Stars

What are stars?

Most important building blocks of the universe: stars

Proper definition:

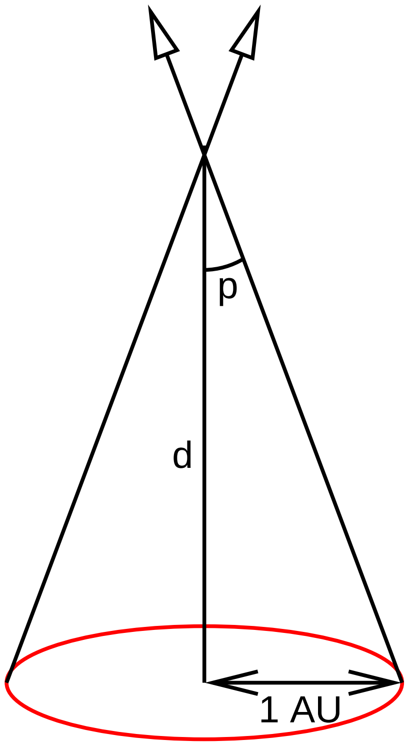
> Stars are gas balls consisting mainly of hydrogen and helium, which produce energy by fusion.

Will now look at observable properties of stars:

1. Distance
2. Luminosity
3. Brightness
4. Masses

... and later deduce how they live from these data.

Distances, I



Direct distance measurements: parallax measurement:

$\Longrightarrow$ Measure stellar position several times over year with respect to background stars.

Parallax angle (small angle approximation):

$$p = \frac{1\,\text{AU}}{d}$$

(p is measured in *radians*)

Typical values for p are arcseconds

$\Longrightarrow$ define distance unit "Parsec" ("parallax second") such that $d = 1\,\text{pc}$ for $p = 1''$:

The parsec is the distance at which 1 AU subtends an angle of $1''$.

Zeilik & Gregory use π instead of p for the parallax...

Distances, II

How far is one parsec?

From $p = 1\,\text{AU}/d$ follows with $p = 1''$:

$$d = 1\,\text{pc} = \frac{1\,\text{AU}}{1''} = \frac{1\,\text{AU}}{\pi/(180 \cdot 3600)} = 206264\,\text{AU} = 3.086 \times 10^{16}\,\text{m} \sim 3.26\,\text{ly}$$

Note: If parallax p is known and given in arcseconds, then distance can be immediately computed:

$$\frac{d}{1\,\text{pc}} = \frac{1}{p/1''} \quad \text{or (sloppy notation)} \quad d = \frac{1}{p}$$

Today: positional accuracy $\sim$0.01$''$ from ground, and better than 1 mas ($10^{-3}{}''$) from space $\Longrightarrow$ can measure parallax out to $\sim$1 kpc

further out: "secondary distance estimators" $\Longrightarrow$ see later lectures

Luminosity

Definition: Luminosity of a star:

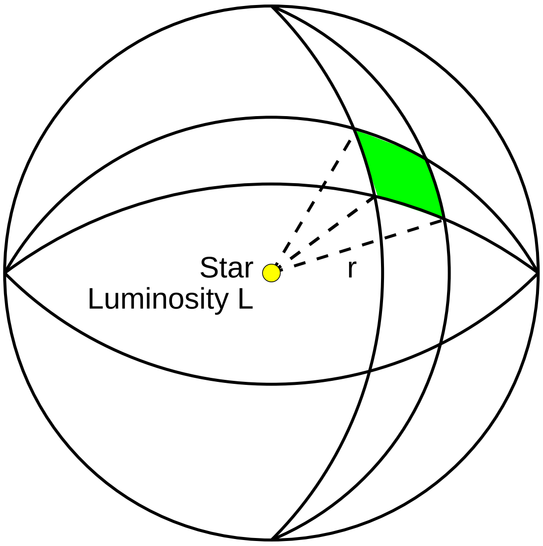
The total energy emitted by a star per second is called its luminosity.

(=the luminosity is a power)

In astronomy, luminosities are often measured in units of the solar luminosity,

$$L_\odot = 3.90 \times 10^{26}\,\mathrm{J\,s^{-1}} = 3.90 \times 10^{26}\,\mathrm{W}$$

Flux, I



Assumption: star emits its radiation isotropically.

Flux: energy passing per second through area of $1\,\mathrm{m}^2$ at distance r:

$$F = \frac{L}{4\pi r^2}$$

(unit: $\mathrm{W\,m^{-2}}$).

Flux, II

Fluxes from stars (apart from the Sun) are *very small*.

Example: α Centauri (closest star to the Sun).
- distance: 1.3 pc $\sim 4 \times 10^{16}$ m
- luminosity: similar to the Sun (4×10^{26} W).

Magnitudes, I

Hipparchus
(??– ∼127 BC)

First classification of stars:

- Stars of "magnitude 1": brightest (visible) stars
- Stars of "magnitude 6": faintest (visible) stars

http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Hipparchus.html

Magnitudes, V

Pogson (1865): Eye sensitivity is logarithmic, such that

A brightness *difference* of 5 magnitudes corresponds to a *ratio* of 100 in detected flux

So, if magnitudes of two stars are m_1 and m_2, then

$$\frac{f_1}{f_2} = 100^{(m_2-m_1)/5}$$

This means:

$$\log_{10}(f_1/f_2) = \frac{m_2 - m_1}{5} \log_{10} 100 = \frac{2}{5}(m_2 - m_1)$$

or

$$m_2 - m_1 = 2.5 \log_{10}(f_1/f_2) = -2.5 \log_{10}(f_2/f_1)$$

Note: Larger Magnitude = *FAINTER* Stars

Luminosity (revisited)

Inverse square law links flux f at distance d to flux F measured at another distance D:

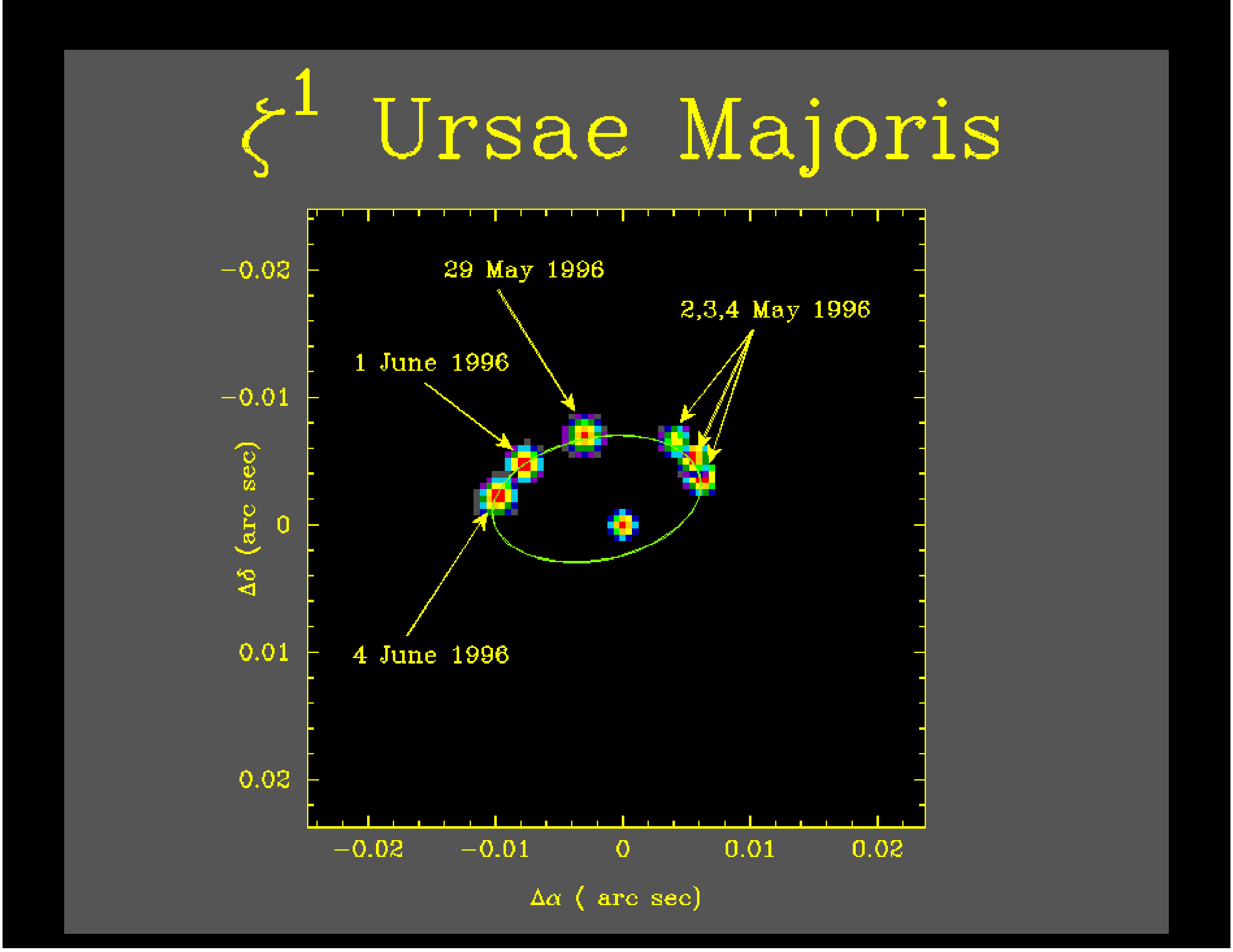
$$\frac{F}{f} = \frac{L/4\pi D^2}{L/4\pi d^2} = \left(\frac{d}{D}\right)^2$$

Convention: to describe luminosity of a star, use the absolute magnitude M, defined as magnitude measured at distance $D = 10\,\mathrm{pc}$.

Therefore,

$$m - M = 2.5 \log(F/f) = 2.5 \log(d/10\,\mathrm{pc})^2 = 5 \log d - 5$$

$m - M$ is called the distance modulus, d is measured in pc.



Masses, XIII

Mizar A and B are rather typical stars:

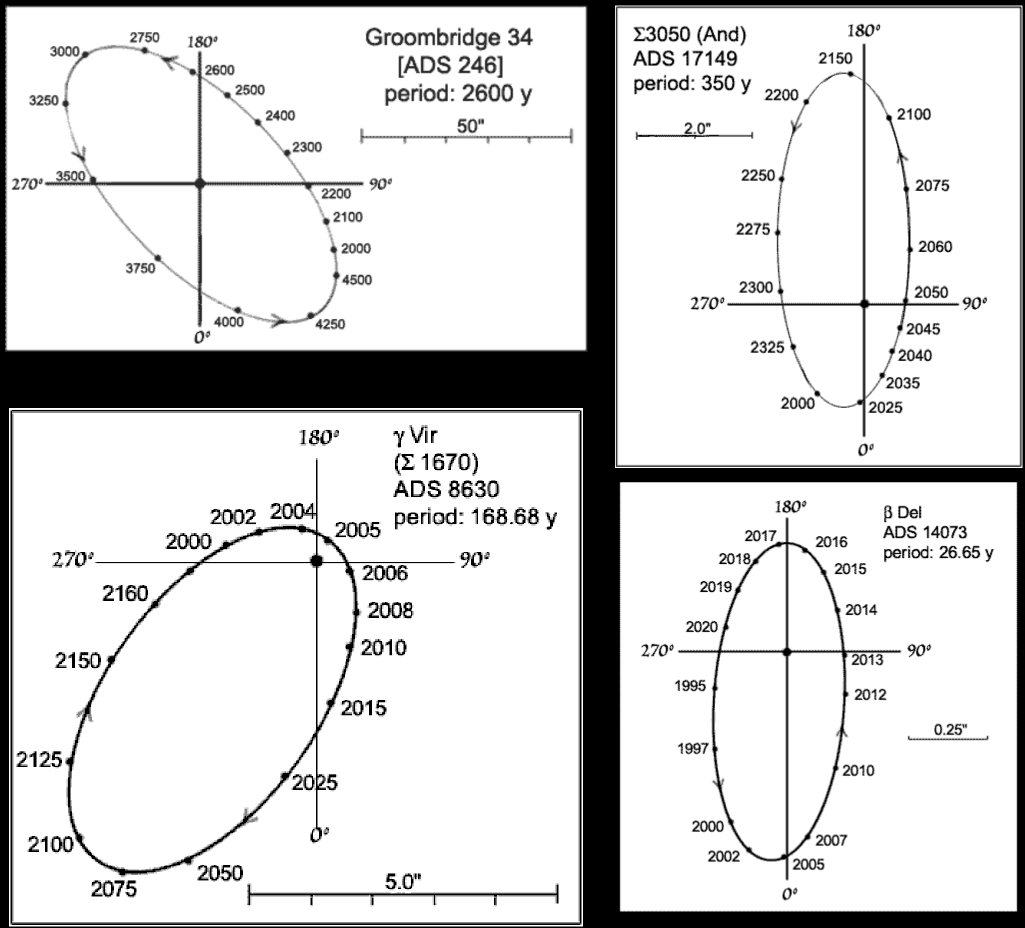
> 50% – 80% of all stars in the solar neighbourhood belong to multiple systems.

Rough classification:

apparent binaries: stars are *not* physically associated, just happen to lie along same line of sight ("optical doubles").

