Resit Exam Questions for Module PX 144: "Introduction to Astronomy"

Useful physical constants:

Solar mass:	$M_\odot = 2 \times 10^{30}\,\mathrm{kg}$
Solar absolute magnitude:	$M_{abs,\odot} = 4.77\,\mathrm{mag}$
speed of light:	$c = 300000\,\mathrm{km\,s^{-1}}$
Gravitational constant:	$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Hubble parameter:	$H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
Parsec:	$1\,\mathrm{pc} = 206265\,\mathrm{AU}$

Question 1 (Solar System):

a) (6 marks): With 3.3 years, Encke's comet has the shortest of all cometary periods.

i) What is the semi-major axis of the comet's orbit in units of AU {3}?

Solution: The semi-major axis is given from Kepler's 3rd law,

$$P^2 = a^3$$

(in solar units), such that $a = P^{2/3} = 2.2\,\mathrm{AU}$.

ii) The eccentricity of its orbit is $e = 0.847$. Compute its perihelion and aphelion distance {3}.

Solution: The distances are given by

$$d_{\mathrm{perihelion}} = (1-e)a_{\mathrm{comet}} = 0.34\,\mathrm{AU} \quad (1)$$

$$d_{\mathrm{aphelion}} = (1+e)a_{\mathrm{comet}} = 4.06\,\mathrm{AU} \quad (2)$$

b) (6 marks): A planet is observed to move on a circular orbit around a star. The period of the orbit is 120 d and the radius of the orbit is measured to be 200 Million km. Estimate the mass of the star (give the mass in both, kg and solar units).

Solution: Kepler's 3rd law is

$$P^2 = \frac{4\pi^2 a^3}{G(M_\star + m_{\mathrm{Planet}})} \quad (3)$$

Since $M_\star \gg m_{\mathrm{Planet}}$, the planet's mass can be ignored, such that

$$M_\star = \frac{4\pi^2 a^3}{GP^2} \quad (4)$$

with $a = 2 \times 10^{11}\,\mathrm{m}$ and $P = 10^7\,\mathrm{s}$, one finds $M_\star = 4.7 \times 10^{31}\,\mathrm{kg} \sim 24\,M_\odot$.

c) (13 marks): i) Explain why Mars is generally considered as the planet that is most similar to our Earth {7}.

Solution: Important points to consider the two planets as similar are:

- both have geological history, including volcanism, possible rift valleys, etc.
- polar caps
- Mars has seasons
- atmosphere shows climatic variations
- water has influence on geology, i.e., moderate wind and water erosion has been observed

but note that there are also clear differences:

- atmospheric composition
- surface gravity
- presence of oceans on Earth
- life has dramatically changed Earth

ii) Extrasolar planets are normally found using spectroscopic methods and not by direct imaging. Describe the basis of these spectroscopic methods and list the major reasons why direct imaging is not as sensitive as spectroscopy {6}.

Solution: Spectroscopic methods use the Doppler effect to get velocity variations caused by motions of star and planet around their common centre of mass. This requires many measurements of the stellar radial velocity over the course of more than one revolution of the planet around the star. Spectroscopic measurements are difficult since the velocity amplitude is very small (metres/sec), so they require high precision velocity standards and wavelength determination. Direct imaging has great problems because the brightness contrast between the star and the planet is very large (10^9 and more), so they have not yet been successful (but with coronographs they might work in the future).

Question 2 (Stars):

a) (17 marks): Two stars are observed with their fluxes differing by a factor of 100. Both stars are known to be main sequence stars and have the same temperature. The brighter of these stars is known to have an apparent magnitude of $m = 5\,\mathrm{mag}$.

i) What is the magnitude of the fainter of these two stars {3}?

Solution: The relationship between the fluxes and the magnitudes of two stars is given by

$$m_2 - m_1 = -2.5 \log_{10}\left(\frac{f_2}{f_1}\right)$$

The fainter star thus has a magnitude of $m = 10\,\mathrm{mag}$.

ii) The fainter star has a distance of 100 pc, what is the distance of the brighter star{3}?

Solution: Since the stars have the same temperature and are both on the main sequence, they will have the same luminosity. Since

$$f = \frac{L}{4\pi r^2}$$

the ratio of the fluxes of the stars is given by

$$\frac{f_1}{f_2} = \left(\frac{r_2}{r_1}\right)^2$$

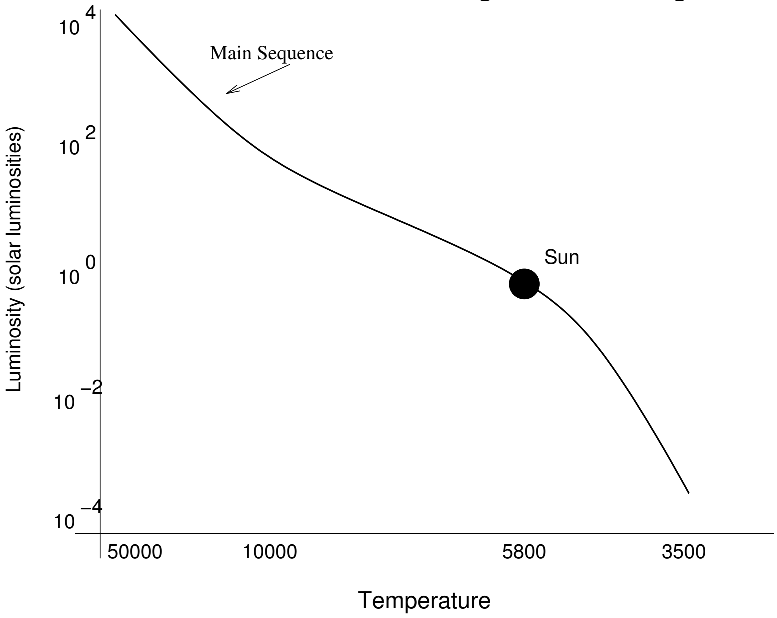
such that the distance of the brighter star is $r = 100\,\mathrm{pc}/\sqrt{100} = 10\,\mathrm{pc}$.

