightarrow Later...

Motivation



Introduction

Constituents of Ω_m :

- Radiation (CMBR)
 - **Neutrinos**
 - Baryons ("normal matter", Ω_b)
 - Other, non-radiating, gravitating material ("dark matter")
- Radiation:** From temperature of CMBR, using $u = \rho c^2 = a_{\text{rad}} T^4$:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \quad (13.4)$$

for $h = 0.72$, $\Omega_\gamma = 4.8 \times 10^{-5}$

Massless Neutrinos have

$$\Omega_\nu = 3 \cdot \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \Omega_\gamma = 0.68 \Omega_\gamma \quad (13.5)$$

Photons and massless neutrinos are unimportant for today's Ω .

Determination of Ω_m



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Massive Neutrinos

Sudbury Neutrino Observatory (SNO) and Super-Kamiokande: Neutrinos are not massless.

From neutrino decoupling and expansion:

Current neutrino density: 113 neutrinos cm^{-3} per neutrino family.

In terms of Ω :

$$\Omega_\nu h^2 = \frac{\sum m_i}{93.2 \text{ eV}} = \frac{M_\nu}{93.2 \text{ eV}} \quad (13.6)$$

\Rightarrow For $h = 0.72$, $m \sim 16 \text{ eV}$ would be sufficient to close universe

Current mass limits:

- cosmological studies (structure formation): $M_\nu < 0.17 \text{ eV}$
- laboratory experiments: $m_{\nu_e} < 2.2 \text{ eV}$, and solar neutrino oscillations imply $\Delta m = m_{\nu_\mu} - m_{\nu_e} \lesssim 10^{-4} \text{ eV}$.

See Steidl (2009) for a recent review on experiments to measure neutrino masses.

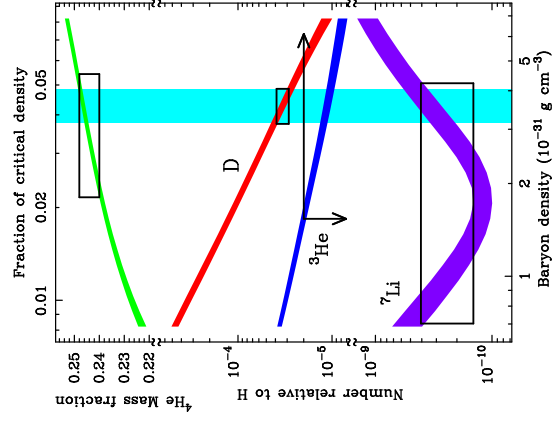
Determination of Ω_m

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13-6

Baryons



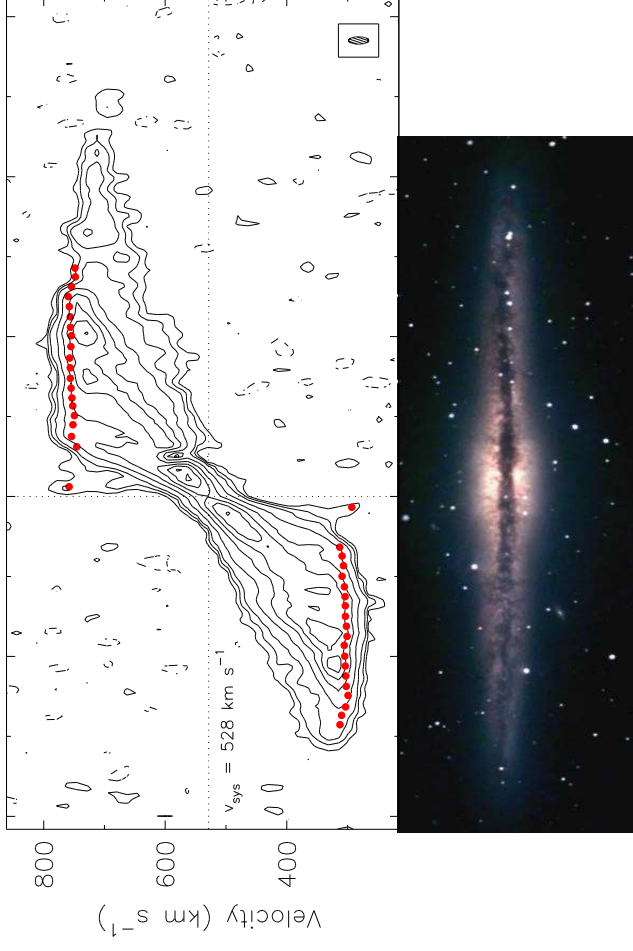
Best evidence for mass in baryons, Ω_b : primordial nucleosynthesis.

$$\Omega_b h^2 = 0.02 \pm 0.002 \quad (13.7)$$

(Burles et al., 1999, Fig. 1)

Determination of Ω_m

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NGC 891 (Swaters et al., 1997, ApJ 491, 140 / Paul LeFevre, S&T Nov. 2002)



13-8

Galaxy Rotation Curves



Reminder: galaxies have flat rotation curves. Analysis of mass required to describe motion leads to dark matter halos and to

$$\frac{M}{L} \Big|_{\text{galaxies}} = 10 \dots 20 \frac{M_\odot}{L_\odot} \quad (13.8)$$

Only about 10% of the gravitating matter in universe radiates.

NGC 891, KPNO 1.3m
Barentine & Esquerdo

Determination of Ω_m

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Coma Cluster (O. Lopez-Cruz, I. K. Shelton, & KPNO)



13-10

Galaxy Clusters

To determine mass of galaxy clusters, use the virial theorem: In statistical equilibrium:

$$E_{\text{kin}} = -E_{\text{pot}}/2 \quad (13.9)$$

Measurement: assume isotropy, such that

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle = 3 \langle v_{\parallel}^2 \rangle \quad (13.10)$$

Assuming that the velocity dispersion is independent of m_i gives:

$$E_{\text{kin}} = \frac{1}{2} \sum_i m_i v_i^2 = \frac{3}{2} M \langle v_{\parallel}^2 \rangle \quad (13.11)$$

where M is the total mass.

If the cluster is spherically symmetric \implies Define weighted mean separation R_{cl} , such that

$$E_{\text{pot}} = \frac{GM^2}{R_{\text{cl}}} \quad (13.12)$$

From Eqs. (13.11) and (13.12):

$$M = \frac{3}{G} \langle v_{\parallel}^2 \rangle R_{\text{cl}} \quad (13.13)$$

E.g.: $v_{\parallel} \sim 1000 \text{ km s}^{-1}$, $R \sim 1 \text{ Mpc} \implies M = 1.4 \times 10^{46} \text{ g} = 7 \times 10^{14} M_{\odot}$ (MW: $6 \times 10^{11} M_{\odot}$).

Determination of Ω_m

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13-10

Derivation of the Virial Theorem

Assume system of particles, each with mass m_i . Acceleration on particle i :

$$\ddot{\mathbf{r}}_i = \sum_{j \neq i} \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \quad (13.14)$$

... scalar product with $m_i \mathbf{r}_i$:

$$m_i \mathbf{r}_i \cdot \ddot{\mathbf{r}}_i = \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \quad (13.15)$$

... since

$$\frac{1}{2} \frac{d^2 r_i^2}{dt^2} = \mathbf{r}_i \cdot \ddot{\mathbf{r}}_i = \mathbf{r}_i \cdot \mathbf{r}_i + \dot{\mathbf{r}}_i \cdot \dot{\mathbf{r}}_i \quad (13.16)$$

... therefore Eq. (13.15)

$$\frac{1}{2} \frac{d^2}{dt^2} (m_i r_i^2) - m_i \dot{\mathbf{r}}_i^2 = \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \quad (13.17)$$

