Department of Physics
1st Year Examples 2005/2006
Term 2, Week 20–24

THE UNIVERSITY OF WARWICK

PX144: Introduction to Astronomy

Academic Week 22/Term Week 18: Stars

Question 1: *The Habitable Zone – This question will be marked for credit*
Current ideas about the possibility of life in extrasolar planetary systems assume that one of the major conditions to be fulfilled is the existence of liquid water in these planetary systems. This idea has led to the concept of "habitable zones" in solar systems. In this question we will use this concept to investigate where life is possible in our own solar system. As a caveat, please note that the concept of habitable zones is heavily debated within the "astrobiology community". For a dissenting opinion, see, e.g., " What Does a Martian Look Like?: The Science of Extraterrestrial Life" by Jack Cohen and Ian Steward. Note also that we will only be looking at producing liquid water from keeping a planet hot enough with irradiated solar energy and that we ignore the possibility of other sources of energy such as the tidal heating of Europa discussed in the lectures.
Note: In questions asking you for quantitative answers, in order to get full marks it is imperative that you not only show how you obtain the value of your answer, but also its units. Just guessing the units will not be sufficient, you will have to prove that your result has indeed the correct units.

a) Assume a spherical planet with planetary radius r situated at a distance d from a star with luminosity L. The total power available for heating the planetary surface is given by the toal power received by the sun-facing side of the planet, P_{tot}, minus that immediately reflected away by clouds in the atmosphere. The planetary reflectivity is usually characterised by the "albedo", a, defined as the fraction of power received that is reflected away. Therefore, the power received on the surface is $P_{\text{abs}} = (1-a)P_{\text{tot}}$. Assume that the planetary atmosphere isolates the planet well enough that a temperature equilibrium sets in over the whole planetary surface. This is reached once the total power received is equal to the power emitted by the planet (mainly as infra-red radiation).

   Derive a formula for the average planetary temperature by setting the power received by the irradiated surface equal to that radiated away over the whole planetary surface. The power emitted per square metre by a planetary or stellar surface of temperature T is given by Stefan-Boltzmann's law for a blackbody, modified by the *emissivity* ϵ, a measure for the efficiency of a radiating body:

$$P_{\text{em}} = \epsilon\sigma_{\text{SB}}T^4$$

   where the Stefan-Boltzmann constant is given by $\sigma_{\text{SB}} = 5.7 \times 10^{-8}\,\text{W m}^{-2}\,\text{K}^{-4}$.
   (*Answer:* $T = \left\{((1-a)L)/(16\pi d^2 \epsilon\sigma_{\text{SB}})\right\}^{1/4}$).

b) Use the above equation to estimate the average temperature on the Earth ($d = 1\,\text{AU} = 150 \times 10^6\,\text{km}$), assuming a surface averaged albedo of $a_\oplus = 0.3$. Assume that the Earth radiates like a black body, i.e., $\epsilon = 1$. Give the temperature in both, Kelvins and Centigrade. The solar luminosity is $L = 4 \times 10^{26}\,\text{W}$.

c) The temperature you found in the previous question is too low – the average temperature on Earth's surface is about +17°C. The reason is the Greenhouse effect: a large fraction of the infra red radiation emitted by the Earth's surface is absorbed in the atmosphere. This absorbed radiation heats up the atmosphere. For symmetry reasons, however, only half of the thermal radiation emitted by the atmosphere is radiated into space, the rest of the energy remains trapped and the overall temperature of the Earth increases. For simplicity we can treat the greenhouse effect as if the Earth's emissivity is $\epsilon = 0.6$, resulting in a predicted surface temperature of approximately 19°C.

   On Earth, life is observed in regions with annual average temperatures between −10°C and +30°C. We can use this temperature range to define the "habitable zone" of our solar system. To make our estimate more reliable, note that it took about 4.6 billion years for intelligent life to evolve on Earth. During this time, the solar luminosity increased by approximately 30%. Using this information, compute the inner and outer radii of the habitable zone around the Sun. What is the maximum eccentricity a planet can have to always remain within the habitable zone throughout the evolution of life on Earth? Assume that the greenhouse effect was similar throughout this period (note that this is *not* correct for the real solar system as the Earth's atmospheric composition is the result of the existence of life on Earth).