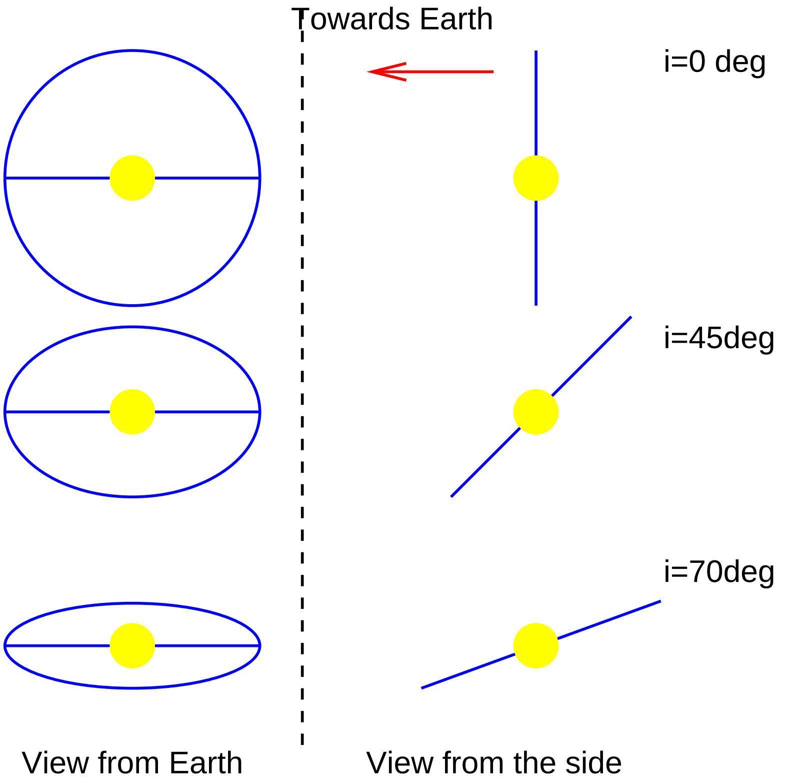
visual binaries: bound system that can be resolved into multiple stars (e.g., Mizar); can image orbital motion, periods typically 1 year to several 1000 years.

spectroscopic binaries: bound systems, cannot resolve image into multiple stars, but see Doppler effect in stellar spectrum; often short periods (hours. . . months).



Potpourri of observed binary star orbits.

Masses, XV



Problem when analysing orbits: orientation of orbit in space: "inclination"

In simplest case: real semimajor axis:

$$a_{\text{observed}} = a_{\text{real}} \cos i$$

Inclination typically found using Kepler's 2nd law plus geometry...

Masses, XVII

To determine stellar masses, use Kepler's 3rd law:

$$\frac{a^3}{P^2} = \frac{G}{4\pi^2}(m_1 + m_2)$$

where

- $M_{1,2}$: masses
- P: period
- a semimajor axis

Observational quantities:

- P – directly measurable
- a – measurable from image *if and only if* distance to binary and the inclination is known

Masses, XVIII

From Kepler's 3rd law, can determine $M_1 + M_2$.

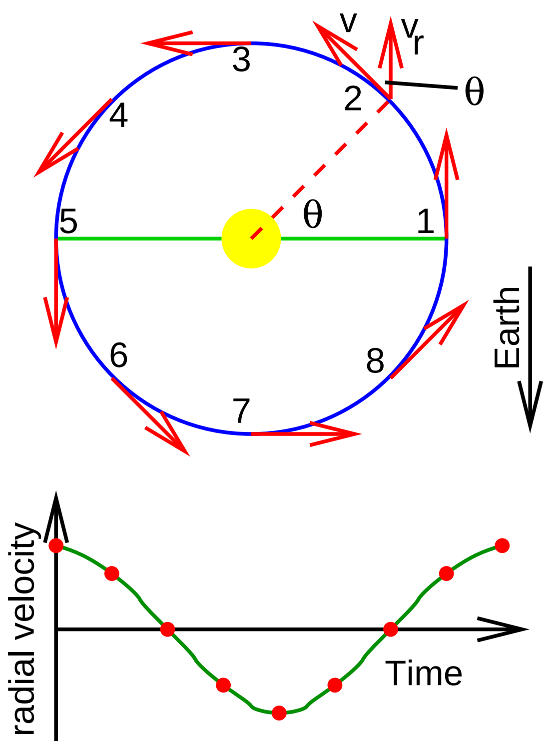
Need to determine individual masses, M_1 and M_2:

$\Longrightarrow$ use center of mass (CM):

$$M_1 a_1 = M_2 a_2 \quad \text{such that} \quad \frac{M_1}{M_2} = \frac{a_2}{a_1}$$

where a_1, a_2: semi-major axes of orbits around CM (observable from imaging).

Spectroscopic Binaries, I



For spectroscopic binaries: can only measure radial velocity along line of sight

For circular orbit, angle θ on orbit:

$$\theta = \omega t$$

where $\omega = 2\pi/P$.

Observed radial velocity:

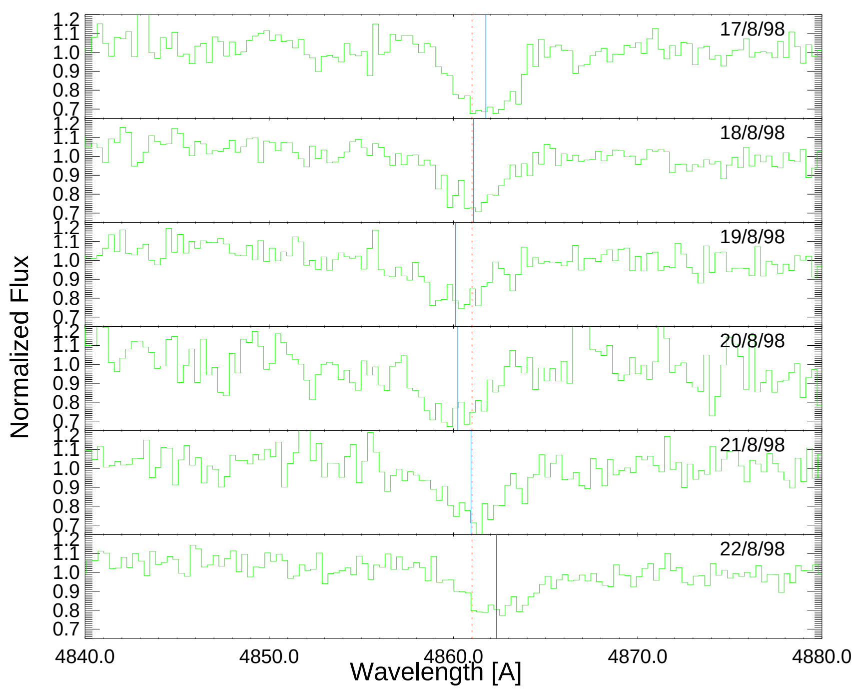
$$v_\mathrm{r} = v\cos(\omega t)$$

If orbit has inclination i, then

$$v_\mathrm{r}(t) = v \sin i \cos(\omega t)$$

From observation of $v_\mathrm{r}(t) \Longrightarrow v \sin i$.
("velocity amplitude")

Spectroscopic Binaries, II



HDE 226868/Cyg X-1; Pottschmidt (2001)

Motion of star visible through Doppler shift in stellar spectrum:

$$\frac{\Delta\lambda}{\lambda} = \frac{v_\mathrm{r}}{c} = \frac{v}{c}\sin i \cos \omega t$$

For almost all stars, classical Doppler effect is enough; once $v \gtrsim 0.1c$, however, use relativistic Doppler effect,

$$\nu_\mathrm{obs} = \nu_\mathrm{em}\sqrt{\frac{1+v/c}{1-v/c}}$$

Mass Function

If only one star visible: can only determine limits for mass: mass function

$$\frac{Pv_{\mathrm{obs}}^3}{2\pi G} = \frac{M_2^3 \sin^3 i}{(M_1+M_2)^2} =: f_{\mathrm{M}}$$

(see handout)

with observables:

- v_{obs}: (velocity amplitude of M_1)
- P: period

and unknowns:

- M_1: mass of "primary star"
- M_2: mass of (unseen) "secondary star"
- i: inclination

$\Longrightarrow f_{\mathrm{M}}$ is lower limit for M_2, since for $M_1 = 0$, $M_2 = f_{\mathrm{M}}/\sin^3 i \geq f_{\mathrm{M}}$

Often used for neutron star and black hole binaries...

Note that the following is for your information only, you will not be tested on your ability to memorize this lengthy derivation...

To derive the mass function, we start as usual with Kepler's 3rd law,

$$\frac{G}{4\pi^2}(M_1+M_2) = \frac{R^3}{P^2}$$

In the following, we will assume that we observe the spectral lines from star number 1 only.

Because of the center of mass definition,

$$M_1 r_1 = M_2 r_2$$

such that

$$R = r_1 + r_2 = r_1\left(1 + \frac{r_2}{r_1}\right) = r_1\left(1 + \frac{M_1}{M_2}\right)$$

In the case that the orbits are circular, the velocity of the star whose spectrum we see is

$$v_1 = \frac{2\pi r_1^2}{P}$$

However, due to the unknown inclination, we only observe the radial velocity component, that is

$$v_{\mathrm{obs}} = v_1 \sin i$$

In terms of the observables, r_1 is

$$r_1 = \frac{P}{2\pi} v_1 = \frac{P}{2\pi}\frac{v_{\mathrm{obs}}}{\sin i}$$

such that finally

$$R = r_1\left(1 + \frac{M_1}{M_2}\right) = \frac{P}{2\pi}\frac{v_{\mathrm{obs}}}{\sin i}\left(1 + \frac{M_1}{M_2}\right)$$

We can now insert R into Kepler's 3rd law:

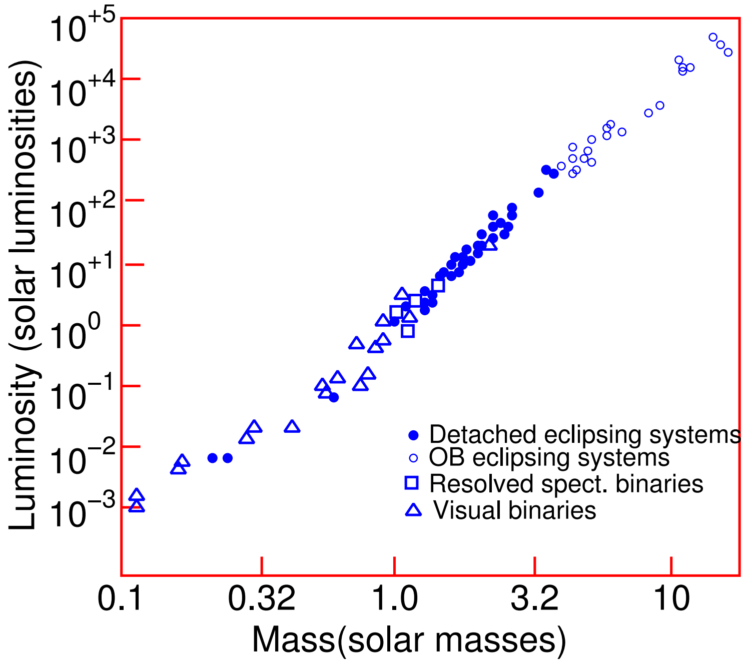
$$\frac{G}{4\pi^2}(M_1+M_2) = \frac{1}{P^2}\frac{P^3}{(2\pi)^3}\frac{v_{\mathrm{obs}}^3}{\sin^3 i}\left(1 + \frac{M_1}{M_2}\right)^3$$

and obtain after some straightforward algebra

$$\frac{M_2^3}{(M_1+M_2)^2}\sin^3 i = \frac{Pv_{\mathrm{obs}}^3}{2\pi G}$$

the mass function. On the right side are the observables P and v_{obs}, on the left hand side the unknowns i, M_1, and M_2.

Application: Mass-Luminosity Relation

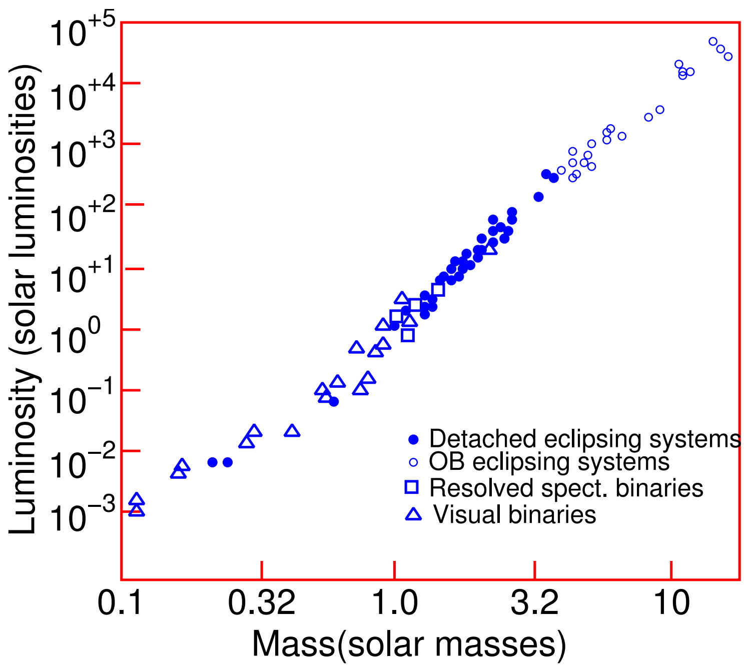


Can now look at solar neighborhood:

- apparent magnitude m *and* distance $\Longrightarrow$ luminosity
- mass from binary stars

$\Longrightarrow$ determine mass-luminosity relationship

Application: Mass-Luminosity Relation



Empirical result:

$$\frac{L}{L_\odot} = \begin{cases} 0.23 \left(\frac{M}{M_\odot}\right)^{2.3} & (M < 0.43\,M_\odot) \\ \left(\frac{M}{M_\odot}\right)^{4.0} & (M \geq 0.43\,M_\odot) \end{cases}$$

$\Longrightarrow$ more massive stars have extremely higher luminosities!

(factor 2 in $M \rightarrow$ factor 8 in L).

Direct consequence:

More massive stars live much shorter lives

sometimes, one also sees $L \propto M^{3.3}\ldots$

Introduction

Final observable of stars: Temperature

Obtained using spectroscopy

In the following: rough outline, as stellar spectroscopy is rather complicated
$\Longrightarrow$ see module "stars" for the g(l)ory details...

Outline:

1. Planck's Radiation Laws
2. Stellar Continuum Spectra
3. Spectral Classification

...unfortunately, need to be a little bit formal first

Planck's Radiation Law, I

Max Planck (1858–1947)

Stars are big glowing gas balls.

