

The Planets

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The aim of this chapter is to introduce the physics of planetary motion and the general properties of the planets.

Useful background reading includes:

- Young & Freedman:
 - section 12.1 (Newton's Law of Gravitation),
 - section 12.3 (Gravitational Potential Energy),
 - section 12.4 (The Motion of Satellites),
 - section 12.5 (Kepler's Laws and the Motion of Planets)
- Zeilik & Gregory:
 - chapter P1 (Orbits in the Solar System),
 - chapter 1 (Celestial Mechanics and the Solar System),
 - chapter 2 (The Solar System in Perspective),
 - section 4-3 (Interiors),
 - section 4-5 (Atmospheres),
 - chapter 5 (The Terrestrial Planets),
 - chapter 6 (The Jovian Planets and Pluto).
- Kutner:
 - chapter 22 (Overview of the Solar System),
 - section 23.3 (The atmosphere),
 - chapter 24 (The inner planets, especially section 24.3),
 - chapter 25 (The outer planets).



Relative sizes of the Sun and the planets

Venus Transit, 2004 June 8





The Inner Planets (SSE, NASA)



Planets: Properties

		a [AU]	$P_{orb}\left[yr ight]$	<i>i</i> [°]	e	$P_{\sf rot}$	M/M_{\oplus}	R/R_{\oplus}
Mercury	Ą	0.387	0.241	7.00	0.205	58.8 d	0.055	0.383
Venus	Ŷ	0.723	0.615	3.40	0.007	-243.0d	0.815	0.949
Earth	\oplus	1.000	1.000	0.00	0.017	23.9 h	1.000	1.00
Mars	0 ⁷	1.52	1.88	1.90	0.094	24.6 h	0.107	0.533
Jupiter	24	5.20	11.9	1.30	0.049	9.9 h	318	11.2
Saturn	ħ	9.58	29.4	2.50	0.057	10.7 h	95.2	9.45
Uranus	Ô	19.2	83.7	0.78	0.046	-17.2h	14.5	4.01
Neptune	Ψ̈́	30.1	163.7	1.78	0.011	16.1 h	17.1	3.88
(Pluto	Р	39.2	248	17.2	0.244	6.39 d	0.002	0.19)

After Kutner, Appendix D;

a: semi-major axis P_{orb} : orbital period *i*: orbital inclination (wrt Earth's orbit)

e: eccentricity of the orbit P_{rot} : rotational period M: mass

R: equatorial radius

 $1\,\text{AU} = 1.496 \times 10^{11}\,\text{m}.$

Introduction

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Structure

Questions that we will deal with:

1. How do the planets move?

Kepler's laws and their physical interpretation

- 2. What do planetary surfaces look like? craters, plate tectonics, volcanism
- 3. How do planetary atmospheres work? hydrostatic structure
- 4. What is the internal structure of the planets? hydrostatic structure (again)
- 5. Is the solar system normal?

Are there planets elsewhere?

Introduction

Johannes Kepler: Motion of planets governed by three laws:

- 1. Each planet moves in an elliptical orbit, with the Sun at one focus of the ellipse. ("Astronomia Nova", 1609)
- A line from the Sun to a given planet sweeps out equal areas in equal times. ("Astronomia Nova", 1609)
- 3. The square of the orbital periods of the planets is proportional to the cube of the major axes. ("Harmonice Mundi", 1619)

Isaac Newton ("Principia", 1687): Kepler's laws are consequence of gravitational interaction between planets and the Sun, and the gravitational force is

$$\mathbf{F}_{1} = -\frac{Gm_{1}m_{2}}{r_{12}^{2}}\frac{\mathbf{r}_{21}}{r_{12}}$$

where \mathbf{F}_1 is the gravitational force exerted on object 1, m_1 , m_2 are the masses of the interacting objects, r their distance, and \mathbf{r}_{21}/r_{12} the unit vector joining the objects, $\mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1$, $\mathbf{r}_{12} = -\mathbf{r}_{21}$ and $r_{12} = |\mathbf{r}_{12}| = |\mathbf{r}_{21}|$.



Kepler's 1st Law: The orbits of the planets are ellipses and the Sun is at one focus of the ellipse.

For the planets of the solar system, the ellipses are almost circular, for comets they can be very eccentric.





