Cosmology: Distances

Introduction

Distances are required to determine properties such as the luminosity or the size of an astronomical object.

Only *direct* method:

1. Trigonometric parallax

Most other methods based on "standard candles", i.e., use known absolute magnitude of an object to derive distance via distance modulus.

Examples dealt with here:

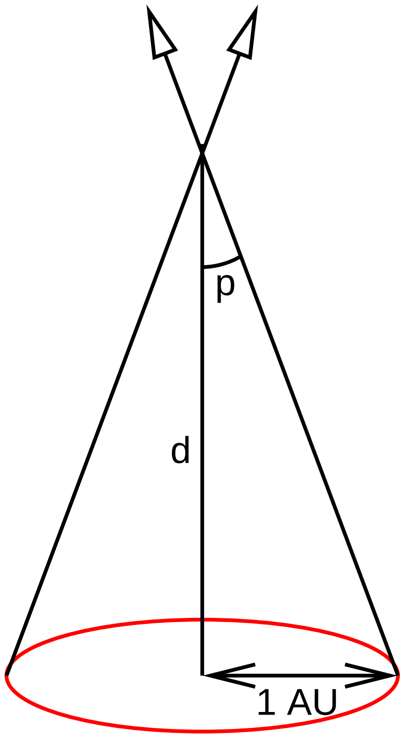
2. Main Sequence Fitting
3. Variable stars: RR Lyrae and Cepheids
4. Type Ia Supernovae

For the farthest objects, can also use expansion of universe:

5. Hubble's law

> Methods are calibrated using distances from the previous step of the distance ladder.

Trigonometric Parallax



Motion of Earth around Sun $\Longrightarrow$ Parallax
Produces apparent motion of star; projected on sky see angular motion, opening angle

$$\tan p \sim p = \frac{r_{\text{Earth}}}{d} = \frac{1\,\text{AU}}{d}$$

p is called the trigonometric parallax.

Note: requires several at several positions of the Earth

Measurement difficult: $\pi \lesssim 0.76''$ (α Cen).

Define unit for distance:
Parsec: Distance where 1 AU has $p = 1''$.

$$\boxed{1\,\text{pc} = 206265\,\text{AU} = 3.086 \times 10^{16}\,\text{m} = 3.26\,\text{ly}}$$

Trigonometric Parallax

Best measurements to date: Hipparcos satellite (1989–1993)

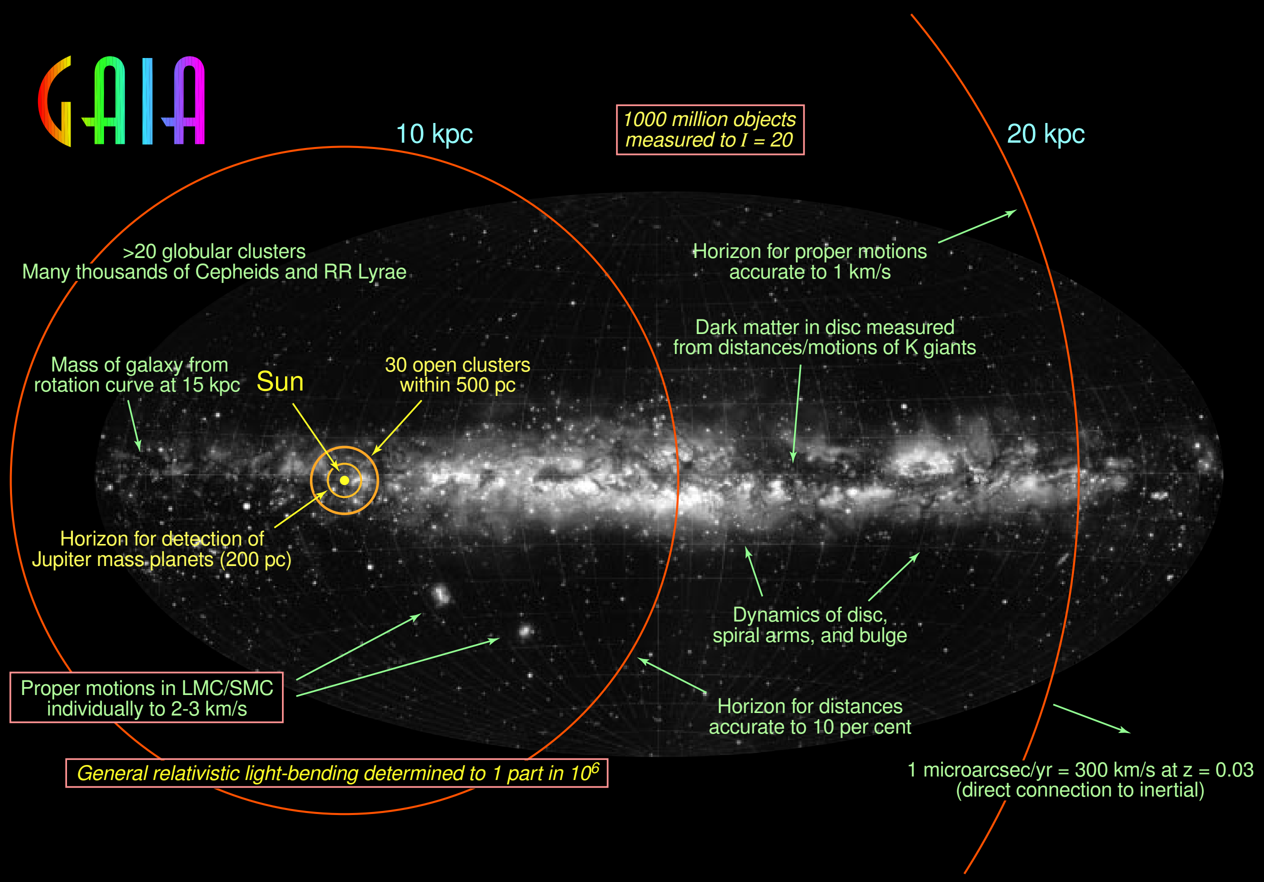
- systematic error of position: ~0.1 mas
- effective distance limit: 1 kpc
- standard error of proper motion: ~1 mas/yr
- photometry
- magnitude limit: 12
- complete to mag: 7.3–9.0