iii) Compute the stars absolute magnitude {3}

Solution: This is a giveaway. The absolute magnitude is the magnitude as seen from a distance of 10 pc, i.e., it is $M = 5\,\mathrm{mag}$.

iv) Draw a clearly labeled Hertzsprung-Russell Diagram, identifying the location of the Sun and of the Main Sequence {6}

Solution: A sketch of the HRD is, e.g., the following



v) Given the absolute magnitude of the stars, what do you think is their spectral type{2}?

Solution: Since the absolute magnitude of the sun is 4.77 mag, the star is very similar to the Sun, although somewhat fainter (5 mag), so its spectral type is late G or early K.

b) (8 marks): The mass-luminosity relationship is $L \propto M^4$, where the luminosity L and the mass M are both measured in solar units.

i) The total time the Sun spends on the Main Sequence is 10^{10} years. Estimate the time a star with a mass of 30 $M_\odot$ lives on the main sequence {3}!

Solution: The lifetime of stars depends on the rate with which they consume their fuel, i.e., the luminosity, and it depends on the amount of fuel available. According to the mass luminosity relationship, the luminosity of the star is $8 \times 10^5 L_\odot$, but the star has only 30× more fuel available. Consequently its predicted lifetime is only 370000 years.

ii) Stars with 30 $M_\odot$ end their life in a supernova, leaving a black hole. Describe the main properties of black holes, including one way on how black holes can be detected {5}.

Solution: The only two properties astrophysical black holes have are their mass and their angular momentum. Their size is given by the Schwarzschild radius, which is 3 km for a solar mass black hole. They can be detected through the X-rays emitted when material is accreted onto the black hole, by eliminating the alternative compact objects (white dwarf or neutron star) through mass measurement.

Question 3 (Galaxies):

a) (10 marks) Supernovae of type Ia have absolute magnitudes of $M = -18.33\,\text{mag}$ at the maximum of their outburst. A supernova is observed in a spiral galaxy to have the magnitude $m = 15\,\text{mag}$ at the peak of its outburst.

i) Compute the distance module and the distance to the galaxy {3}.

Solution: The distance module is

$$\text{DM} = m - M = 33.33\,\text{mag}$$

and the distance is given from

$$\text{DM} = m - M = 5\log_{10}\left(\frac{d}{10\,\text{pc}}\right)$$

such that

$$d = 10\,\text{pc} \cdot 10^{\text{DM}/5}$$

Inserting numbers gives $d = 4.6 \times 10^{7}\,\text{pc} = 46\,\text{Mpc}$

ii) Using Hubble's law, compute the recession velocity of the galaxy {2}.

Solution: Hubble's law is $v_{\text{rec}} = H_0 d$, such that $v_{\text{rec}} \sim 3200\,\text{km}\,\text{s}^{-1}$.

iii) Spiral galaxies have radii of 10 kpc. Compute the angular diameter of the galaxy. Assuming Earth bound telescopes are limited to an angular resolution of $0.1''$, would you be able to determine its angular diameter? {5}

Solution: Using the small angle approximation, the angular *diameter* is

$$\theta = 2\frac{r}{d}$$

where r is the radius and d the distance. Inserting numbers gives

$$\theta = 2\frac{10\,\text{kpc}}{46000\,\text{kpc}} = 4.4 \times 10^{-4}\,\text{rad} = 90''$$

and thus the diameter can be easily measured.

b) (5 marks) Galaxies come in different types, first classified by Edwin Hubble. Draw the "Hubble tuning fork diagram" and briefly describe the properties of elliptical galaxies.

Solution: For a good version of the tuning fork diagram, see the online notes of the PX144 module.

Elliptical galaxies are rather dust free objects. Since the star formation rate in these systems is small, they consist mainly of old and evolved stars, giving them a reddish colour. They generally have low mass (only a few million $M_\odot$), although there are some very massive elliptical galaxies in the centres of galaxy clusters.

c) (10 marks) One of the fundamental claims of modern cosmology is that the universe is homogeneous and isotropic, that is, the properties of the universe today do not depend on where they are measured and the properties are independent on the direction of the observation. Describe how you would observationally test this assumption.

Solution: Ways to determine the isotropy and homogeneity of the universe are to perform galaxy surveys, i.e., to go out and determine the distribution of galaxies as a function of the distance from our Milky Way. In order to do so one has to measure the redshifts from a large sample of galaxies and then plot their three-dimensional distribution. If this distribution looks the same along the line of sight, one can convince oneself that the universe is homogeneous. In order to determine the isotropy one looks at the distribution of galaxies on the sky.