Summing over all particles in the system gives

$$\begin{aligned} \frac{1}{2} \sum_i \frac{d^2}{dt^2} (m_i r_i^2) - \sum_i m_i \dot{\mathbf{r}}_i^2 &= \sum_i \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \\ &= \frac{1}{2} \left(\sum_i \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j \sum_{i \neq j} \frac{Gm_j m_i \mathbf{r}_j \cdot (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \right) \\ &= \frac{1}{2} \left(\sum_i \sum_{j \neq i} \frac{Gm_i m_j \mathbf{r}_i \cdot \mathbf{r}_j - r_i^2}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j \sum_{i \neq j} \frac{Gm_j m_i \mathbf{r}_j \cdot \mathbf{r}_i - r_j^2}{|\mathbf{r}_j - \mathbf{r}_i|^3} \right) \\ &= -\frac{1}{2} \sum_{i \neq j} \frac{Gm_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|} \end{aligned} \quad (13.18) \quad (13.19) \quad (13.20) \quad (13.21)$$

Thus, identifying the total kinetic energy, T , and the gravitational potential energy, U , gives

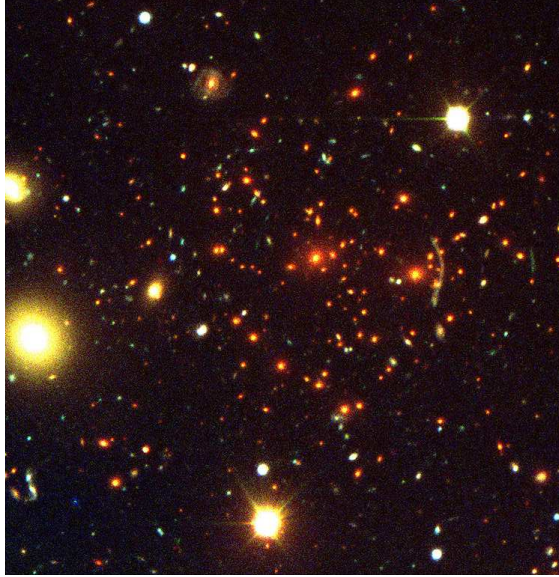
$$2T - U = \frac{1}{2} \frac{d^2}{dt^2} \sum_i m_i r_i^2 = 0 \quad (13.22)$$

in statistical equilibrium.

Thus we find the virial theorem: $T = \frac{1}{2} |U|$



Galaxy Clusters



More detailed analysis using more complicated mass models gives (Merritt, 1987):

$$\frac{M}{L} \sim 350 h^{-1} \frac{M_{\odot}}{L_{\odot}} \quad (13.23)$$

while we would have expected $M/L = 10 \dots 20$ as for galaxies

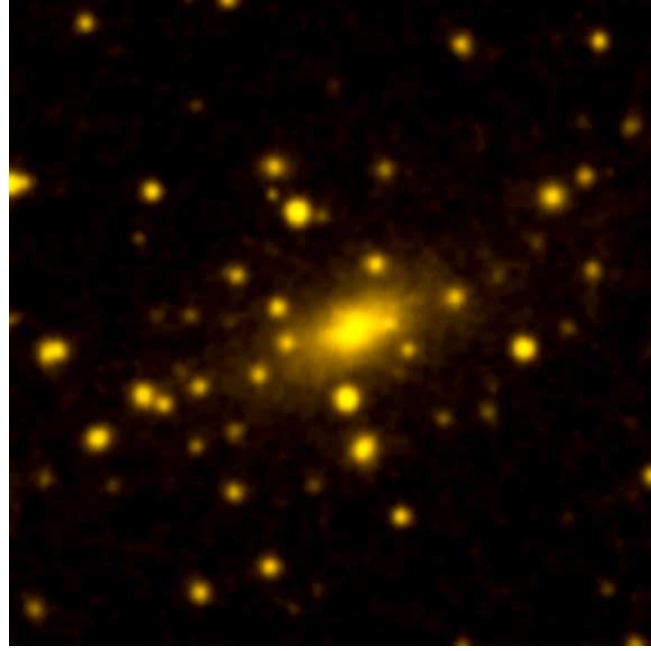
Dark matter is an important constituent in galaxy clusters

Abell 370 (VLT UT1+FORs)

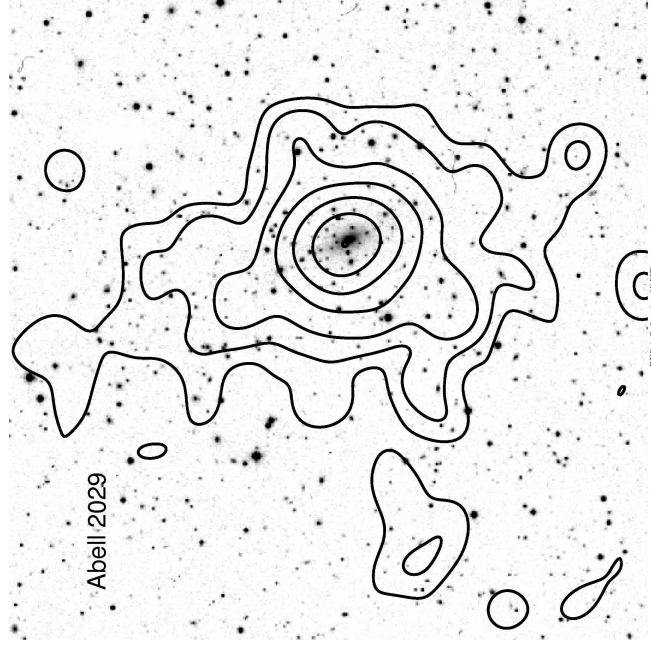
Determination of Ω_m



Abell 2029, Soft X-rays (Chandra; NASA/CXC/UCI/A. Lewis et al.)



Abell 2029, Palomar Schmidt [DSS]



Abell 2029, Optical and X-rays (XMM-Newton; Andy Read [Leicester]/DSS/ESA; larger FoV)

X-ray emission

X-ray emission from galaxy clusters gives mass to higher precision:

Assume gas in potential of galaxy cluster. If gas is in hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2} \quad (13.24)$$

where the pressure P can be determined from the equation of state:

$$P = nkT = \frac{\rho kT}{\mu m_H} \quad (13.25)$$

where m_H : mass of H-atom, μ mean molecular weight of gas ($\mu = 0.6$ for fully ionized).

Differentiating Eq. (13.25) wrt r gives

$$\frac{dP}{dr} = \frac{k}{\mu m_H} \left(T \frac{d\rho}{dr} + \rho \frac{dT}{dr} \right) = \frac{\rho kT}{\mu m_H} \left(\frac{d \log \rho}{dr} + \frac{d \log T}{dr} \right) \quad (13.26)$$

Inserting dP/dr into Eq. (13.24) and solving for M_r gives

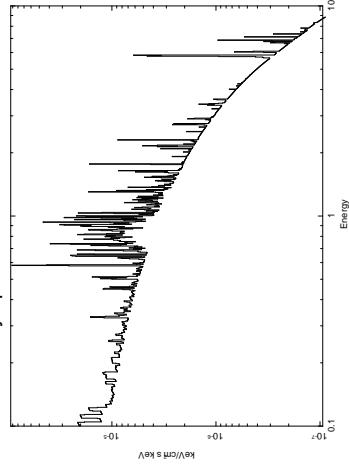
$$M_r = -\frac{kT r^2}{G \mu m_H} \left(\frac{d \log \rho}{dr} + \frac{d \log T}{dr} \right) \quad (13.27)$$

Determination of Ω_m

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X-ray emission

To determine M_r , we need to measure $T(r)$ and $\rho(r)$. These quantities can be obtained from the observed X-ray spectrum:



Cluster gas mainly radiates by bremsstrahlung emission, with a spectral continuum shape

$$\epsilon(E) \propto \left(\frac{m_e}{kT} \right)^{1/2} g(E, T) n n_e \exp \left(-\frac{E}{kT} \right) \quad (13.28)$$

where

n : number density of nuclei,

n_e : number density of electrons,

$g(E, T)$: Gaunt factor (QM correction factor, roughly constant).

plus emission lines...

