Department of Physics
1st Year Examples 2005/2006
Term 2, Week 20–24

THE UNIVERSITY OF WARWICK

PX144: Introduction to Astronomy

Academic Week 20/Term Week 16: Solar System

Question 1: *Keplerian Motion and Satellites*
In this question we treat some of the practical problems faced by space industry in getting satellites into orbit.

a) Using energy conservation arguments or otherwise, derive a *general* equation for the energy needed to put a satellite of mass m into a circular orbit of radius r around Earth from a launch site such as the European Space Agency's Kourou launch site, which is situated at the equator. Take into account that the Earth rotates, i.e., the satellite has already some kinetic energy before launch when looked at in a nonrotating frame of reference. You may assume that the satellite is launched due east, i.e., in the rotational direction of the Earth.

b) Using the above general equation, compute the energy needed to launch one kg of material into a low Earth orbit which has a height of 300 km above ground. The Earth has a mass $M_\oplus = 6 \times 10^{24}$ kg, a radius of $r_\oplus = 6378$ km, and rotates around its axis in 23 hours 56 minutes 4.1 seconds ($t_\oplus = 86164$ sec). The gravitational constant is $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

Question 2: *Orbits, Properties of Ellipses*

a) The semi-major axis of Pluto's orbit is $a_♇ = 39.44$ AU, and its eccentricity is $e_♇ = 0.249$, whilst the orbital parameters of Neptune are $a_♆ = 30.06$ and $e_♆ = 0.009$. In 1989, Pluto was at perihelion. Compute the perihelion distance of Pluto. Was Pluto within or outside Neptune's orbit in 1989?

b) Sketch the orbits of Pluto and Neptune. To get the shape of Pluto's orbit right, apart from the information already computed in the previous question, you need to convince yourself that the semi-minor axis b of an ellipse is given by

$$b = a\sqrt{1-e^2}$$

(reading chapter 1-2 of Zeilik & Gregory will greatly help you in answering this question).

Question 3: *Keplerian Motion, Law of Gravitation – This question will be marked for credit*
In class we showed that Kepler's 2nd law is a consequence of the conservation of angular momentum. Specifically, it was shown that

$$\frac{dA}{dt} = \frac{L}{2m} = \text{const.} \tag{3.1}$$

where dA/dt was the sector velocity, i.e. where dA is the area covered by the radius vector of a planet during the time interval dt.

a) By integrating dA/dt over one revolution of duration P of a planet of mass m around the Sun, show that the angular momentum per unit mass is given by

$$\frac{L}{m} = \frac{2\pi ab}{P} \tag{3.2}$$

You might need to know that the area of an ellipse is $A = \pi ab$ where a and b are its semi-major and semi-minor axes.

b) Use Eq. (3.2) to show that the perihelion and aphelion speeds of a planet are given by

$$v_{\text{perihelion}} = \frac{2\pi a}{P}\sqrt{\frac{1+e}{1-e}} \quad \text{and} \quad v_{\text{aphelion}} = \frac{2\pi a}{P}\sqrt{\frac{1-e}{1+e}} \tag{3.1}$$

c) Compare the perihelion and aphelion speeds of the Earth and Mercury. Mercury's orbit has a semi-major axis $a_{☿} = 0.387\,\mathrm{AU}$, an eccentricity $e_{☿} = 0.206$, and an orbital period of $P_{☿} = 87.98\,\mathrm{days}$, the data for the Earth are $a_{\oplus} = 1\,\mathrm{AU}$, $e_{\oplus} = 0.017$, and $P_{\oplus} = 365.26\,\mathrm{days}$. $1\,\mathrm{AU} = 1.496 \times 10^{11}\,\mathrm{m}$.

Question 4: *Comments on this week's lectures*

In order to improve the teaching and to enable myself to react to problems you might have with the module, I would like to hear your opinion on my teaching as early as possible. I would appreciate it if you would voice any problems and criticisms as soon as possible, e.g., on the speed with which I talk about the subjects of the lectures, the overall difficulty level of the class and the homework, the quality and contents of the handouts, and so on.

Please write these comments on a separate sheet of paper and give them to me: Either put the paper on the lectern before class or put it in my "pigeon hole" in the mailboxes on the 5th floor of the physics building, close to the physics undergraduate office. Feel free to remain anonymous, if you deem this necessary. You can also ask questions or post comments by using the discussion board for this module at `http://forums.warwick.ac.uk/wf/browse/forum.jsp?fid=912` or by sending email to `j.wilms@warwick.ac.uk` (I will post answers to emailed questions on the discussion board, if they are of sufficient interest for others).