In zeroth order: thermodynamic equilibrium.

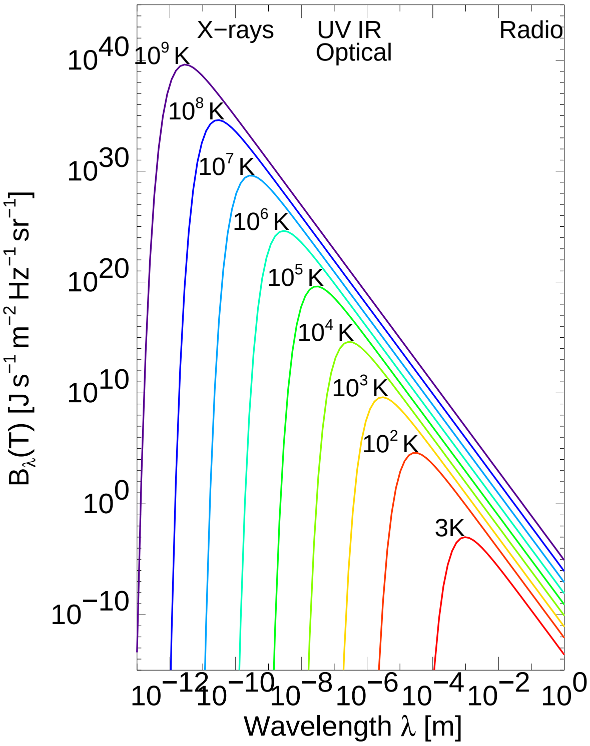
Max Planck: under these circumstances: emitted spectrum is blackbody radiation:

$$F_\lambda = \frac{2hc^2/\lambda^5}{\exp(hc/\lambda kT) - 1}$$

F_λ: Energy emitted per second and wavelength interval

- $h = 6.623 \times 10^{-34}\,\mathrm{J\,s}$: Planck's constant
- $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$: Boltzmann constant

Planck's Radiation Law, II



Without proof, the following two important relationships hold for blackbody radiation:

Stefan-Boltzmann law: Power emitted per square-metre surface of a blackbody:

$$P = \sigma T^4$$

where $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

"hotter bodies have a much higher luminosity"

Wien's displacement law: Wavelength of maximum blackbody emission:

$$\lambda_{\mathrm{max}} T = 2.898 \times 10^{-3}\,\mathrm{m\,K}$$

"hotter bodies radiate higher energetic radiation"

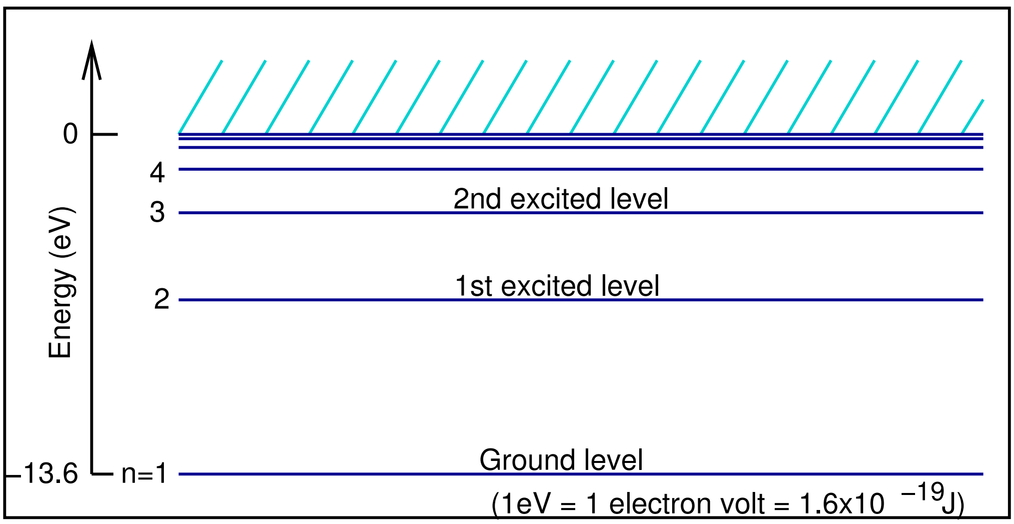
Spectroscopy, I

Quantum mechanics: atoms have discrete energy levels

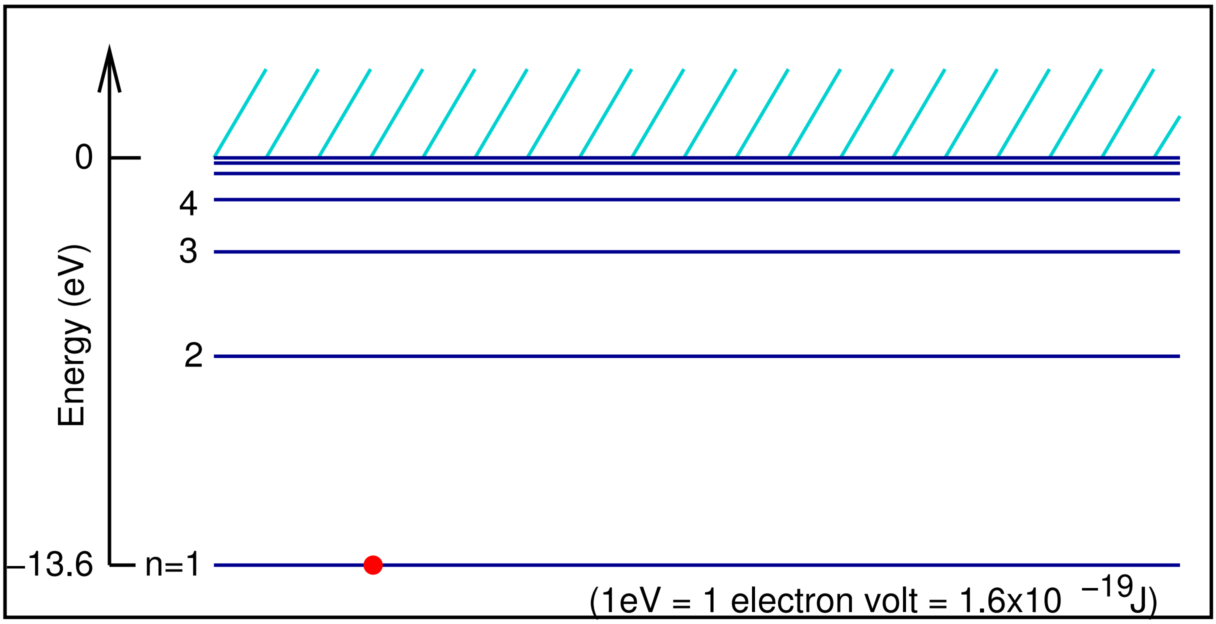
Energy levels in Hydrogen:

$$E_n = -\frac{2\pi^2 \mu e^4}{\hbar^2} \cdot \frac{1}{n^2} \propto -\frac{1}{n^2}$$

($n \in \mathbb{N}$; Balmer formula)



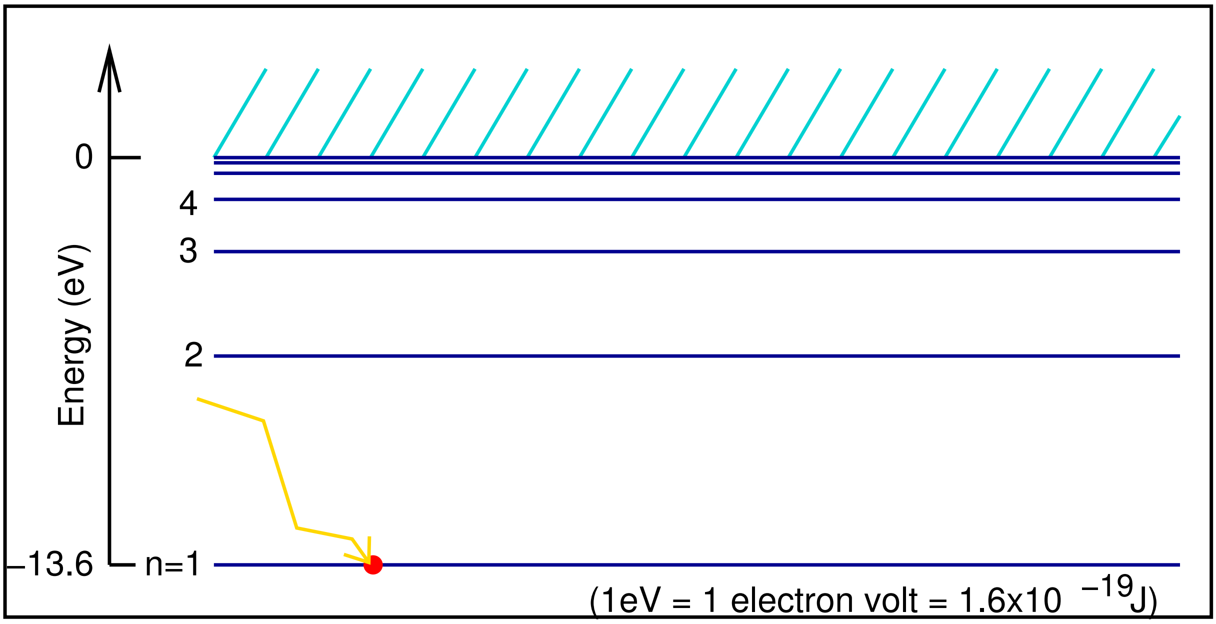
Spectroscopy, II



In hydrogen atom: electrons typically found in ground state.

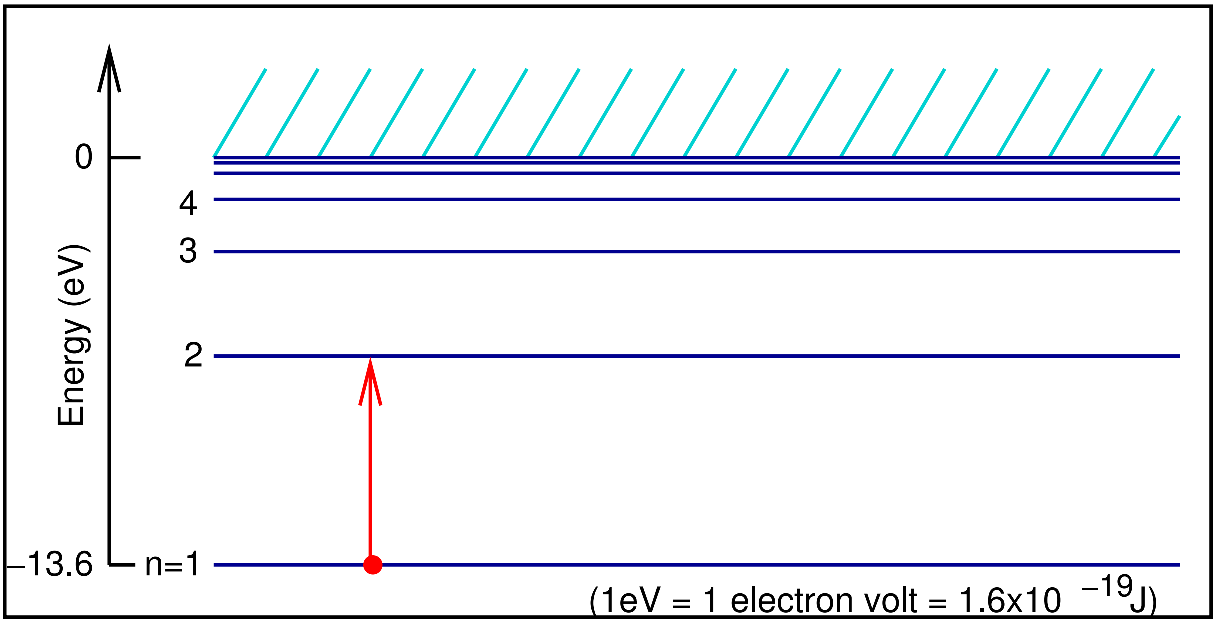
if temperature is higher, can also be in 1st excited state, but the physical principles following remain the same. . . .

Spectroscopy, III



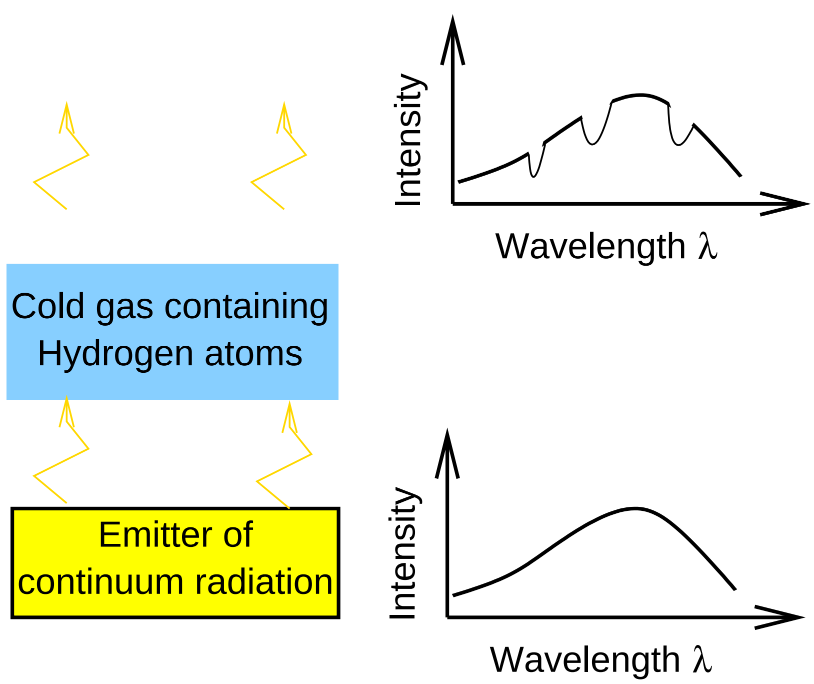
Photon hitting atom has energy $E_{\mathrm{phot}} = h\nu = hc/\lambda$. If $E_{\mathrm{phot}} = E_2 - E_1$, then photon can be absorbed. . .

