Introduction to Astronomy I

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Part I. The Solar System

1. The Planets

1.1. Overview

	Terrestrial	Giant
mean orbital distance	0.4-1.5 AU	5.0-30.0 AU
equatorial radius ($R_\oplus$)	0.4-1.0	3.9-11.5
mass ($M_\oplus$)	0.055-1.0	14.5-320
mean density in kg/m	3930-5520	690-1640
sideral rotation period	24h-243d	9.9h-17.2h
number of moons	0-2	13-63
ring system?	no	yes
surface temperature (K)	215-730	70-165

Eccentricities: almost zero, exceptions: Mercury 0.2 (+Pluto 0.2)
Inclination angles to the ecliptic: very small (Pluto 17°)
Def.: Planet:

- orbit around sun
- hydrostatic equilibrium (=roundshape)
- cleared neighborhood around its orbit

1.2. Celestrial Mechanics

1.2.1. Laws

Kepler's Laws:

1. Each planet moves around the sun in an ellipse, with the sun at one focus.
2. The radius vector from the sun to the planet sweeps out equal areas in equal intervals of time.
3. The squares of the periods of any two planets are proportional to the cubes of the semimajor axes of their respective orbits: $\mathrm{T} \sim \mathrm{a}^{\frac{3}{2}}$.

Law of Gravitation:

$$\mathbf{F}_1 = -\frac{G\, m_1\, m_2}{r_{12}^2}\,\frac{\mathbf{r}_{21}}{r_{12}} \tag{1}$$

where F_1 is the force exerted on object 1.

1.2.2. Properties of Ellipses

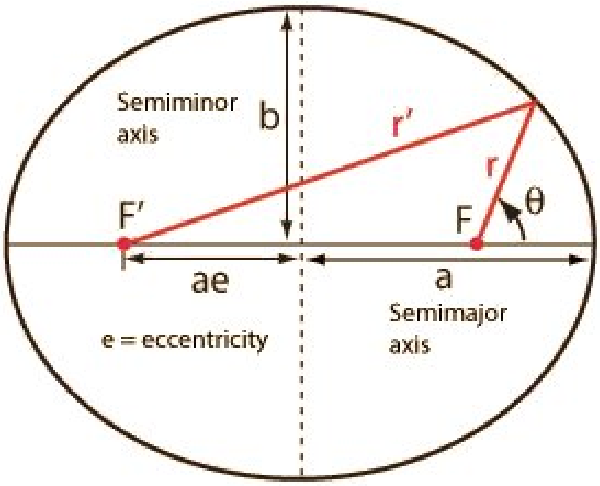
Def.: Ellipse:

$$r + r' = 2a \tag{2}$$

Def.: Eccentricity e:

$$e = \frac{a\, e}{a} \tag{3}$$

Ratio between distance from center of ellipse to focal point and semimajor axis.



Polar coordinate form:

$$r = \frac{a\,(1-e^2)}{1+e\,cos\theta} \tag{4}$$

where θ is called true anomaly.
For planets and comets/asteroids:

- nearest point from focus is called perihelion
- farthest point from focus is called aphelion

For stars:

- periastron
- apastron

Calculation:

$$\boxed{d_{peri} \;= a\,(1-e) \qquad \text{and} \qquad d_{ap} = a\,(1+e)} \tag{5}$$

1.2.3. Derivation of Kepler 2

In polar coordinates the radius vector is $\mathbf{r} = r\,\hat{\mathbf{e}}_r$. If the planet moves with angular velocity $\dot{\theta}$ the direction of $\hat{\mathbf{e}}_r$ changes also at the same rate:

$$\dot{\hat{\mathbf{e}}}_r = \dot{\theta}\,\hat{\mathbf{e}}_\theta \tag{6}$$

⇒ velocity of the planet:

$$\dot{\mathbf{r}} = \dot{r}\,\hat{\mathbf{e}}_r + r\,\dot{\hat{e}}_r = \dot{r}\,\hat{\mathbf{e}}_r + r\,\dot{\theta}\,\hat{\mathbf{e}}_\theta \tag{7}$$