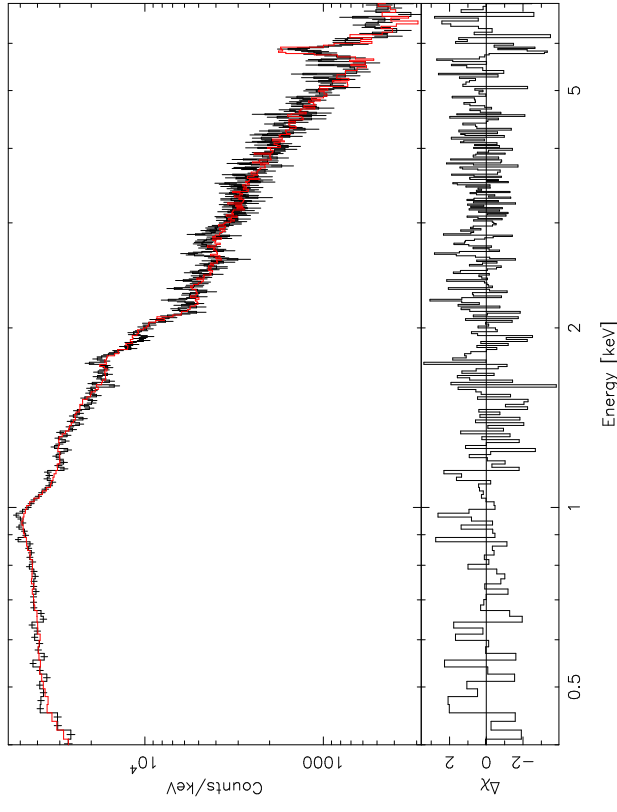
Theoretical X-ray spectrum of a cluster.

$\Rightarrow T(r)$ can be obtained from the X-ray spectral shape, n and n_e from the measured flux

$\Rightarrow M_r$.

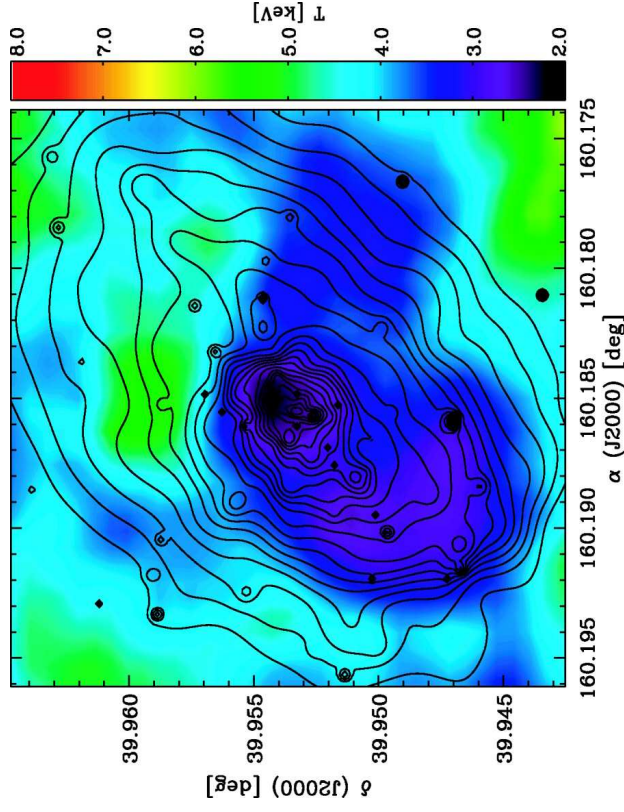
Determination of Ω_m

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(Wise et al., 2004, Fig. 2)

X-ray spectrum of A1068 obtained from Chandra

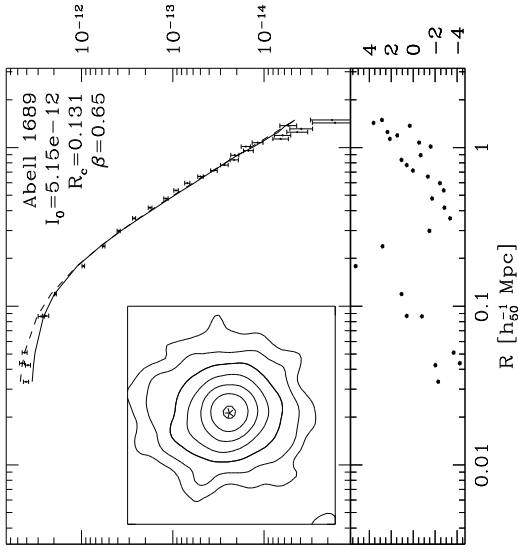


(Wise et al., 2004, Fig. 8)

Temperature distribution in A1068 obtained with Chandra



X-ray emission



Generally: assume intensity profile from β -model,

$$\frac{I(r)}{I_0} = \left(1 + \left(\frac{r}{R_c}\right)^2\right)^{-3\beta + \frac{1}{2}} \quad (13.30)$$

and obtain T from fitting X-ray spectra to "shells" \implies technically complicated...

Summary for X-ray mass determination for 45 clusters (Mohr et al., 1999):

$$f_{\text{gas}} = (0.07 \pm 0.002) h^{-3/2} \quad (13.31)$$

resulting in

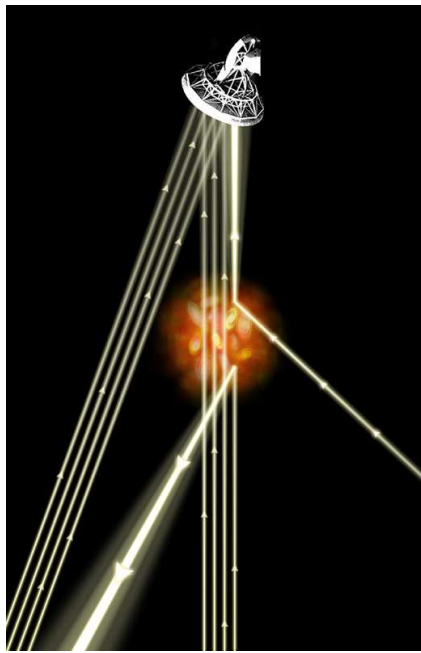
$$\Omega_m = \Omega_b / f_{\text{gas}} = (0.3 \pm 0.05) h^{-1/2} \quad (13.32)$$

(Mohr et al., 1999)

Determination of Ω_m



Sunyaev-Zeldovich

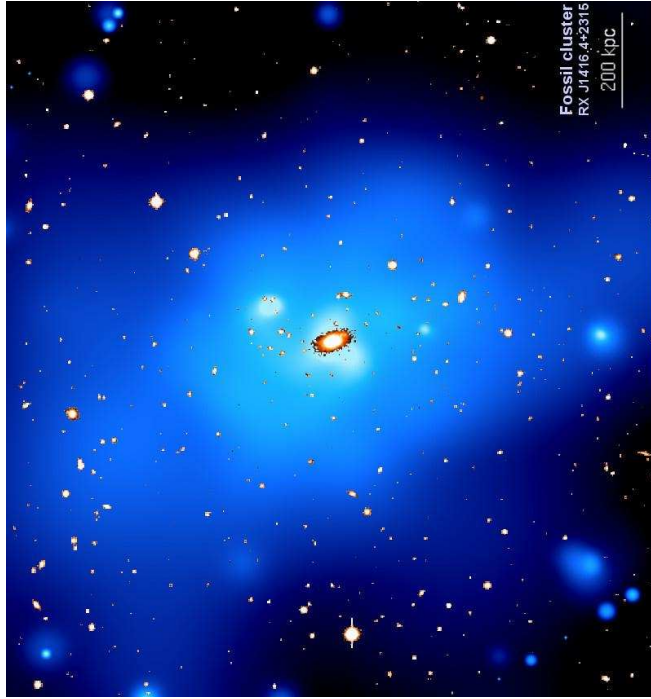


NASAXC/M. Weiss

Gas in cooling flow influences CMBR by Compton upscattering

\implies Sunyaev-Zeldovich effect (1970).