Spectroscopy, IV



Photon hitting atom has energy $E_{\text{phot}} = h\nu = hc/\lambda$. If $E_{\text{phot}} = E_2 - E_1$, then photon can be absorbed. . . and electron has higher energy (is excited).

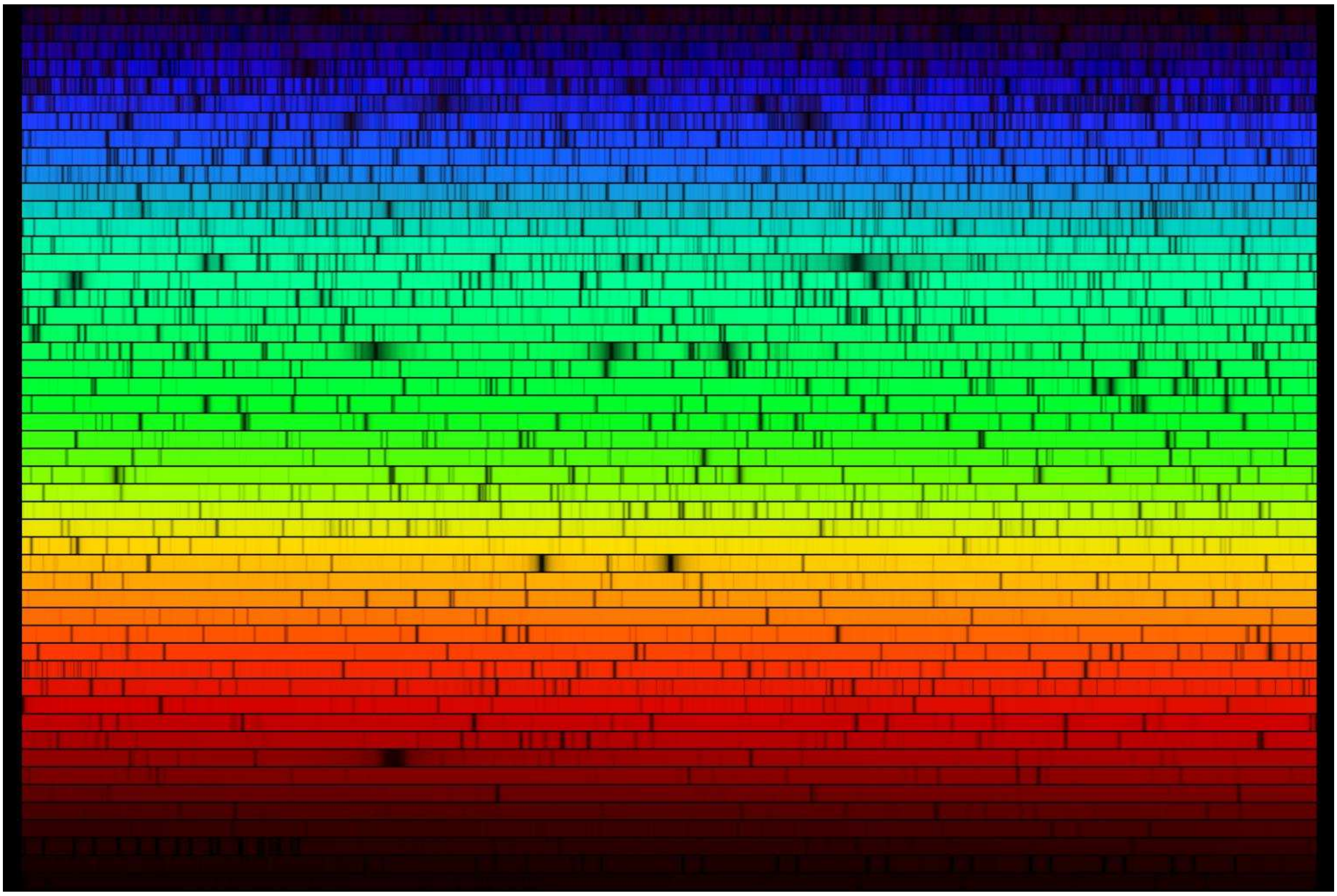
Spectroscopy, VIII



1. Assume stellar surface has continuum spectrum (Planck).
2. Assume surface is below atmosphere of colder gas.

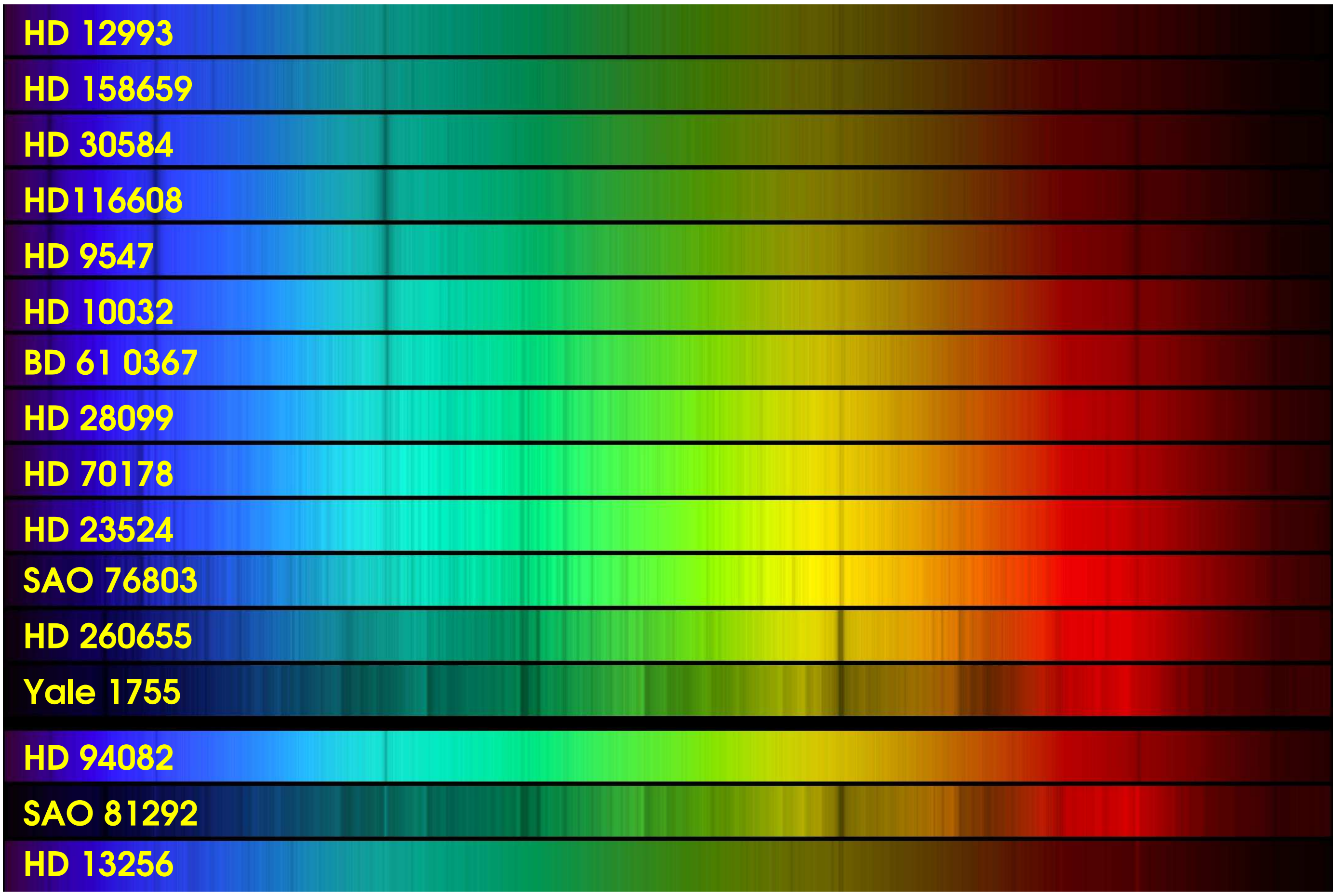
$\Longrightarrow$ Atmosphere absorbs photons at wavelengths characteristic for the elements present in stellar atmosphere.

$\Longrightarrow$ Formation of absorption line spectrum.



N.A.Sharp, NOAO/NSO/Kitt Peak FTS/AURA/NSF

Absorption line spectrum of the Sun: Fraunhofer Lines



Annie Cannon (around 1890): Stars have different spectra.

NOAO

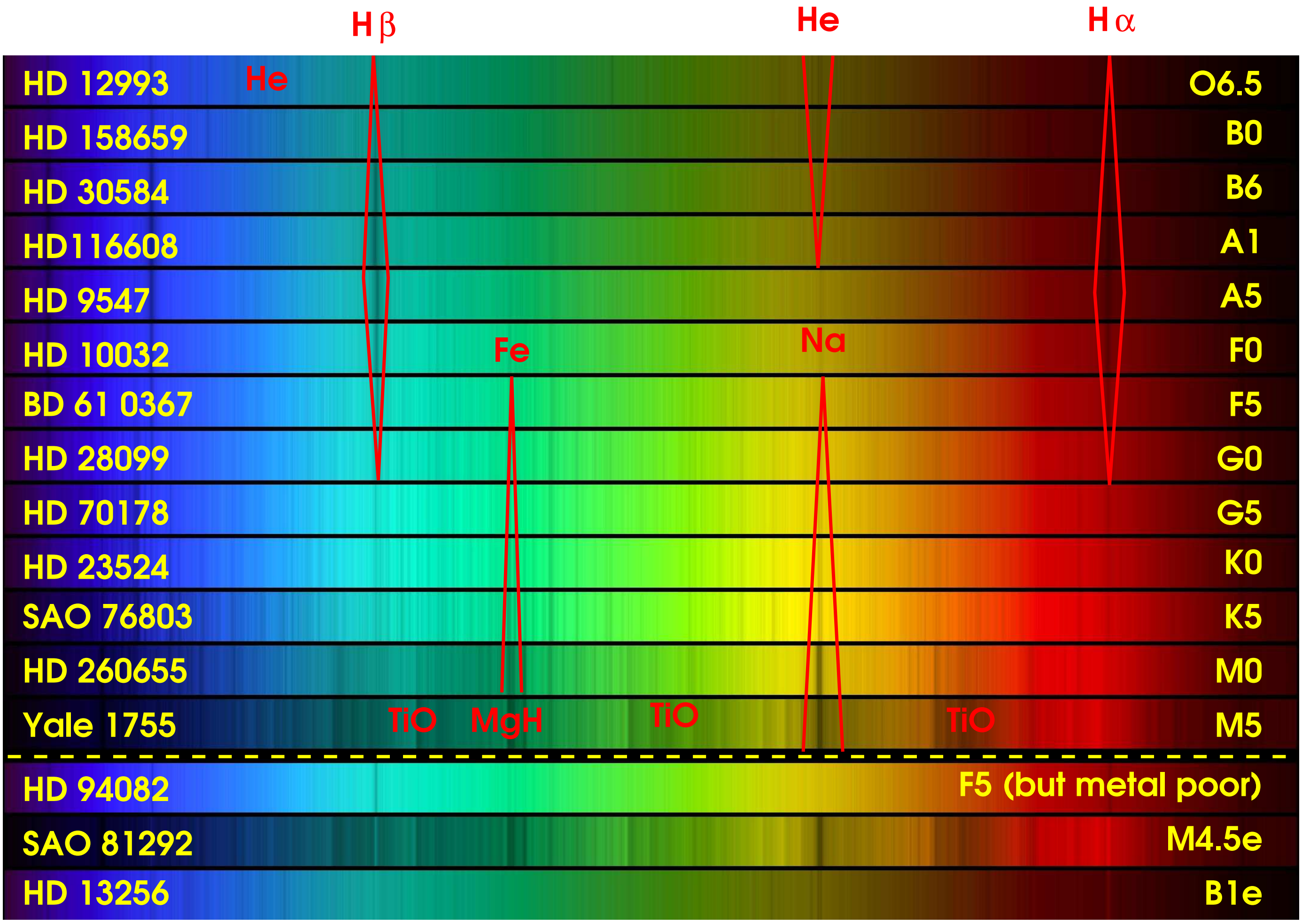
Stellar Spectra

Annie Jump Cannon (1863–1941)

Biography: `http://www.sdsc.edu/ScienceWomen/cannon.html`

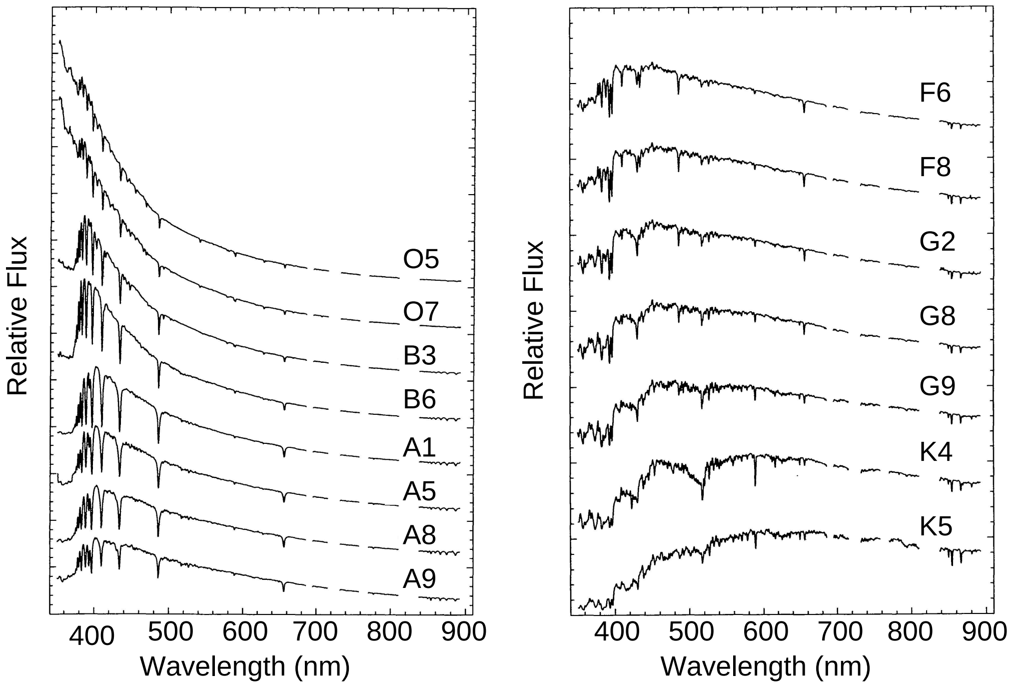
Annie Jump Cannon: There are spectral types.

