Example Exam Questions for Module PX 144: "Introduction to Astronomy"

Example Solutions

Useful physical constants:

Solar mass:	$M_\odot = 2 \times 10^{30}$ kg
Earth mass:	$M_{\rm Earth} = 6 \times 10^{24}$ kg
Solar luminosity:	$L_\odot = 3.9 \times 10^{26}$ W
speed of light:	$c = 300000\,{\rm km\,s^{-1}}$
Gravitational constant:	$G = 6.67 \times 10^{-11}\,{\rm N\,m^2\,kg^{-2}}$
Hubble parameter:	$H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$
Parsec:	$1\,{\rm pc} = 206265\,{\rm AU}$

Concerning the following solutions, please note that I sketch one way on how you can get to a desired solution, which does not have to be the most perfect one. So alternative ways are obviously not forbidden! For the questions asking you to "discuss" something, you can also do this, e.g., by sketching something where appropriate. Also, whether you have a bullet-point approach or write plain text is up to you.

Finally, giving more information always helps me to pick out more things for which I might award a point. To maximise your grade, I would suggest to take the points to be awarded for a question as a rule of thumb on the time you ought to spend on any question: you will have one hour to work on two of the three questions, so to spend on average one minute per point seems appropriate. If you get stuck, at least write down your ansatz (such that I can search for anything that might give you a point!) and continue with the next question. You can always try to attack the question again once you're done with the remainder of the exam…

Question 1 (Solar System):

a) (6 points): Ceres has an orbital period of 4.6 years

i) What is Ceres the semi-major axis in units of AU? {3}

Solution: Kepler's third law in solar units is

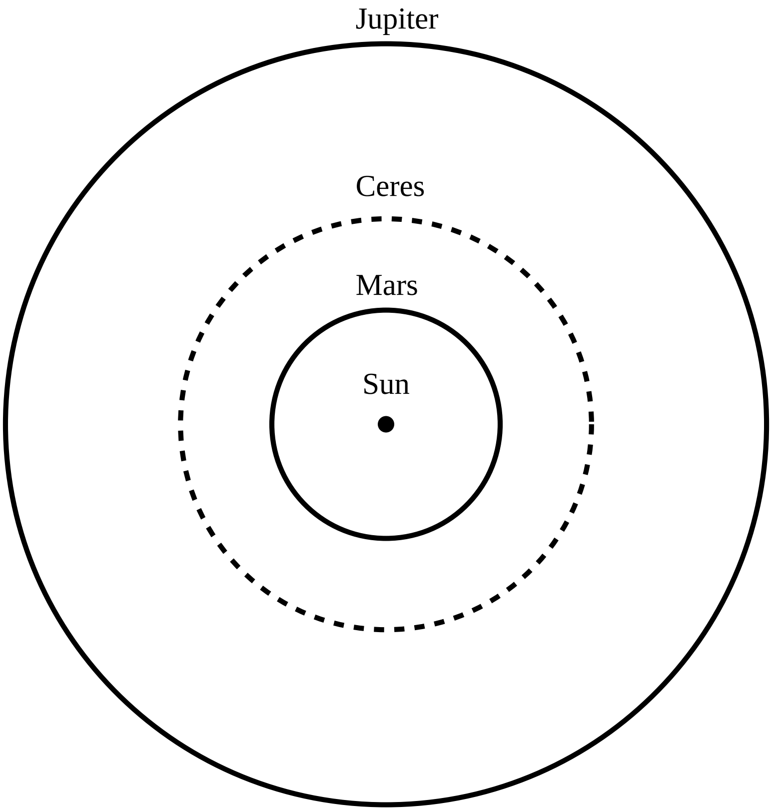
$$\frac{a^3}{P^2} = 1$$

since Ceres mass can be ignored. The semi-major axis is thus

$$a = P^{2/3} = 2.77\ \mathrm{AU}$$

ii) Sketch the location of Ceres orbit in the solar system. {3}

Solution: Ceres is between the orbits of Mars ($a = 1.5$ AU) and Jupiter ($a = 5$ AU). For the purposes of this exercise, both can be assumed circular, so the orbit would look roughly like this:



b) (6 points): Geostationary satellites have an orbital period of 24 h such that they are always situated over the same point over the Earth's equator.