Determination of Ω_m



XMM-Newton explores the fossil galaxy cluster RX J1416.4+2315
Image courtesy of Habib Ebraheemali (University of Birmingham)

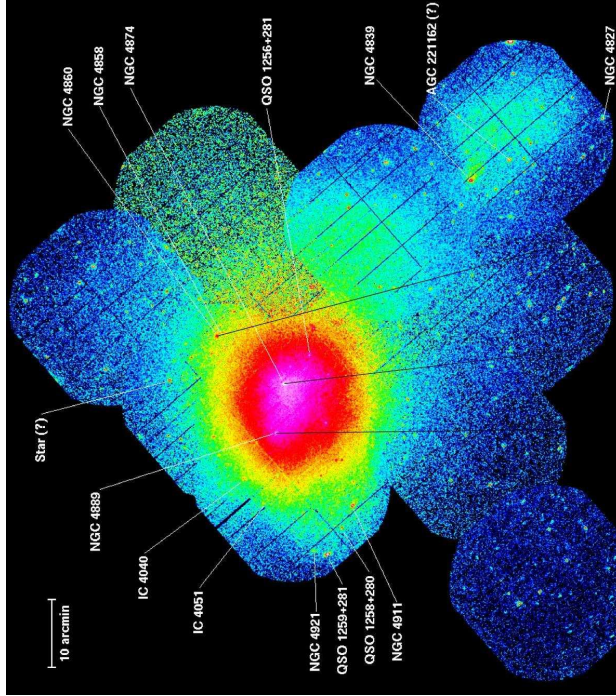
European Space Agency

Technical problems:

- see through cluster \implies integrate over line of sight, assuming spherical geometry.
- spherical geometry is assumed
- it is unclear whether gas is in hydrostatic equilibrium (cooling flows? – but note, there is sparse evidence for a "flow")

XMM-Newton, EPIC-pn

$$(13.29)$$

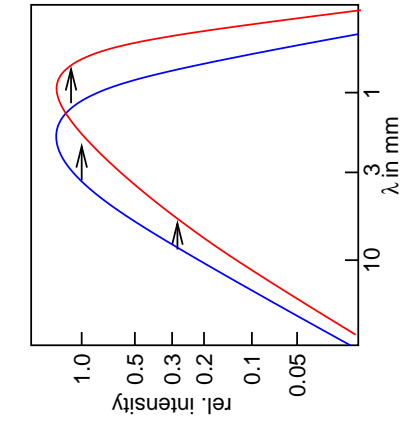


Result for Coma:

$$\frac{M_B}{M_{\text{tot}}} = 0.01 + 0.05 h^{-3/2} \sim 0.18$$

Sunyaev-Zeldovich

The quantitative derivation of the SZ-effect is difficult, basically, one sets up the Fokker-Planck equation for the photon gas and from this derives the so-called Kompaneets equation, see, e.g., Peacock (1999, p. 375ff.).



after Schneider

The basic ingredients are the optical depth for Compton scattering (Compton y -parameter):

$$y = \int \left(\frac{kT_e}{m_e c^2} \right) \sigma_T N_e dl \quad (13.33)$$

From this follows in the Rayleigh-Jeans regime that the intensity due to Compton upscattering changes as follows:

$$\frac{\Delta I}{I} = -2y \sim 10^{-4} \quad (13.34)$$

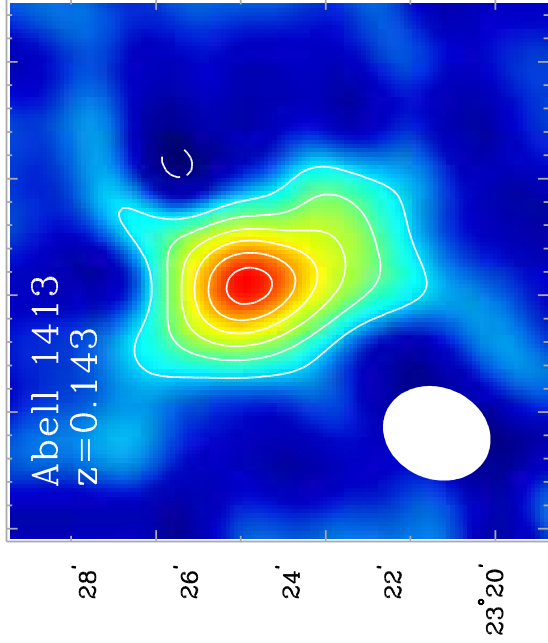
(for typical parameters).

$\Rightarrow \Delta I$ allows to measure of $\int N_e T_e dl$

\Rightarrow Mass!

T is known from X-ray spectrum.

Determination of Ω_m



11 55 36 30 24 18 12 6 55 0

(temperature decrement from 3 K background, Carlstrom et al., 2000, Fig. 3)

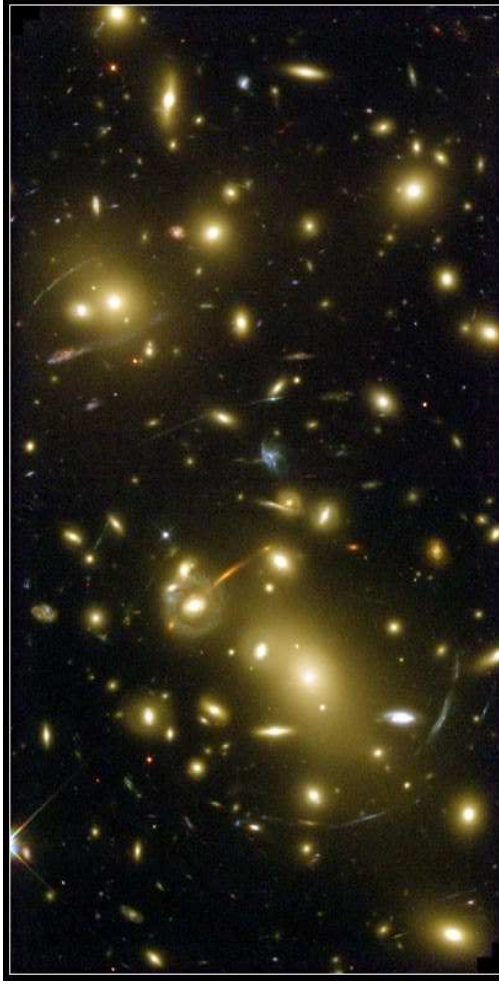
SZ analysis gives gas fraction for 27 clusters

$$f_{\text{gas}} = (0.06 \pm 0.006) h^{-3/2} \quad (13.35)$$

remarkably similar to X-ray result \Rightarrow clumping of gas does not influence results! (SZ only traces real gas...)

f_{gas} translates to

$$\Omega_m = (0.25 \pm 0.04) h^{-1} \quad (13.36)$$



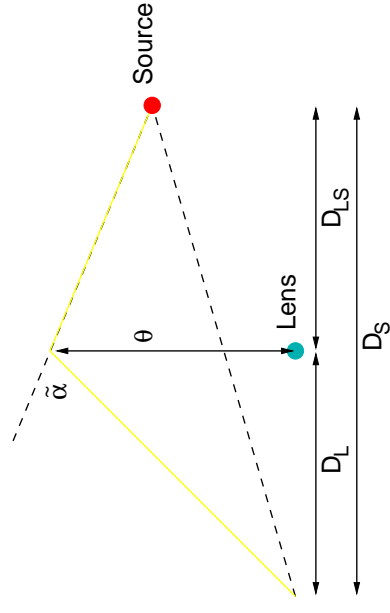
Galaxy Cluster Abell 2218

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC000-08

HST • WFPC2



Gravitational Lenses



(after Longair, 1998, Fig. 4.8a)

GR: Angular deflection of light due to presence of mass M :

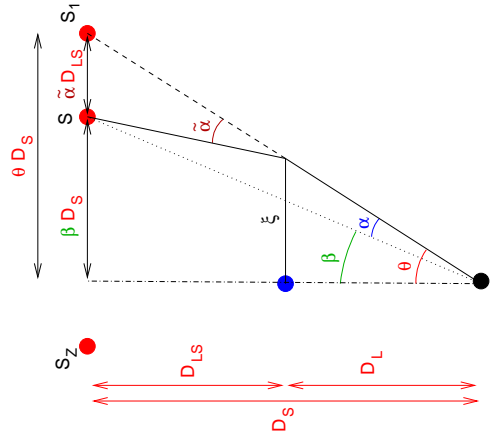
$$\tilde{\alpha} = -\frac{4GM}{\theta c^2} = \frac{2}{c^2} \cdot \frac{2GM}{\theta} \quad (13.37)$$

where θ : distance of closest approach (twice the classical result).