Definition: Ellipse = Sum of distances r, r' from any point on ellipse to two fixed points (foci, singular: focus), F, F', is constant:

$$r + r' = 2a$$

where a is called the semi-major axis of the ellipse.



Definition: Eccentricity e: ratio between distance from centre of ellipse to focal point and semi-major axis.

So circles have e = 0.







use r + r' = 2a and solve for r to find the polar coordinate form of the ellipse:

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

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Check this for yourself! θ is called the *true anomaly*.



Finally, we need the closest and farthest point from a focus:

closest point :
$$d_{\text{perihelion}} = a - ae = a(1 - e)$$

farthest point : $d_{\text{aphelion}} = a + ae = a(1 + e)$

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for stars: periastron and apastron,

for satellites circling the Earth: perigee and apogee.



Kepler's 2nd Law: The radius vector to a planet sweeps out equal areas in equal intervals of time.

- 1. Kepler's 2nd Law is also called the *law of areas*.
- 2. perihelion: planet nearest to Sun \implies planet is fastest
- 3. aphelion: planet farthest from Sun \implies planet is slowest

Kepler's Laws



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Kepler's 2nd law is a direct consequence of the conservation of angular momentum. Remember that angular momentum is defined as

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v} \tag{3.1}$$

and its absolute value is



To interpret the angular momentum, look at the figure at the left. Note that $v \sin \phi$ is the projection of the velocity vector perpendicular to the radius vector r, and the distance traveled by the planet in an infinitesimally short time Δt is given by $\Delta x = \Delta t \cdot v \sin \phi$. Therefore, the area of the triangle ABC is given by

$$\Delta A = \frac{1}{2}r\Delta x = \frac{1}{2}r\Delta tv\sin\phi = \frac{L}{2m}\Delta t$$

Kepler's 2nd law states that the "sector velocity" dA/dt is constant with time:

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \frac{L}{2m} = \text{const.}$$

To confirm that this claim is true, we need to prove that

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{1}{2m}\frac{\mathrm{d}L}{\mathrm{d}t} = 0$$

But
$$dL/dt$$
 is given by

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} \times \mathbf{p} + \mathbf{r} \times \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \mathbf{v} \times \mathbf{p} + \mathbf{r} \times \mathbf{F} = \mathbf{v} \times m\mathbf{v} + \mathbf{r} \times \frac{GMm}{r^2} \frac{\mathbf{r}}{r} = \mathbf{0}$$

since the cross product of a vector with itself is zero. Therefore, Kepler's 2nd law is true and is a consequence of the conservation of angular momentum for a central field.



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3rd Law

Kepler's 3rd Law: The squares of the periods of the planets, P, are proportional to the cubes of the semimajor axes, a, of their orbits: $P^2 \propto a^3$.



Computing the motion of two bodies of mass m_1 and m_2 in gives Newton's form of Kepler's third law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} R^3$$

where $r_1 + r_2 = R$ (for elliptical orbits: R is the semi-major axis).



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Circular Motion



canceling m_1 and m_2 gives

Dividing these two equations by each other results in

For an interpretation of Kepler's third law, consider the motion of two bodies with masses m_1 and m_2 on circular orbits with radii r_1 and r_2 around a point CM (see figure).

The reason for doing the computation with circular orbits is that the following discussion will be *much* easier, however, all results from this section also apply to the general case of elliptical motion.

The attractive force between the two points is given by Newton's law:

$$F_{\rm grav} = G \frac{m_1 m_2}{R^2} = G \frac{m_1 m_2}{(r_1 + r_2)^2}$$

In order to keep the two bodies on circular orbits, the gravitational force needs to be equal the centripetal force keeping each body on its circular orbit.

The centripetal force is

$$F_{\text{cent}}, \mathbf{1} = \frac{m_1 v_1^2}{r_1} = \frac{4\pi^2 m_1 r_1}{P^2}$$
$$F_{\text{cent}}, \mathbf{2} = \frac{m_2 v_2^2}{r_2} = \frac{4\pi^2 m_2 r_2}{P^2}$$

where I used $v = 2\pi r/P$ to compute the velocity of each of the bodies. Setting the centripetal force equal to the gravitational force then gives

$$\frac{4\pi^2 m_1 r_1}{P^2} = G \frac{m_1 m_2}{(r_1 + r_2)^2}$$
$$\frac{4\pi^2 m_2 r_2}{P^2} = G \frac{m_1 m_2}{(r_1 + r_2)^2}$$

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$$\frac{4\pi^2 r_2}{P^2} = G \frac{m_1}{(r_1 + r_2)^2}$$

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$
 or $m_1r_1 = m_2r_2$

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This is the definition of the *center of mass*.

