The Planets: Dynamics

Introduction

Johannes Kepler: Motion of planets governed by three laws:

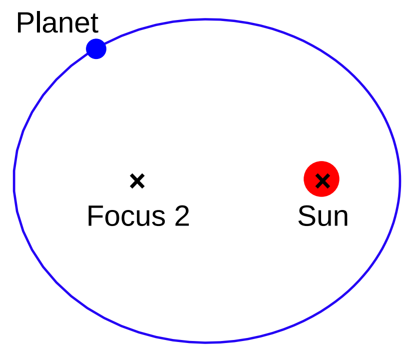
1. Each planet moves in an elliptical orbit, with the Sun at one focus of the ellipse. (“Astronomia Nova”, 1609)
2. A line from the Sun to a given planet sweeps out equal areas in equal times. (“Astronomia Nova”, 1609)
3. The square of the orbital periods of the planets is proportional to the cube of the major axes. (“Harmonice Mundi”, 1619)

Isaac Newton (“Principia”, 1687): Kepler’s laws are consequence of gravitational interaction between planets and the Sun. The gravitational force is

$$\boldsymbol{F}_1 = -\frac{G m_1 m_2}{r_{12}^2} \frac{\boldsymbol{r}_{21}}{r_{12}} \tag{4.1}$$

where $\boldsymbol{F}_1$ is the gravitational force exerted on object 1, m_1, m_2 are the masses of the interacting objects, r their distance, and $\boldsymbol{r}_{21}/r_{12}$ the unit vector joining the objects, $\boldsymbol{r}_{21} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, $\boldsymbol{r}_{12} = -\boldsymbol{r}_{21}$ and $r_{12} = |\boldsymbol{r}_{12}| = |\boldsymbol{r}_{21}|$.

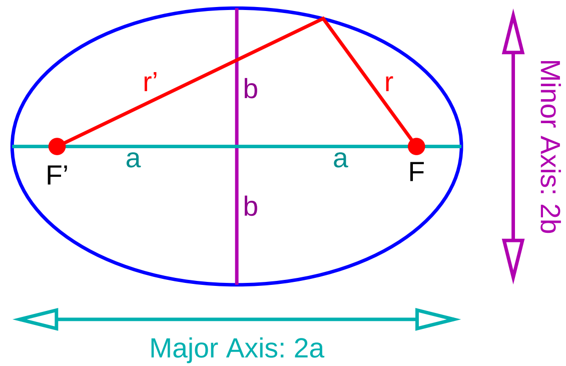
Kepler’s 1st Law



Kepler’s 1[st] Law: The orbits of the planets are ellipses and the Sun is at one focus of the ellipse.

For the planets of the solar system, the ellipses are almost circular, for comets they can be very eccentric.

Kepler’s 1st Law

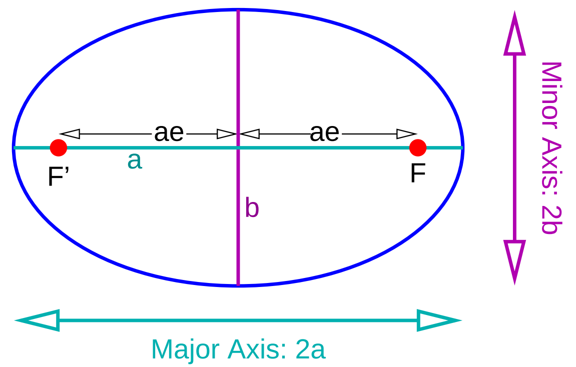


Definition: Ellipse = Sum of distances r, r' from any point on ellipse to two fixed points (foci, singular: focus), F, F', is constant:

$$r + r' = 2a \tag{4.2}$$

where a is called the semi-major axis of the ellipse.

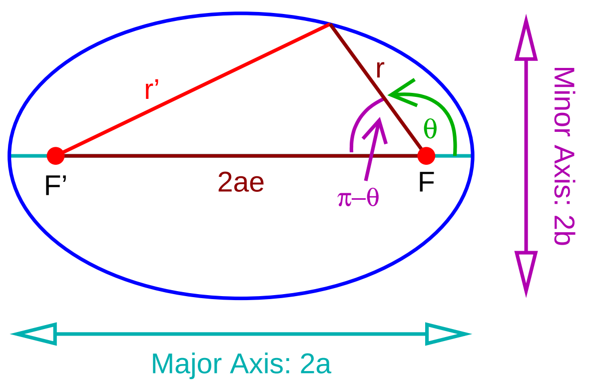
Kepler's 1st Law



Definition: Eccentricity e: ratio between distance from center of ellipse to focal point and semi-major axis.

So circles have $e = 0$.

Kepler's 1st Law



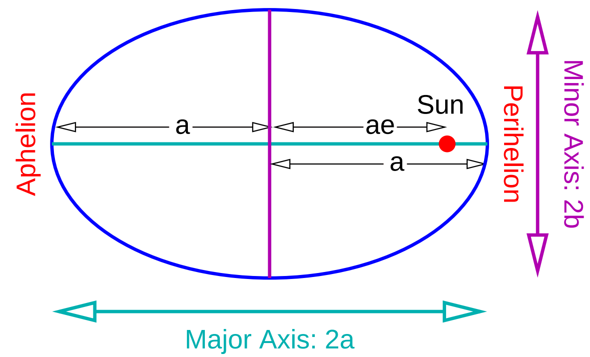
Law of cosines: $r'^2 = r^2 + (2ae)^2 - 2 \cdot r \cdot 2ae \cdot \cos(\pi - \theta)$

use $r + r' = 2a$ and solve for r to find the polar coordinate form of the ellipse:

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta} \tag{4.3}$$

Check this for yourself! θ is called the *true anomaly*.

Kepler's 1st Law



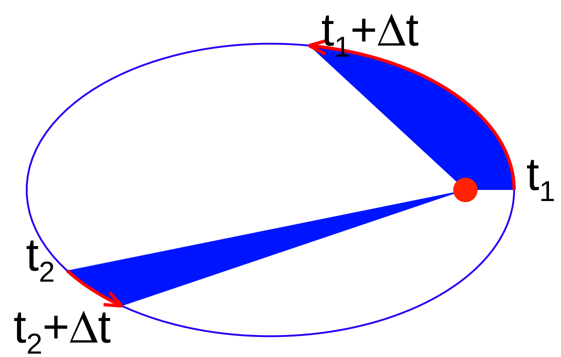
Finally, we need the closest and farthest point from a focus:

$$\begin{aligned} \text{closest point} : d_{\text{perihelion}} &= a - ae = a(1 - e) \\ \text{farthest point} : d_{\text{aphelion}} &= a + ae = a(1 + e) \end{aligned} \tag{4.4}$$

for stars: periastron and apastron,

for satellites circling the Earth: perigee and apogee.

2nd Law

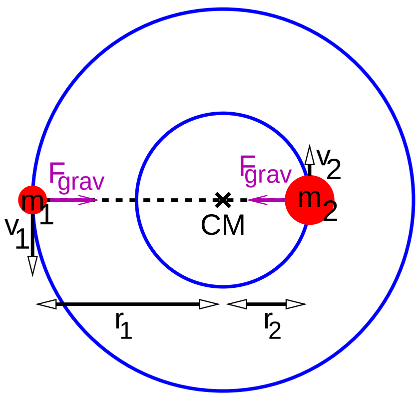


Kepler's 2nd Law: The radius vector to a planet sweeps out equal areas in equal intervals of time.

1. Kepler's 2nd Law is also called the *law of areas*.
2. perihelion: planet nearest to Sun $\Longrightarrow$ planet is fastest
3. aphelion: planet farthest from Sun $\Longrightarrow$ planet is slowest

3rd Law

Kepler's 3rd Law: The squares of the periods of the planets, P, are proportional to the cubes of the semimajor axes, a, of their orbits: $P^2 \propto a^3$.

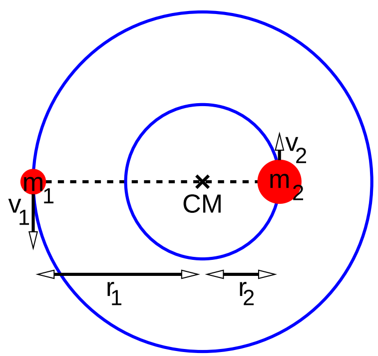


Calculating the motion of two bodies of mass m_1 and m_2 gives Newton's form of Kepler's third law:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)}R^3 \tag{4.5}$$

where $r_1 + r_2 = R$ (for elliptical orbits: R is the semi-major axis).

Circular Motion



For an interpretation of Kepler's third law, consider the motion of two bodies with masses m_1 and m_2 on circular orbits with radii r_1 and r_2 around a point CM (see figure).

The reason for doing the computation with circular orbits is that the following discussion will be easier, however, all results from this section also apply to the general case of elliptical motion. This will be proven later in the lectures on Theoretical Mechanics.

The attractive force between the two points is given by Newton's law:

$$F_{\text{grav}} = G\frac{m_1 m_2}{R^2} = G\frac{m_1 m_2}{(r_1+r_2)^2} \tag{4.6}$$

In order to keep the two bodies on circular orbits, the gravitational force needs to be equal the centripetal force keeping each body on its circular orbit.

The centripetal force is

$$F_{\text{cent},1} = \frac{m_1 v_1^2}{r_1} = \frac{4\pi^2 m_1 r_1}{P^2}$$
$$F_{\text{cent},2} = \frac{m_2 v_2^2}{r_2} = \frac{4\pi^2 m_2 r_2}{P^2} \tag{4.7}$$

where $v = 2\pi r/P$ was used to determine the velocity of each of the bodies. Setting the centripetal force equal to the gravitational force gives

$$\frac{4\pi^2 m_1 r_1}{P^2} = G\frac{m_1 m_2}{(r_1+r_2)^2}$$
$$\frac{4\pi^2 m_2 r_2}{P^2} = G\frac{m_1 m_2}{(r_1+r_2)^2} \tag{4.8}$$

canceling m_1 and m_2 results in

$$\frac{4\pi^2 r_1}{P^2} = G\frac{m_2}{(r_1+r_2)^2}$$
$$\frac{4\pi^2 r_2}{P^2} = G\frac{m_1}{(r_1+r_2)^2} \tag{4.9}$$

Dividing these two equations by each other gives

$$\frac{r_1}{r_2} = \frac{m_2}{m_1} \quad \text{or} \quad m_1 r_1 = m_2 r_2 \tag{4.10}$$

This is the definition of the *center of mass*.

The total distance between the two bodies is

$$R = r_1 + r_2 = r_1 + \frac{m_1}{m_2} r_1 = r_1\left(1 + \frac{m_1}{m_2}\right) \tag{4.11}$$

Inserting into one of the equations 4.9 gives

$$\frac{4\pi^2}{P^2} \cdot R \cdot \frac{m_2}{m_1+m_2} = \frac{Gm_2}{R^2} \tag{4.12}$$

such that

$$\frac{4\pi^2}{P^2} = \frac{G(m_1+m_2)}{R^3} \quad \text{or} \quad P^2 = \frac{4\pi^2}{G(m_1+m_2)}R^3 \tag{4.13}$$

This is *Newton's form of Kepler's 3rd law*.

3rd Law

Newton's form of Kepler's 3rd law is the most general form of the law.

However, often shortcuts are possible.

Assume one central body dominates, $m_1 = M \gg m_2$:

$$\frac{P^2}{a^3} = \frac{4\pi^2}{GM} = \text{const.} = k \tag{4.14}$$

So, if we know P and a for one body moving around m_1, can calculate k.

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So, if we know P and a for one body moving around m_1, can calculate k.

For the Solar System, use Earth:

- $P_\oplus = 1$ year (by definition!)
- $a_\oplus = 1$ AU (Astronomical Unit, 1 AU $= 149.6 \times 10^6$ km)

$\Longrightarrow k = 1\,\text{yr}^2\,\text{AU}^{-3}$

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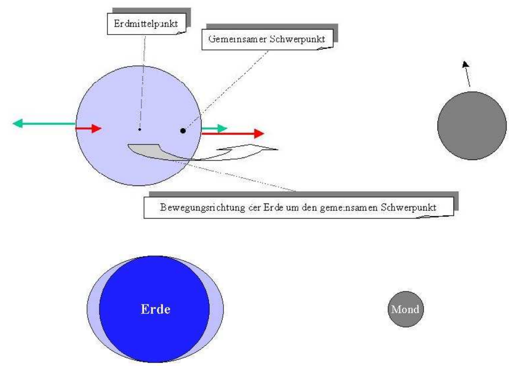
Jupiter: $a_{♃} = 5.2$ AU. What is its period?