Results available at `http://astro.estec.esa.nl/Hipparcos/`:

Hipparcos catalogue: 120000 objects with milliarcsecond precision.

Tycho catalogue: 10^6 stars with 20–30 mas precision, two-band photometry

Plans for the future: GAIA (ESA mission, launch 2010, observations 2011–2016):



GAIA: $\sim$ 4μarcsec precision, 4 color to $V = 20$ mag, 10^9 objects.

Standard Candles

Assuming isotropic emission, the flux measured at distance d from object with luminosity L is given by the "inverse square law",

$$f(d) = \frac{L}{4\pi d^2}$$

note that f is a function of the d.

Remember that the magnitude is defined through comparing two fluxes,

$$m_2 - m_1 = 2.5 \log_{10}(f_1/f_2) = -2.5 \log_{10}(f_2/f_1)$$

To allow the comparison of sources at different distances, define

absolute magnitude M = magnitude if star were at distance 10 pc

Because of this

$$M - m = -2.5 \log_{10}\left(f(10\,\text{pc})/f(d)\right) = -2.5 \log_{10}\left(\frac{L/(4\pi(10\,\text{pc})^2)}{L/(4\pi d^2)}\right) = -2.5 \log_{10}\left(\frac{d}{10\,\text{pc}}\right)^2$$

The difference $m - M$ is called the distance modulus,

$$m - M = 5 \log_{10}\left(\frac{d}{10\,\text{pc}}\right)$$

Standard Candles

To obtain distance, use standard candles

Standard candles are defined to be objects for which their absolute magnitude is known.

Requirements:

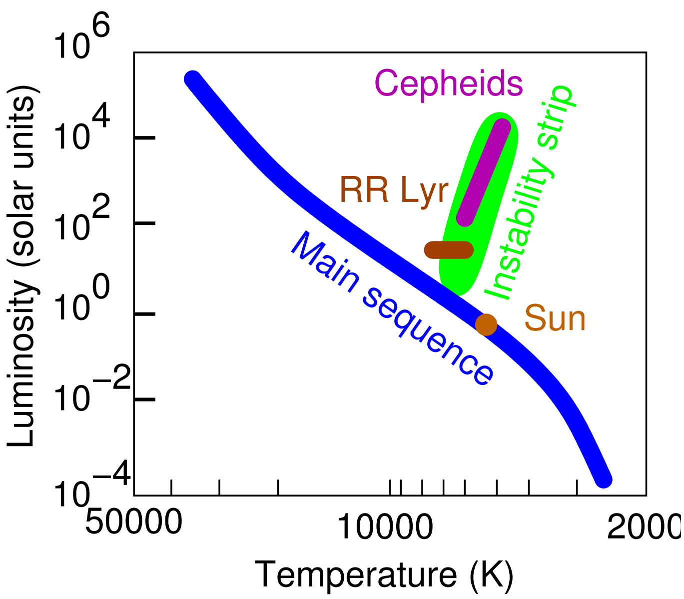
- physics of standard candle well understood (i.e., need to know *why* object has certain luminosity).
- absolute magnitude of standard candle needs to be calibrated, e.g., by measuring its distance by other means (this is a *big problem*)