Determination of Ω_m



Gravitational Lenses



In the small angle approximation:

$$\theta D_S = \beta D_S + \tilde{\alpha} D_{LS} \quad (13.38)$$

such that

$$\beta = \theta - \frac{D_{LS} \tilde{\alpha}}{D_S} \quad (13.39)$$

Defining the reduced deflection angle,

$$\alpha = \frac{D_{LS} \tilde{\alpha}}{D_S} = \frac{D_{LS}}{D_S} \cdot \frac{2}{c^2} \cdot \frac{GM}{\xi} \quad (13.40)$$

then gives the lens equation

$$\beta = \theta - \alpha = \theta - \frac{D_{LS}}{D_S} \cdot \frac{4GM}{c^2 \xi} = \theta - \frac{1}{D} \cdot \frac{4GM}{c^2 \theta} \quad (13.41)$$

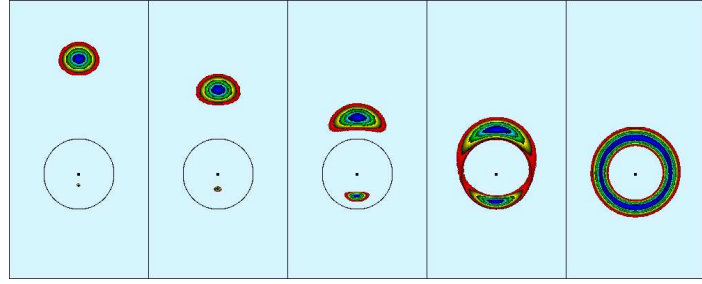
where

$$D = \frac{D_L D_S}{D_{LS}} \quad (13.42)$$

(last expression valid for a point-mass)

after Wambsganss (1998, Fig. 3)

Determination of Ω_m



Einstein ring: source directly behind lens,

Obtain radius by setting $\beta = 0$ in lens-equation

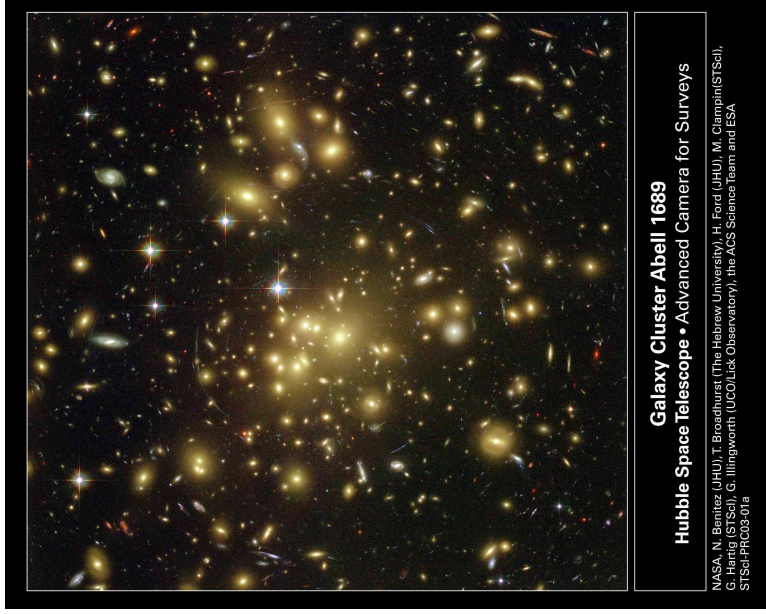
(Eq. 13.41):

$$\theta_E^2 = \frac{4GM}{c^2} \frac{1}{D} \quad (13.43)$$

i.e.,

$$\theta_E = 98.9'' \left(\frac{M}{10^{15} M_\odot} \right)^{1/2} \frac{1}{(D/1 \text{ Gpc})^{1/2}} \quad (13.44)$$

Mass measurements possible by observing “giant luminous arcs” and Einstein rings.



Galaxy Cluster Abell 1689
Hubble Space Telescope • Advanced Camera for Surveys

NASA, N. Benitez (JHU), T. Broadhurst (The Hebrew University), H. Ford (JHU), M. Clampin (STScI), G. Hartig (STScI), G. Illingworth (UCO/Lick Observatory), the ACS Science Team and ESA STScI-PRC03-01a

General results of mass determinations from lensing agree with other methods.



Summary

So far, we have seen:

Photons:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \quad (13.45)$$

Neutrinos:

$$\Omega_\nu h^2 = 1.69 \times 10^{-5} \quad (13.46)$$

Baryons (from nucleosynthesis):

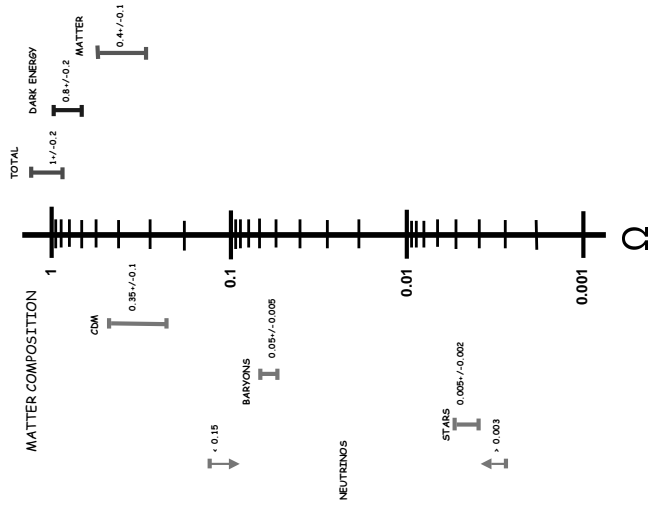
$$\Omega_b h^2 = 0.02 \quad \text{where} \quad \Omega_{\text{stars}} \sim 0.005 \dots 0.01 \quad (13.47)$$

Baryons+dark matter (from clusters):

$$\Omega_m \sim 0.25 \quad (13.48)$$

(of which $\sim 10\%$ in baryons)

If we believe in $\Omega_{\text{total}} \equiv 1 \implies \Omega_\Lambda \sim 0.7$.



(Turner, 1999, Fig. 1, numbers slightly different to ours...)



Introduction

Clusters and galaxies: $\Omega_m \sim 0.3$, but for baryons $\Omega_b \sim 0.02$

⇒ Rest of gravitating material is dark matter.

⇒ There are two dark matter problems:

$$\Omega_m \leftarrow \begin{matrix} \text{nonbaryonic dark matter} \\ \text{baryonic dark matter} \end{matrix} \leftarrow \Omega_{\text{stars}}$$

baryonic dark matter= undetected baryons:

- diffuse hot gas?
- MACHOs (Massive compact halo objects; white dwarfs, neutron stars, black holes, brown dwarfs, Jupiters...)

nonbaryonic dark matter= exotic stuff:

- massive neutrinos
- axions
- neutralinos

Baryonic Dark Matter

Intra Cluster Gas:

Pro:

1. same location where the hot gas in clusters also found,
2. structure formation suggests most baryons are *not* in structures today

Contra:

1. 90% of the universe is *not* in clusters. ...
2. gas has not been detected at any wavelength

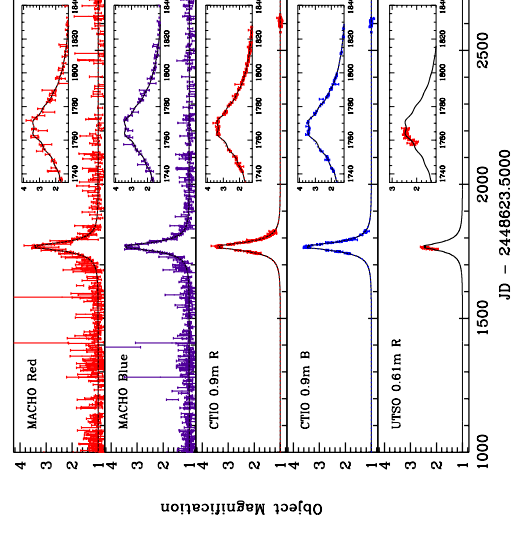
If gas cold enough, would not expect it to be detectable, so point 2 is not really valid.