Question 2: *The Doppler Effect*
This question is based on problem 8–3 from Zeilik & Gregory, reading through section 8-1 of the book will greatly help you with answering this question.
At what wavelength will the following spectral lines be observed?

a) A line emitted at 500 nm by a star moving towards us at $100\,\mathrm{km\,s^{-1}}$.

b) A line emitted by Calcium at $\lambda = 397\,\mathrm{nm}$ by a galaxy receding at $60\,000\,\mathrm{km\,s^{-1}}$.

c) A cloud of neutral hydrogen emitting a radio line with a frequency of $\nu = 1420.4\,\mathrm{MHz}$ while moving away at $200\,\mathrm{km\,s^{-1}}$ (because $\lambda \sim 21\,\mathrm{cm}$ this line is often called the "21 cm line" of hydrogen and is of great importance of observational radio astronomy) . What is the *frequency* at which the line is observed?

d) By how much can the Hydrogen Hα line ($\lambda = 656.3\,\mathrm{nm}$) of an astronomical object maximally vary due to the motion of the Earth around the Sun?

Question 3: *Properties of Stars*
A good rule of thumb is that a star remains on the main sequence until ~15% of the available hydrogen has been converted to helium. Population I stars with a composition similar to that of the Sun consist of 73% hydrogen, 25% helium, and 2% heavier elements (by mass). The energy released during the nuclear fusion of four hydrogen atoms to helium is $\epsilon = 4 \times 10^{-12}\,\mathrm{J}$. The current luminosity of the Sun is $L_\odot = 4 \times 10^{26}\,\mathrm{W}$, its mass is $M_\odot = 2 \times 10^{30}\,\mathrm{kg}$, and the mass of a hydrogen atom is $m_\mathrm{H} = 1.67 \times 10^{-27}\,\mathrm{kg}$.

a) Compute the rate of $4\mathrm{H} \longrightarrow \mathrm{He}$ required to sustain the current luminosity of the Sun. How long does it take to convert 15% of the solar hydrogen to helium? Since the subsequent phases of stellar evolution are much faster, this is a good estimate for the total lifetime of a star (*Answer: 10 billion years*).

b) The solution of the question above should have convinced you that the life time, $t_\star$, of a star is proportional to the ratio between the energy available to be spent, and the rate at which this energy is spent, i.e., proportional to $M_\star/L_\star$. For the main sequence, there exists an empirical relationship between the mass of a star, $M_\star$, and its luminosity, $L_\star$, which has the following form:

$$\frac{L_\star}{L_\odot} = \left(\frac{M_\star}{M_\odot}\right)^{3.3}$$

Using your result from above, compute the lifetime of a very massive star with $M_\star = 25\,M_\odot$, and for a dwarf star with a mass of $M_\star = 0.1\,M_\odot$.

Question 4: *Comments on this week's lectures*
In order to improve the teaching and to enable myself to react to problems you might have with the module, I would like to hear your opinion on my teaching as early as possible. I would appreciate it if you would voice any problems and criticisms as soon as possible, e.g., on the speed with which I talk about the subjects of the lectures, the overall difficulty level of the class and the homework, the quality and contents of the handouts, and so on.
Please write these comments on a separate sheet of paper and give them to me: Either put the paper on the lectern before class or put it in my "pigeon hole" in the mailboxes on the 5th floor of the physics building, close to the physics undergraduate office. Feel free to remain anonymous, if you deem this necessary. You can also ask questions or post comments by using the discussion board for this module at `http://forums.warwick.ac.uk/wf/browse/forum.jsp?fid=912` or by sending email to `j.wilms@warwick.ac.uk` (I will post answers to emailed questions on the discussion board, if they are of sufficient interest for others).