The total distance between the two bodies is

$$R = r_1 + r_2 = r_1 + \frac{m_1}{m_2}r_1 = r_1\left(1 + \frac{m_1}{m_2}\right)$$

Inserting into one of the above equations gives

such that

$$\frac{4\pi^2}{P^2}\cdot R\cdot \frac{m_2}{m_1+m_2}=\frac{Gm_2}{R^2}$$

$$\frac{4\pi^2}{P^2} = \frac{G(m_1 + m_2)}{R^3} \quad \text{or} \quad P^2 = \frac{4\pi^2}{G(m_1 + m_2)}R^3$$

This is Newton's form of Kepler's 3rd law.



Newton's form of Kepler's 3rd law is the most general form of the law.

However, often shortcuts are possible.

Assume one central body dominates, $m_1 = M \gg m_2$:

$$\frac{P^2}{a^3} = \frac{4\pi^2}{GM} = \text{const.} = k$$

So, if we know P and a for one body moving around m_1 , can compute k.







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For the Solar System, use Earth:

- $P_{\oplus} = 1$ year (by definition!)
- $a_{\oplus} = 1 \text{ AU}$ (Astronomical Unit, $1 \text{ AU} = 149.6 \times 10^6 \text{ km}$)

 $\implies k = 1 \text{ yr}^2 \text{ AU}^{-3}$



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Jupiter: $a_{2_{+}} = 5.2 \text{ AU}$. What is its period?





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Jupiter:
$$a_{2+} = 5.2$$
 AU. What is its period?
Answer: $P_{2+}^2 = 1$ yr² AU⁻³ · 5.2³ AU³ ~ 140 yr², or $P_{2+} \sim 12$ years
(with pocket calculator: $P_{2+} = 11.86$ years)

Kepler's Laws

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Overview

Division of Solar System into two major types of planets:

- 1. Inner "Terrestrial" Planets: Mercury, Venus, Earth/Moon, Mars:
 - \implies all similar to Earth ("rocks").
 - \implies no moons (Earth/Moon better called "twins")
 - \implies Moons of
- 2. Outer Planets: Jupiter, Saturn, Uranus, Neptune:
 - \implies "gas giants"
 - \implies all have extensive moon systems

Although not planets (i.e., motion not around Sun), large moons of gas giants are very similar in structure to terrestrial planets. Plan for this and next lecture:

- 1. Surfaces / Interiors of terrestrial planets
- 2. Atmospheric structure of gas giants (and terrestrial planets)

Planets: Overview



The Inner Planets (SSE, NASA)



Introduction, I



Structure of terrestrial planets:

 Core: high-density material (Fe)

The Inner Planets



Introduction, II



Structure of terrestrial planets:

- Core: high-density material (Fe)
- Mantle: plastic materials, hot (e.g., Earth: molten rocks)

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Introduction, III



Structure of terrestrial planets:

- Core: high-density material (Fe)
- Mantle: plastic materials, hot (e.g., Earth: molten rocks)
- Lithosphere: rigid material, e.g., Silicates

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Introduction, IV



Structure of terrestrial planets:

- Core: high-density material (Fe)
- Mantle: plastic materials, hot (e.g., Earth: molten rocks)
- Lithosphere: rigid material, e.g., Silicates

Knowledge of structure important for, e.g.,

- origin of magnetic fields (thought to be caused by molten core \implies currents \implies *B*-field ("dynamo"). Details unknown).
- atmospheric composition (molten mantle \implies volcanism \implies CO₂, CH₄,...)

The Inner Planets

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Mercury:

- not much larger than Moon
- densest of all terrestrial planets
- no evidence for atmosphere
- Rotation period: 59 d, 2/3 of orbital period.
- surface: impact craters and tectonics
- Only information available is from Mariner 10 (three flybys, 1974/1975)
- ESA Mission Bepi Colombo, planned for \sim 2012
- NASA mission "Messenger"

(launched 2004 August 3, flyby 2008 and 2009, in orbit from 2011 on)

Major landforms: Large Structures



Caloris Basin (1300 km diameter) close to sub-solar point at perihelion \Rightarrow hot ($T > 400^{\circ}$ C on day, $T \sim -170^{\circ}$ C during night) result of *large* impact event

Major landforms: Large Structures



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Robinson, NWU / NASA
 Hilly/lineated terrain antipodal to
 Caloris (120 km across)
 ⇒ effect of shock from Caloris impact.