i) Why do geostationary orbits have to be circular? {1}

Solution: According to Kepler's 2nd law (the law of areas) the angular speed changes with position on elliptical orbits, so in order to be stationary with respect to a point on the Earth's surface, we need an orbit with constant angular speed – a circular orbit.

ii) Compute the orbital radius for geostationary satellites. {5}

Solution: Balancing the centripetal force and the gravitational force gives

$$G\frac{m_{\text{Earth}}m_{\text{Sat}}}{R^2} = \frac{4\pi^2 m_{\text{Sat}} R}{P^2}$$

such that

$$R = \left(\frac{GMP^2}{4\pi^2}\right)^{1/3} = 4.23 \times 10^7 \text{ m} = 42300 \text{ km}$$

c) (13 points): i) Several planets in the solar system show volcanic activity or have shown so in the past. Looking at the shield volcanoes on Earth and Mars:

(a) List their major similarities and differences. {4}

Solution:

1. rather shallow slopes
2. much bigger on Mars than on Earth (Mars: 30 km height, Earth: 5 km height), and this despite Mars having a much smaller radius
3. Earth: still active, Mars: inactive

4. Earth: also other kinds of vulcanoes observed, Mars: seems that shield volcanoes are the only ones

(b) What is the reason for the apparent differences? {4}

Solution: Earth still has active plate tectonics. This means that the planetary crust is moving over the hotspot $\Longrightarrow$ volcanoes cannot get very big. On Mars it seems that there is no plate tectonics, as a result the crust stayed fixed over the hotspot and the vulcanoes could become much bigger. Concerning the activity: Mars vulcanoes are extinct as the planet is now too cold to still have a molten interior, while the Earth is still molten in its centre (apart from vulcanoes, other evidence is e.g., that the Earth has a magnetic field, produced by convection streams in the core).

ii) NASA and ESA are planning to search for extrasolar planets by directly imaging them. Briefly describe the major observational problems that are to be solved before these missions can succeed. {5}

Solution: The major observational problems are that the angular distance of the planets to the stars is expected to be very small. For example, for a star at a distance of 10 pc, a planet with an orbital radius of 1 AU will have an angular distance of $0.1''$. This is of the order or smaller than the typical resolution due to astronomical seeing, so it is imperative that the observations are done from satellites, outside of the Earth's atmosphere. A second observational problem is the large difference in flux expected from the host star and the planet, with a typical flux ratio of $1 : 10^6$ and worse. This means that effective techniques to suppress the host star ("coronagraphs") are required.

Question 2 (Stars):

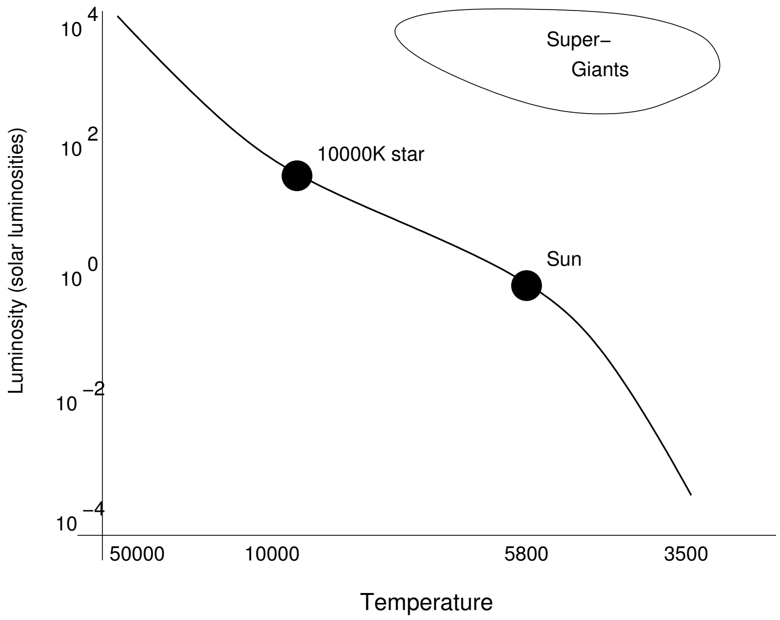
a) (18 points): An astronomer observes a star and deduces that it is a main sequence star with a surface temperature of 10000 K.