Answer: $P_{♃}^2 = 1\,\text{yr}^2\,\text{AU}^{-3} \cdot 5.2^3\,\text{AU}^3 \sim 140\,\text{yr}^2$, or $P_{♃} \sim 12$ years

(with pocket calculator: $P_{♃} = 11.86$ years)

Tidal forces: The Earth–Moon system



(Einsteins-Erben)

gravitational force of the Moon onto Earth

centrifugal force of Earth around center of gravity

Tidal forces: The Earth–Moon system

Gravitational acceleration for center of Earth and for the point closest to a gravitating body:

$$a_{\text{center}} = \frac{GM}{r^2} \quad \text{and} \quad a_{\text{closest}} = \frac{GM}{(r-R_\oplus)^2} \tag{4.18}$$

such that difference in acceleration:

$$\Delta a = a_{\text{closest}} - a_{\text{center}} = \frac{GM}{(r-R_\oplus)^2} - \frac{GM}{r^2} = \frac{GM}{r^2}\left(\frac{1}{(1-\frac{R_\oplus}{r})^2} - 1\right) \sim \frac{GM}{r^2}\left(1 + 2\frac{R_\oplus}{r} - 1\right) = \frac{2GMR_\oplus}{r^3} \tag{4.19}$$

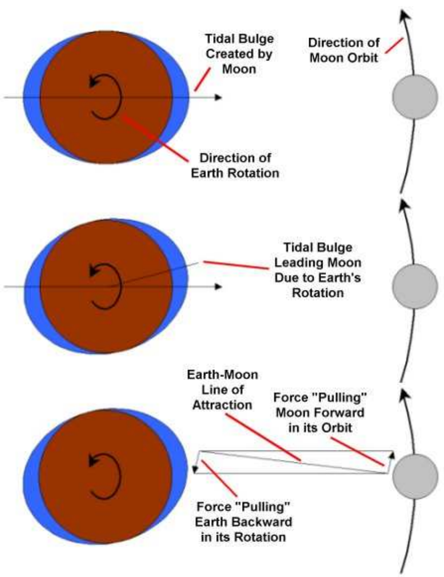
Therefore the tides due to the Moon and Sun are

$$\Delta a_{☾} = \frac{2GM_{☾} R_\oplus}{r_{☾}^3} \quad \text{and} \quad \Delta a_\odot = \frac{2GM_\odot R_\oplus}{(1\,\text{AU})^3} \tag{4.20}$$

Since $M_{☾} \sim M_\oplus/81$ and $r_{☾} \sim 60 \times R_\oplus$

$\Longrightarrow \Delta a_\odot / \Delta a_{☾} = 0.46$

$\Longrightarrow$ Moon twice as important as Sun; two sets of tidal bulges, spring-tides



- Friction Earth–Ocean: Tidal bulges preceed rotation by 2.5 h
- Rotation of Earth slows down $\Longrightarrow$ length of day increases by 1.6 ms century^{-1}

  confirmed by historic solar eclipse observations (e.g., Thales, 585 B.C., eclipse observations by Chinese astronomers)
- 370 Mio. years ago (Devon): 1 Year∼400 days (from coral growth)
- Moon is accelerated, its distance increases by 4 cm yr^{-1}
- length of the month grows
- friction ceases when 1 day = 1 month (synodic) = 50 days (now)
- equilibrium, i.e., bound rotation, is reached in ∼10^{12} years
- Moon's rotation bound to orbital motion by tidal friction on the liquid interior of the Moon soon after formation
- Other moons are also synchronized (due to work on planetary body): e.g. Galilean moons of Jupiter
- Pluto/Charon both synchronized

Stability of Satellites

Tidal forces also important for the stability of small bodies (e.g., moons) moving around a central body at a distance r.

Notation: mass $M_{\text{p,s}}$, radius $R_{\text{p,s}}$, density $\rho_{\text{p,s}}$ where p: planet, s: satellite.

Satellite is bound by its own gravitational force. Satellite is not ripped apart, if binding force $F_{\text{G}} >$ tidal force F_{T}:

Binding force is ∼ mutual attraction of two halfs of the satellite separated by R_{s}:

$$F_{\text{G}} = \frac{GM_{\text{s}}M_{\text{s}}}{4R_{\text{s}}^2} > \frac{GM_{\text{p}}M_{\text{s}}R_{\text{s}}}{r^3} = F_{\text{T}} \tag{4.21}$$

where binding force ∼ gravitational attractions of two hemispheres of the satellite onto each other.

$$\Longrightarrow \frac{GM_{\text{s}}M_{\text{s}}}{4R_{\text{s}}^2} > \frac{GM_{\text{p}}M_{\text{s}}R_{\text{s}}}{r^3} \quad \Longrightarrow \quad \frac{M_{\text{s}}}{4R_{\text{s}}^3} > \frac{M_{\text{p}}}{r^3} \quad \text{or} \quad \frac{\rho_{\text{s}}}{4} > \frac{R^3\rho_{\text{p}}}{r^3} \tag{4.22}$$

This means there is a critical distance (Roche, 1850):

$$\frac{r_{\text{c}}}{R} = \sqrt[3]{\frac{4\rho_{\text{p}}}{\rho_{\text{s}}}} \qquad \frac{r_{\text{c}}}{R} = 2.44\sqrt[3]{\frac{\rho_{\text{p}}}{\rho_{\text{s}}}} \tag{4.23}$$

$\Longrightarrow$ If of same density, the moon has to be more distant than 2.44 planetary radii from the planet in order to avoid disruption. This is important, e.g., for the formation of rings.

Precession and Nutation

Earth is ∼ rotational ellipsoid, orbits of Sun and Moon are *not* in plane of equator

(Earth's axis has tilt of ∼23.5°, Moon's orbit tilted by 7° against ecliptic)

$\Longrightarrow$ Sun and Moon excert torques onto Earth

Earth's rotational axis is not stable in space.

Two major effects:

luni-solar precession: Earth's axis rotates around pole of ecliptic once every 25800 years (∼50″ per year).

Already discovered by Hipparcos in ∼200 BC!

nutation: "Wobble" with ∼18 year periodicity caused by short-term perturbations caused by Moon and Sun.

N-Body Problem

Apart from Sun, motion of planets also influenced by forces between planets:

Total equation of motion for the i-th object:

$$m_i \ddot{\boldsymbol{r}}_i = -\sum_{k=1}^{N} \frac{G m_i m_k}{r_{i,k}^2} \frac{\boldsymbol{r}_i - \boldsymbol{r}_k}{r_{i,k}} \qquad (4.24)$$

$\Longrightarrow$ $3N$ differential equations of 2nd order, requiring $6N$ integrations for their solution.

Closed solution only possible for 10 of these (6: from motion of center of mass, 3: conservation of angular momentum, 1: conservation of energy).

Analytic solution: “Perturbation theory”:

1. Assume two body motion around Sun for all planets
2. Evaluate force based on this motion.
3. Update positions with this “perturbation”.
4. Iterate (i.e., goto step 2)

N-Body Problem

Perturbation theory yields two kinds of perturbations:

periodic perturbations: Terms containing time in $\sin$- and $\cos$-functions.

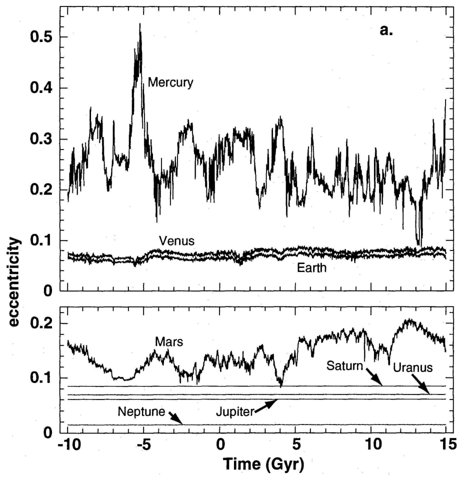
secular perturbations: Long term changes which depend on time (usually as a polynomial).

Analytical approach is very important for understanding the underlying physics, but mathematically *very* tedious. Series do not converge on long timescales (1000’s of years).

$\Longrightarrow$ New high precision calculations are all based on numerical simulations, i.e., direct solution of equation of motion on computers.

Today’s standard: DE102, DE405, DE414 from Jet Propulsion Laboratory, Pasadena, and INPOP06 from Laskar et al., IMCCE, Observatoire de Paris.

Long-Term Evolution of the Solar System



(Laskar, 1994)

Numerical simulations allow to obtain good information about behavior of solar system for timescales of a few 10 million years around the present $\Longrightarrow$ Important, e.g., for paleoclimatology.

Laskar (1989, 1990): Motion of inner planets is chaotic.

“Chaotic”: Initial errors get amplified exponentially, here by factor of 10 on time scales of ~10 million years.

Important, e.g., for climate variations on Earth (“Milankovitch cycles”).

Also found with different methods by Wisdom and Suskind.

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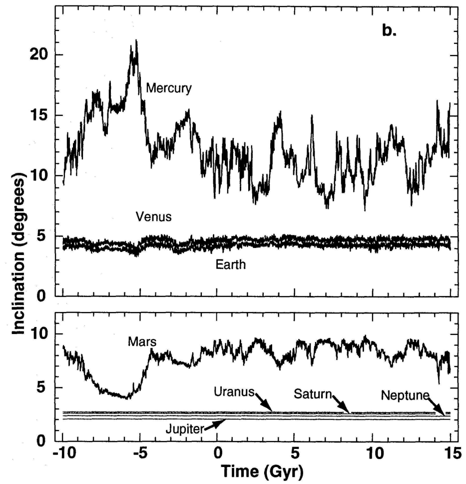

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Laskar, J. 1989, Nature, 338, 237 Laskar, J. 1990, Icarus, 88, 266
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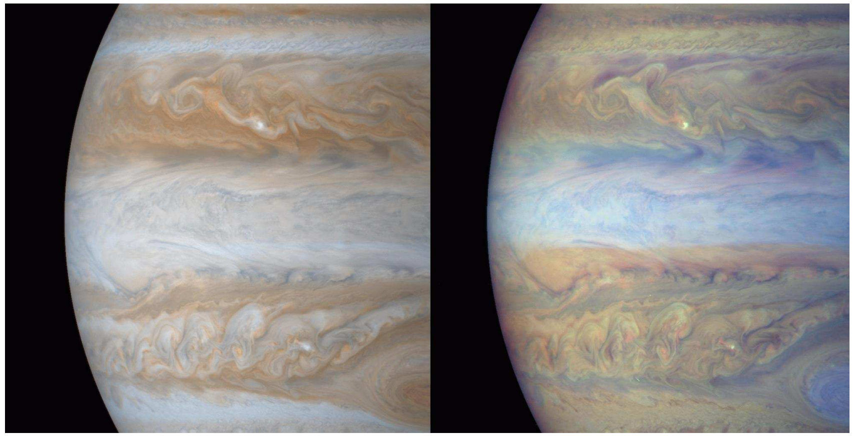
Planets: Atmospheres

Mars: thin atmosphere (pressure on surface 1% Earth), but real seasonal variations.

2005 Feb 7, NASA/Malin Space Systems



NASA/JPL, Cassini, 2000 Dec 31

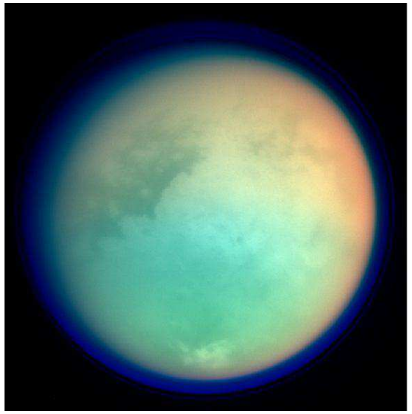
Jupiter: true color image; colors likely from trace content of organic compounds in atmosphere

false color image, red: waterclouds, dark spots: deep hot spots

Overall atmospheric structure: three layers:

Ammonia – ammonia hydrosulfide (NH_4HS) – water ice/water (deepest)

NASA Voyager

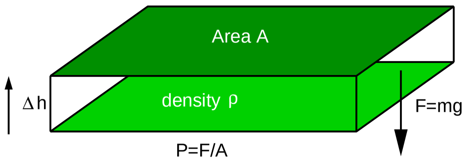


27.10.2004, false colour IR/UV; NASA/ESA

Titan: dense atmosphere, 99% nitrogen, 1% methane, some hydrocarbons, thought to be similar to primordial atmosphere of Earth.
Radius: 2575 km ($\sim$ Mercury!)
ESA probe *Huygens* landed on Titan on 2005 January 14

Hydrostatic Equilibrium

Structure of atmosphere defined through hydrostatic equilibrium:



Force on area A by slab of gas of area A and density ρ:

$$F = mg = \rho V g = Ah\rho g \qquad (5.1)$$

With $P = F/A$ one finds:

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\rho g$$

where g gravitational acceleration.

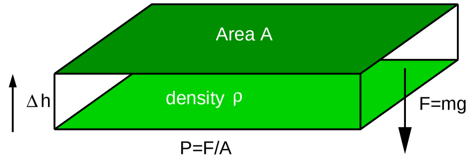
Assuming ideal gas, $P = (\rho/\mu)kT$, and isothermal atmosphere:

$$P(h) = P_0 \exp\left(-\frac{\mu g}{kT}\cdot h\right) = P_0 \exp\left(-\frac{h}{H}\right)$$

The pressure in the atmosphere thus decreases exponentially, the characteristic height scale of the decrease is given by the scale height H.

On Earth, $H \sim 9$ km.

The structure of an atmosphere is defined by the concept of hydrostatic equilibrium:



The force excerted by gas with density ρ sitting on top of an area A is given by

$$F = mg = \rho V g = Ah\rho g \qquad (5.2)$$

Such that pressure becomes

$$P = \frac{F}{A} = \rho h g \qquad (5.3)$$

where g is the gravitational acceleration.
For a thin atmosphere (g constant): Decrease of P when going upwards by Δh:

$$\Delta P = -\rho g \Delta h \quad \text{and for } \lim_{\Delta h \to 0}: \quad \frac{\mathrm{d}P}{\mathrm{d}h} = -\rho g \qquad (5.4)$$

To solve this differential equation, we need a relationship between density and pressure ("equation of state"). For an "ideal gas", this relationship is given by

$$P = (\rho/\mu)kT \qquad (5.5)$$

where T is the Temperature (K!), μ the average mass of a gas particle, and k is Boltzmann's constant ($k = 1.38 \times 10^{-23}$ J K^{-1}). Therefore:

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\left(\frac{\mu g}{kT}\right)P \qquad (5.6)$$

In order to obtain P as a function of height, h, we need to solve this differential equation with the boundary condition that for $h = 0$, $P = P_0$. This can be done easily using the technique of "separation of variables", assuming that T does not change.