Henry Draper catalogues (Cannon plus ∼10 female "computers"): 225000 spectral classifications.



Annie Cannon: Strength of absorption lines varies with spectral type. NOAO

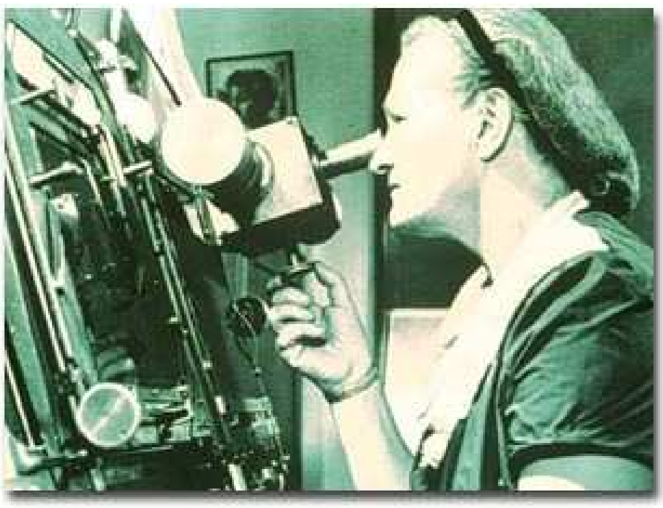
Stellar Spectra



Silva & Cornell, 1992, ApJ Suppl. 81, 865

Cecilia Payne-Gaposchkin: Spectral sequence is temperature sequence.

Stellar Spectra



Cecilia Payne-Gaposchkin (1900–1979)

Biography: http://www.harvardsquarelibrary.org/unitarians/payne2.html

BSc, Cambridge, left UK because of situation of women in astronomy

1st person to obtain PhD in Astronomy at Harvard: “Stellar Atmospheres, A Contribution to the Observational Study of High Temperature in the Reversing Layers of Stars”

Otto Struve: “undoubtedly the most brilliant Ph.D. thesis ever written in astronomy.”

Spectral types are a temperature sequence.

later: 1st female full professor at Harvard

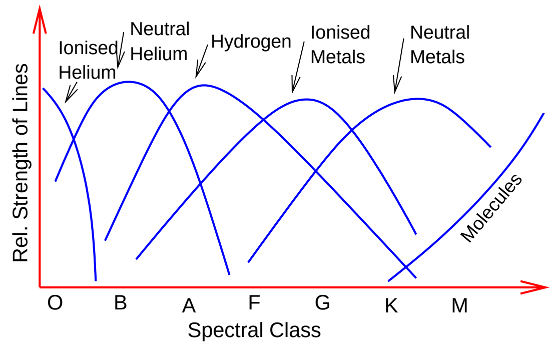
Stellar Spectra

Summary spectral classes as a temperature sequence.

O - B - A - F - G - K - M

30000 K   3000 K

"early type"   "late type"

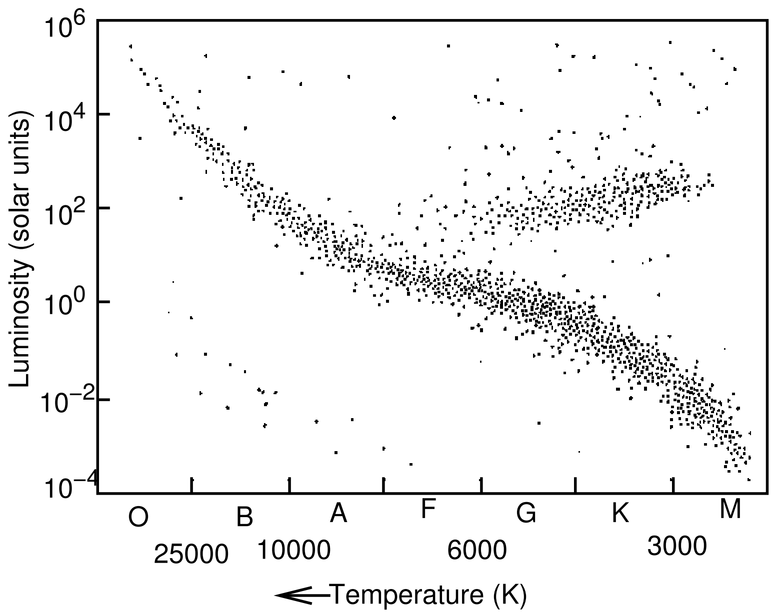


plus subtypes: B0. . . B9,A0. . . A9, etc.

Sun is G2.

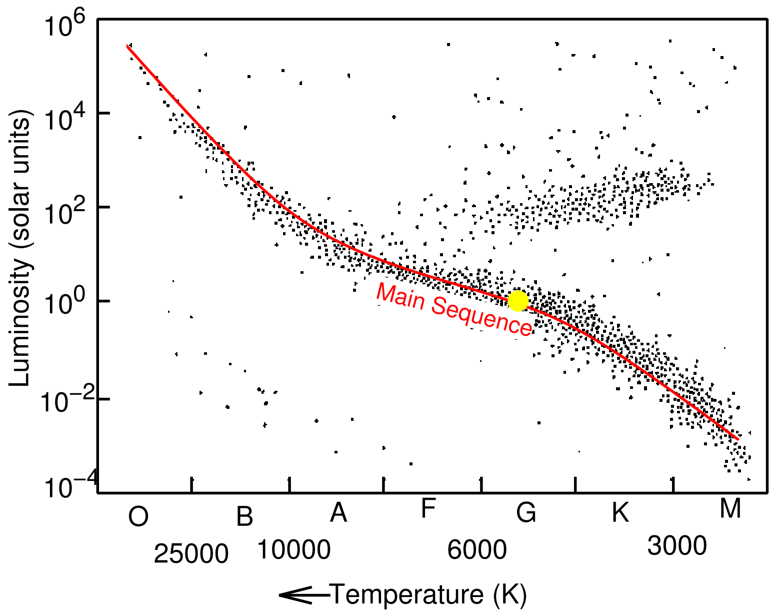
Note: "early" and "late" has *nothing* to do with age!

HRD



Hertzsprung - Russell Diagram (HRD): Stellar temperature versus stellar luminosity

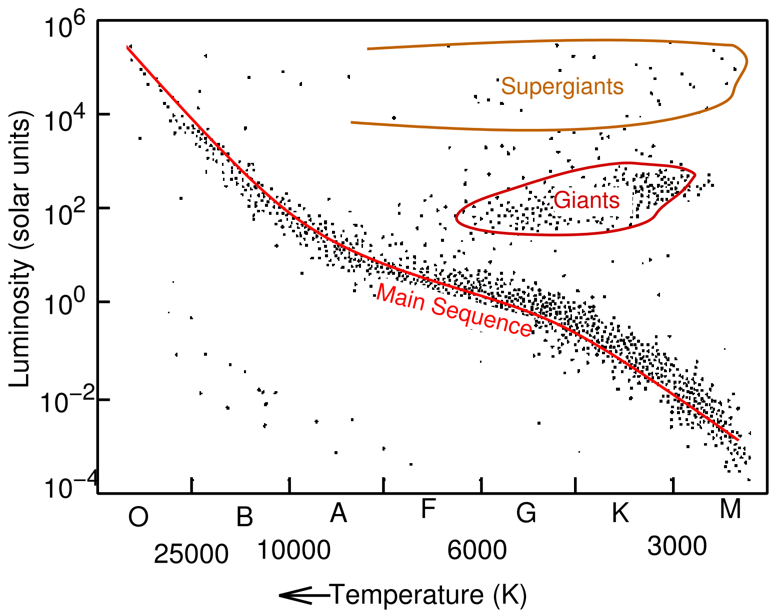
HRD



- Most stars on Main Sequence (“dwarfs”)

Hertzsprung - Russell Diagram (HRD): Stellar temperature versus stellar luminosity

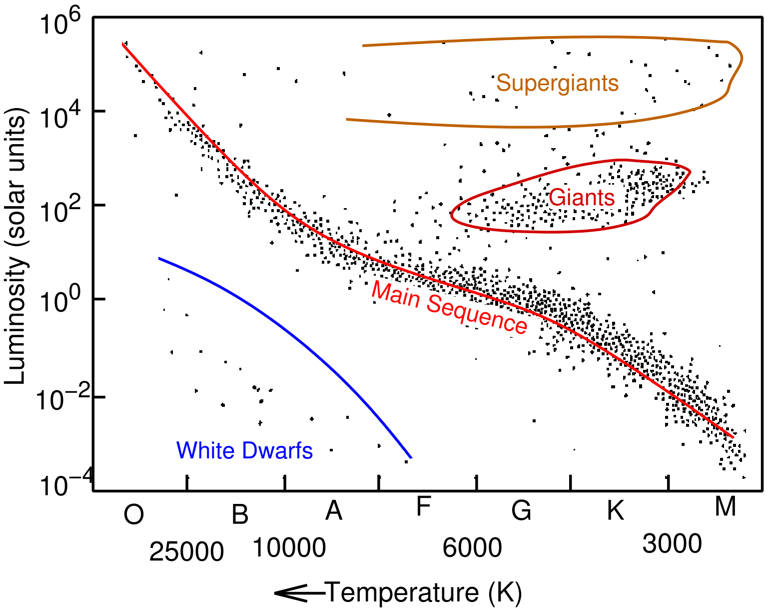
HRD



- Most stars on Main Sequence (“dwarfs”)
- Stellar Luminosity: $L = 4\pi R^2 \sigma T^4 \propto R^2 T^4$ $\Longrightarrow$ cold, luminous stars are *BIG* $\Longrightarrow$ “giants”

Hertzsprung - Russell Diagram (HRD): Stellar temperature versus stellar luminosity

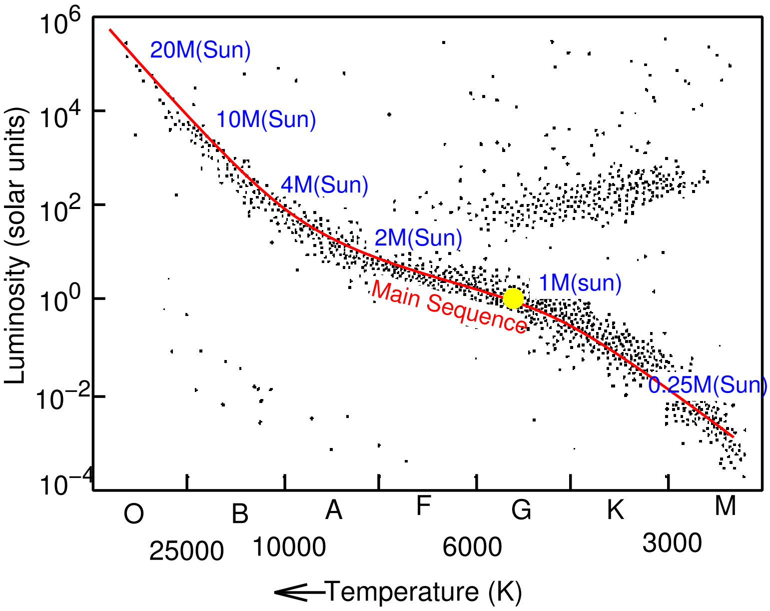
HRD



- Most stars on Main Sequence ("dwarfs")
- Stellar Luminosity: $L = 4\pi R^2 \sigma T^4 \propto R^2 T^4$
  $\Longrightarrow$ cold, luminous stars are *BIG*
  $\Longrightarrow$ "giants"
- Hot, underluminous stars are small: "white dwarfs"

Hertzsprung - Russell Diagram (HRD): Stellar temperature versus stellar luminosity