Dark Matter



Baryonic Dark Matter

MACHO Event 96-LMC-2



MACHOS:

Pro:

1. detected by microlensing towards SMC and LMC (see figure) ⇒ MW halo consists of 50% WD

Contra:

1. possible "self-lensing" (by stars in MW or SMC/LMC; confirmed for a few cases)
2. where are white dwarfs?
3. WD formation rate too high ($100 \text{ year}^{-1} \text{ Mpc}^{-3}$)

(Alcock et al., 2001, Fig. 2)

Dark Matter



Nonbaryonic Dark Matter

Nonbaryonic dark matter:

Requirements: must be gravitating and non-interacting with baryons

⇒ Grab-box of elementary particle physics:

1. Neutrinos with non-zero mass

Pro: It exists, mass limits are a few eV, need only $\langle m_\nu \rangle \sim 10 \text{ eV}$

Contra: ν are relativistic ⇒ Hot dark matter ⇒ Forces top down structure formation, contrary to what is believed to have happened.

2. Axion

(=Goldstone boson from QCD, invented to prevent strong CP violation in QCD; $m \sim 10^{-5} \dots -2 \text{ eV}$)

Pro: It *could* exist, would be in Bose-Einstein condensate due to inflation (⇒ Cold dark matter!), might be detectable in the next 10 years

Contra: We do not know it exists...

3. Neutralino or other WIMPs (weakly interacting massive particles; masses

$m \sim \text{GeV}$)

Pro: Also is CDM

Contra: We do not know they exist...

Dark Matter



Friedmann with $\Lambda \neq 0$

Since $\Omega_m \sim 0.3$, if $\Omega_{\text{tot}} = 1$, then $\Omega_\Lambda \sim 0.7$ ⇒ We need to study cosmology with $\Lambda \neq 0$.

Reviews: Carroll et al. (1992), Carroll (2001)

Friedmann equation with $\Lambda \neq 0$:

$$H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3} \quad (12.31)$$

And define the Ω 's (Eqs. 7.42, 12.15):

$$\Omega_m = \frac{8\pi G\rho_m}{3H_0^2}, \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}, \quad \Omega_k = -\frac{kc^2}{R_0^2 H_0^2} \quad (13.49)$$

Because of Eq. (12.31),

$$\Omega_m + \Omega_\Lambda + \Omega_k = \Omega + \Omega_k = 1 \quad (13.50)$$

Friedmann with nonzero Lambda



Friedmann with $\Lambda \neq 0$

It is easier to set $c = 1$ and to work with the dimensionless scale factor,

$$a = \frac{R_i(t)}{R_0} \quad (7.29)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \frac{\rho_{m,0}}{a^3} - \frac{k}{a^2 R_0^2} + \frac{\Lambda}{3} \quad (13.51)$$

since $\rho_m = \rho_{m,0} a^{-3}$ (Eq. 7.63).

Inserting the Ω 's

$$\left(\frac{\dot{a}/H_0}{a}\right)^2 = \frac{\Omega_m}{a^3} + \frac{1 - \Omega_m - \Omega_\Lambda}{a^2} + \Omega_\Lambda \quad (13.52)$$

Substituting the time in units of today's Hubble time,

$$\tau = H_0 \cdot t \quad (13.53)$$

results in

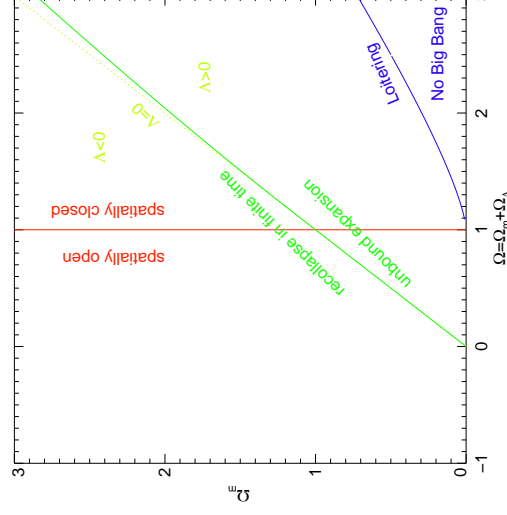
$$\left(\frac{d\tau}{d\tau}\right)^2 = 1 + \Omega_m \left(\frac{1}{a} - 1\right) + \Omega_\Lambda (a^2 - 1) \quad \text{where } a(\tau = 1) = 1 \quad \text{and} \quad \left.\frac{d\tau}{d\tau}\right|_{\tau=1} = 1 \quad (13.54)$$

For most combinations of Ω_m and Ω_Λ , we need to solve the equations numerically.

Friedmann with nonzero Lambda



Friedmann with $\Lambda \neq 0$



(after Carroll et al., 1992, Fig. 1)

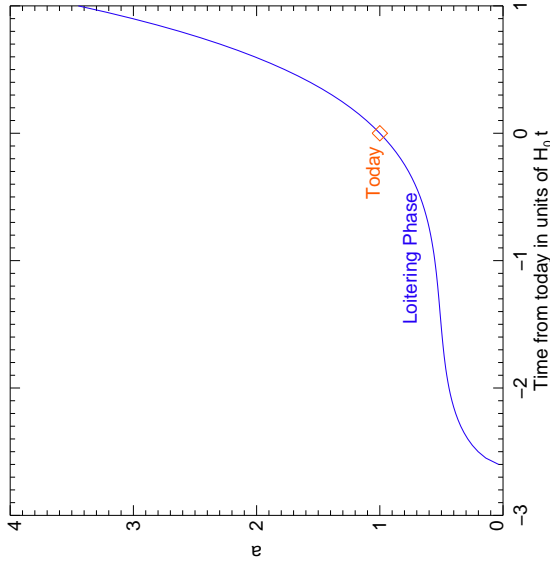
With Λ , evolution of universe is more complicated than without:

- unbound expansion possible for $\Omega < 1$,
- For Ω_Λ large: no big bang!
- For Ω_Λ large: possible "loitering phase"

Friedmann with nonzero Lambda



$$\Omega_\Lambda > 1$$



For large Ω_Λ : contraction from $+\infty$ and reexpansion
 \implies no big bang.
 For slightly smaller Ω_Λ : phase where $\dot{a} \sim 0$ in the past
 \implies loitering universe.

"Loitering universe" with $\Omega_m = 0.55$,
 $\Omega_\Lambda = 2.055$

Friedmann with nonzero Lambda

4



$$\Omega_\Lambda > 1$$



QSO at $z = 5.82$, courtesy SDSS

Since $1 + z = 1/a$ (Eq. 7.53), existence of turning-point \implies maximal possible z :

$$z \leq 2C_\kappa \left(\frac{1 - \Omega_m}{3C_\kappa} \right)^{-1} - 1 \quad (13.56)$$

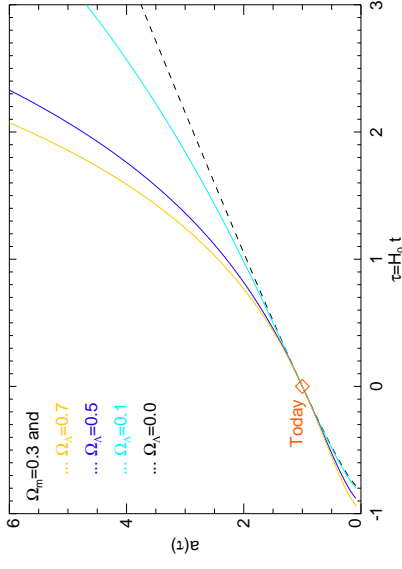
(Carroll et al., 1992, Eq. 14). Since quasars are observed with $z > 5.82$, this means that $\Omega_m < 0.007$, clearly not what is observed $\implies \Omega_\Lambda < 1$.