Major landforms: Craters



NASA/JPL Terraced craters, with central mountains.

Major landforms: Craters



NASA/JPL Terraced craters, with central mountains. S-Pole; NASA/JPL 50 km diam craters with rays (remains from impact)



Impact Craters, I

Physics of impact cratering:

Kinetic energy:

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \cdot \frac{4}{3}\pi r^3 \rho v^2 = \frac{\pi d^3 \rho v^2}{12}$$

Important numbers:

- Velocity of impact: several times orbital speed of planet
- Impacting body: rock or Fe, several meters to kilometers in size

Example:

E.g., $v = 10 \text{ km s}^{-1}$, d = 25 m, $\rho = 7900 \text{ kg m}^{-3}$ $\implies E = 3 \times 10^{15} \text{ J}$ (~1 Megaton of TNT)

1 Megaton TNT is typical strength of US nuclear bombs [B-83 bomb], UK's Trident bombs correspond to \sim 0.3–100 kilotons TNT.

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French, 1998, LPI Cont. 954






French, 1998, LPI Cont. 954







French, 1998, LPI Cont. 954







French, 1998, LPI Cont. 954







French, 1998, LPI Cont. 954







Impact Craters, VII



French, 1998, LPI Cont. 954



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French, 1998, Traces of Catastrophe, LPI Contribution 954, p. 21



Venus:

- similar size to Earth, similar structure
- \bullet insolation ${\sim}\text{2}{\times}$ Earth
- very slow rotation (243 d, retrograde; \implies no *B*-field)
- very dense atmosphere: surface pressure \sim 90 \times Earth
- atmosphere: 96.5% CO₂, 3.5% N
 - \implies strong greenhouse effect
 - \implies surface temperature \sim 460°C.
- acid rain (yes, sulphuric acid!)

NASA, Pioneer Venus

Information mainly from radar surveying from Earth and from Magellan (1990–1994), plus images from Pioneer Venus Probe (1979). Several landings (Venera, 1975/1981)

Major landforms



NASA, Magellan

 $440 \times 350 \text{ km}^2$ area in Eistla Regio, shows basic stratigraphy (sequence of geologic events): right half: old highlands, fractured structure (~15% of surface), left part: lowlands, younger area, origin in former volcanism? Craters (note: strong erosion \implies fewer craters overall)



Eistla Regio; heights exagerrated by factor 22.5



Gula Mons; heights exagerrated by factor 22.5

Gula Mons; real heights

Venus surface images:



Venera 13 (3 March 1982): images from color TV camera



courtesy D.P. Mitchell

Venera 13 (3 March 1982): reanalysed image without camera distortion



Venera 14 (5 May 1982)



Earth:

- double planet system
- Earth surface: *dominated by plate tectonics, erosion*
- atmosphere: 80% N₂, 20%O₂ \implies moderate greenhouse effect
 - \implies surface temperature $>0^{\circ}C$.
- water present
- Moon:
 - very similar to Mercury, overall
 - Mariae (plains from massive impacts) and impact craters
 - Rotation synchronous to orbit around Earth

Atmospheric features © Eumetsat, 2001



Earth: Wolf Creek Crater, Australia Currently 172 confirmed impact structures on Earth



Glaciers on Greenland; (2002 April, J. Wilms)





Evidence for plate tectonics (few craters!)



Evidence for plate tectonics (few craters!), volcanism,...



Evidence for life – note light pollution



Earth's Moon



Earth's Moon : surface dominated by mariae (large, dark lava basins)





Moon: Crater Copernicus



Moon: Apollo 16, 1972 Apr, Descartes Highlands



Moon, Apollo 17, 1972 Dec, Taurus-Littrow valley



Mars:

- smaller than Earth
- ullet very low density ($\langle
 ho
 angle \sim$ 3 g cm $^{-3}$)
 - \implies small core, probably Fe and Fe_xS_y,
- polar caps, seasons
- thin atmosphere, clouds, fog,...
- water sublimes
 - \implies no liquid water today
- atmosphere: 95% CO₂
 - \implies weak greenhouse effect
- two moons (captured asteroids)

NASA, Mars Global Surveyor

Early Exploration through Mariner missions and Viking 1 and Viking 2 orbiters and landers in 1970s, recently, strong interest (NASA Mars Global Surveyor [MGS], ESA Mars Express, plus several landers). Currently best surveyed planet except for Earth.