i) Briefly outline how she might have measured the star's surface temperature. {3}

Solution: To obtain the temperature, the astronomer will have to measure the star's spectrum. She can then compare the continuum with a black body spectrum, or she can look at the spectral lines and determine the spectral type of the star (since stars of the same spectral type all have the same surface temperature, once the spectral type is known, obtaining the temperature is just a matter of looking things up in a book).

ii) Sketch a Hertzsprung-Russell Diagram (HRD), showing the location of the star, the Sun, the main sequence, and the supergiants in the diagram. {6}

Solution:



iii) From the HRD, what spectral type and what approximate mass do you expect the star to have? {3}

Solution: From the HRD, the star has a luminosity of about $10\,L_\odot$. Since G stars have 5800 K and O/B stars have > 30000 K one would expect the star to be of spectral type A or F (indeed, 10000 K is the canonical temperature of A stars). The mass is best estimated from the mass-luminosity relation, $L \propto M^4$, such that $M = 10^{1/4}\,M_\odot = 1.8\,M_\odot$ (guesstimates of $2\,M_\odot$ would be o.k. as well, but anything giving it a smaller mass than the Sun, or a much larger mass, e.g., $10\,M_\odot$, would not have been accepted).

iv) Describe how the mass of the star can be measured. {4}

Solution: The mass of the star can be measured by using Kepler's 3rd law in a binary system. For the measurement one has to determine the radial velocity curve from spectra taken over one orbit of the star. Alternatively, if the star is in a wide binary, direct imaging of the orbit would also have been possible.

v) Is this mass measurement possible for all stars? {2}

Solution: No, it is only possible if the star is in a binary system.

b) (7 points): i) Compute the parallax angle for a star at a distance of 15 pc. {2}

Solution: The parallax angle p in arcseconds is $p = 1/d$ where d is measured in parsecs. Thus $p = 0.067''$.

ii) A star has an absolute magnitude of $M = 3$ mag, what is its apparent magnitude at a distance of 15 pc? {3}

Solution: The distance modulus is

$$m - M = 5\log_{10} d - 5$$

where d is in parsecs. Plugging in the numbers gives $m = 3.9$ mag.

iii) What is the luminosity of this star in terms of the solar luminosity, given that the absolute magnitude of the Sun is 5 mag? {2}

Solution: The definition of the magnitude is

$$m_2 - m_1 = -2.5 \log_{10}\left(\frac{f_2}{f_1}\right)$$

For stars at the same distance (in this case: 10 pc), since $f = L/4\pi d^2$, this equation is equivalent to

$$M_2 - M_1 = -2.5 \log_{10}\left(\frac{L_2}{L_1}\right)$$

such that

$$\frac{L_2}{L_1} = 10^{(M_2-M_1)/(-2.5)} = 10^{(2/5)(M_1-M-2)}$$

Plugging in numbers gives $L_{\rm star} = 6.3 L_\odot$.

Question 3 (Galaxies):

a) (4 points) In 1998, the Sloan Digital Sky Survey discovered a quasar with a redshift of $z = 5$.

i) Using Hubble's law, compute the distance fo this quasar in Mpc and lightyears. Comment on the result. {2}

Solution: Hubble's law is

$$v = H_0 d \quad \text{where } v = cz$$

such that $d = cz/H_0 = 21400$ Mpc = 21.4 Gpc or about 70 Gly.
The results shows that for these redshifts it is *not* appropriate to use Hubble's law in its simplest form, since other cosmological effects need to be taken into account. Indeed, a distance of 70 Gly implies that light took 70 billion years from the galaxy to us, which is a problem, given that the age of the universe is 15 billion years (you would have gotten the point if you had just pointed out that 70 gigayears is older than the universe).

ii) One of the strongest emission lines of hydrogen is the Lyman α line, which is emitted at $\lambda = 121.567$ nm in the ultraviolet. Given the high redshift of the quasar, would you be able to observe this line in the optical regime (assume the optical waveband is the 300–1000 nm band). {2}

Solution: The redshift is defined by

$$z = \frac{\lambda_{\rm observed} - \lambda_{\rm emitted}}{\lambda_{\rm emitted}}$$

such that

$$\lambda_{\rm observed} = \lambda_{\rm emitted}(1+z) = 730\,\text{nm}$$

smack on in the optical band.

b) (7 points) The astronomical distance ladder is based by determining the absolute magnitude of standard candles using their distances as measured with other distance indicators. Currently, the major distance scale outside of the Milky Way is based on δ Cepheid variables in the Large Magellanic Cloud (LMC), for which two values of the distance modulus are currently being discussed: $m - M = 18.3$ mag and $m - M = 18.7$ mag.