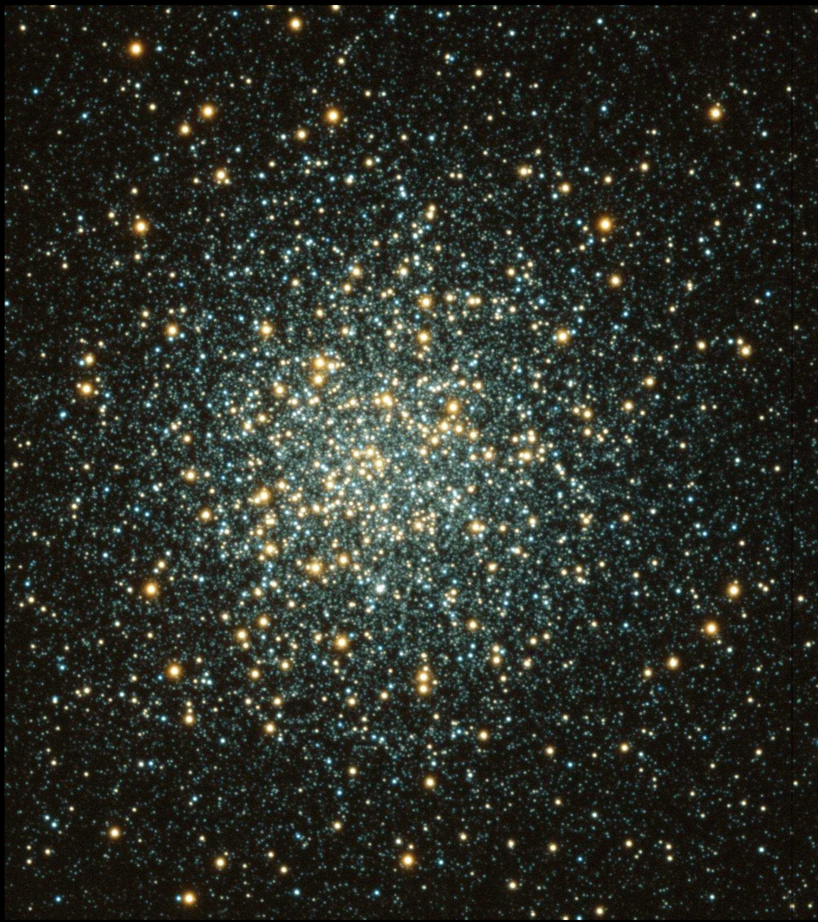
HRD

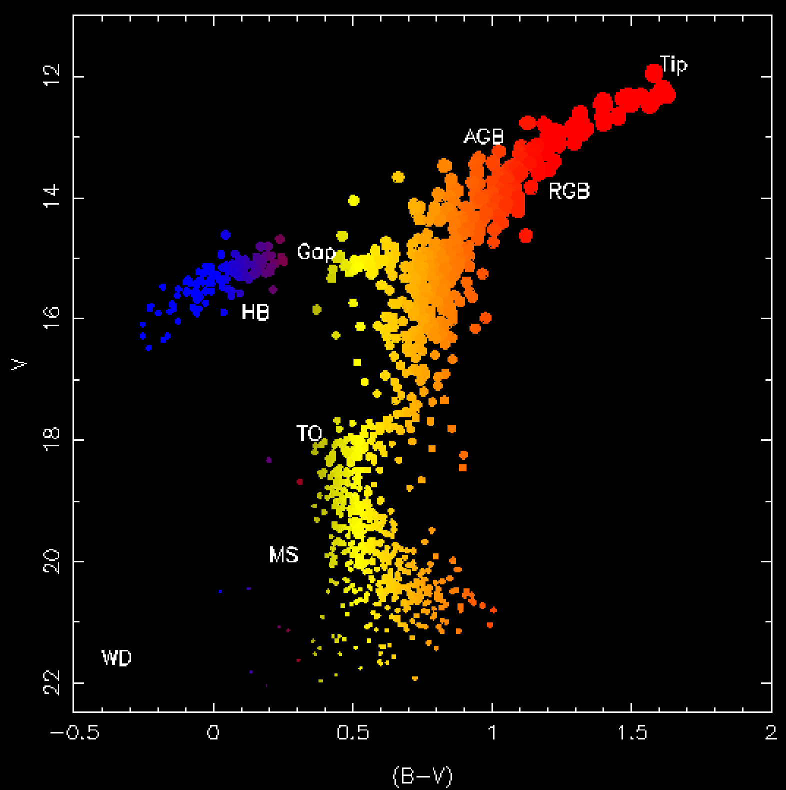


Combining Mass-Luminosity Relationship and HRD:

Main Sequence is a Mass Sequence

- M-Dwarfs have $M \lesssim 0.25 M_\odot$
- G-Stars are similar to Sun and have $M \sim M_\odot$
- O- and B-Stars are very massive ($M \gtrsim 20 M_\odot$)





HRD of Globular Cluster M5 (UNSW, Sydney)
(B-V: $\sim$ spectral class; V is a magnitude)

Globular Clusters: HRD is *very* different of solar neighbourhood

MS: Main Sequence

TO: Turn-Over point

HB: Horizontal Branch

RGB: Red Giant Branch

AGB: Asymptotic Giant Branch

WD: White Dwarfs

All stars in globular cluster born at the same time

$\Longrightarrow$ HRD shows evidence for stellar evolution

Stellar Evolution

Orion Nebula; R. Gendler

Optical View of B68 (ESO; VLT/FORS1)

Stellar Birth, V

Stars are born in “Giant Molecular Clouds”.

Pieces of cloud collapse, if mass within radius of cloud is larger than Jeans mass:

$$M_{\mathrm{J}} \sim \frac{4\pi}{3} R_{\mathrm{J}}^3 \rho$$

...which has typical values of 50–100 $M_\odot$.

After this:

$\Longrightarrow$ collapsing piece fragments into smaller pieces of $\sim$1 $M_\odot$ mass (takes $< 10^6$ years)

$\Longrightarrow$ density in centre of each piece increases

$\Longrightarrow$ $T \gtrsim 4 \times 10^6$ K

$\Longrightarrow$ nuclear fusion starts

$\Longrightarrow$ star is formed

The following is for your information only and will not be assessed in any way:

In order to derive the Jean's mass, let's look at a simple model where a cloud in the interstellar medium is kept together by its own gravitation. Because the cloud has a temperature above absolute zero, thermal motion from the gas particles will try to disperse the cloud. The cloud is thus gravitationally only if the total energy of the gas particles, i.e., the sum of the particle kinetic energy and the gravitational binding energy, is negative:

$$\frac{3}{2}\frac{M}{m_\mathrm{p}}kT - \frac{3}{5}\frac{GM^2}{R} \leq 0$$

If the energy is less than zero, the cloud is not in hydrostatic equilibrium and will collapse. This is the case if

$$\frac{M}{R} \geq \frac{5}{2}\frac{kT}{Gm_\mathrm{p}} \quad \text{or} \quad \frac{4\pi}{3}\rho R^2 \geq \frac{5}{2}\frac{kT}{Gm_\mathrm{p}}$$

This is true for radii greater than the Jeans length,

$$R_\mathrm{J} = \sqrt{\frac{15kT}{8\pi G m_\mathrm{p}\rho}} \sim \sqrt{\frac{kT}{G m_\mathrm{p}\rho}}$$

Plugging in typical values, i.e., $T \sim 50\,\mathrm{K}$, particle density $n = 10^5\,\mathrm{H\text{-}atoms\,cm^{-3}}$ (that is a mass density of $\rho = nm_\mathrm{p} \sim 1.7 \times 10^{-9}\,\mathrm{g\,cm^{-3}}$) gives $R_\mathrm{J} \sim 0.2\,\mathrm{pc}$, corresponding to a Jeans mass of around 70 solar masses.

Main Sequence, I

Once star has collapsed and nuclear fusion has started: main sequence

The Main Sequence is the result of steady state fusion (“burning”) of hydrogen into helium in stellar centres.

. . . longest phase of stellar evolution (10 billion years for Sun)

Stellar structure defined by balance between pressure inwards due to gravitation and pressure outwards due to energy release (“hydrostatic equilibrium”).

Main Sequence, II

Nuclear fusion:

$$4\text{p} \longrightarrow {}^4_2\text{He} + E$$

How much energy is gained?
Particle physics: express mass as "rest energy equivalent" via $E = mc^2$ (and call it "mass"...).

usually use energy units of MeV, 1 MeV $= 1.602 \times 10^{-13}$ J

	mass of 4 protons (4×938 MeV):	3752 MeV
$-$	mass of ${}^4_2$He:	3727 MeV
	mass defect Δmc^2:	25 MeV

In the fusion of hydrogen to helium, 0.7% of the available rest mass energy is converted to energy.

Nuclear physics: efficiency of H-burning strongly depends on temperature
$\Longrightarrow$ explanation for mass-luminosity relation (massive stars have hotter cores)

Main Sequence, III

Stellar structure governed by four coupled differential equations:

Mass structure
(mass conservation)

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

Pressure structure
(hydrostatic equilibrium)

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\frac{GM(r)}{r^2}$$

Temperature structure
(energy transport)

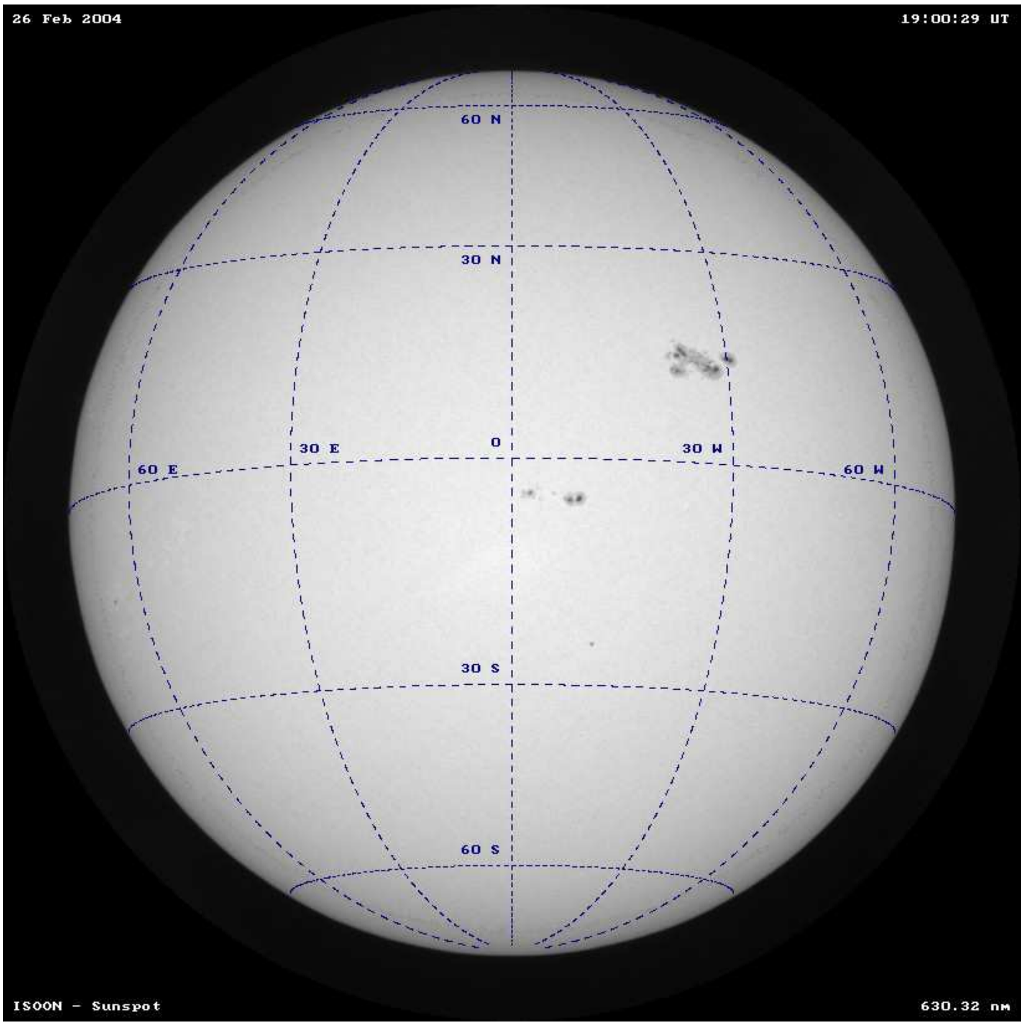
$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{k\rho(r)}{T^3}\frac{L(r)}{4\pi r^2}$$

Energy conservation
(energy transport)

$$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho(r)\epsilon(r)$$

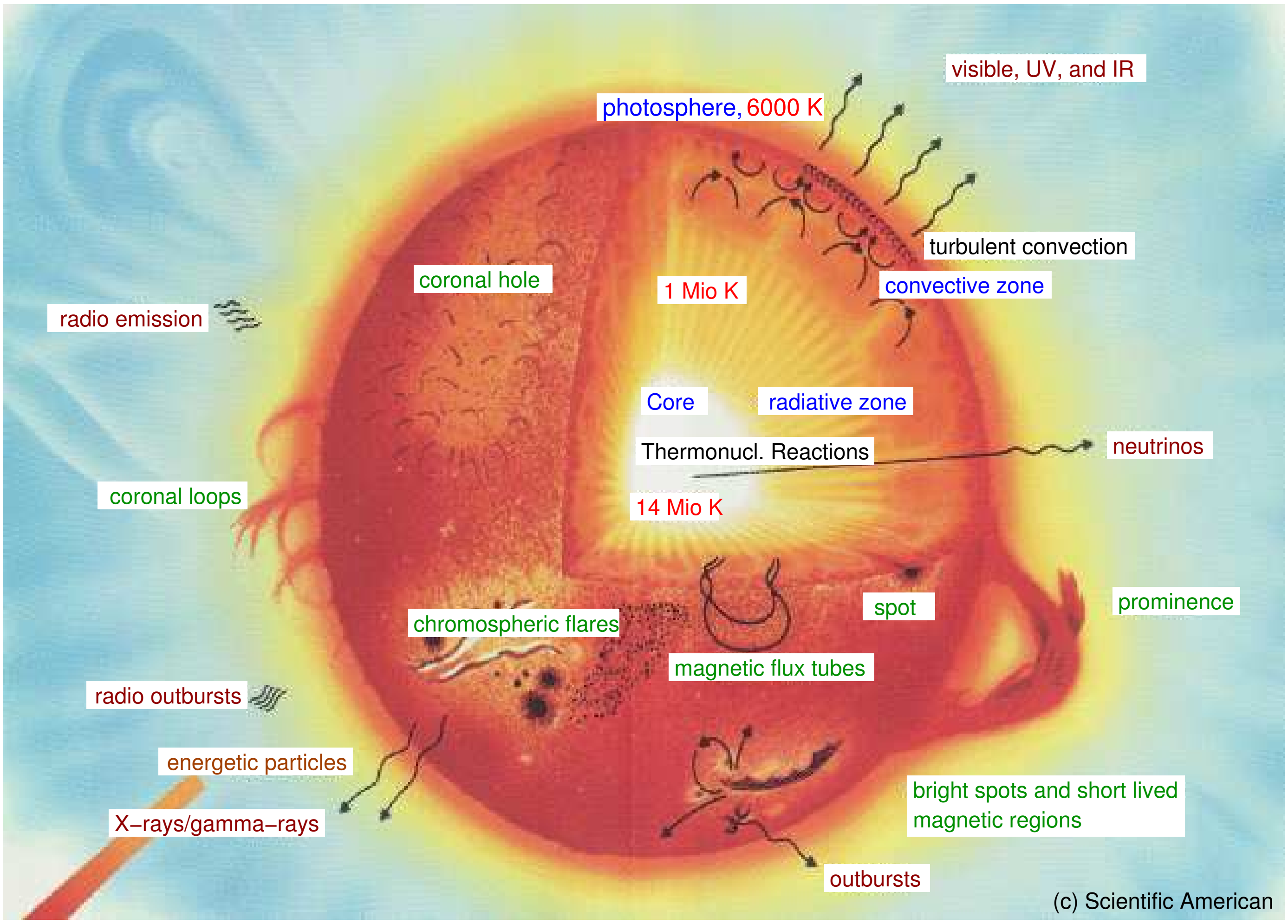
plus "equation of state" ($P = P(T, \rho)$), energy generation ($\epsilon = \epsilon(T, \rho, Z)$),...