Friedmann with nonzero Lambda

5



$$\Omega_\Lambda < 1$$



For $\Omega_\Lambda < 1$ evolution has two parts:

- matter domination, similar to earlier results
- Λ domination, exponential rise.

Exponential rise called by some workers the "second inflationary phase"...

Friedmann with nonzero Lambda

6



$$\Omega_\Lambda < 1$$

Calculation of age of universe is similar to $\Omega_\Lambda = 0$ case (see, e.g., Eq. 7.81), but generally only possible numerically.

Result:

Universes with $\Omega_\Lambda > 0$ are *older* than those with $\Omega_\Lambda = 0$.

This solves the age problem, that some globular clusters have age comparable to age of universe if $\Omega_\Lambda = 0$.

Analytical formula for age (Carroll et al., 1992, Eq. 17):

$$t = \frac{2}{3H_0} \frac{\sinh^{-1} \left(\sqrt{(1 - \Omega_0)/\Omega_0} \right)}{\sqrt{1 - \Omega_0}} \quad (13.57)$$

for $\Omega_0 < 1$, where

$$\Omega_0 = 0.7\Omega_m + 0.3(1 - \Omega_\Lambda) \quad (13.58)$$

For $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$: $t = 13.5 \text{ Gyr}$.
 Remember that for $\Omega_m = 1$, $t = 2/(3H_0)$!

Friedmann with nonzero Lambda

7



Luminosity Distance

Influence of Λ is most prominent at large distances!

- ⇒ Expect **influence on Hubble Diagram**.
- ⇒ Need to find relation between measured flux, emitted luminosity, and redshift.

Assume source with luminosity L at comoving coordinate r , emitting isotropically into 4π sr.

- At time of detection today, photons are
- on sphere with proper radius $R_0 r$,
 - redshifted by factor $1+z$,
 - spread in time by factor $1+z$.

⇒ observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} \quad (13.59)$$

Determination of Ω_Λ



Luminosity Distance

Because the observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} \quad (13.59)$$

in analogy to the inverse square law one defines the luminosity distance as

$$d_L = R_0 \cdot r \cdot (1+z) \quad (13.60)$$

The calculation of d_L is somewhat technical, one can show that (Carroll et al., 1992):

$$d_L = \frac{c}{H_0} |\Omega_k|^{-1/2} \cdot S_{-\text{sgn}(\Omega_k)} \left\{ |\Omega_k|^{1/2} \int_0^z [(1+z)^2 (1 + \Omega_m z) - z(2+z)\Omega_\Lambda]^{1/2} dz \right\} \quad (13.61)$$

Remember: $S_{-\text{sgn}(\Omega_k)}(x) = x$ for $\Omega_k = 0$ (Eq. 7.89).

Determination of Ω_Λ



Supernovae

Best way to determine Ω_Λ :

Type Ia supernovae

Remember: SN Ia = CO WD collapse... (Hoyle, Fowler, Colgate, Wheeler, ...)

The distance modulus is

$$m - M = 5 \log \left(\frac{d_L}{1 \text{ Mpc}} \right) + 25 \quad (13.62)$$

Use SNe as standard candles ⇒ Deviations from $d_L \propto z$ indicative of Λ .

Two projects:

- High- z Supernova Team (STSCI, Riess et al.)
- Supernova Cosmology Project (LBNL, Perlmutter et al.)

Both find SNe out to $z \sim 1$.

Present mainly Perlmutter et al. results here, Riess et al. (1998) are similar.

Newest analysis of the data set: Kowalski et al. (2008)

Determination of Ω_Λ



Supernovae

Basic observations: easy:

- Detect SN in rise ⇒ CTIO 4 m
- Follow SN for $\sim 2-3$ months with 2-4 m class telescopes, HST, Keck...

More technical problems in data analysis: Conversion into source frame:

- Correction of photometric flux for redshift: "K-correction"
- Correct for time dilatation in SN light curve

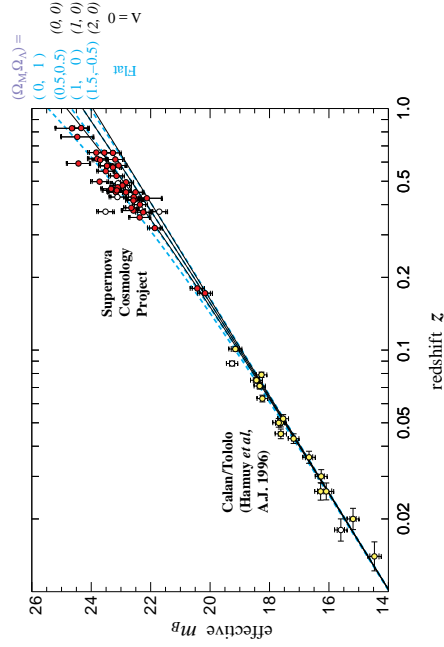
Further things to check

- SN internal extinction
- Galactic extinction
- Galactic reddening
- Photometric cross calibration
- Peculiar motion of SN

Determination of Ω_Λ



Supernovae

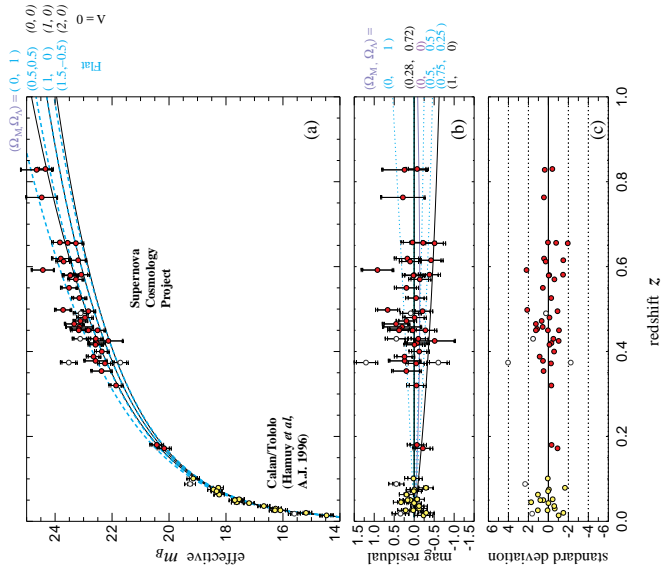


Vertical error bars:
measurement
uncertainty plus
0.17 mag intrinsic mag.
dispersion

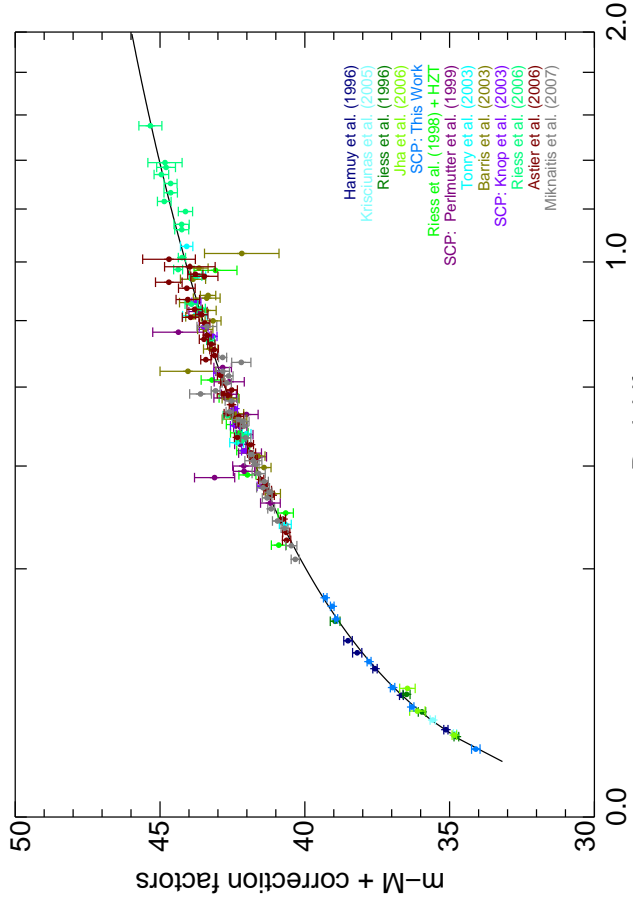
Horizontal error bars:
300 km s⁻¹ peculiar
velocity uncertainty

(Perlmutter et al., 1999, Fig. 1)
42 SNe from SCP, 18 low redshift from Calán/Tololo SN Survey

Determination of Ω_Λ



Best fit: $\Omega_{m, flat} = 0.28^{+0.09}_{-0.08}$
 $\chi^2/DOF = 56/50$
 corresponding best free fit:
 $(\Omega_m, \Omega_\Lambda) = (0.73, 1.32)$.