To determine distance to astronomical object:

1. find standard candle(s) in object,
2. measure their m
3. determine $m - M$ from known M of standard candle
4. compute distance d

Often, distances are given in terms of $m - M$, and not in pc, so last step is not always performed.

Introduction, I



Instability strip in the Hertzsprung-Russell Diagram

Certain regions of HRD: stars prone to instability:

Ionisation of Helium: transparency of outer parts of star changes

$\Longrightarrow$ size of star changes

$\Longrightarrow$ surface temperature and luminosity variations

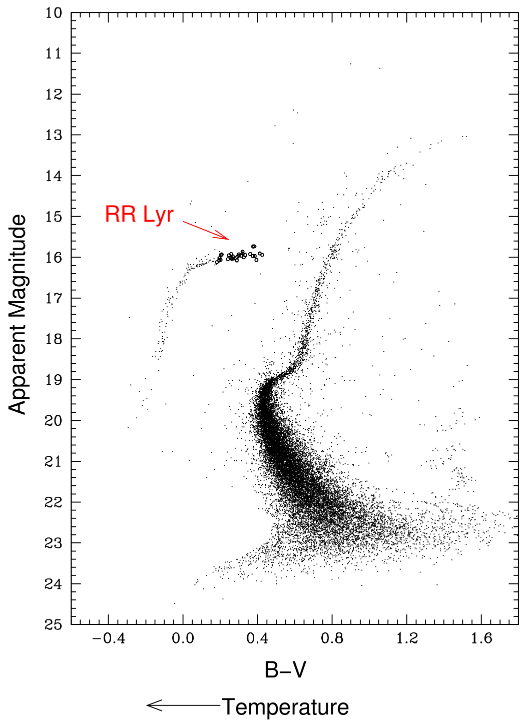
Most important variables of this kind:

1. RR Lyr variables

   mainly in globular clusters: lower metallicity of clusters (“population II”) allows stars to enter instability strip

2. δ Cepheids

RR Lyrae



HRD of Globular Cluster M2
(after Lee et al., 1999, Fig. 2)