⇒ angular momentum:

$$\mathbf{L} = \mu(\mathbf{r}\times\dot{\mathbf{r}}) = \mu\,r^2\,\dot{\theta}\,\hat{\mathbf{e}}_z \tag{8}$$

where $\hat{\mathbf{e}}_z$ is unit vector perpendicular to orbital plane.
⇒ Magnitude of $\mathbf{L}$:

$$L = \mu\,r^2\,\dot{\theta} \tag{9}$$

an infinitesimal area in polar coordinates is:

$$\mathrm{d}A = \mathrm{d}r\,(r\,\mathrm{d}\theta) = r\,r\mathrm{d}r\,\mathrm{d}\theta \tag{10}$$

⇒ integration from focus to distance r ⇒ area swept out when there is an infinitesimal change in θ is:

$$\mathrm{d}A = \frac{1}{2}\,r^2\,\mathrm{d}\theta \tag{11}$$

⇒ time rate of change:

$$\dot{A} = \frac{1}{2} r^2 \dot{\theta} \tag{12}$$

comparison with L:

$$\dot{A} = \frac{1}{2\mu} L \tag{13}$$

Since L is constant in a conservative force field the swept out area is constant!
Because the radius vector to the planet varies the velocity does so too. The planet is the fastest at perihelion and the slowest at aphelion.

1.2.4. Derivation of Kepler 3

$$\dot{A} = \frac{1}{2\mu} L \;\Leftrightarrow\; \mathrm{d}A = \frac{1}{2\mu} L \,\mathrm{d}t \tag{14}$$

To get the area swept over in one orbit integration is necessary:

$$\int \mathrm{d}A = \frac{1}{2\mu} L \int_0^P \mathrm{d}t \tag{15}$$

since the area of an ellipse is

$$\pi\, a\, b = \pi\, a^2 \sqrt{1-e^2} \tag{16}$$

⇒

$$\pi\, a^2 \sqrt{1-e^2} = \frac{1}{2\mu} L\, P \tag{17}$$

⇒ squared:

$$\pi^2\, a^4\, (1-e^2) = \frac{1}{4\mu^2} L^2\, P^2 \tag{18}$$

For a closed planetary orbit L is:

$$L = \mu \sqrt{G\, M\, a\, (1-e^2)} \tag{19}$$

where M is the total mass
⇒ Inserting L:

$$P^2 = \frac{\pi^2\, 4\, a^3}{M} \;\Leftrightarrow\; P^2 = \frac{4\,\pi^2}{G\,(m_1+m_2)}\, a^3 \tag{20}$$

if $m_1 \gg m_2$:

$$\boxed{\frac{P^2}{a^3} = \frac{4\,\pi^2}{G\, m_1}} \tag{21}$$

1.3. Long-Term Evolution

Numerical simulations show that the motion of the inner planets is chaotic.

1.4. Tidal forces

Not only the Sun has a gravitational force onto the Earth but also the Moon. Forces between Earth and Moon are called “Tidal Forces”.
The gravitational acceleration caused by a moon at the center of a planet is

$$a_{\text{center}} = \frac{GM}{r^2} \tag{22}$$

whereas the acceleration at the surface point of the planet closest to the moon is

$$a_{\text{closest}} = \frac{GM}{(r - R_{\oplus})^2} \tag{23}$$

such that there is a difference in acceleration:

$$\Delta a = a_{\text{closest}} - a_{\text{center}} = \frac{GM}{(r-R_\oplus)^2} - \frac{GM}{r^2} = \frac{GM}{r^2}\left(\frac{1}{(1-\frac{R_\oplus}{r})^2} - 1\right) \sim \frac{GM}{r^2}\left(1 + 2\frac{R_\oplus}{r} - 1\right) = \frac{2GMR_\oplus}{r^3} \quad (24)$$