First, divide by P and integrate both sides of the equation with respect to height:

$$\int_0^h \frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}h}\,\mathrm{d}h = -\int_0^h \left(\frac{\mu g}{kT}\right)\mathrm{d}h$$

We can now substitute $P(h)$ for h on the left hand side. Using the chain rule gives

$$\int_{P_0}^{P(h)} \frac{\mathrm{d}P'}{P'} = -\int_0^h \left(\frac{\mu g}{kT}\right)\mathrm{d}h$$

such that

$$\ln\left(\frac{P(h)}{P_0}\right) = -\left(\frac{\mu g}{kT}\right)h$$

and exponentiating then gives

$$P(h) = P_0 \exp\left(-\frac{\mu g}{kT}\cdot h\right) = P_0 \exp\left(-\frac{h}{H}\right) \qquad (5.7)$$

The pressure in the atmosphere thus decreases exponentially, the characteristic height scale of the decrease is given by the scale height,

$$H = \frac{kT}{\mu g} \qquad (5.8)$$

Typical values for the planets are for Earth: $H \sim 9$ km.

Remark: The method employed above is called "separation of variables" since people often jump from the first (linear) equation to the third one in one step, by "separating the dependent from the independent variable":

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\left(\frac{mg}{kT}\right)P \quad \Longrightarrow \quad \frac{\mathrm{d}P}{P} = -\left(\frac{mg}{kT}\right) \quad \Longrightarrow \quad \int_{P_0}^{P(h)} \frac{\mathrm{d}P'}{P'} = -\int_0^h \left(\frac{mg}{kT}\right)\mathrm{d}h$$

Atmospheric Composition

Atmospheric composition of selected terrestrial objects

	Venus	Earth	Mars	Titan
O_2	0.0	21.0	0.0	0.0
H_2O	50ppm	1.0	$<$ 100ppm	0.4ppb
CO_2	96.5	0.0	95.3	10ppb
N_2	3.5	78.0	2.7	90
Ar	0.0	0.9	1.6	

Values are given as percentages by volume.

Titan has atmospheric structure similar to Earth!

after de Pater and Lissauer

Atmospheric Composition

Typical H for the inner planets (Karttunen)

Gas	μ	Venus km	Earth km	Mars km
H_2	2	360	120	290
O_2	32	23	7	81
H_2O	18	40	13	32
CO_2	44	16	5	13
N_2	28	26	8	20
T [K]		750	275	260
g [m s^{-2}]		8.61	9.81	3.77

Atmospheric Composition

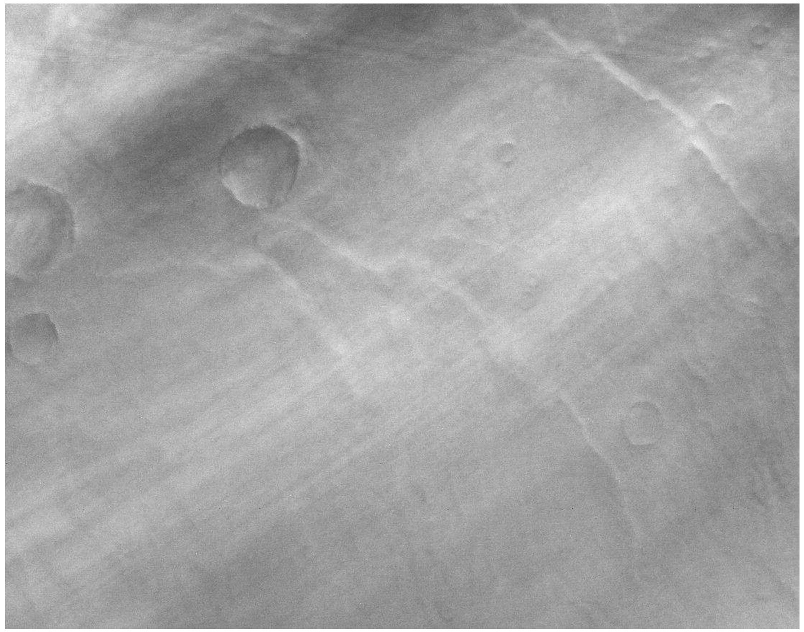
Atmospheric composition of gas giants and the Sun

	Sun	Jupiter	Saturn	Uranus	Neptune
H_2	83.5	86.4	96.3	85±5	85±5
He	19.5	15.7	3.4	18±5	18±5

H: volume percent relative to total atmosphere
He: volume percent relative to H

Gas giants have approximately same composition as the Sun.

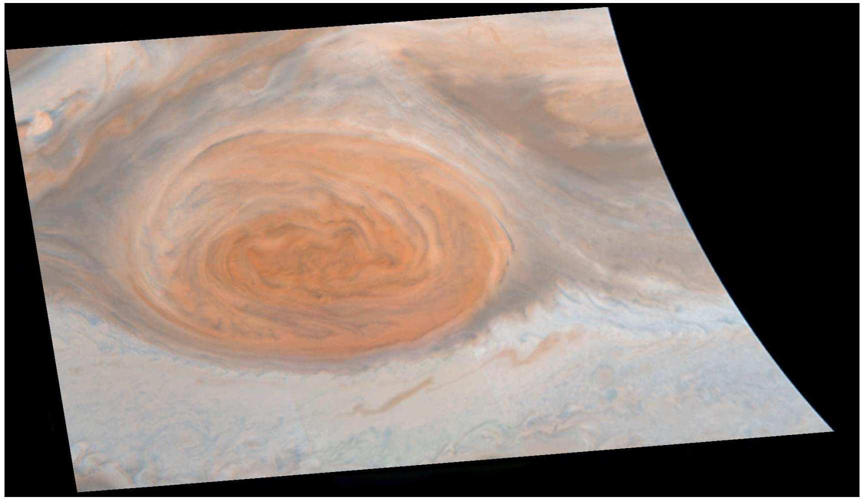
after de Pater and Lissauer



NASA/C.J.Hamilton

Mars: Streaky clouds

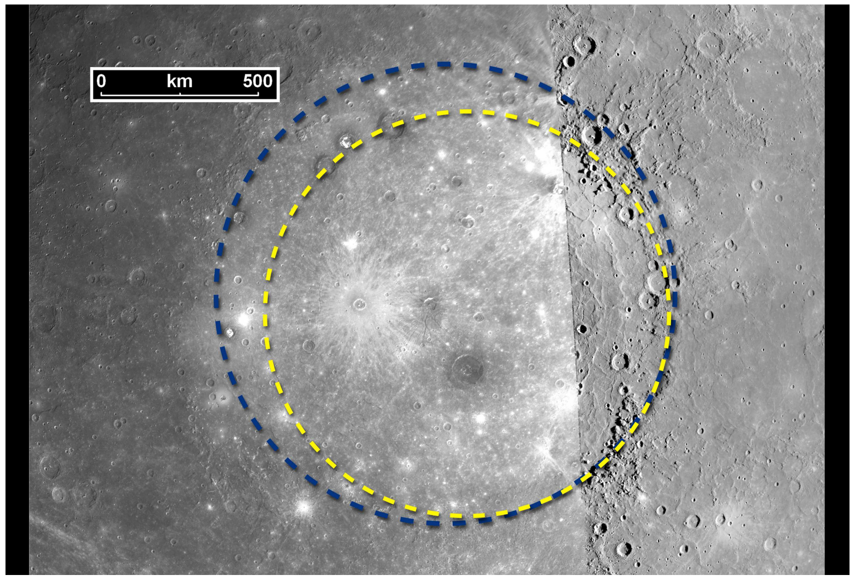
Planets: Surfaces and Interiors



Great Red Spot
NASA Galileo, 1996 June 26

Jupiter's Great Red Spot: Storm System, $\sim 2\times$ Earth diameter, exists since more than 300 years, 8 km above and 10° cooler than surrounding region (rising high pressure region), rotates counterclockwise (Coriolis force on Southern hemisphere).

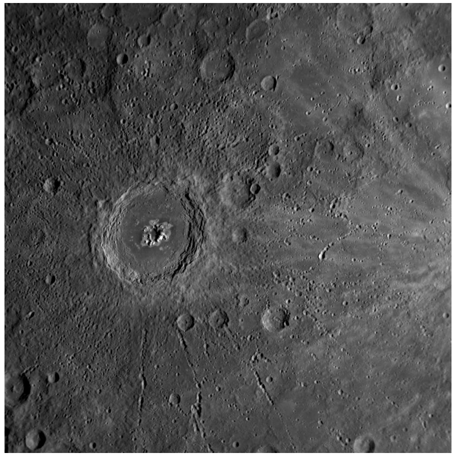
HST Image (image enhanced) of Uranus ring system, plus evidence for banded atmosphere and clouds



NASA/MESSENGER

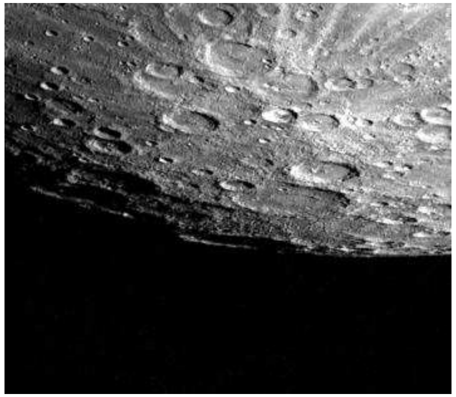
Mercury: Caloris Basin (1300 km diameter)
close to sub-solar point at perihelion $\Longrightarrow$ hot! ($T > 400°$C on day, $T \sim -170°$C during night)
Basin: result of *large* impact event

Major landforms: Craters



NASA/MESSENGER

Terraced craters, with central mountains.



S-Pole; NASA/JPL (Mariner)

50 km diam craters with rays (remains from impact)

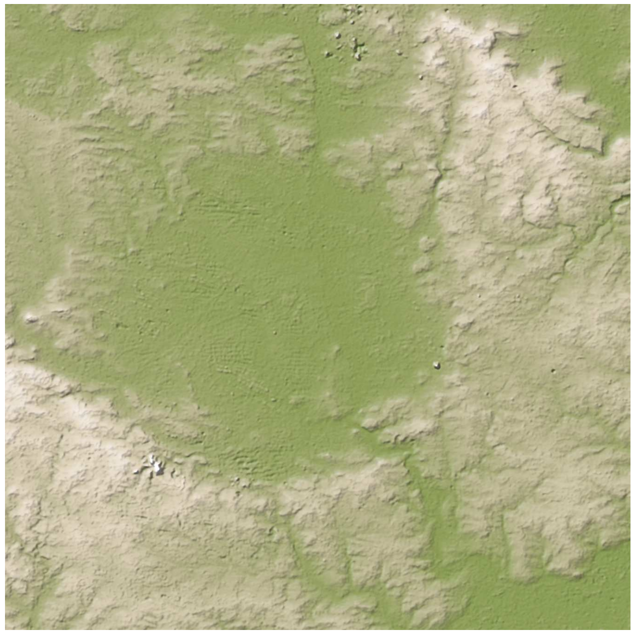
Earth: Nördlinger Ries

Impact crater identified by Shoemaker in 1960s

V. L. Sharpton

Earth: Wolf Creek Crater, Australia

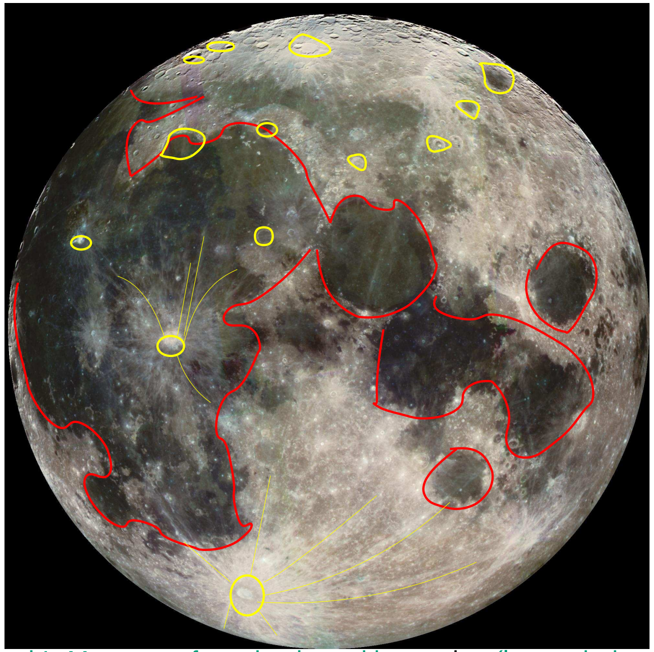
Currently 178 confirmed impact structures on Earth



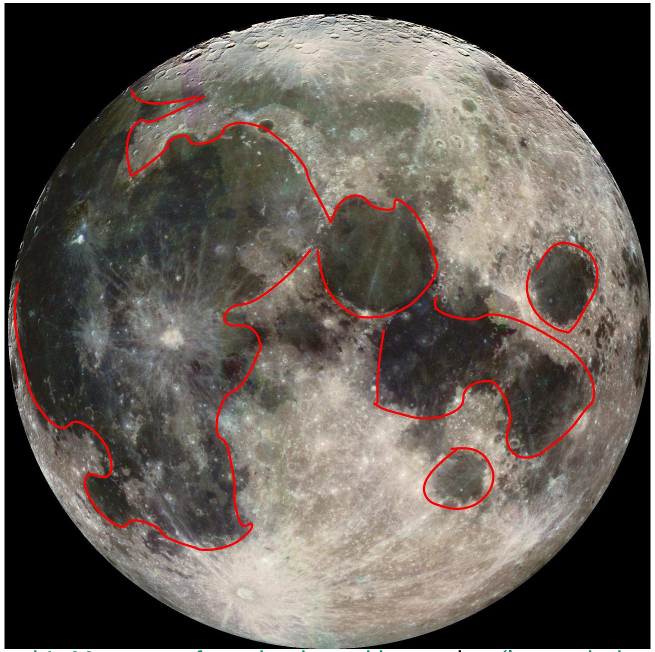
Earth: Elevation image Nördlinger Ries

Thermal Emission and Reflection Radiometer on NASA Terra satellite, credit: NASA

Earth's Moon



Earth's Moon : surface dominated by mariae (large, dark lava basins) and craters (only most prominent shown).