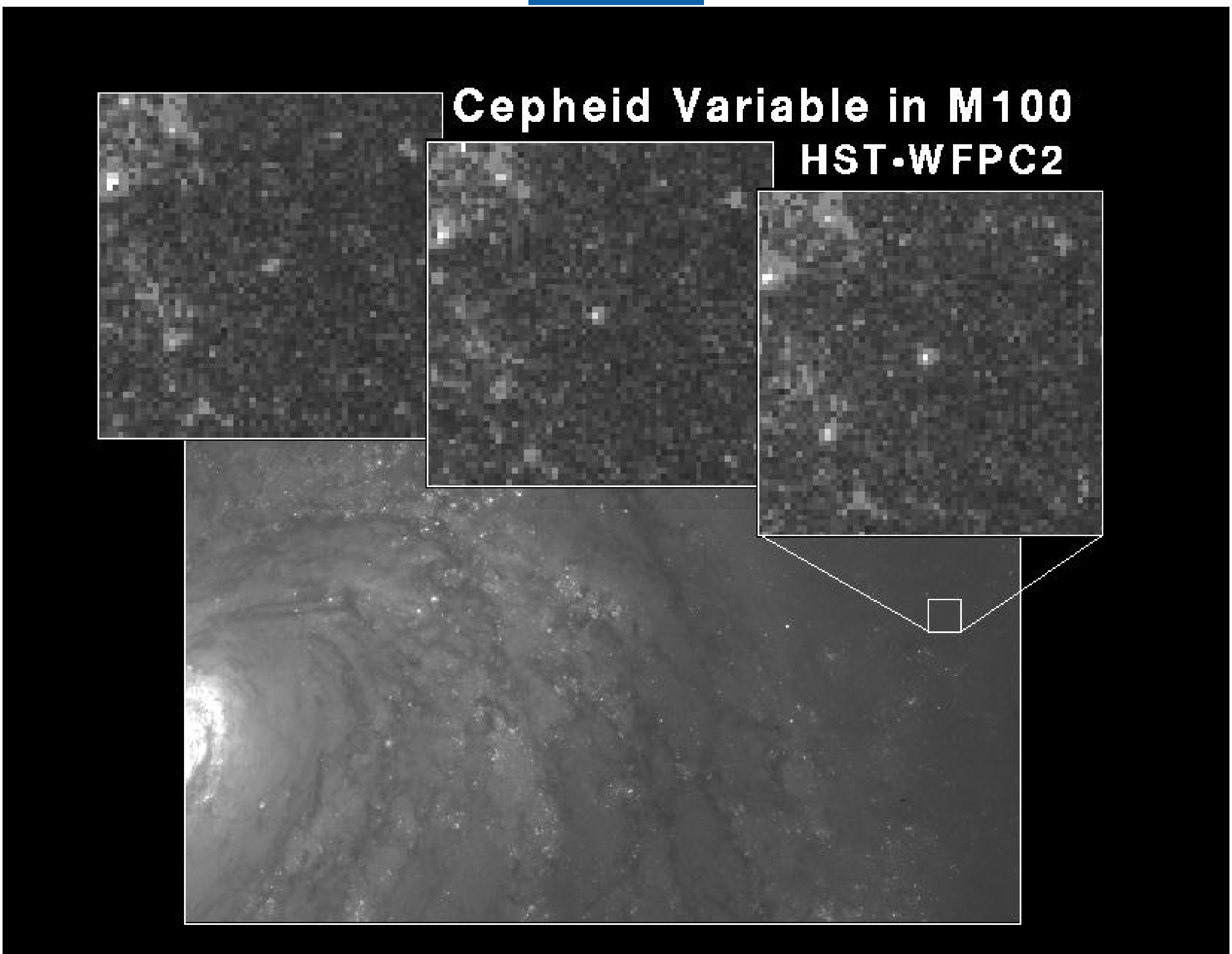
RR Lyrae variables:

- Variability ($P \sim 0.2 \ldots 1\,\mathrm{d}$)
- Mainly temperature change
- RR Lyr gap clearly observable in globular cluster HRD

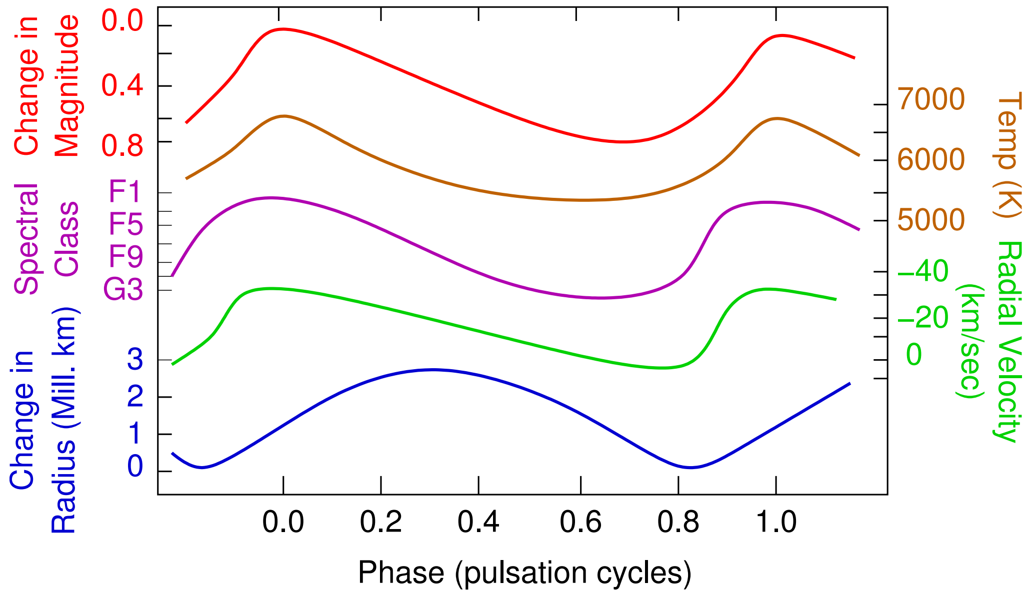
Absolute magnitude of RR Lyr gap:
$M_{\mathrm{V}} = 0.6$, $M_{\mathrm{B}} = 0.8$, i.e., $L_{\mathrm{RR}} \sim 50\,L_{\odot}$).