Therefore the tides due to the Moon and Sun are

$$\Delta a_{☾} = \frac{2GM_{☾} R_\oplus}{r_{☾}^3} \quad (25)$$

and

$$\Delta a_\odot = \frac{2GM_\odot R_\oplus}{(1\,\text{AU})^3} \quad (26)$$

With $M_{☾} \sim M_\oplus/81$ and $r_{☾} \sim 60 \times R_\oplus$ we get

$$\frac{\Delta a_\odot}{\Delta a_{☾}} = 0.46 \quad (27)$$

→ The Moon is twice as important as the Sun, there are two sets of tidal bulges and there are spring tides.

1.5. Atmospheres: The Hydrostatic Equilibrium

Force of gas (above the area) with ρ on an area A is given by:

$$F = m \cdot g = A \cdot h \cdot \rho \cdot g \Rightarrow \text{Pressure: } p = \frac{F}{A} = \rho \cdot h \cdot g \quad (28)$$

for thin atmospheres $g = \text{const.}!$ →p decreases when going upwards by Δh:

$$\Delta p = -\rho \cdot g \cdot \Delta h \rightarrow \text{infinitesimal} \lim_{\Delta h \to 0} : \frac{\mathrm{d}p}{\mathrm{d}h} = -\rho \cdot g \quad (29)$$

because density and pressure aren't independent; need relationship between density and pressure ("equation of state").
→"Ideal Gas":

$$p = \frac{\rho}{\mu} \cdot k \cdot T \quad (30)$$

where μ is the average mass of a gas particle.

$$\rightarrow \frac{\mathrm{d}p}{\mathrm{d}h} = -\left(\frac{\mu\, g}{k\, T}\right) \cdot p \quad (31)$$

at the beginning $h = 0$ and $p = p_0$. Assumption: $T = \text{const.}!$
Seperation of Variables:

$$\int_{p_0}^{p(h)} \frac{1}{p}\mathrm{d}p = -\int_0^h \left(\frac{\mu\, g}{k\, T}\right)\mathrm{d}h \quad (32)$$

$$\rightarrow \ln\left(\frac{p(h)}{p(0)}\right) = -\frac{\mu\, g}{k\, T} \cdot h \quad (33)$$

$$\boxed{p(h) = p_0 \cdot e^{-\left(\frac{\mu\, g}{k\, T}\cdot h\right)} = p_0 \cdot e^{\left(-\frac{h}{H}\right)} \text{with } H = \frac{k\, T}{\mu\, g}} \quad (34)$$

H is called scale height ($H_\oplus \approx 9$km).
⇒ only for isothermal atmospheres + ideal gases

1.6. The Terrestrial Planets

1.6.1. Mercury

- rotation period:orbital period = 3:2 resonance ⇒ because doesn't obey Kepler's Laws corretly due to distance to Sun
- pocked with craters but large smooth plains in between (→ lava flows) e.g. Caloris Basin 1300km diameter → large impact event → other side: hilly, jumbled area because of seismic waves
- not much larger than the moon (a bit similar but larger smooth areas)
- densiest planet → 3/4 of diameter iron
- no atmosphere
- rotation + iron core + not a lot of liquid metal (otherwise stronger B-field) ⇒ weak magnetic field

1.6.2. Venus

- similar to earth (size, mass, average density...)
- very slow rotation ; retrograde → no B-field
- surface temperature very high because IR-waves get locked in + high reflection of atmosphere (96.5 % CO_2, 3.5% N_2) strong Greenhouse effect → surface temperature ≈ 460° C
- 90x higher surface pressure than at Earth
- acid rain (H_2SO_4)
- few craters because of strong erosion (acid!)
- young surface → probably volcanism just "short" time ago (lava flows)
- observation: mainly radar

1.6.3. Earth and Moon

Erde

- double system
- surface: almost no craters → dominated by plate tectonics , erosion, volcanos
- atmosphere: 80% N_2, 20% O_2 → moderate Greenhouse effect → T > 0° C
- water
- varying B-field