Stellar model: numerical solution of stellar structure equations.



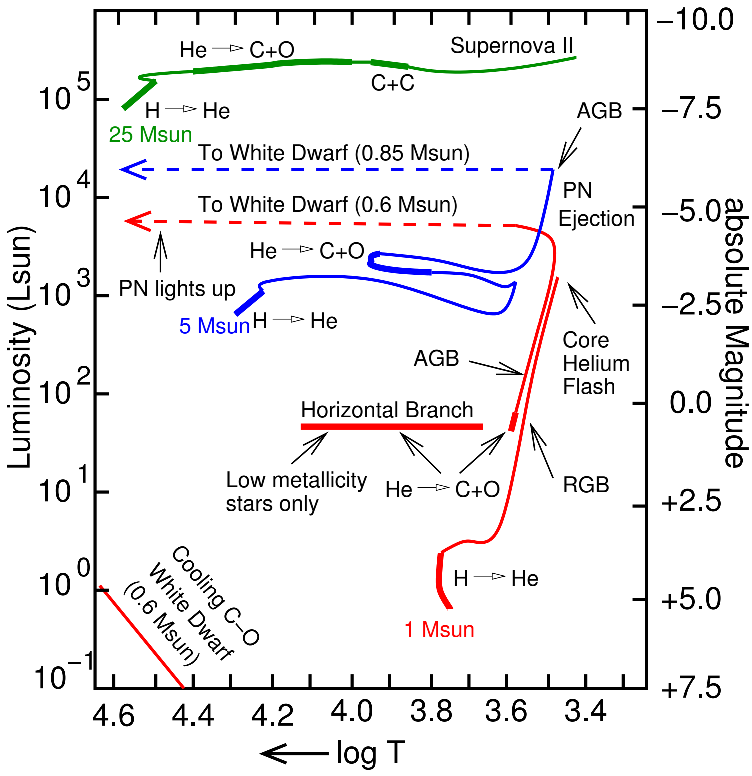
26 Feb 2004
19:00:29 UT
60 N
30 N
30 E
0
30 W
60 E
60 W
30 S
60 S
ISOON - Sunspot
630.32 nm

X



visible, UV, and IR
photosphere, 6000 K
turbulent convection
coronal hole
1 Mio K
convective zone
radio emission
Core
radiative zone
Thermonucl. Reactions
neutrinos
coronal loops
14 Mio K
spot
prominence
chromospheric flares
magnetic flux tubes
radio outbursts
energetic particles
bright spots and short lived magnetic regions
X–rays/gamma–rays
outbursts
(c) Scientific American

Main Sequence, VI



after Iben, 1991

Evolution of stars in the HRD from main sequence to death

Typical timescales (units of 10^6 yr; Schaller et al. 1992):

	$1\,M_\odot$	$5\,M_\odot$	$25\,M_\odot$
H→He	10000	94	6.4
He→C		12	0.6
C+C			0.01
PN	$\lesssim 0.01$	$\lesssim 0.01$	N/A
WD	∞	∞	N/A

Post-H-burning burning: need higher core temperatures (Coulomb barrier!), less energy release $\Longrightarrow$ last much shorter than hydrogen burning.

Abell 39 (WIYN, AURA, NOAO, NSF)

planetary nebulae: material ejected during AGB phase, photoionized once central star emits UV photons; after PN phase: white dwarf

End result of stellar evolution for stars with $M \leq 5\,M_\odot$.

White Dwarfs

Reminder: stars: hydrostatic equilibrium, inwards gravitational pressure balanced by outwards gas pressure

For gas pressure ($P = nkT$): energy source needed to heat gas (=fusion).

End of stellar life: energy source ceases to work $\Longrightarrow$ gravitational collapse!

BUT:

collapse cannot continue indefinitely:
increased density $\Longrightarrow$ quantum mechanical effects become important

Different ways to write the equation of state of an ideal gas

Among the more confusing subjects of thermodynamics are the many different ways in which the ideal gas equation can be written.

The one I prefer for astronomy is

$$P = nkT$$

where

- P: Pressure (measured in $N\,\mathrm{m}^{-1}$)
- n: particle density (i.e., number of particles per cubic metre, unit: m^{-3})
- $k = 1.38066 \times 10^{-23}\,\mathrm{J\,K}^{-1}$: Boltzmann constant
- T: Temperature (measured in Kelvins)

This equation has the advantage that it counts all particles individually (thus using n). If you know the mass of the gas particles, m_{gas} then another way of writing the ideal gas equation is

$$P = \frac{n m_{\mathrm{gas}}}{m_{\mathrm{gas}}} kT = \rho kT \frac{1}{m_{\mathrm{gas}}}$$

illustrating that for an ideal gas, $P \propto \rho$, where ρ is the mass density.

Another way to write the ideal gas equation is in terms of the total number of gas molecules, $N = nV$, where V is the volume. The ideal gas equation then is

$$P = \frac{N}{V} kT \quad \Longleftrightarrow PV = NkT$$

This version has the problem, however, that the number of gar molecules is typically rather large (there are 6×10^{23} molecules in a volume of 22.4 litres of gas, this number of particles is called one *mole*). Because working with smaller numbers is generally better, chemists prefer to work with moles. Per definition, the unit of particle number here is the Avogadro number $N_{\mathrm{A}} = 6.0221 \times 10^{23}$. So, if you want to work with moles, then the above equation becomes

$$PV = \frac{N}{N_{\mathrm{A}}} AkT = N_{\mathrm{mol}} RT$$

where

- N_{mol}: the number of moles of the gas in the volume V,
- $R = N_{\mathrm{A}} k 8.3145\,\mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$: the universal gas constant

To summarise, each of these equations has its own uses, and which one you want to use, really depends on the circumstances of the problem you are solving. For your future life as physicists, try to remember one of them, and then understand how you get from this one to the others, instead of memorising all four ones. This approach will need less memory and lead to a better understanding of what is really going on behind the scenes.

QM interlude, I

Quantum mechanics: One of the weirder phenomena in QM is the Pauli exclusion principle:

> For particles such as electrons (“Fermions”), at least one of their quantum numbers must be different.

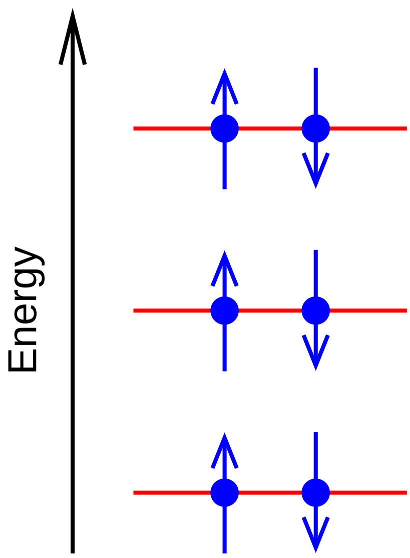
Quantum numbers are, e.g.,

- position (x, y, z),
- momentum (mv_x, mv_y, mv_z),
- angular momentum,
- spin (s)

All of these numbers are “quantized”, i.e., can only have discrete values (e.g., spin: $+1/2$, $-1/2$).

In typical gas, this is not a problem (“phase space is (almost) empty”), but once it becomes dense $\Longrightarrow$ exclusion principle kicks in.

QM interlude, VIII



Energy of electrons at the same position in space

Effect of high density on electron energy:

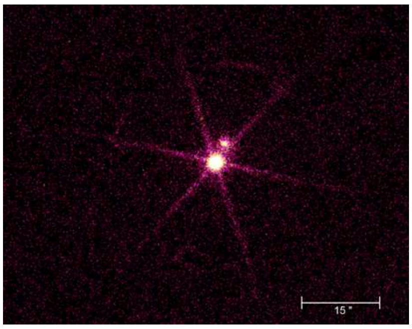
> In degenerate electron gases, electrons have much higher energies than in thermal gas.

Interaction of electrons results in degeneracy pressure:

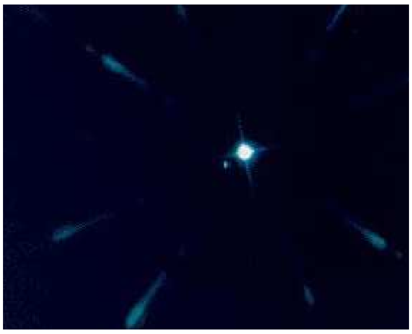
$$P = \frac{\hbar^2}{m_\mathrm{e}} n_\mathrm{e}^{5/3} \propto \rho^{5/3}$$

Note: The degeneracy pressure is independent of the temperature!

White Dwarfs: Summary



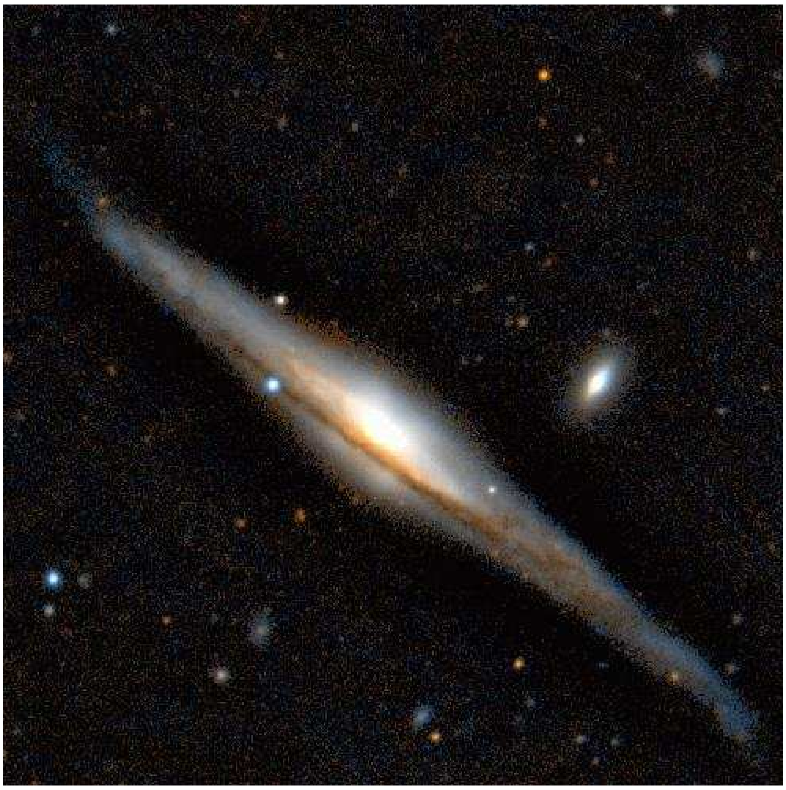
Sirius A+B: *Chandra*
(X-rays; *WD is bright*)



McDonalds Observatory
(optical; *WD is faint*)

White Dwarfs:

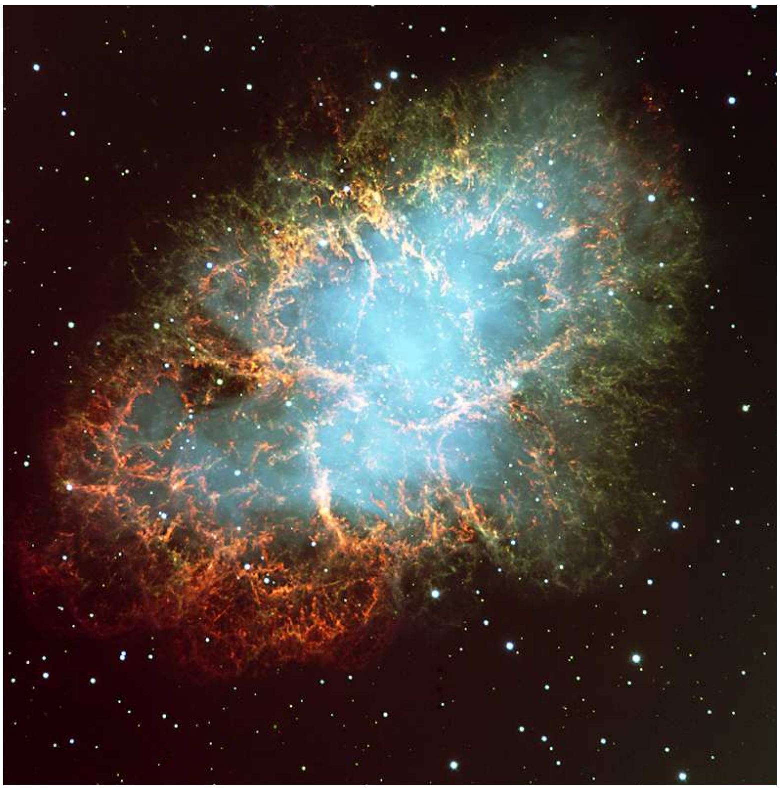
1. End stages of evolution of stars with $M \lesssim 10\,M_\odot$ on main sequence
2. typically $M \sim 0.8\,M_\odot$, and always $M < 1.44\,M_\odot$ (Chandrasekhar mass; above that: relativistic degenerate gas ($P \propto \rho^{4/3}$), can show that under these circumstances WD is not stable.
3. mainly consist of C and O
4. Radius $\sim$ Earth
5. Typical density $\rho \sim 10^6\,\mathrm{g\,cm^{-3}}$
6. interior temperature $\sim 10^7\,\mathrm{K}$, atmosphere $\sim 10^4\,\mathrm{K}$, slowly cooling down (observable for $\gtrsim 10^9$ years).