Works out to LMC ($d \sim 50\,\mathrm{kpc}$) and other dwarf galaxies of local group, mainly used for globular clusters and local group.

Example: M5: gap at $m = 16\,\mathrm{mag} \Longrightarrow m - M = 15.4\,\mathrm{mag}$
$\Longrightarrow d = 12\,\mathrm{kpc}$.



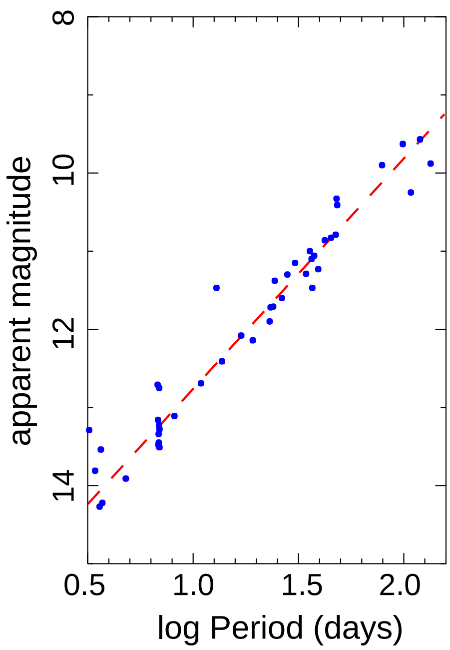
Cepheids



after http://csep10.phys.utk.edu/astr162/lect/index.html

Cepheids: Luminous stars ($L \sim 1000\,L_\odot$) in instability strip with large luminosity amplitude variation, $P \sim 2\ldots150\,\mathrm{d}$ (easily measurable).

Cepheids



Period-Luminosity relation for the LMC Cepheids

after Mould et al. (2000, Fig. 2)

Henrietta Leavitt (1907):

> Cepheids have a period luminosity relationship: $M \propto -\log P$

Low luminosity Cepheids have lower period

Observations find:

$$M = -2.76 \log P - 1.40$$

(P in days)

Calibrated from observing Large Magellanic Cloud Cepheids (see figure), and determining LMC distance from other means (MS fitting, RR Lyr,...) to find absolute magnitudes...

With HST: works out to Virgo cluster ($d = 18.5\,\mathrm{Mpc}$).

The origin of the Period-Luminosity relationship is in the Helium ionisation instability discussed before. The details of this are rather messy, however, it is easy to see that a Period-Luminosity relationship as that observed for the Cepheids is a simple consequence of the fact that the pulsating star is not disrupted by its oscillation.

For the outer parts of the star to remain bound, the kinetic energy of the pulsating outer parts of the stars has to remain smaller than their binding energy:

$$\frac{1}{2}mv^2 \lesssim \frac{GMm}{R}$$

But we know that for the velocity

$$v < \frac{2R}{P}$$

where P is the period of the star and R its radius at maximum extension (we observe the star to expand to a radius R once every P seconds, so the maximum distance the expanding material can go during that time is $2R$). Inserting v into the above equation gives

$$\frac{1}{2}\frac{4R^2}{P^2} \lesssim \frac{GM}{R} \quad \iff \quad P^2 \gtrsim \frac{2}{G}\frac{R^3}{M} = \frac{2}{G}\frac{1}{M/R^3}$$

If we assume that the pulsation is close to the break-up speed, and noting that M/R^3 is proportional to the average density of the star, then it is easy to see that