Mars: thin atmosphere (pressure on surface 1% Earth), but real seasonal variations.

Mars, 2005 Feb 7, NASA/Malin Space Systems



NASA/C.J.Hamilton
Atmospheric Features: Streaky clouds





Valles Marineris on Mars: largest canon in the solar system (3000 km long, 8 km deep)



Rim of Valles Marineris Sedimentary rocks, steep slope caused by faulting, possible location of fault two-thirds down the slope.

NASA MGS, 6 September 2003, image: 3 km wide



Rim of Valles Marineris Similar structure as previous picture, but in West Candor

NASA MGS, 24 May 2003



Tharsis vulcanos: Large shield vulcanos, now extinct \implies no plate tectonics \implies Mars interior is colder than Earth.


Olympus Mons: highest volcano in solar system (25 km above surrounding plain; but slope only 2° to 5°).



ESA/Mars Express, HRSC, 11.02.2004



ESA/Mars Express, HRSC, 11.02.2004



ESA/Mars Express, HRSC, 11.02.2004



Mars: Surface panorama, Exploration Rover "Opportunity" looks back to lander (2004 Feb 09)



NASA/JPL/Cornell



NASA/JPL/Cornell

Mars: Crater Endurance



NASA/JPL/Cornell

Mars: Crater Endurance



Mars: "Spirit" rolls towards Columbia Hills (2004 June)



Mars: "Spirit" looks from Columbia Hills towards Gusev crater (2004 Aug)



Don Davis/NASA

Viking lander 2: frost (water ice) in early morning (*very* thin layer [< 0.1 mm])



Mars' Polar Caps: Mainly CO_2 ice ("dry ice"), grows and shrinks with seasons.



The Outer Planets (SSE, NASA)



NASA/ESA, Cassini-Huyghens

Jupiter:

- Largest planet in solar system
- rapid rotation =>> severely flattened, banded atmosphere (Coriolis force), Great Red Spot
- strong magnetic field (strong radio emission)
- atmosphere: 75% H, 24% He (by mass), very close to solar
- differential rotation (rotation period 9h50m on equator, 9h55m on poles).
- strong magnetic field
- four major "Galilean" moons plus 59 small ones (as of Jan. 2005; all are captured asteroids)

Early Exploration 1970s through Pioneer 11 and 12, and then through the Voyager probes. Extensively studied by NASA's Galileo project (ended 2003 Sep 14).





Jupiter: true color image; colors likely from trace content of organic compounds in atmosphere false color image, red: waterclouds, dark spots: deep hot spots

Overall atmospheric structure: three layers: Ammonia – ammonia hydrosulfide (NH₄HS) – water ice/water (deepest)



Great Red Spot NASA Galileo, 1996 June 26

Great Red Spot: Storm System, $\sim 2 \times$ Earth diameter, exists since more than 300 years, 8 km above and 10° cooler than surrounding region (rising high pressure region), rotates counterclockwise (Coriolis force on Southern hemisphere).

Jupiter, IV

Structure of atmosphere defined through hydrostatic equilibrium:



Force on area A by slab of gas of area A and density ρ :

$$F = mg = \rho Vg = Ah\rho g$$

Such that pressure becomes

$$P = \frac{F}{A} = \rho hg$$

where g gravitational acceleration.

For thin atmosphere (g constant): Decrease of P when going upwards by Δh :

$$\Delta P = -\rho g \Delta h$$
 and for $\lim_{\Delta h \to \mathbf{0}} : \frac{\mathrm{d}P}{\mathrm{d}h} = -\rho g$

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Jupiter, V

For (ideal) gas: relationship between density and pressure ("equation of state") given by

$P = (\rho/m)kT$

where *T*: Temperature (K!), *m*: average mass per gas particle, *k*: Boltzmann's constant $k = 1.38 \times 10^{-23} \,\text{J}\,\text{K}^{-1}$. Therefore:

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\left(\frac{mg}{kT}\right)P$$

For isothermal atmosphere separation of variables (see handout) gives

$$P(h) = P_{0} \exp\left(-\frac{mg}{kT} \cdot h\right) = P_{0} \exp\left(-\frac{h}{H}\right)$$

The pressure in the atmosphere thus decreases exponentially, the characteristic height scale of the decrease is given by the scale height H. On Earth, $H \sim 9$ km.