A Cepheid is observed in the Virgo cluster with $m = 25.7$ mag, its period suggests that $M = -6$ mag.

i) What is the distance modulus of the Virgo Cluster? {1}

Solution: This is the easiest point of the whole exam... The distance modulus is $m - M = 31.7$ mag.

ii) The Cepheid's absolute magnitude was inferred assuming a LMC modulus of 18.3, what is the relative error of the Virgo distance if the LMC's distance modulus were $m - M = 18.7$ mag instead? {6}

Solution: The distance modulus is

$$\mathrm{DM} = m - M = 5 \log_{10} d - 5$$

such that

$$d = 10^{(\mathrm{DM}+5)/5}$$

With a distance modulus of 31.7, $d = 2.2 \times 10^7$ pc $= 22$ Mpc.
Had the distance modulus to the LMC been 18.7 instead of 18.3, then the absolute magnitude of the Cepheids would have been estimated to be too bright by 0.4 mag, such that the real distance modulus of the Virgo cluster would have been DM $= 32.1$ mag, translating into a distance of $d = 2.6 \times 10^7$ pc. in other words, the uncertainty of the Virgo distance is 18%, just because we do not know the distance to the Large Magellanic Cloud!

c) (6 points) A black hole with a mass of $10\,M_\odot$ accretes $10^{-7}\,M_\odot$ of material per year.

i) What is the Schwarzschild radius $2GM/c^2$ of the black hole? {1}

Solution: Either by plugging in the numbers or by remembering $r_\mathrm{S} = 3\,\mathrm{km}(M/M_\odot)$, one finds that the Schwarzschild radius is 30 km.

ii) Assuming the material falls in from infinity, compute the amount of potential energy released before the accreted material reaches the Schwarzschild radius, and compare this with the luminosity of the Sun.

Solution: Since the potential energy at infinity is 0, the amount of energy a particle of mass m releases by falling from infinity to a radius r_S is given by

$$E = E_\infty - E(r_\mathrm{S}) = 0 - \left(-\frac{GMm}{r_\mathrm{s}}\right) = \frac{GMm}{r_\mathrm{s}}$$

The energy released per second (i.e., the luminosity) is given by

$$L = \frac{E}{T} = \frac{GMm}{r_\mathrm{s} T}$$

where $T = 1\,\text{year} = 365.25 \cdot 86400\,\text{s} = 3.16 \times 10^7\,\text{s}$ and where $m/T = 10^{-7}\,M_\odot/T = 6.3 \times 10^{15}\,\text{kg}\,\text{s}^{-1}$. Plugging in the numbers gives a luminosity of $L = 2.8 \times 10^{32}\,\text{J}\,\text{s}^{-1} = 7 \times 10^5 L_\odot$, much brighter than the Sun.

d) (8 points) The basic assumption of most modern cosmological models is that the universe is isotropic and homogeneous. Discuss what is meant by these two assumptions and describe what types of observations can be used to test whether both assumptions are correct. {8}

Solution: That the universe is "isotropic" means that it looks the same in all directions. Observationally, the isotropy can be tested by imaging several areas of the sky and then looking at the properties of the sources seen in these directions. Care has to be taken to not throw in foreground sources (e.g., stars in our Galaxy), which could bias this sample. If the distribution of the sources looks the same in the different images, then the sky can be described to be isotropic.

Homogeneity means that the universe looks the same from all places. The best test for homogeneity is to measure the distances to many sources on the sky. This allows one to prepare a 3D map of the universe. If that map has the same statistical properties at all places in space, then the universe can be assumed to be homogeneous.