Department of Physics
1st Year Examples 2005/2006
Term 2, Week 20–24

THE UNIVERSITY OF WARWICK

# PX144: Introduction to Astronomy

## Academic Week 24/Term Week 20: Galaxies and Cosmology

**Question 1:** *Accretion in X-ray Binaries and Active Galactic Nuclei*
The high luminosities of both, X-ray binaries and Active Galactic Nuclei (AGN), is produced by material falling from somewhere far away onto a compact object, i.e., a white dwarf, neutron star, or a black hole in the case of X-ray binaries, or a supermassive black hole with $M \gtrsim 10^6\,M_\odot$ in the case of AGN. In this exercise you will prove that these objects are indeed the most luminous objects in the Universe.

a) Using energy conservation arguments, derive a formula for the energy released by a mass $m$ that is accreted from infinity to a distance $r$ from a compact object of mass $M$. You may assume that $m$ is stationary at infinity. At distance $r$, the mass is moving around the compact object on a circular orbit. (*N.B.:* For the purposes of this question it is sufficient to assume the Newtonian physics holds throughout, although properly spoken the general theory of relativity would have to be used; *Answer:* $E = GMm/2r$)

b) Use your result from the previous question to determine the energy released by one kilogram of material that is accreted from infinity onto the surface of a $1.44\,M_\odot$ neutron star with radius $r_{\mathrm{NS}} = 10\,\mathrm{km}$. To appreciate the magnitude of the energy released, give it not only in joules but also in Megatons TNT, where $1\,\mathrm{MT} = 3 \times 10^{15}\,\mathrm{J}$, and is a typical strength for todays nuclear bombs ($G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$, $1\,M_\odot = 2 \times 10^{30}\,\mathrm{kg}$).

c) As shown in the lectures the luminosity is defined by the power released by an astronomical object, i.e., by the energy released per time interval.
   1. Convince yourself that this definition is equivalent to $L = \mathrm{d}E/\mathrm{d}t$ and use this equation to derive a formula for the total luminosity of an accreting black hole with a *mass accretion rate* $\dot{m}$, where $\dot{m} = \mathrm{d}m/\mathrm{d}t$ is the amount of mass accreted, d$m$ per time interval d$t$ (the total luminosity is the energy released by accretion to the Schwarzschild radius, $r_{\mathrm{S}} = 2GM/c^2$). Does the luminosity depend on the mass of the black hole?
   2. Estimate the mass accretion rate for a supermassive black hole in an Active Galactic Nucleus with a luminosity of $10^{13}\,L_\odot$. Appropriate units for your answer are solar masses per year ($L_\odot = 4 \times 10^{26}\,\mathrm{W}$, $c = 300000\,\mathrm{km\,s^{-1}}$, 1 year=365.25 days, 1 day=86400 seconds; note that your answer will differ from the numbers given in the lecture notes due to several different assumptions used here).

**Question 2:** *Cepheids and Distance Determination – This question will be marked for credit*

a) The most crucial tool for estimating distances in the universe is the period-luminosity relationship of Cepheid variables. In this question we use rough approximations to show that we do expect such a relationship from a pulsating gas ball (formally, the approach sketched out below is called "dimensional analysis" and used with good success especially in hydrodynamics):
   1. Convince yourself that $(G\rho)^{-1/2}$ has the units ("dimension") of a period, i.e., s. Here $\rho$ is the average density of the star and $G = 6.67 \times 10^{-11}\mathrm{N\,m^2\,kg^{-2}}$.
   2. Using the proportionality just found in the previous subquestion, $P \propto \rho^{-1/2}$, where $P$ is the period, and the mass-radius dependency of the density of the star, $\rho \propto M/R^3$, show that $\log P = c_1 + c_2 \log L$ where $c_1$ and $c_2$ are constants for the given star. Use this dependence to show that $\log P = c_3 - c_4 m$ where $m$ is the observed magnitude of the Cepheid and $c_3$ and $c_4$ are constants. To solve this quesion, first use the relationship between the star's luminosity, $L$, and its temperature $T$ and radius $R$, $L \propto R^2 T^4$, to determine a relationship between the period and the star's luminosity, and then express the luminosity in terms of a magnitude (this subquestion is a longer version of Kuttner's problem 10.4).