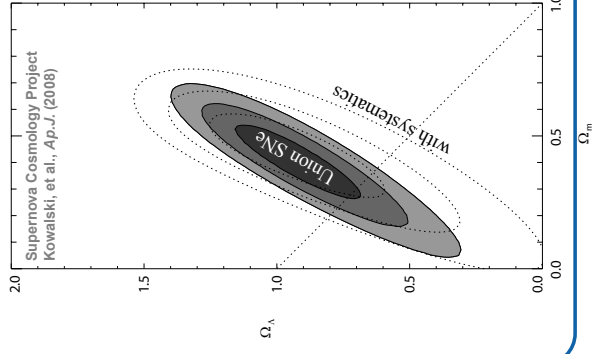


Kowalski et al. (2008): Union2 compilation of 414 SNe (307 usable)

<http://www.supernova.lbl.gov/Union/>



Supernovae

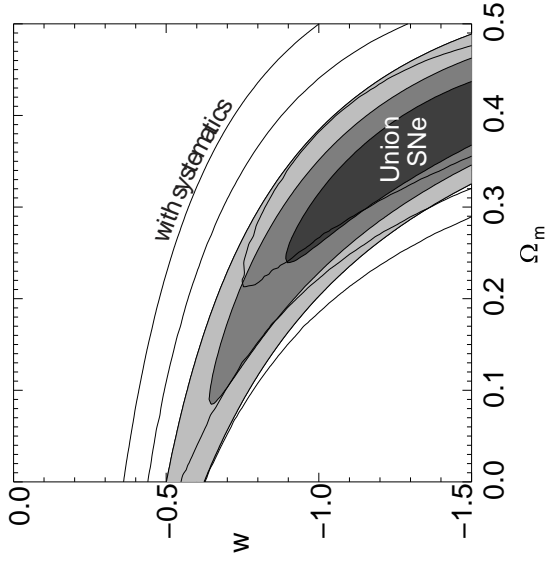


Best fit to SN data to an Ω_Λ - Λ -universe *without* any prior assumptions gives results consistent with a flat universe.
 Assuming a flat universe:

$$\Omega_\Lambda = 0.713^{+0.026}_{-0.029}(\text{stat})^{+0.036}_{-0.039}(\text{syst}) \quad (13.63)$$

(Kowalski et al., 2008)

Supernovae



Data are now good enough to measure "equation of state" of whatever is Λ .

One possible model to solve the cosmological constant problem (=explain what Λ is) is quintessence or dark energy, with the equation of state

$$p_q = w \rho_q c^2 \quad (13.64)$$

where $w = -1$ for Einstein's Λ .

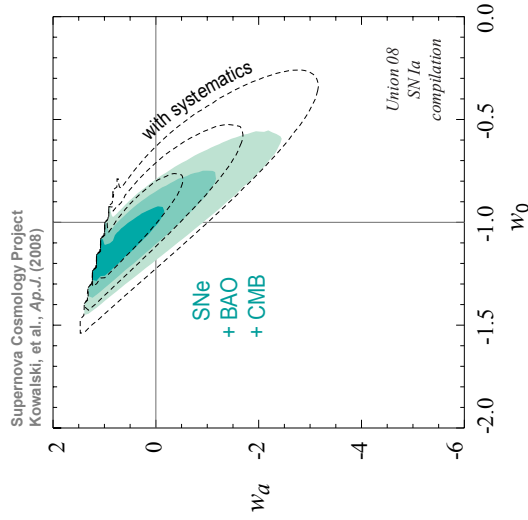
Current data are consistent with $w = -1$.

(Kowalski et al., 2008)

Determination of Ω_Λ

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Supernovae



Physical models for quintessence are derived from string theory, M-theory, and other alternative gravities, which predict w to be time dependent (Linder & Miquel, 2004):

$$w(a) = w_0 + w_a(1 - a) \quad (13.65)$$

where $a = R/R_0$.

Note that since $a = (1+z)^{-1}$ this implies

$$w(a) = w_0 + w_a \frac{z}{1-z} \quad (13.66)$$

Current data are still consistent with $w_0 = -1$ and $w_a = 0$.

A better determination of w_a will be possible in the next years.

(Kowalski et al., 2008)

Determination of Ω_Λ

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Summary

For all practical purposes, currently the best values of Ω_m and Ω_Λ are

$$\Omega_m \sim 0.3 \quad \text{and} \quad \Omega_\Lambda = 0.7$$

Even if $\Omega \neq 1$:

$$\Omega_\Lambda \neq 0$$

And therefore

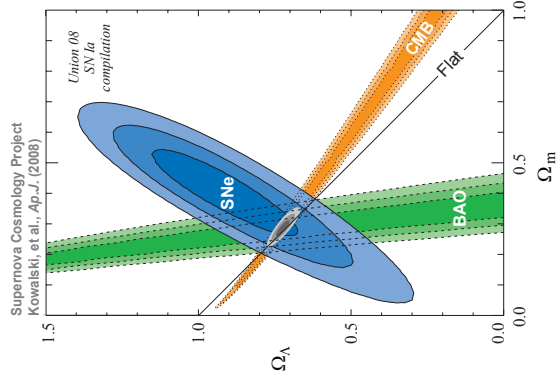
Baryons are an energetically unimportant constituent of the universe.

"The dark side of the force..." :-)

Summary

1

Outlook



Even better constraints come from combination of SNe data with structure formation.

Summary

2

- Alcock, C., Alsman, R. A., Alves, D. R., et al. 2001, *ApJ*, 552, 259
- Burles, S., Nollett, K. M., & Turner, M. S. 1999, Big Bang Nucleosynthesis: Linking Inner Space and Outer Space, APS Centennial Exhibit, astro-ph/9903300
- Carlfstrom, J. E., Joy, M. K., Grego, L., et al. 2000, *Phys. Scr.*, T85, 148
- Carroll, S. M., 2001, *Living Rev. Rel.*, 4, 2001
- Carroll, S. M., Press, W. H., & Turner, E. L. 1992, *ARA&A*, 50, 499
- Kowalski, M., Rubin, D., Aldering, G., et al. 2008, *ApJ*, 666, 749
- Linder, E. V., & Maguei, R. 2004, *Phys. Rev. D*, 70, 123516
- Merritt, D., 1987, *ApJ*, 313, 121
- Mohr, J. J., Mathiesen, B., & Eyrard, A. E., 1999, *ApJ*, 517, 627
- Peacock, J. A., 1999, *Cosmological Physics*, (Cambridge Univ. Press)
- Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, *ApJ*, 517, 565
- Riess, A. G., Filippenko, A. V., Challinor, P., et al. 1998, *ApJ*, 116, 1009
- Steidl, M., 2009, in *Proc. Heavy Quarks and Leptons*, (arXiv:0906.0464)
- Turner, M. S., 1999, in *The Third Stromlo Symposium: The Galactic Halo*, ed. B. K. Gibson, T. S. Axelrod, M. E. Purnamm, *Astron. Soc. Pacific, Conf. Ser., ASP*, in press (astro-ph/9811454)
- Wambsganss, J., 1998, *Living Rev. Rel.*, 1, 12
- Wise, M. W., McNamara, B. R., & Murray, S. S. 2004, *ApJ*, 601, 184