The Outer Planets

Separation of variables is a standard technique for solving boundary conditions such as the equation of hydrostatic equilibrium,

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\left(\frac{mg}{kT}\right)P$$

In order to obtain P as a function of height, h, we need to solve this differential equation with the boundary condition that for h = 0, $P = P_0$. First, divide by P and integrate both sides of the equation with respect to height:

$$\int_{0}^{h} \frac{1}{P} \frac{\mathrm{d}P}{\mathrm{d}h} \,\mathrm{d}h = -\int_{0}^{h} \left(\frac{mg}{kT}\right) \mathrm{d}h$$

we can now substitute ${\cal P}(h)$ for h on the left hand side. Using the chain rule gives

$$\int_{P_0}^{P(h)} \frac{\mathrm{d}P'}{P'} = -\int_0^h \left(\frac{mg}{kT}\right) \mathrm{d}h$$

such that

$$\ln\left(\frac{P(\mathbf{0})}{P(h)}\right) = -\left(\frac{mg}{kT}\right)h$$

 $P(h) = P_0 \mathbf{e}^{-(mg/kT)h}$

and exponentiating then gives

This method is called "separation of variables" since people often jump from the first (linear) equation to the third one in one step, by "separating the dependent from the independent variable":

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\left(\frac{mg}{kT}\right)P \quad \Longrightarrow \quad \frac{\mathrm{d}P}{P} = -\left(\frac{mg}{kT}\right) \quad \Longrightarrow \quad \int_{P_0}^{P(h)} \frac{\mathrm{d}P'}{P'} = -\int_0^h \left(\frac{mg}{kT}\right) \mathrm{d}h$$

Jupiter, VI

In general, gas giants have very different properties from terrestrial planets:

- average density low, e.g.,
 - Jupiter: $\langle \rho
 angle \sim$ 1.3 g cm $^{-3}$
 - Saturn: $\langle
 ho
 angle \sim 0.7\,{
 m g\,cm^{-3}}$

(compare to terrestrial planets: $\langle \rho \rangle \sim 5.5\,{
m g\,cm^{-3}}$; water has $ho=1\,{
m g\,cm^{-3}}$).

- elemental composition similar to stars (by mass):
 - 75% H
 - 24% He
 - 1% rest ("metals")
- \implies expect fundamentally different internal structure!

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Jupiter, VII

Structure of a gas giant from equation of hydrostatic equilibrium:



To solve, need to know $\rho(r)$, $M(r)\Longrightarrow$ complicated, but doable if properties of material are known.

To guesstimate the central pressure, one can show for a planet of radius R:

$$P_{\text{central}} = \frac{2\pi}{3} G \langle \rho \rangle^2 R^2$$

(see handout for derivation).

Plug in numbers for Jupiter: R = 70000 km, $\langle \rho \rangle = 1.3$ g cm⁻³, get $P_{\text{central}} = 1.2 \times 10^{12}$ Pa (10× Earth).

At this pressure: existence of metallic hydrogen (i.e., electrons can move freely around).

more detailed computations: metallic hydrogen from 14000-45000 km away from center

The Outer Planets



Note: relative sizes of planets not to scale! Also rotational flattening not taken into account.

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The Outer Planets

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The following is for your education only and its knowledge will not be assessed in any way!

To obtain information on the pressure structure of any gravitationally supported static body we can again use the *concept of hydrostatic equilibrium* already used for estimating the structure of atmospheres,

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)g(r)$$

here, r is now the radial distance from the planetary centre. In contrast to atmospheres, the acceleration g depends on the position, g = g(r). It is easy to show that

$$g(r) = \frac{GM(r)}{r^2}$$

where M(r) is the mass of the planet contained within the radius r:

$$M(r) = \int_0^r \frac{4\pi}{3} \rho(r) r^2 \mathrm{d}r$$

(interpretation: integrate over onion shells of thickness dr and density $\rho(r)$; the mass in each of these shells is $(4\pi/3)\rho(r)dr$, summing over all onion shells gives the above answer).