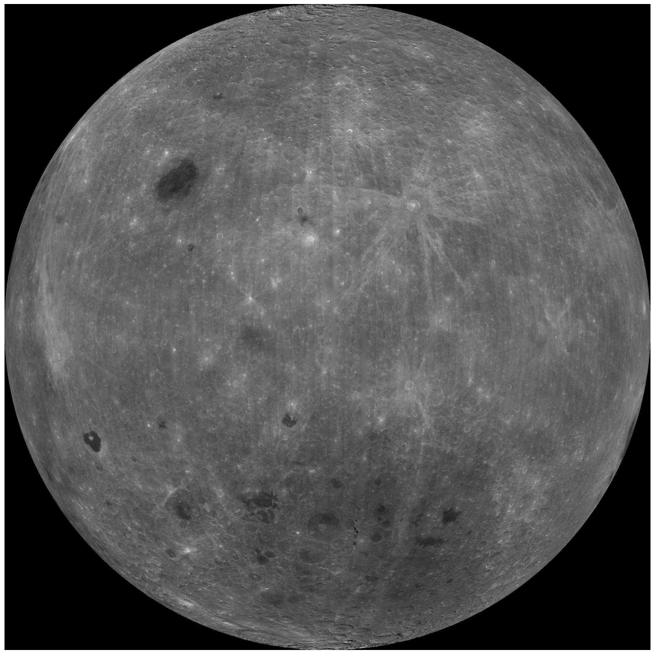


Earth's Moon : surface dominated by mariae (large, dark lava basins)

Moon: Crater Copernicus

Moon: Apollo 16, 1972 Apr, Descartes Highlands

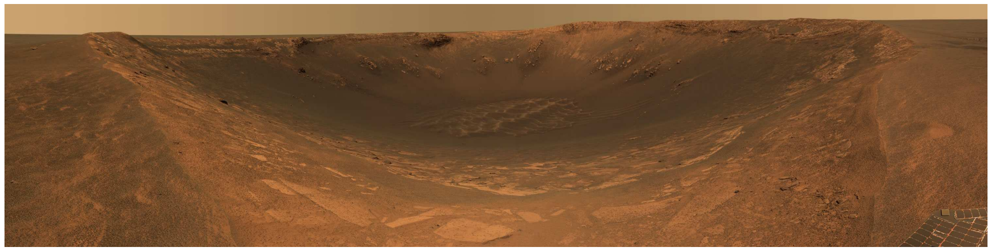
Mars: Surface panorama, Exploration Rover “Opportunity” looks back to lander (2004 Feb 09)



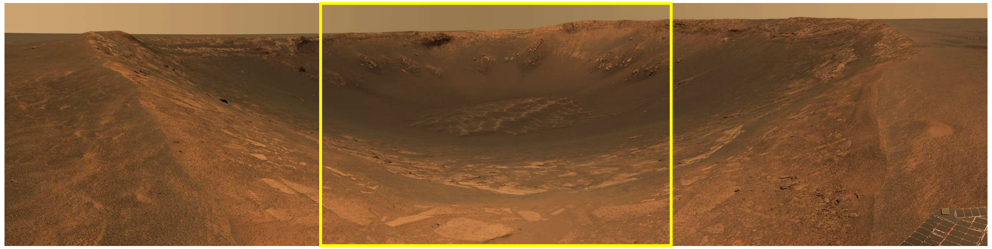
Far side of the Moon:

- few maria
- stronger relief (16 km roughness, compared to 5–6 km on near side)
- Aitkin basin near South pole: 12 km deep

NASA/Clementine

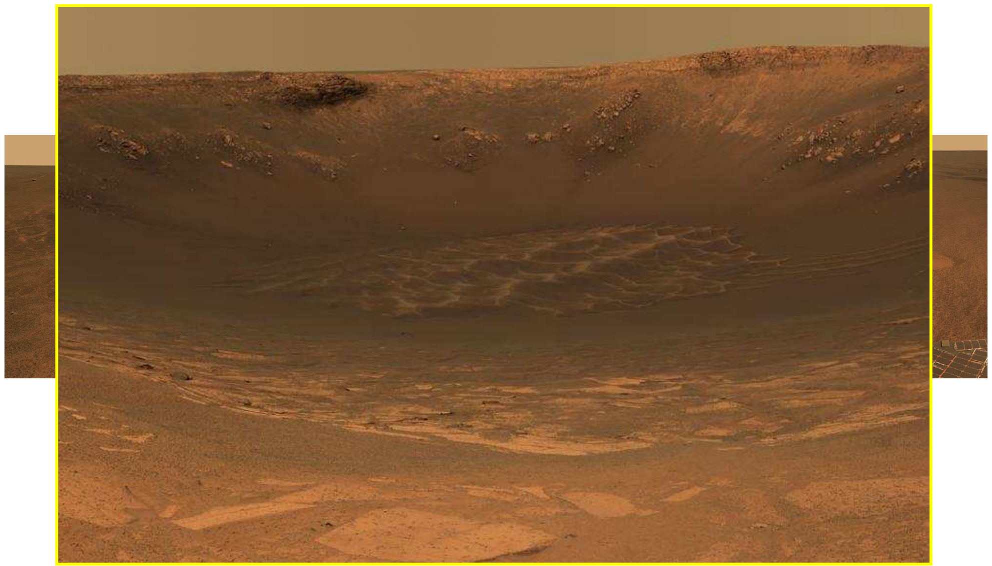


NASA/JPL/Cornell



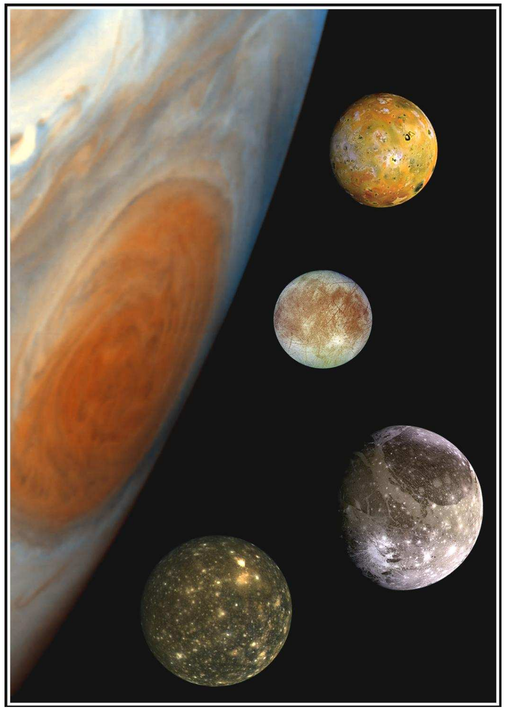
NASA/JPL/Cornell

Mars: Crater Endurance



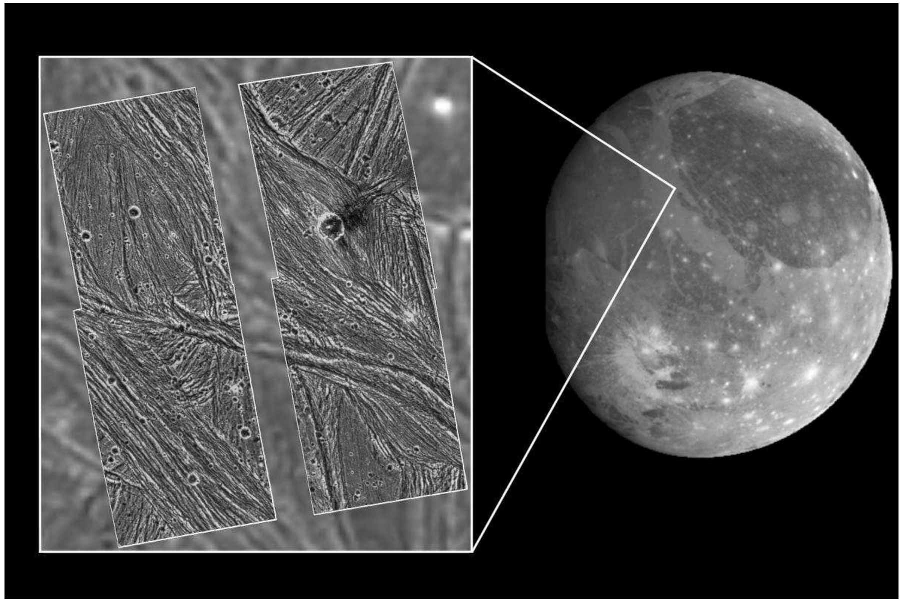
NASA/JPL/Cornell

Mars: Crater Endurance



Montage of Jupiter and Galilean Moons: top to bottom: Io, Europa, Ganymede and Callisto.

(N.B.: All Galilean moons tidally locked to Jupiter – always same side is facing Jupiter)

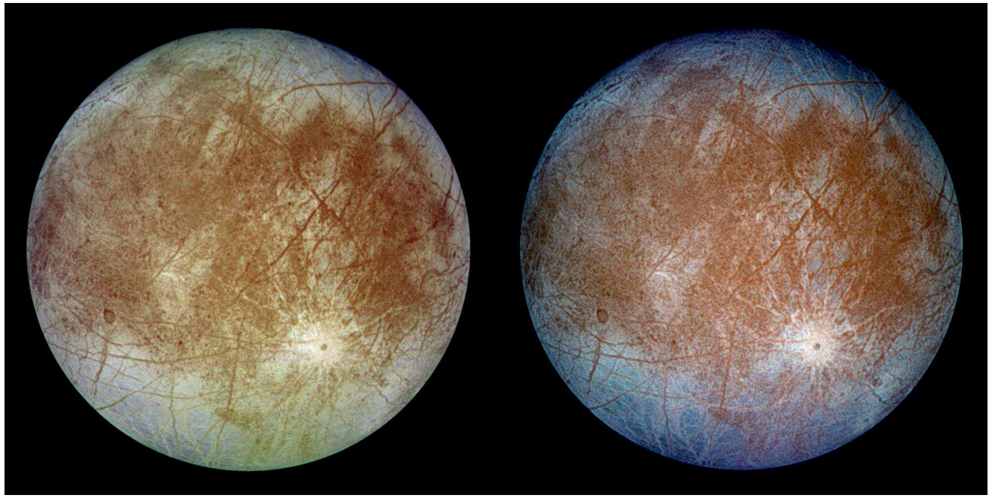


NASA Galileo / DLR, inset: 120×110 km

Ganymede – icy surface, ice hills and valleys, craters
Radius: 2634 km (largest moon in solar system, larger than Mercury!)

Callisto: “pock faced”, mainly impact craters.
white: ice
dark: ice-poor material

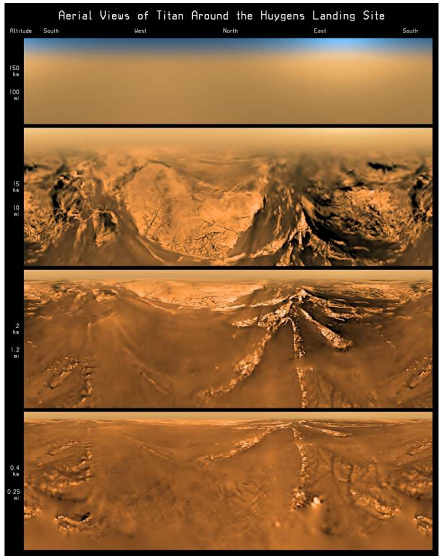
Radius: 2406 km (similar to Mercury!)



NASA Galileo / DLR, 1996 September 7

Europa – icy surface with ridges (colors: different kinds of ice)
Radius: 1565 km ($\sim$ Earth Moon)
possibility of water ocean below surface

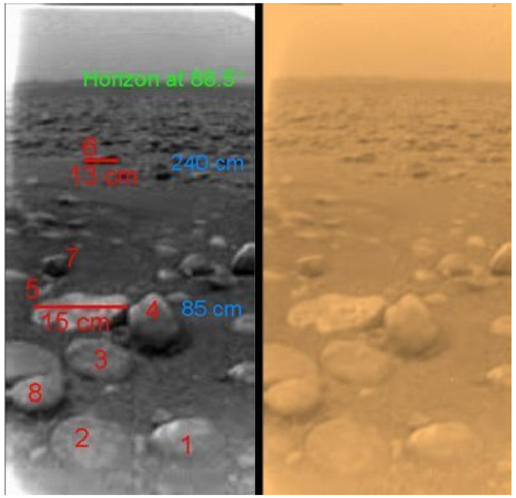
Saturn's moons



Titan:
- Huygens lander: pictures from descend.
- Radius: 2575 km second largest moon, larger than Mercury!