Moon

- very similar to Mercury
- rotation synchronous to orbit around Earth
- Mariae (dark-colored, smooth surfaced ← plains from massive impacts)
- lots of impact craters of all sorts ⇒ history of Moon/Solar System can be seen in craters, no changes since Moon became rigid
- some craters have rays
- far side of the Moon: few Mariae because of thicker crust, one large crater ⇒ otherwise more smaller craters

1.6.4. Mars

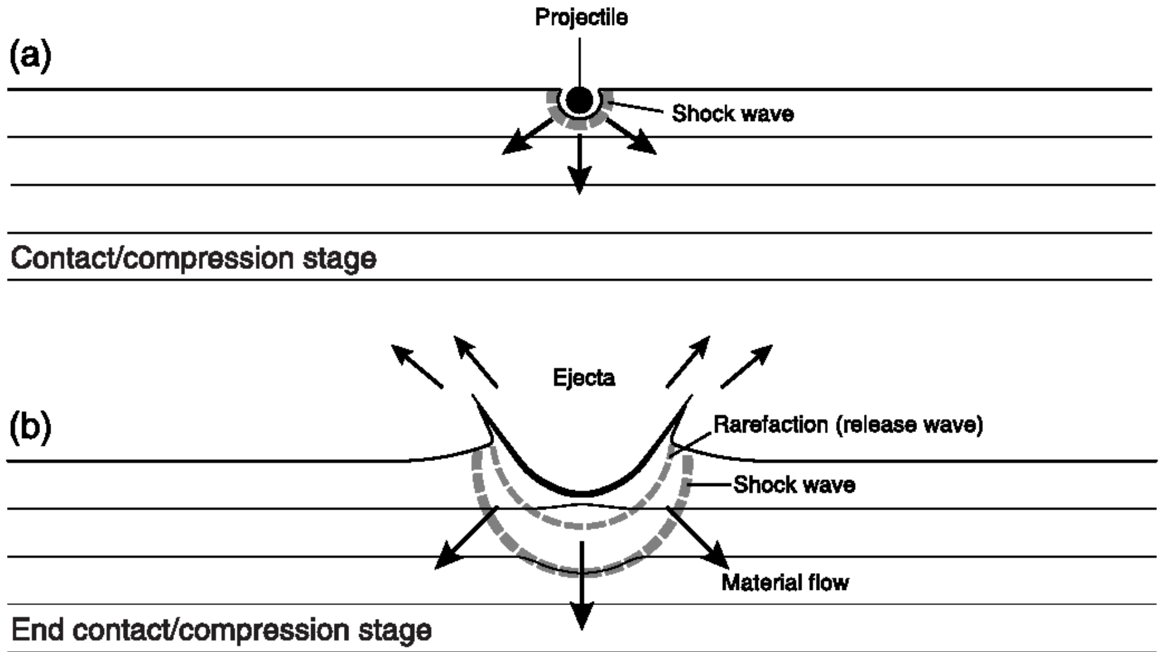
- smaller than Earth
- Polar Caps (dry ice) → seasons because of 25° tilt → ice sublimates in summer and freezes again in winter (but: small water ice cap remains frozen)
- thin atmosphere: 95% CO_2 → weak Greenhouse effect (surface pressure 1% of Earth surface pressure)
- very low density → small or no core of Fe → no B-field
- water sublimates → no liquid water
- two moons (captured asteroids)
- Olympus Mons (highest mountain in Solar System: 24 km) → shield volcano (all volcanos extinct)
- no plate tectonics
- sometimes dust storms cover whole planet → some are seasonal → erosion