To solve the equation of the hydrostatic equilibrium one needs to know the equation of state. Unfortunately, this equation of state is generally much more complicated than for gases and often only roughly known. One can estimate, however, the order of magnitude for the pressure within a planet. In order to do so, we assume that the density is the same throughout the planet, and that it equals the planet's average density $\rho(r) = \langle \rho \rangle = \text{const.}$. This is o.k. to an order of magnitude. Under this assumption,

$$M(r) = (4/3)\pi r^3 \langle \rho \rangle$$

such that the equation of hydrostatic equilibrium reads

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \langle \rho \rangle^2 G(4/3)\pi r$$

Differential equations looking like this are called separable. They can be solved "separation of variables", as we already did when computing the structure of an isothermal atmosphere.

First integrate both sides of the equation from r = 0 to the surface of the planet at r = R:

$$\int_0^R \frac{\mathrm{d}P}{\mathrm{d}r} \,\mathrm{d}r = \int_0^R \langle \rho \rangle^2 G(4/3) \pi r \,\mathrm{d}r$$

To integrate the left hand side of the equation, substitute $r \longrightarrow P(r)$ where P(r) is an unknown function (the pressure as a function of radius r). Luckily enough, we only need to know its values at r = 0 and r = R (the "boundary conditions"). By definition of the surface of the planet, the pressure at r = R will be P(R) = 0 to very good

accuracy, while the pressure at r = 0 is the (unknown) central pressure, $P(0) = P_c$. Therefore

$$\int_0^R \frac{\mathrm{d}P}{\mathrm{d}r} \,\mathrm{d}r = \int_{P_{\rm c}}^0 \mathrm{d}P = -P_{\rm c}$$

The right hand side of the equation is easily found as well:

$$\int_0^R \langle \rho \rangle^2 G(4/3) \pi r \, \mathrm{d}r = -\langle \rho \rangle^2 (4\pi/3) G \int_0^R r \, \mathrm{d}r = -\langle \rho \rangle^2 (4\pi/3) G R^2/2 = -\frac{2\pi}{3} \langle \rho \rangle^2 R^2$$

Putting everything together gives

$$P_{\rm c} = \frac{2\pi}{3} \langle \rho \rangle^2 R^2$$

As a rule of thumb, this formula gives central pressures that are correct to better than a factor of 10 compared to the detailed theory.





Montage of Jupiter and Galilean Moons: top to bottom: Io, Europa, Ganymede and Callisto.

(N.B.: All Galilean moons tidally locked to Jupiter – always same side is facing Jupiter)



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Jupiter's moon Io - the vulcano moon (Diam. 1821 km [Earth moon: 1738 km])



Active vulcanoes on lo (interior heated by tidal forces from Jupiter), color due to large contents of sulphur and sulphur oxides in lava. Height of vulcanoes: 6 km or higher



Io — Tvashtar Catena

125 (26 Nov 1999) + C21 low-resolution color

127 (22 Feb 2000)

visible wavelength data + IR data of active lava flow



curtains of lava fountains [white: overexposed] NASA Galileo, 1999 Nov 26

High temperature volcanism (2000 K; hotter than on Earth [1700 K]!)



NASA Galileo / DLR, inset: 120×110 km Ganymede – icy surface, ice hills and valleys, craters Radius: 2634 km (~ Mercury!)



NASA Galileo / DLR, 1996 September 7



Callisto: "pock faced", mainly impact craters. white: ice dark: ice-poor material

Radius: 2406 km (similar to Mercury!)



Structure of Jupiter's Galilean Moons similar to terrestrial planets (but some also have very thick ice layer on top)





Saturn:

- similar to Jupiter, slightly smaller
- rapid rotation =>> flattened, banded atmosphere
- atmosphere: 75% H, 24% He (by mass), molecules etc. similar to Jupiter
- Rings!
- six major moons plus 27 small ones (as of Jan. 2005; mainly captured asteroids)

NASA Cassini, 2003 Dec.