Credit: ESA/NASA/JPL

Saturn's moons

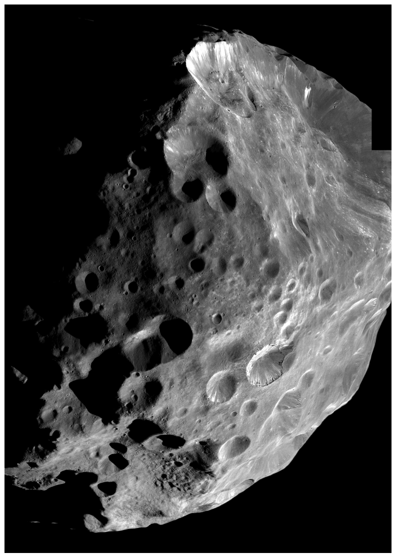


Titan:

- Huygens lander: pictures from landing site.
- Methan/water ices
- River bed?

Credit: ESA/NASA/JPL

Saturn's moons



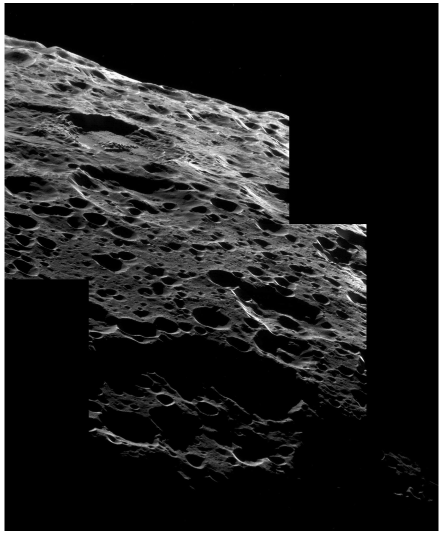
Phoebe:

- small moon of Saturn: radius 220 km.
- dark surface: albedo=0.08

orbits inside the enormous dust ring of Saturn

Surface details from Cassini space craft.
Credit: ESA/NASA/JPL

Saturn's moons



Iapetus:

- Radius: 1436km
- Two hemispheres:
  Cassini Regio: very dark (albedo≈0.05)
  Roncevaux Terra: bright like Europa (albedo≈0.5)
- low density: 1.27 g/cm^3
  large fraction of water ice?
- Dark material accreted from Saturn's dust ring?

Surface details from Cassini space craft.
Credit: ESA/NASA/JPL

Impact Craters

Physics of impact cratering:

Kinetic energy:

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\cdot\frac{4}{3}\pi r^3 \rho v^2 = \frac{\pi d^3 \rho v^2}{12}$$

Important numbers:

- Velocity of impact: several times orbital speed of planet
- Impacting body: rock or Fe, several meters to kilometers in size

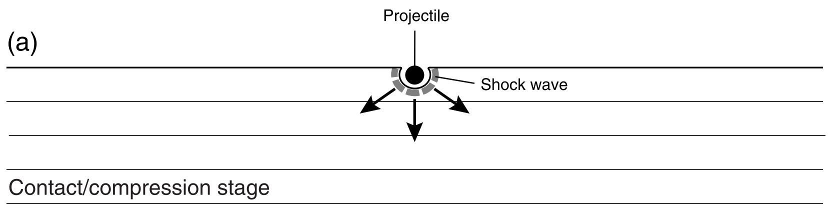
Example:

E.g., $v = 10\,\mathrm{km\,s^{-1}}$, $d = 25\,\mathrm{m}$, $\rho = 7900\,\mathrm{kg\,m^{-3}}$

$\Longrightarrow E = 3\times 10^{15}\,\mathrm{J}$ ($\sim$1 Megaton of TNT)

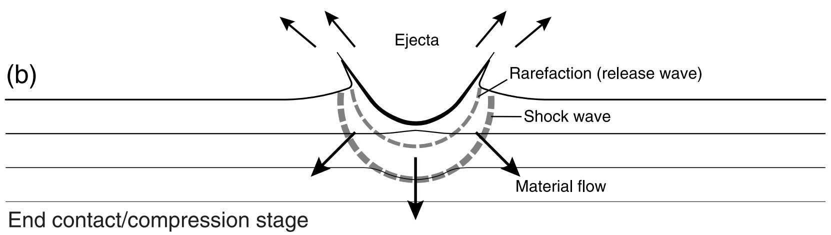
1 Megaton TNT is typical strength of US nuclear bombs [B-83 bomb]

Impact Craters



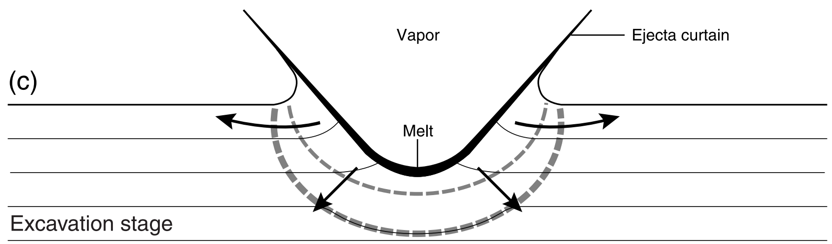
French, 1998, LPI Cont. 954

Impact Craters



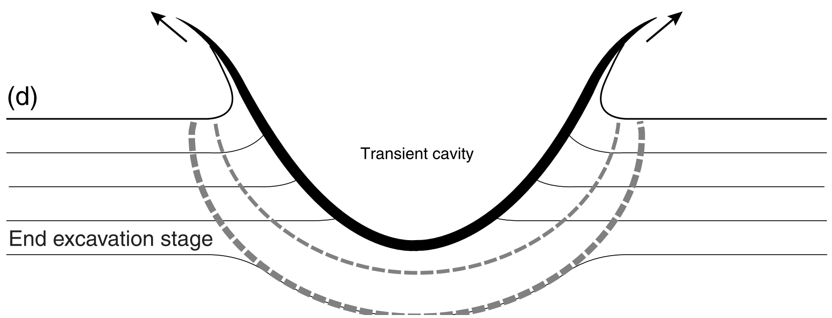
French, 1998, LPI Cont. 954

Impact Craters



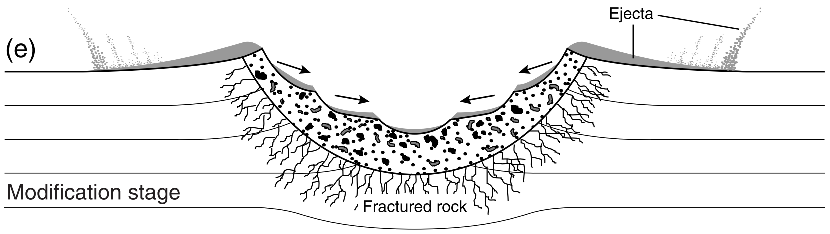
French, 1998, LPI Cont. 954

Impact Craters



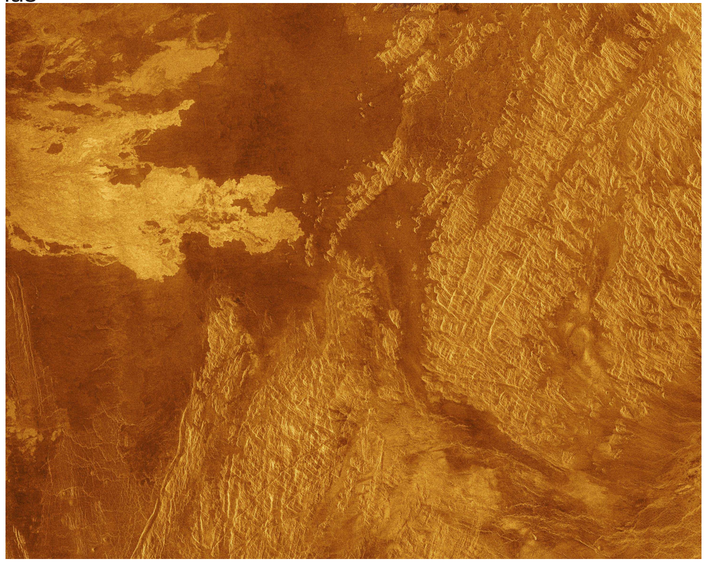
French, 1998, LPI Cont. 954

Impact Craters



French, 1998, LPI Cont. 954

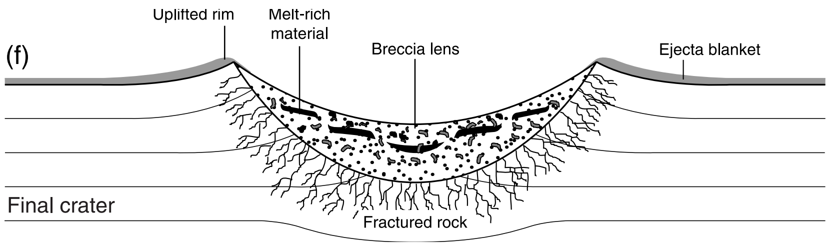
Venus



NASA, Magellan

440 $\times$ 350 km^2 area in Eistla Regio, shows basic stratigraphy (sequence of geologic events): right half: old highlands, fractured structure ($\sim$15% of surface), left part: lowlands, younger area, origin in former volcanism?
Craters (note: strong erosion $\Longrightarrow$ fewer craters overall)

Impact Craters



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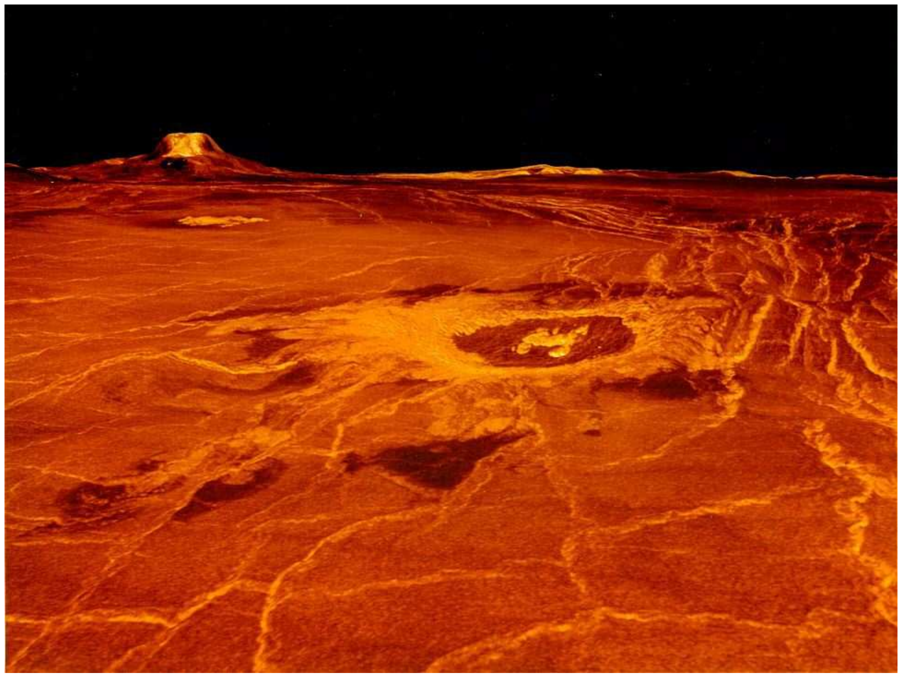
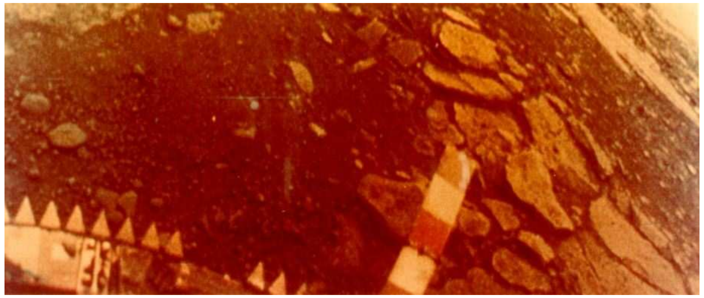


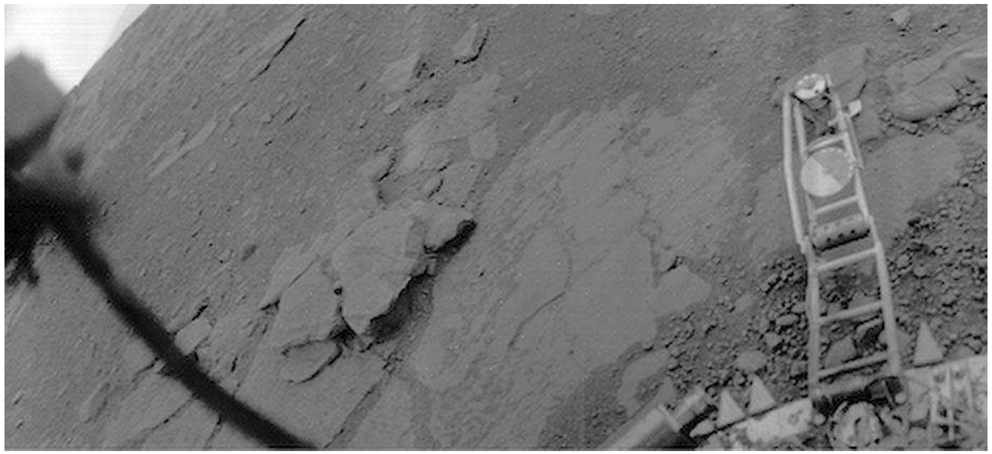
Eistla Regio; heights exagerrated by factor 22.5