b) When Edwin Hubble first determined distances using Cepheids, he was not yet aware that there are two types of pulsating variables, W Virginis stars and $\delta$ Cepheids. These two types of objects have a similar period-luminosity relationship, with the major difference being that W Virginis stars are 1.5 magnitudes fainter than $\delta$ Cepheids. When Hubble determined the distance, he calibrated the period-luminosity relationship with W Virginis stars, and then unfortunately used observations of $\delta$ Cepheids to determine the distance to other galaxies. When this mistake was corrected by Walter Baade in 1952, by how much did the assumed distances change?

c) Observations of a quasar reveal that the H$\alpha$ line is seen at a wavelength of 700 nm. Estimate its distance and its distance modulus (the rest frame wavelength of H$\alpha$ is $\lambda = 656.28\,\mathrm{nm}$; assume $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$).

**Question 3:** *3K Radiation and the Big Bang*

a) The 3K background radiation is seen as one of the strongest pieces of evidence for a hot big bang. According to Wien's displacement law, the wavelength of maximum emission from a black body is given by

$$\lambda_{\max} T = 2.898 \times 10^{-3}\,\mathrm{m\,K}$$

It is believed that the 3K radiation decoupled from the rest of the universe when the temperature of the universe fell below 3000 K. Since the 3K radiation is the remainder of this primordial radiation, calculate the redshift of the Cosmic Microwave Background.

b) For a black body, the energy density of the radiation is given by:

$$u = aT^4 \quad \text{where } a = 7.564 \times 10^{-16}\,\mathrm{W\,m^{-3}\,K^{-4}}$$

(*N.B.:* this equation is related to the Stefan-Boltzmann law encountered elsewhere in this module).

1. Use Einstein's formula $E = mc^2$ to convert the energy density of the 3K radiation to the equivalent of a mass density, and compare this "radiation mass density" with the density of luminous matter, $\rho_\mathrm{m} = 4 \times 10^{-28}\,\mathrm{kg\,m^{-3}}$ ($c = 300000\,\mathrm{km\,s^{-1}}$).
2. The energy per photon is given by $E = h\nu = hc/\lambda$ where $h = 6.6 \times 10^{-34}\,\mathrm{J\,s}$ is Planck's constant, $\nu$ the frequency of the photon and $\lambda$ its wavelength. Using the energy density of radiation, derive the number density of 3K photons, i.e., the number of 3K photons per cubic metre. For simplicity, assume that all 3K photons have a wavelength corresponding to the peak of the 3K radiation as calculated in the previous question. Compare this number density with the number density of normal matter, which you can assume to consist only of protons (with a mass of $m_\mathrm{p} = 1.7 \times 10^{-27}\,\mathrm{kg}$ per proton).

**Question 4:** *Comments on this week's lectures*
At the time when this homework is due, the module will be over, and you will have already filled out the module questionnaires. This means that it will be too late for me to change anything in the current module, however, I would still like to hear about any other suggestions you might have.
Please write these comments on a separate sheet of paper and give them to me: Either put the paper on the lectern before class or put it in my "pigeon hole" in the mailboxes on the 5th floor of the physics building, close to the physics undergraduate office. Feel free to remain anonymous, if you deem this necessary. You can also ask questions or post comments by using the discussion board for this module at `http://forums.warwick.ac.uk/wf/browse/forum.jsp?fid=912` or by sending email to `j.wilms@warwick.ac.uk` (I will post answers to emailed questions on the discussion board, if they are of sufficient interest for others).