Early Exploration 1970s through Pioneer 11 and 12, and then through the Voyager probes. Studied since 2004 July 1 by NASA/ESA Cassini-Huygens project (duration: four years)



NASA/ESA Cassini, 2005 July (polarised IR light) Saturn: Similar atmospheric structure as Jupiter


NASA HST, 1999 October

Rings: equatorial plane, thin (few km high, 71000–140000 km from centre); gaps due to gravitational effects of outer moons (widest gap: Cassini's division); speed of rings agrees with 3rd Kepler (⇒⇒ individual particles!)



NASA Voyager 2, 1981 August 22

Rings: equatorial plane, thin (few km high, 71000–140000 km from centre); gaps due to gravitational effects of outer moons (widest gap: Cassini's division); speed of rings agrees with 3rd Kepler (⇒⇒ individual particles!)





Dione, Rings (edge on), and ring shadows on Saturn



NASA Voyager (montage)

Six major moons, typically $\langle \rho \rangle \sim 2 \, \mathrm{g \, cm^{-3}}$ \implies mainly ice (60–70%), with smallish rocky cores As with Jupiter, small moons are captured asteroids.





Dione (surface detail, 23 km wide, 2005 Oct.) – note craters and *fractures*) Saturn's moons are icy moons, similar to Jupiter's Galilean moons



Tethys (2005 September; surface detail, \sim 500 km wide) Density of Tethys \sim Water ice, so (if existent) its rocky core is small



NASA Voyager

27.10.2004, false colour IR/UV; NASA/ESA

Titan: dense atmosphere, 99% nitrogen, 1% methane, some hydrocarbons, thought to be similar to primordial atmosphere of Earth. Radius: 2575 km (~ Mercury!) ESA probe *Huygens* landed on Titan on 2005 January 14



Surface of Titan (2005 January 14): methane ice rocks strewn over icy surface.



Uranus:

- atmosphere cold (59 K = -214° C) \implies ammonia has frozen out
- methane, hydrogen, and helium detected so far (less He than expected from Jupiter and Saturn!)
 ⇒ bluish color
- inclination of rotation axis: 98° ("rolling on ecliptic plane").
- small ring system
- five major moons in equatorial plane plus 22 small ones (as of Jan. 2004; captured asteroids)

NASA Voyager 2, 1986 Jan 10

Flyby of Voyager 2 in 1986 January, since then only remote sensing via Hubble Space Telescope (HST) and ground based instruments.



HST Image (image enhanced) of Uranus ring system, plus evidence for banded atmosphere and clouds



major satellites have $\langle \rho \rangle \sim$ 1.3–2.7 g cm $^{-3}$ \Longrightarrow rocks and ice



NASA Voyager 2

Neptune:

- atmosphere similar to Uranus, but more active; bright methane clouds above general cloud layer
- ring system (5 individual rings)
- Two major moons (Triton, 2720 km diameter(!) and Nereid 355 km),
 - 11 captured asteroids

Flyby in 1989 August by Voyager 2, only HST since then (showed in 1995 that dark spot has vanished, detected new storm system)



Triton: ice cap of frozen methane (freezing point 90 K) and frozen nitrogen (freezing point 60 K). Few impact craters \Longrightarrow young surface \Longrightarrow volcanism (dark spots in picture; nitrogen geysers with $T \sim 70$ K)

Only three volcanically active bodies in solar system: Earth, Io, and Triton.

NASA/Voyager 2





NASA/ESA HST

Pluto/Charon:

- discovered 1930
- double planet system (Pluto: D = 2320 km, Charon: D = 1270 km), 2 smaller moons
- planet nature debated

Kuiper belt: similar to asteroid belt, >70000 objects outside Neptune in 30–50 AU region; largest further members currently known: Quaoar ($D = 1200 \pm 200$ km), Ixion ($D = 1060 \pm 165$ km), Varuna ($D = 900 \pm 140$ km), 2002 AW197 ($D = 890 \pm 120$ km), see http://www.ifa.hawaii.edu/faculty/jewitt/kb.html



M. Brown et al. (Caltech)

$\begin{array}{l} \mbox{2003 UB}_{313} \mbox{, discovered 2005: distance} \sim 100 \mbox{ AU,} \\ \mbox{ brightness similar to Pluto} \\ \implies \mbox{ has to be larger than Pluto, unless it is 100\% reflective (unlikely)!} \end{array}$



2003 UB₃₁₃: View of the Solar System (drawing courtesy M. Brown et al./NSF/NASA)