Venus surface images:



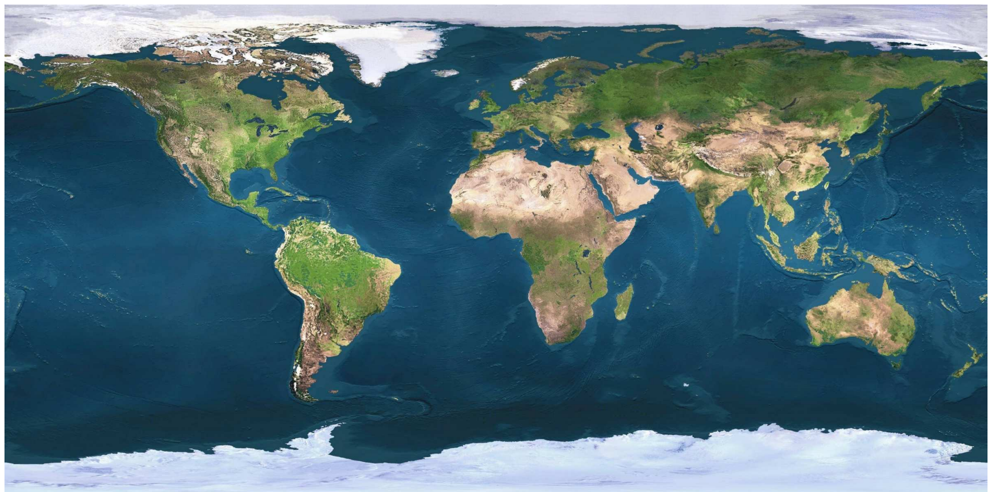
Venera 13 (3 March 1982): images from color TV camera

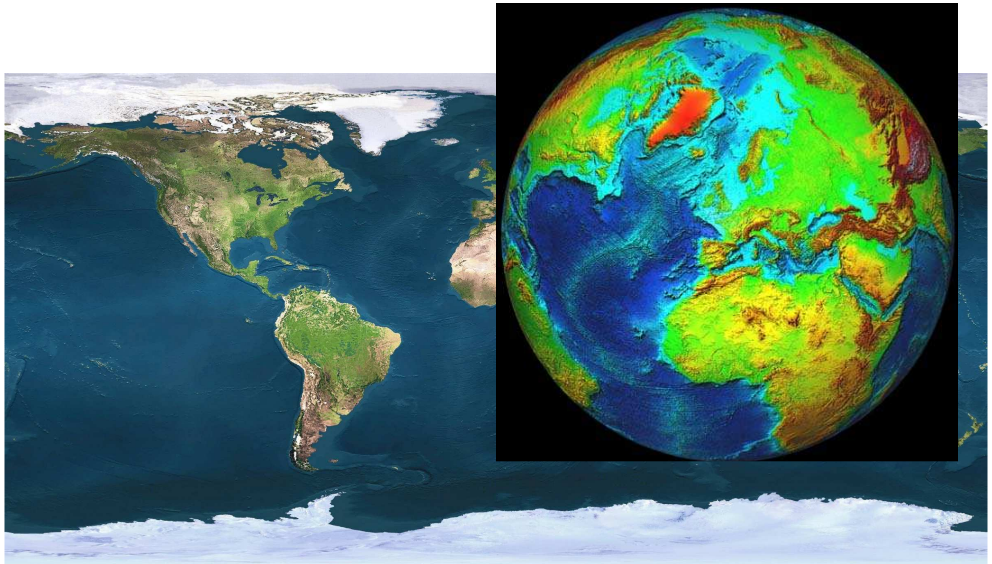
Venera 14 (5 May 1982)



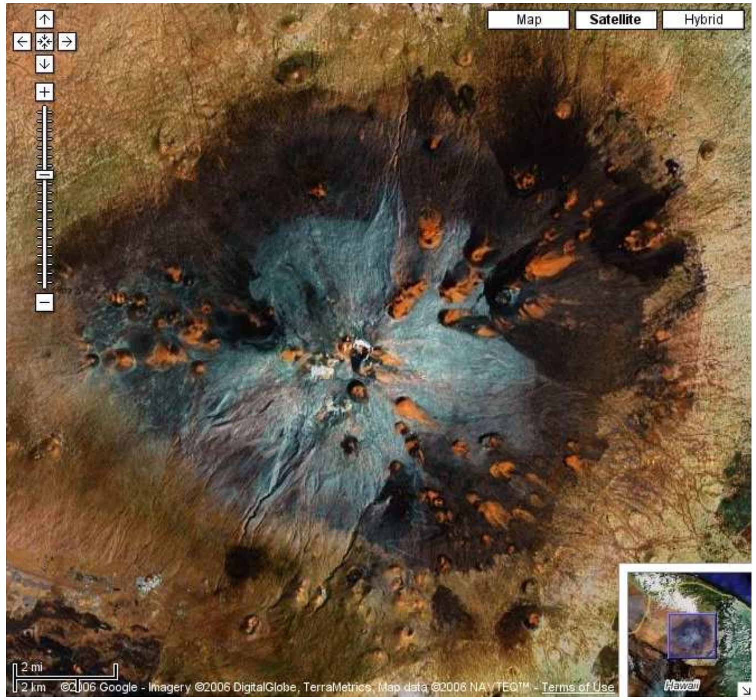
courtesy D.P. Mitchell

Venera 13 (3 March 1982): reanalysed image without camera distortion

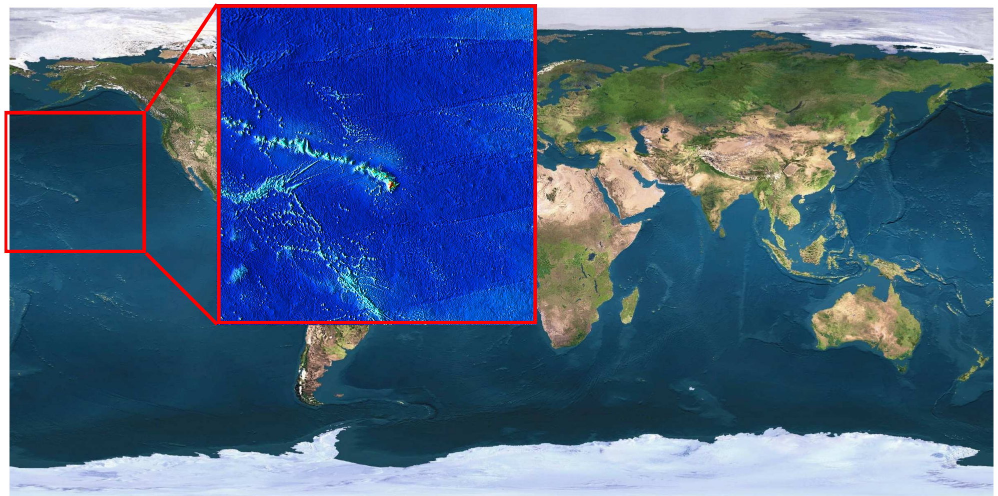




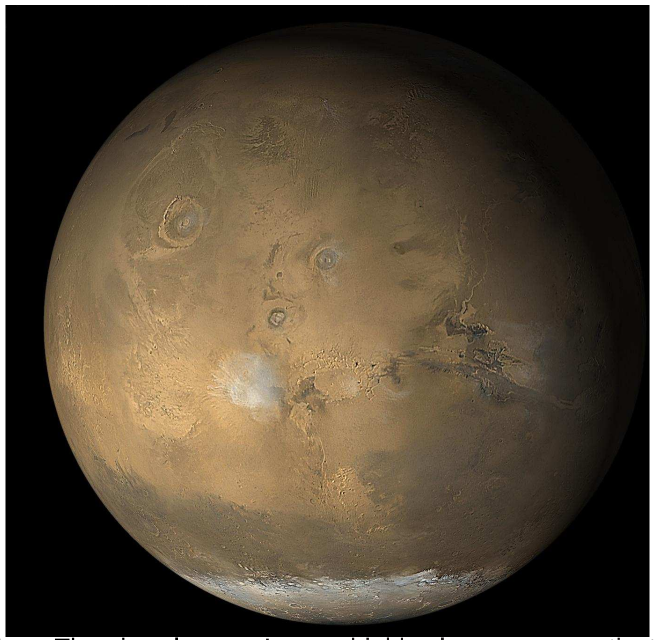
Evidence for plate tectonics (few craters!)



© Google

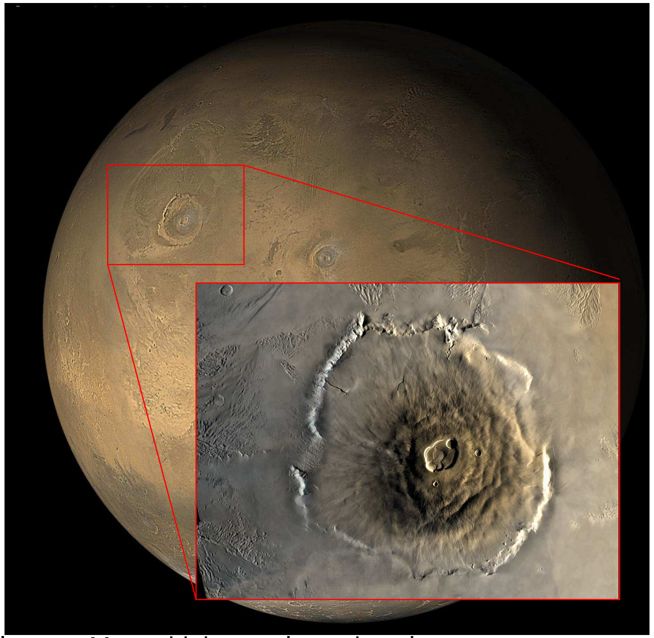


Evidence for plate tectonics (few craters!) , volcanism,. . .

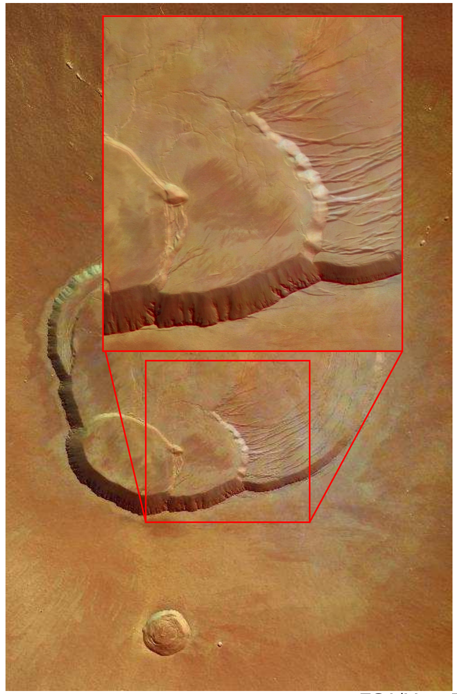


Mars: Tharsis vulcanos: Large shield vulcanos, now extinct

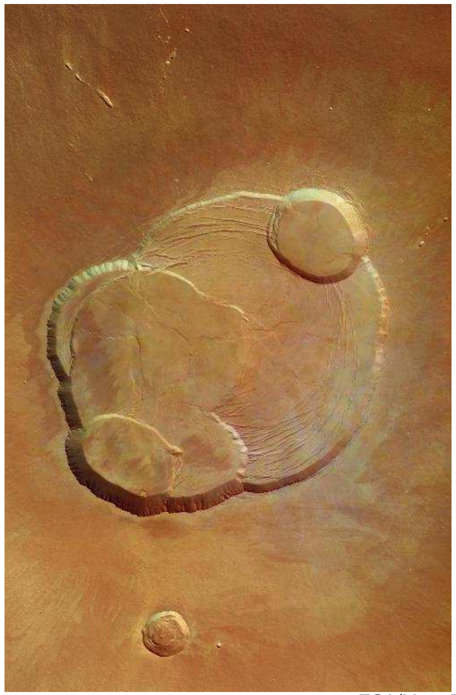
$\Longrightarrow$ no plate tectonics $\Longrightarrow$ Mars interior is colder than Earth.



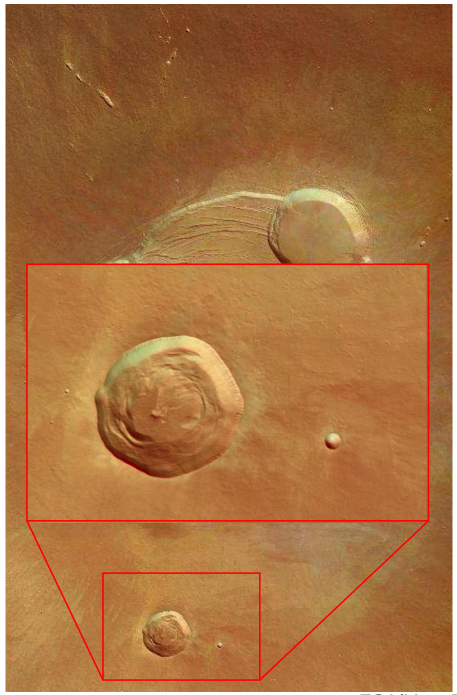
Olympus Mons: highest volcano in solar system
(25 km above surrounding plain; but slope only 2° to 5°).