Type II SN2001cm in NGC5965 (2.56 m NOT, Håkon Dahle; NORDITA)

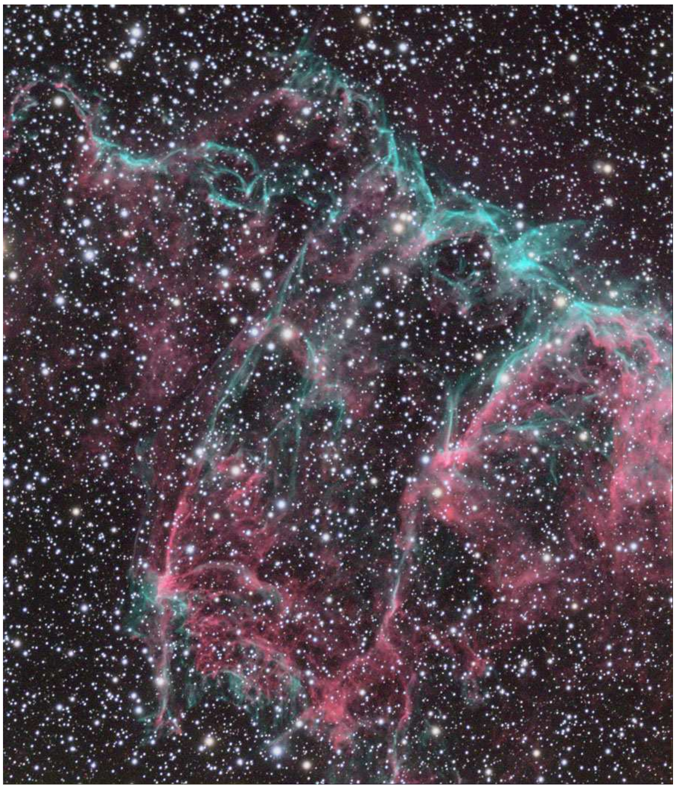
Evolution of more massive stars: fusion up to ^{56}Fe, then no energy gain $\Longrightarrow$ no pressure balance in centre $\Longrightarrow$ supernova explosion of type II.

energy release: 10^{46} W ($10^{20} L_\odot$; about 1% in light, rest in neutrinos)



(ESO VLT/FORS 2)

Crab nebula: young remnant of SN of 1054, observed light due to synchrotron radiation (radiation emitted by electrons accelerated in magnetic field)



5000–10000 year old IC 1340/Veil Nebula/Cygnus Loop (©Loke Kun Tan)

Older supernova remnants: “wispy structure” due to interaction with interstellar medium, radiation (line emission) mainly caused by heating due to shocks.

Neutron Stars

During SN explosion:
Core of exploding star above Chandrasekhar limit $\Longrightarrow$ core collapses

Densities get so high that neutronization sets in:

$$\mathrm{p} + \mathrm{e}^{-} \longrightarrow \mathrm{n} + \nu_{\mathrm{e}}$$

General properties:

- Pressure mainly through degenerate neutrons (similar to degenerate electrons for WD!).
- Typical density: $\rho \sim 10^{14}\,\mathrm{g\,cm^{-3}}$ (nuclear densities)
- Typical radius: 10...15 km (Coventry!)
- surface gravity $\sim 10^{11} \times$ Earth
- Detailed structure not yet fully understood,

Neutron Stars: Rotation, III

During SN collapse, angular momentum is conserved (Explosion: symmetric)
Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

Angular momentum conservation ($J_{\mathrm{before}} = J_{\mathrm{NS}}$):

$$\frac{2}{5}M_{\mathrm{before}}R^2_{\mathrm{before}}\omega_{\mathrm{before}} = \frac{2}{5}M_{\mathrm{NS}}R^2_{\mathrm{NS}}\omega_{\mathrm{NS}}$$

or (assume $M_{\mathrm{NS}} = M_{\mathrm{before}}$):

$$\omega_{\mathrm{NS}} = \left(\frac{R_{\mathrm{before}}}{R_{\mathrm{NS}}}\right)^2 \omega_{\mathrm{before}} \quad \text{or} \quad P_{\mathrm{NS}} = \left(\frac{R_{\mathrm{NS}}}{R_{\mathrm{before}}}\right)^2 P_{\mathrm{before}}$$

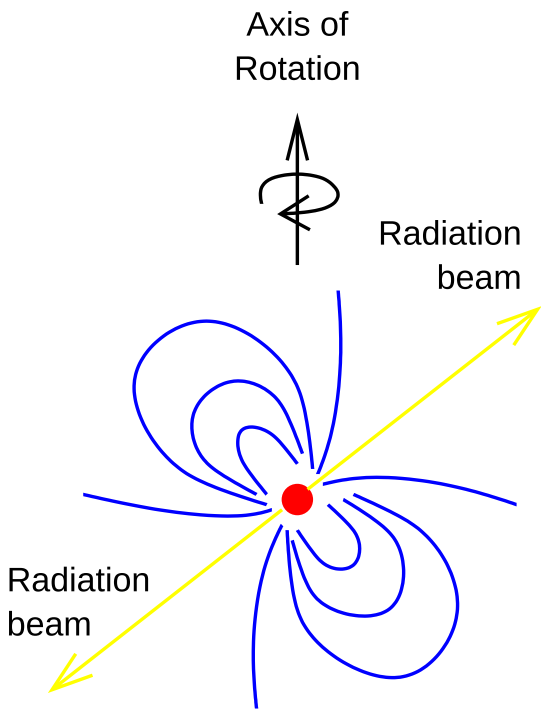
(where P: rotation period)

Example: $R_{\mathrm{before}} = 700000$ km (sun), $R_{\mathrm{NS}} = 15$ km, $P_{\mathrm{Sun}} = 27$ d $\Longrightarrow$ $P_{\mathrm{NS}} = 0.001$ s

Neutron Stars are extremely fast rotators.

close to break-up speed!

Neutron Stars: Pulsars



“Lighthouse model” for pulsars

Another conserved observable:
magnetic flux: $\Phi = BR^2$

magnetic field after SN:

$$B_{\text{NS}} = \left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2 B_{\text{before}}$$

$\Longrightarrow$ neutron stars have strong magnetic fields (typical: $B \sim 10^6 \ldots 10^8$ T)

Radio pulsars are fast rotating neutron stars with strong magnetic fields.

Black Holes, I

Neutron stars also have upper mass limit: Oppenheimer Volkoff limit.

Detailed mass limit unknown, causality considerations give $M \sim 3\,M_\odot$ (for “stiff equation of state” the sound speed becomes greater than speed of light at this mass)

Compact objects with mass above Oppenheimer Volkoff limit: Black Holes

More conservative astronomers: “Black Hole Candidates”.

Black Holes, III

Rev. John Michell: *Phil. Trans. R. Soc. London*, ***74***, *35–57 (1784)*:

42 Mr. Michell *on the Means of diſcovering the*

16. Hence, according to article 10, if the ſemi-diameter of a ſphære of the ſame denſity with the ſun were to exceed that of the ſun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its ſurface a greater velocity than that of light, and conſequently, ſuppoſing light to be attracted by the ſame force in proportion to its vis inertiæ, with other bodies, all light emitted from ſuch a body would be made to return towards it, by its own proper gravity.

Black Holes, IV

In more modern usage:

Total energy of a mass m:

$$E = E_{\text{pot}} + E_{\text{kin}} = -G\frac{Mm}{R} + \frac{1}{2}mv^2$$

Mass m is unbound if $E > 0$, i.e., for

$$v \geq v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

Black Hole: Body of mass M and radius R for which $v_{\text{escape}} > c$, where c is the speed of light.

This is the case if

$$R \leq R_{\text{S}} = \frac{2GM}{c^2} \sim 3\,\text{km}\frac{M}{M_\odot}$$

the Schwarzschild Radius.

Black Holes: Accretion

Astrophysical energy sources:

1. Nuclear fusion

Reactions à la

$$4\mathrm{p} \longrightarrow {}^4\mathrm{He} + \Delta E_{\mathrm{nuc}}$$

Energy released:

$\boxed{\text{Fusion produces } \sim 6 \times 10^{11}\,\mathrm{J\,g^{-1}}}$

(i.e., $\Delta E_{\mathrm{nuc}} \sim 0.007 m_{\mathrm{p}} c^2$)

2. Gravitation

Accretion of mass m from ∞ to R_{S} on black hole with mass M gives

$$\Delta E_{\mathrm{acc}} = \frac{GMm}{R_{\mathrm{S}}} \quad \text{where} \quad R_{\mathrm{S}} = \frac{2GM}{c^2}$$

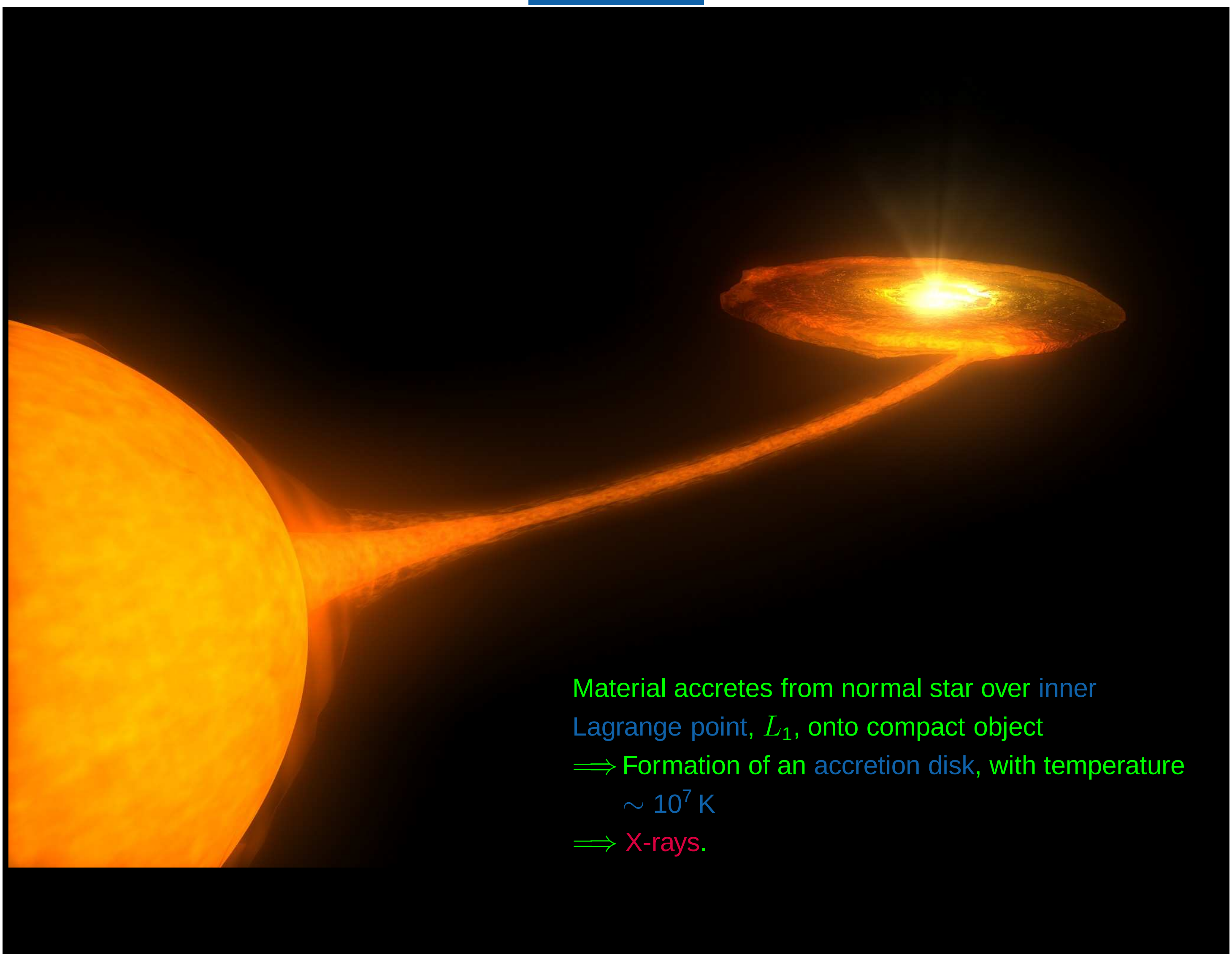
$\boxed{\text{Accretion produces } \sim 10^{13}\,\mathrm{J\,g^{-1}}}$

(i.e., $\Delta E_{\mathrm{acc}} \sim 0.1\, m_{\mathrm{p}} c^2$)

$\Longrightarrow$ Accretion of material is the most efficient astrophysical energy source.

...thus accreting objects are the most luminous in the whole universe.

Note: energy gets radiated away from *outside* the Schwarzschild radius!



Summary

Stars end their lifes as one of three kinds of compact objects:

White Dwarf: $R \sim R_{\text{Earth}}$, $\rho \sim 10^{5\ldots6}\,\text{g}\,\text{cm}^{-3}$
$M < 1.44\,M_\odot$ (Chandrasekhar Limit)
Equilibrium between gravitation and pressure of degenerate electrons

Neutron Star: $R \sim 10\,\text{km}$, $\rho \sim 10^{13}\ldots10^{16}\,\text{g}\,\text{cm}^{-3}$
$1.44\,M_\odot < M \lesssim 3\ldots4\,M_\odot$ (Oppenheimer-Volkoff Limit)
Density implies inv. β-decay ($\text{p} + \text{e}^- \longrightarrow \text{n}$), i.e., star has high neutron content

Black Hole: Above OV-Limit no stable configuration known
$\Longrightarrow$ star collapses
$\Longrightarrow$ Black Hole
$M \gtrsim 4\,M_\odot$
Event horizon at $R_\text{S} = 2GM/c^2 = 3(M/M_\odot)\,\text{km}$ (Schwarzschild radius)