$$P \propto (G\rho)^{-1/2}$$

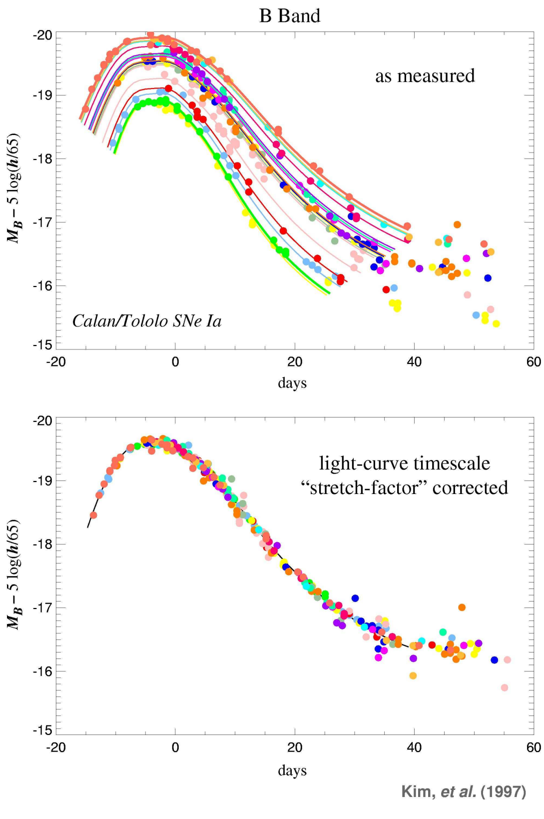
In the homework for this week you are asked to convince yourself that $(G\rho)^{-1/2}$ has the dimension of a period, i.e., for all gas balls oscillating close to the break up speed, we expect that $P \propto \rho^{-1/2}$. To obtain the period luminosity relationship, you need to remember that the emissivity per square-metre of the surface of a star with temperature T is σT^4 (per the Stefan-Boltzmann law), while the surface of the star is proportional to R^2. Therefore, the luminosity of the star is $L \propto R^2T^4$.

This week's homework asks you to use $L \propto R^2T^4$ and $P \propto \rho^{-1/2}$ to show that from these the absolute magnitude of a pulsating star is related to the period through

$$\log P \propto -m$$

as observed for Cepheids.

Supernovae, II



After correction of systematic effects and time dilatation (expansion of the universe, see later):

SN Ia lightcurves all look the same

$\Longrightarrow$ standard candle

Supernovae, III

SN Ia = Explosion of CO white dwarf when pushed over Chandrasekhar limit (1.4 $M_\odot$) (via accretion?).

$\Longrightarrow$ Always similar process

$\Longrightarrow$ Very characteristic light curve: fast rise, rapid fall, exponential decay ("FRED") with half-time of 60 d.

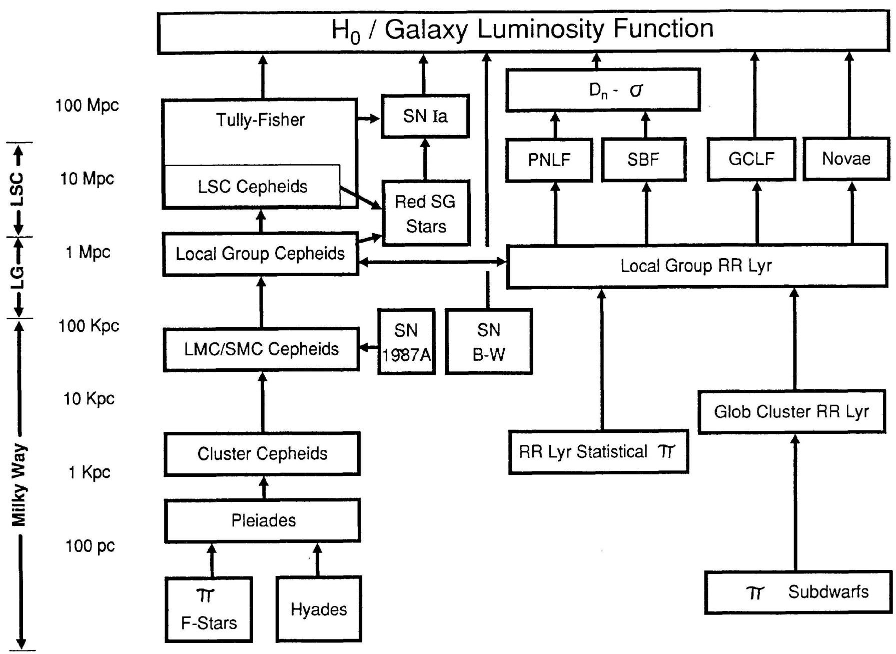
60 d time scale from radioactive decay $\mathrm{Ni}^{56} \rightarrow \mathrm{Co}^{56} \rightarrow \mathrm{Fe}^{56}$
("self calibration" of lightcurve if same amount of Ni^{56} produced everywhere)

Calibration: SNe Ia in nearby galaxies where Cepheid distances known.
At maximum light:

$$M_\mathrm{B} = -18.33 \pm 0.11 \quad \Longleftrightarrow \quad L \sim 10^{9\ldots10}\, L_\odot$$

Observable out to $\gtrsim$1 Gpc $\Longrightarrow$ essentially cover almost the whole universe...

Summary: Distance Ladder



Pathways to Extragalactic Distances

Jacoby (1992, Fig. 1)