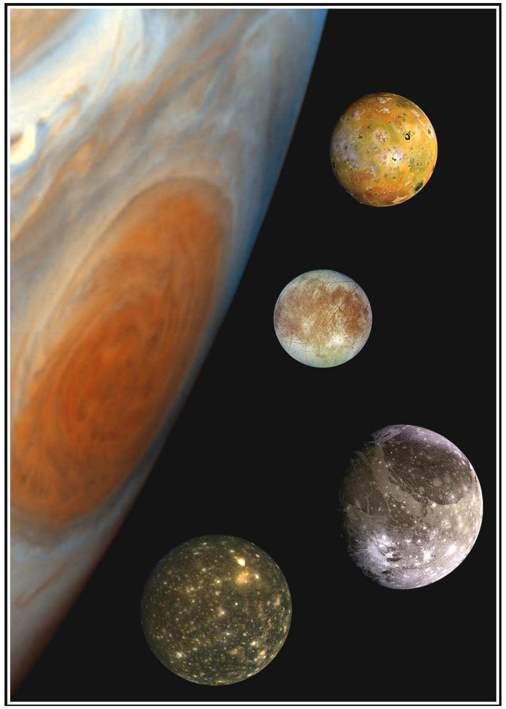
ESA/Mars Express, HRSC, 11.02.2004



ESA/Mars Express, HRSC, 11.02.2004

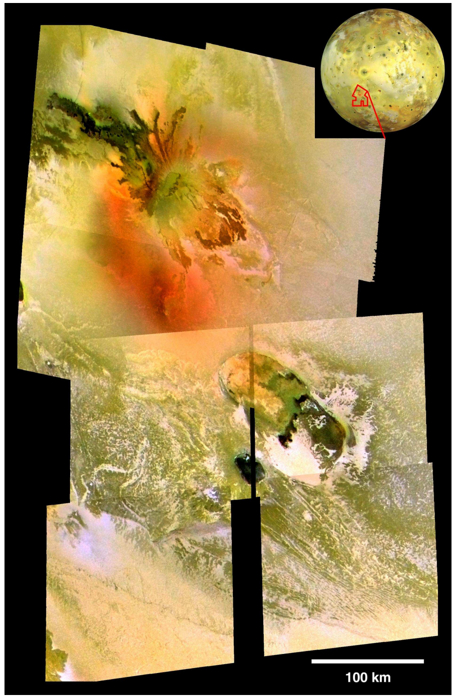


ESA/Mars Express, HRSC, 11.02.2004



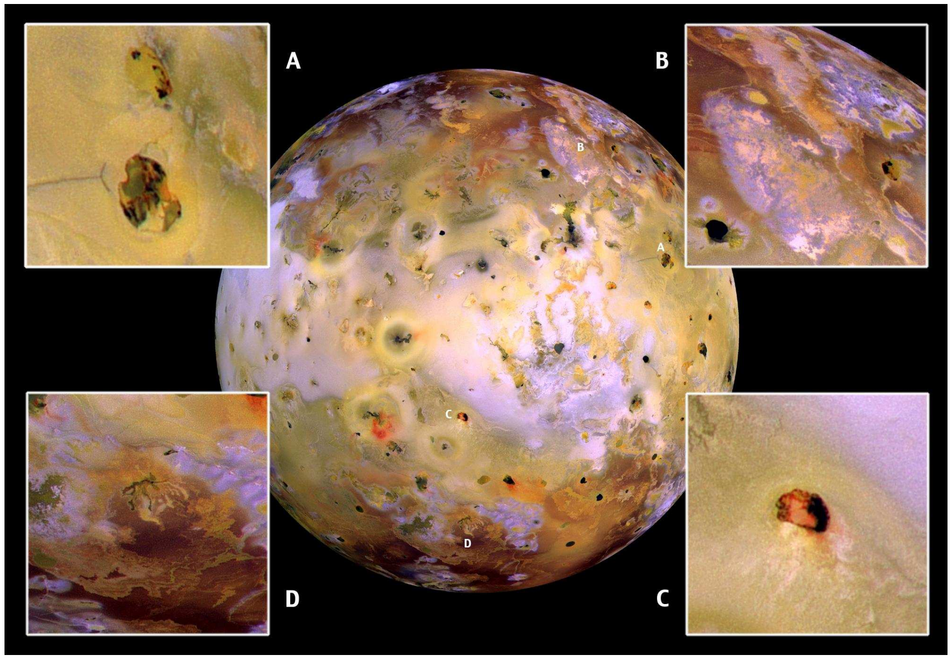
Montage of Jupiter and Galilean Moons: top to bottom: Io, Europa, Ganymede and Callisto.

(N.B.: All Galilean moons tidally locked to Jupiter – always same side is facing Jupiter)

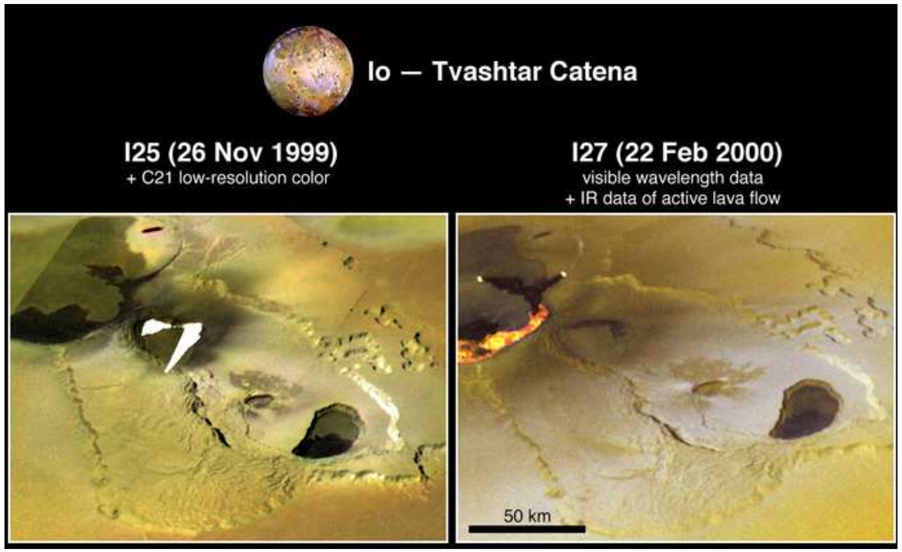


Active vulcanoes on Io (interior heated by tidal forces from Jupiter), color due to large contents of sulphur and sulphur oxides in lava.

Height of vulcanoes: 6 km or higher

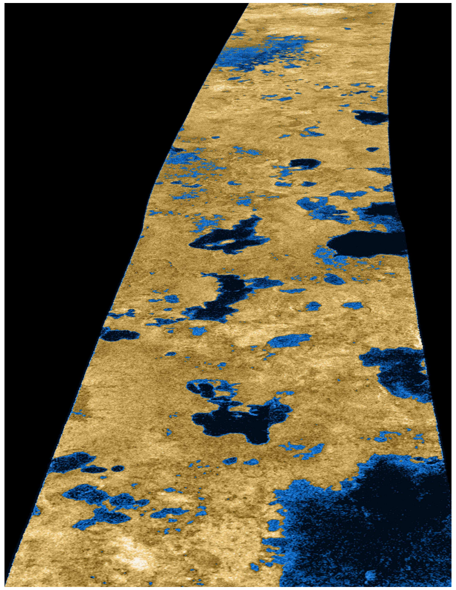


Jupiter's moon Io – the vulcano moon (Diam. 1821 km [Earth moon: 1738 km])

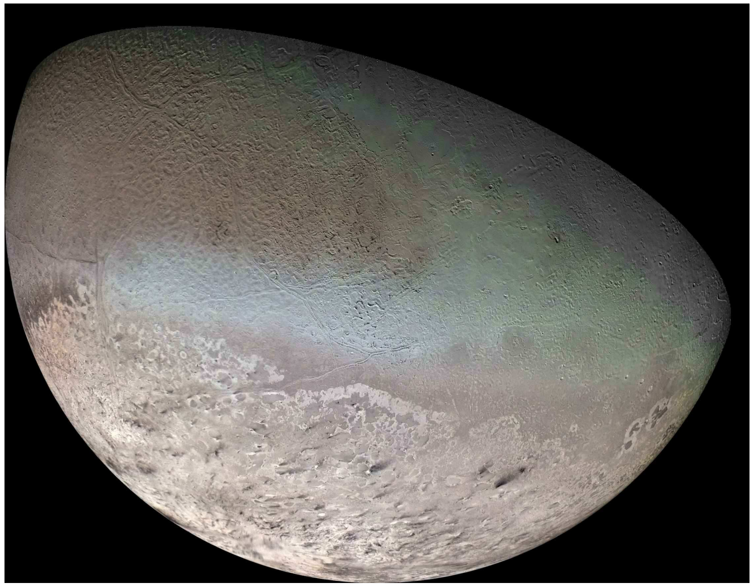


curtains of lava fountains [white: overexposed]
NASA Galileo, 1999 Nov 26

High temperature volcanism (2000 K; hotter than on Earth [1700 K]!)

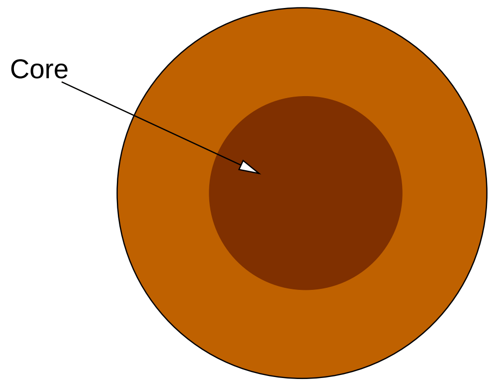

Cassini-Huygens, ESA/NASA/JPL

Saturn's Moon Titan:
Methane lakes.
Cryovolcanism (methane geysers)


NASA/Voyager 2/Calvin J. Hamilton

Neptune's Moon Triton:
ice cap of frozen methane (freezing point 90 K) and frozen nitrogen (freezing point 60 K).
Few impact craters $\Longrightarrow$ young surface $\Longrightarrow$ volcanism (dark spots: nitrogen geysers with $T \sim 70$ K)

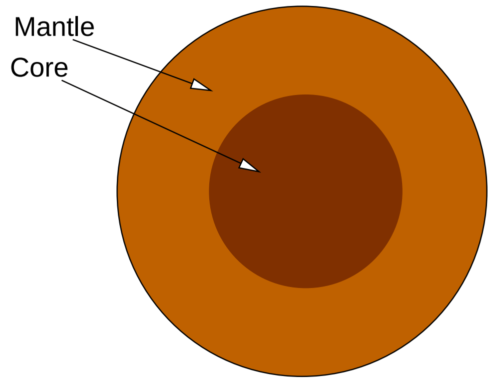
Interiors: Terrestrial Planets



Structure of terrestrial planets:

- Core: high-density material (Fe)

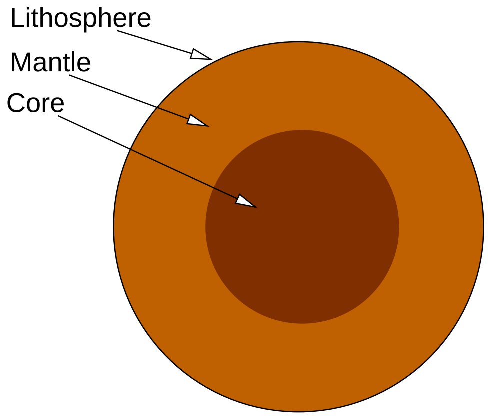
Interiors: Terrestrial Planets



Structure of terrestrial planets:

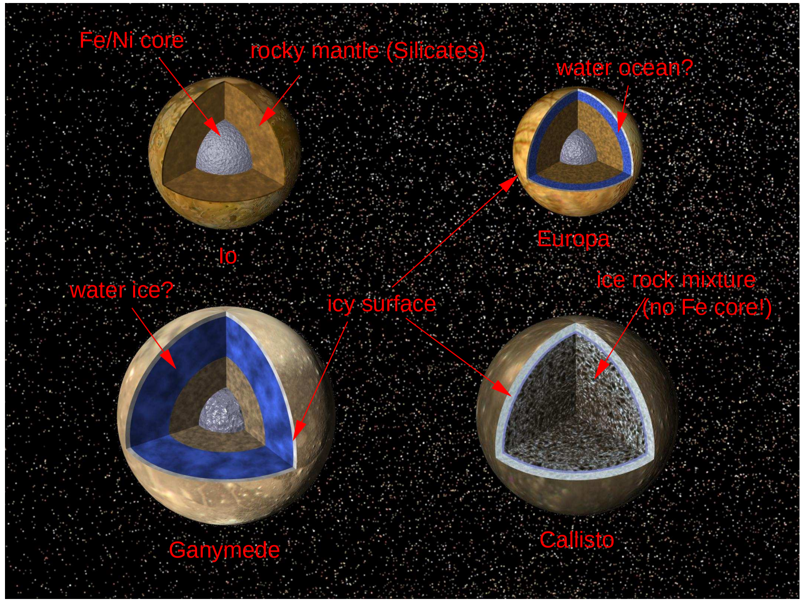
- Core: high-density material (Fe)
- Mantle: plastic materials, hot (e.g., Earth: molten rocks)