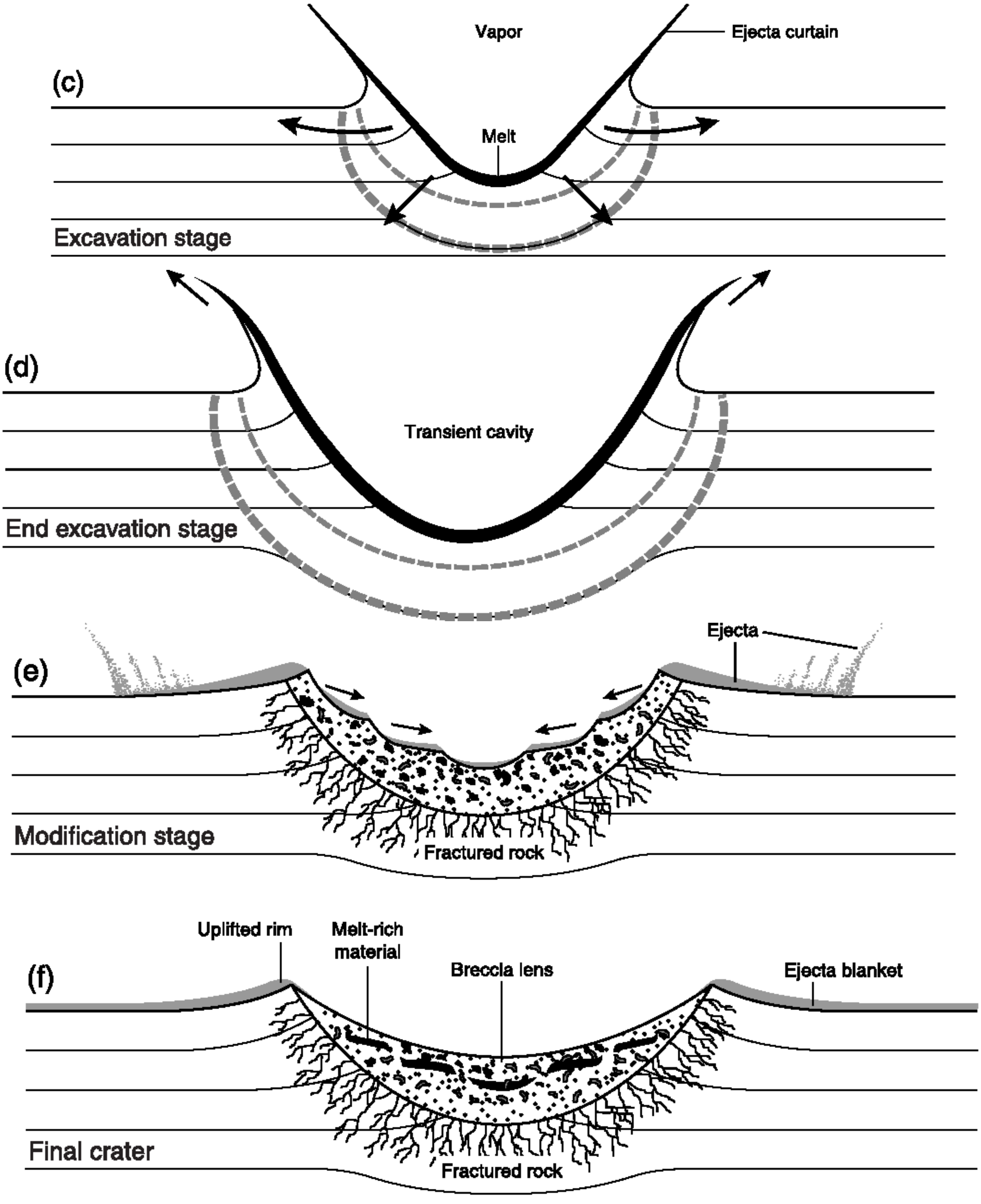
1.6.5. Crater Formation

Kinetic energy:

