Department of Physics
1st Year Examples 2005/2006
Term 2, Week 20–24

THE UNIVERSITY OF WARWICK

PX144: Introduction to Astronomy

Academic Week 21/Term Week 17: Solar System

Question 1: *Kepler's Laws*

a) Given that 1 year on Earth is 365.26 days, compute the duration of one Mars year in days using Kepler's 3rd law. The semi-major axis of Mars is $a_{♂} = 1.524$ AU.

b) For the planets, the derivation of Kepler's 3rd law in the lecture was somewhat overkill. Assume that the mass of one of the bodies is much larger than that of the other, $m_1 \gg m_2$. In this case, the centre of mass is to very good approximation the centre of the body with the larger mass.

- Verify this statement by computing the radius of the orbit of the Sun due to the pull of Jupiter. Express the radius in both, km and in units of the solar radius, $R_\odot = 700000$ km. Assume that Jupiter's orbit is circular with a radius of 5.2 AU, it has a mass of 318 Earth masses, where the mass of the Earth is $m_\oplus = 5.97 \times 10^{24}$ kg. The mass of the Sun is $M_\odot = 1.99 \times 10^{30}$ kg and 1 AU = 149.6×10^6 km.
- Assuming that the massive body is stationary, convince yourself that in this case the force is

$$F_{\text{grav}} = G\frac{m_1 m_2}{r^2} = \frac{m_2 v^2}{r} = F_{\text{cent}}$$

where r is the distance between the two bodies. Show that you can deduce a simplified version of Kepler's 3rd law from this equation.

Question 2: *The Warwick Model of the Solar System*
The aim of this question is to give you a feeling for the emptiness of the Universe. In order to do so, imagine that you shrink the Sun (radius 700000 km) down to the size of an orange with a diameter of 9 cm and that you put this orange in the amphitheatre between the Rootes building and the Student's Union. (*Note:* Given that there are no maps of the campus available that have any information on the map scale, just guess the distances, we don't want to go overboard with precision in this exercise...).

a) Going west, where on campus would you place the Earth and what size would it have? The Earth's semi-major axis is 1 AU = 150×10^6 km, its radius is 6400 km.

b) Where would you place Jupiter ($a_{♃} = 5.2$ AU) and Pluto ($a_{♇} = 39.2$ AU)?

c) Finally, the closest star to our Sun is α Centauri at a distance of 1.31 parsec, where 1 parsec corresponds to 206255 AU. Going east from Warwick, where would you find the orange corresponding to α Cen in this scale model of the Solar System?

Question 3: *Planetary Atmospheres – This question will be marked for credit*
As shown in the lectures, the pressure distribution as a function of height is given by

$$P(h) = P_0 \exp(-h/H) \quad \text{with the scale height} \quad H = \frac{kT}{mg}$$

where T is the temperature, g the surface acceleration, and m the average mass of the molecules making up the atmosphere.
Mars' atmosphere consists mainly of CO_2 with a mean molecular mass of $m = 40 m_\text{p}$ where the proton mass is $m_\text{p} = 1.67 \times 10^{-27}$ kg. Determine the ratio of the atmospheric pressures at the basis and at the top of its highest mountain, the extinct volcano Olympus Mons, ($h = 25$ km). Compare this ratio with the ratio of atmospheric pressures on top of Mt. Everest ($h = 8.8$ km) and at mean sea level (assume $H = 8.7$ km).

N.B. You will have to calculate Mars surface acceleration first. In order to do so note that the gravitational force outside of a spherical mass distribution with total mass M equals that of a point masss M at the centre of the spherical mass. Furthermore, ignore the variation of g with height, the rotation of the planet, and assume the atmosphere is isothermal (i.e., the temperature does not change with height). Appropriate data for Mars are a surface temperature $T = -73°\mathrm{C}$, a mass $M_{♂} = 0.1M_{\oplus}$ and a planetary radius of $r_{♂} = 0.533r_{\oplus}$, for Earth the appropriate values are $M_{\oplus} = 6 \times 10^{24}\,\mathrm{kg}$ and $r_{\oplus} = 6378\,\mathrm{km}$. The gravitational constant is $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$, Boltzmann's constant is $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$.

Question 4: *Comments on this week's lectures*
In order to improve the teaching and to enable myself to react to problems you might have with the module, I would like to hear your opinion on my teaching as early as possible. I would appreciate it if you would voice any problems and criticisms as soon as possible, e.g., on the speed with which I talk about the subjects of the lectures, the overall difficulty level of the class and the homework, the quality and contents of the handouts, and so on.

Please write these comments on a separate sheet of paper and give them to me: Either put the paper on the lectern before class or put it in my "pigeon hole" in the mailboxes on the 5th floor of the physics building, close to the physics undergraduate office. Feel free to remain anonymous, if you deem this necessary. You can also ask questions or post comments by using the discussion board for this module at `http://forums.warwick.ac.uk/wf/browse/forum.jsp?fid=912` or by sending email to `j.wilms@warwick.ac.uk` (I will post answers to emailed questions on the discussion board, if they are of sufficient interest for others).