Interiors: Terrestrial Planets



Structure of terrestrial planets:

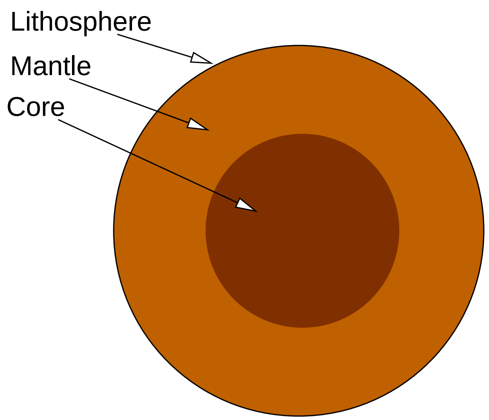
- Core: high-density material (Fe)
- Mantle: plastic materials, hot (e.g., Earth: molten rocks)
- Lithosphere: rigid material, e.g., Silicates



Structure of Jupiter's Galilean Moons similar to terrestrial planets (but some also have very thick ice layer on top)

6–48

Interiors: Terrestrial Planets

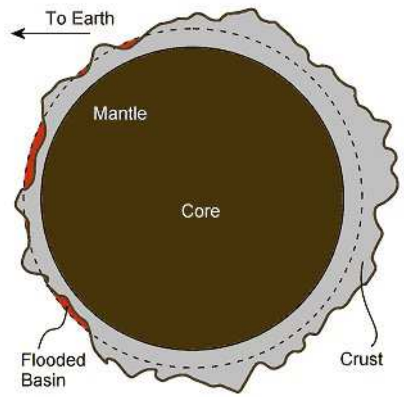


Structure of terrestrial planets:

- Core: high-density material (Fe)
- Mantle: plastic materials, hot (e.g., Earth: molten rocks)
- Lithosphere: rigid material, e.g., Silicates

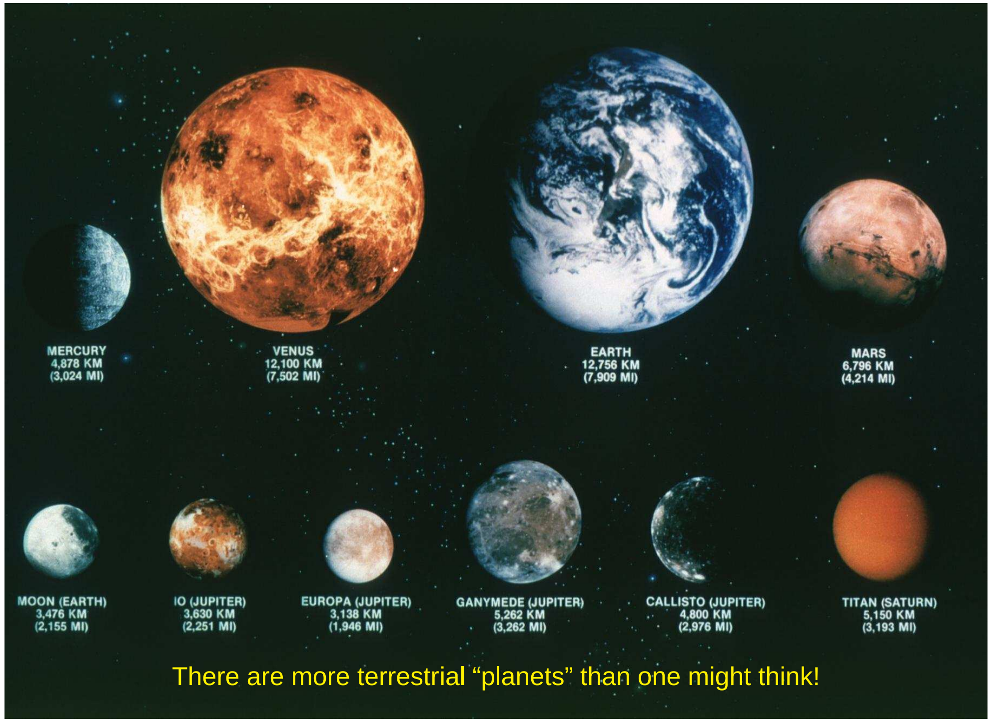
Knowledge of structure important for, e.g.,

- origin of magnetic fields (thought to be caused by molten core $\Longrightarrow$ currents $\Longrightarrow$ B-field ("dynamo"). Details unknown).
- atmospheric composition (molten mantle $\Longrightarrow$ volcanism $\Longrightarrow$ CO_2, CH_4, ...)



Structure of the Moon:

- gravimetric measurements
- seismometry from Apollo 12, 14, 15 & 16: mild moonquakes from 800–1000 km depth; generated by tidal forces?
- center of mass off-set by 2 km from center of sphere
- crust much thicker on the far side (100 km) than on the near side (60 km)
- iron core must be small ($<$400 km) if present (no magnetic field detected)



Structure: Gas Giants

In general, gas giants have very different properties from terrestrial planets:

- average density low, e.g.,
  - Jupiter: $\langle\rho\rangle \sim 1.3\,\mathrm{g\,cm^{-3}}$
  - Saturn: $\langle\rho\rangle \sim 0.7\,\mathrm{g\,cm^{-3}}$

  (compare to terrestrial planets: $\langle\rho\rangle \sim 5.5\,\mathrm{g\,cm^{-3}}$; water has $\rho = 1\,\mathrm{g\,cm^{-3}}$).
- elemental composition similar to stars (by mass):
  - 75% H
  - 24% He
  - 1% rest ("metals")

$\Longrightarrow$ expect fundamentally different internal structure!

Structure: Gas Giants

Structure of a gas giant from equation of hydrostatic equilibrium (like atmospheres!):

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\frac{GM(r)}{r^2} \tag{6.2}$$

To solve, need $\rho(r)$, $M(r) \Longrightarrow$ complicated, but doable if properties of material are known.

To guesstimate the central pressure, one can show for a planet of radius R (see writeup):

$$P_{\text{central}} = \frac{2\pi}{3}G\langle\rho\rangle^2 R^2 \tag{6.1}$$

Plug in numbers for Jupiter: $R = 70000\,\mathrm{km}$, $\langle\rho\rangle = 1.3\,\mathrm{g\,cm^{-3}}$, get $P_{\text{central}} = 1.2 \times 10^{12}\,\mathrm{Pa}$ ($10\times$ Earth).

At this pressure: existence of metallic hydrogen (i.e., electrons can move freely around).

More detailed computations: metallic hydrogen from 14000–45000 km away from center

To obtain information on the pressure structure of any gravitationally supported static body we can use the *concept of hydrostatic equilibrium*, which we also encountered when looking at atmospheric structure and which will also be important later for the structure of stars. As we have seen before, the radial pressure gradient is given by

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)g(r) \tag{6.2}$$

here, r is now the radial distance from the planetary center. In contrast to atmospheres, the acceleration g now depends on the position, $g = g(r)$. It is easy to show that

$$g(r) = \frac{GM(r)}{r^2} \tag{6.3}$$

where $M(r)$ is the mass of the planet contained within a radius r:

$$M(r) = \int_0^r 4\pi\rho(r)r^2\,\mathrm{d}r \tag{6.4}$$

(interpretation: integrate over onion shells of thickness $\mathrm{d}r$ and density $\rho(r)$; the mass in each of these shells is $4\pi\rho(r)\mathrm{d}r$, summing over all onion shells gives the above answer).

To solve the equation of the hydrostatic equilibrium one needs to know the equation of state, i.e., the pressure as a function of the parameters of the material. Unfortunately, this equation of state is generally much more complicated than for gases (where $P = nkT$) and often only roughly known. One can estimate, however, the order of magnitude for the pressure within a planet. In order to do so, we assume that the density is the same throughout the planet, and that it equals the planet's average density $\rho(r) = \langle\rho\rangle = \text{const.}$. This is o.k. to an order of magnitude. Under this assumption,

$$M(r) = (4/3)\pi r^3\langle\rho\rangle \tag{6.5}$$

such that the equation of hydrostatic equilibrium reads

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\langle\rho\rangle^2 G(4/3)\pi r \tag{6.6}$$

Differential equations looking like this are called separable. They can be solved "separation of variables", as we already did when computing the structure of an isothermal atmosphere.

First integrate both sides of the equation from $r = 0$ to the surface of the planet at $r = R$:

$$\int_0^R \frac{\mathrm{d}P}{\mathrm{d}r}\,\mathrm{d}r = -\int_0^R \langle\rho\rangle^2 G(4/3)\pi r\,\mathrm{d}r \tag{6.7}$$

To integrate the left hand side of the equation, substitute $r \longrightarrow P(r)$ where $P(r)$ is an unknown function (the pressure as a function of radius r). Luckily enough, we only need to know its values at $r = 0$ and $r = R$ (the "boundary conditions"). By definition of the surface of the planet, the pressure at $r = R$ will be $P(R) = 0$ to very good accuracy, while the pressure at $r = 0$ is the (unknown) central pressure, $P(0) = P_c$. Therefore

$$\int_0^R \frac{\mathrm{d}P}{\mathrm{d}r}\,\mathrm{d}r = P(R) - P(0) = -P(0) =: -P_c \tag{6.8}$$

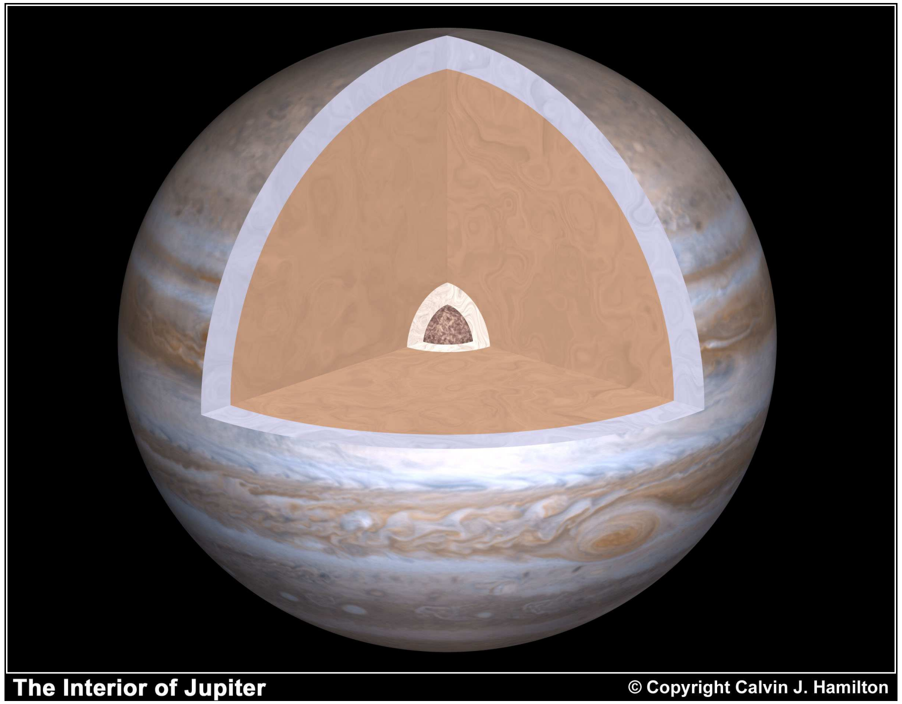
The right hand side of the equation is easily found as well:

$$-\int_0^R \langle\rho\rangle^2 G(4/3)\pi r\,\mathrm{d}r = -\langle\rho\rangle^2(4\pi/3)G\int_0^R r\,\mathrm{d}r = -\langle\rho\rangle^2(4\pi/3)GR^2/2 = -\frac{2\pi}{3}\langle\rho\rangle^2 R^2 \tag{6.9}$$

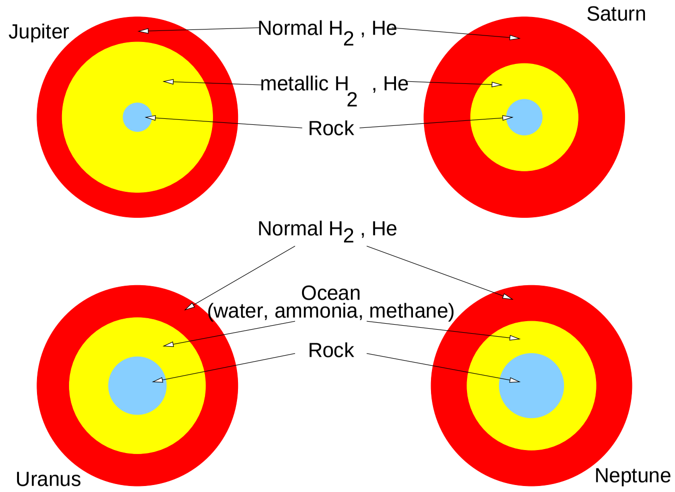
such that

$$P_c = \frac{2\pi}{3}\langle\rho\rangle^2 R^2 \tag{6.10}$$

As a rule of thumb, this formula gives central pressures that are correct to better than a factor of 10 compared to the detailed theory.



Structure: Gas Giants



Note: relative sizes of planets not to scale! Also rotational flattening not taken into account.