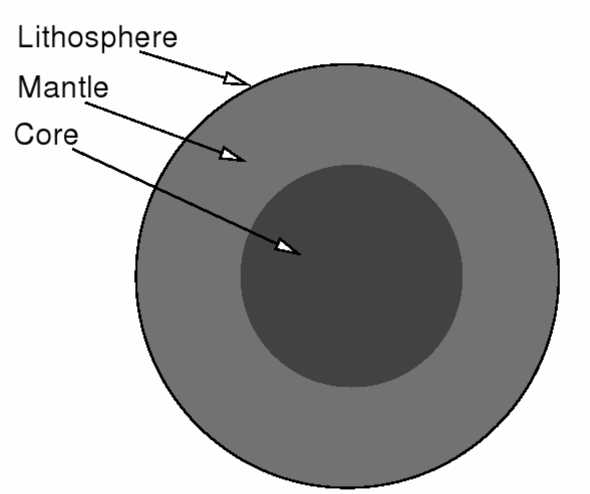
$$E = \frac{1}{2}m \cdot v^2 = \frac{1}{2}\left(\frac{4}{3}\pi r^3 \rho\right) \cdot v^2 = \frac{2\pi d^3 \rho v^2}{3} \tag{35}$$

Important values: velocity of impact, structure, size of body.
Process of Crater Formation:




Vapor
Ejecta curtain
(c)
Melt
Excavation stage
(d)
Transient cavity
End excavation stage
Ejecta
(e)
Modification stage
Fractured rock
Uplifted rim
Melt-rich
material
Breccia lens
Ejecta blanket
(f)
Final crater
Fractured rock

1.6.6. Interiors of the Terrestrial Planets



Core: high density (Fe); Mantle: plastic materials, hot (e.g. Earth: molten rocks); Lithosphere: rigid material, e.g. silicates.

1.7. The Jovian Planets

All have Rings!
Composition ≈ stars : 75% H, 24% He, 1% "metals"

1.7.1. Jupiter

- largest planet
- rapid motion ⟶ flattend, banded atmosphere
- differential rotation (equator: slower rotation)
- strong B-field
- 4 "Galilean" moons

1.7.2. Saturn

- see Jupiter
- Rings!
- six major moons

1.7.3. Uranus

- cold atmosphere ⟶ frozen ammonia
- less He than Gas Giants
- inclination of rotation axis : 98° ("rolling on the ecliptic plane")
- five major moons

1.7.4. Neptune

- atmosphere like Uranus but more active , bright methane clouds + cloud layers
- 2 major moons (Triton, Nereid)
- dark spot → new spot

1.7.5. Atmospheres

- Jupiter:
  - 3 layers: Ammonia, ammonia hydrosulfide, water ice/ water
  - colors from organic stuff → banded atmosphere
  - darker= deeper + hotter spots
  - storms: Great Red Spot (stable)
- Saturn: Atmospheres deeper than at Jupiter + not that dynamic
- Ice Giants:
  - H + He + a few % methane → absorbs red light → blueish color
  - banded atmosphere + clouds

1.7.6. Magnetic fields

differential rotation + metallic hydrogen $\Longrightarrow$ B-field $\Longrightarrow$ synchrotron radiation $\Longrightarrow$ strong radio emission

1.7.7. The Interiors of the Gas Giants: Hydrostatic Equilibrium

Estimation for supported material:

$$\boxed{\frac{\mathrm{d}p}{\mathrm{d}r} = -\rho(r) \cdot g(r)} \tag{36}$$

with: r = radial distance to center and $g(r) = \frac{G \cdot M(r)}{r^2}$.
Where $M(r)$ is the mass of the planet within r:

$$M(r) = \int_0^r 4\pi\, \rho(r)\, r^2\, \mathrm{d}r \tag{37}$$

(=summing up all shells)
⇒ now "equation of state" is needed → BUT: too complex
⇒ Assumption: $\rho(r)$ = const. = $\bar{\rho}$

$$\Rightarrow M(r) = \frac{4}{3}\pi\, r^3\, \bar{\rho} \tag{38}$$

$$\Rightarrow \frac{\mathrm{d}p}{\mathrm{d}r} = -\bar{\rho}^2\, G\, \frac{4}{3}\pi\, r \tag{39}$$

$$\Rightarrow \mathrm{d}p = -\bar{\rho}^2\, G\, \frac{4}{3}\pi\, r\, \mathrm{d}r \tag{40}$$

boundary conditions: $r = 0$, $r = R$, $p(R) = 0$, p_C=center pressure

$$\int_0^R \mathrm{d}p = p(R) - p(0) = 0 - p_c = -p_c \tag{41}$$

$$\Longleftrightarrow -\int_0^R \bar{\rho}^2\, G\, \frac{4}{3}\pi\, r\, dr \;=\; -\bar{\rho}^2\, G\, \frac{2}{3}\pi\, (R^2 - 0) = \;= -\bar{\rho}^2\, G\, \frac{2}{3}\pi\, R^2 \tag{42}$$

$\Longrightarrow$

$$\boxed{p_\mathrm{C} = \bar{\rho}^2\, G\, \frac{2}{3}\pi\, R^2} \tag{43}$$

(≈ factor 10 wrong)

1.7.8. The Moons of the Giants

The Galilean Moons All moons show the same face to Jupiter (bound rotation). They are bild-up similar to terrestrial planets + ice.
Io and Europa: moonsize; Ganymede and Callisto: Mercury sized

Io

- colorful sulfur layer deposited by explosive eruptions from volcanic vents ($\longrightarrow$ somewhat like terrestrial geysers) $\Longleftarrow$ because: interior is heated by tidal forces with Jupiter (gets flexed) $\rightarrow$ volcanism
- very high temperatures!
- because of movement in B-field $\rightarrow$ radio emission

Europa

- composed of rock + covered with smooth ice layer
- no craters but cracks and riffled crust $\rightarrow$ volcanism in the past
- $\longrightarrow$ possibly: water ocean below ice (because of internal heat $\leftarrow$ tidal forces)

Ganymede

- icy surface:
  - areas of dark ancient cratered surface
  - young, heavily grooved, lighter-colored terrain
- probably metallic core $\leftarrow$ because: has strong magnetic field

Callisto

- pocked with craters, no geologic activity and tidal heating
- covered with dark,dusty substance
- no Fe-core!

Titan

- Saturn's largest satelite
- terrestrial structure
- dense atmosphere: 99% N_2, 1% CH_4, some hydrocarbons $\rightarrow$ perhaps: similar to early atmosphere of Earth

Triton

- moon of Neptune
- young icy surface $\rightarrow$ volcanism (not a lot craters) $\rightarrow$ nitrogen geysers
- frozen N_2 + ice cap of frozen methane
- strange orbit (retrograde)
- nitrogen atmosphere (very thin)

2. Small Solar System Bodies (SSSBs)

2.1. Asteroids

- minor planets: between Mercury and Neptune (esp. between Mars and Jupiter)
- Trojans, Greeks in Langrangian Points of Jupiter and Sun
- Types of Asteroids:
  - S-type: 1/6 of known ones (silicaceous) (2 –3.5 AU)
  - M-type: 8% iron and nickel dominated (metals) (2 –3.5 AU)
  - C-type: 75% carbonaceous (2 –4 AU)
  - also P, D (=Trojans)
- special gaps in asteroid belt called Kirkwood Gaps because of special resonances (orbits not very stable)
- diameter < 1000km

2.2. Comets

Components:

- Nucleus: "Dusty Snowball" (1 –50 km) water ice + 15 –20% CO_2, CO
- Coma: 10^4 –10^5 km evaporated gas surrounding nucleus → interacts with sunlight and solar winds → produces long, familiar tailes (up to 1 AU length)
  Coma is surrounded by hydrogen gas (halo) envelope ($\approx 10^{10}$ m diameter)
- Tail consists of 2 parts:
  - Dust Tail: evaporated dust away from nucleus; size $\approx 10^6$-10^7 km; behind comet slightly affected by centrifugal forces
  - Ion Tail: ionized gas, extends up to 10^8 km, often blueish; moves perpendicular to direction of movement

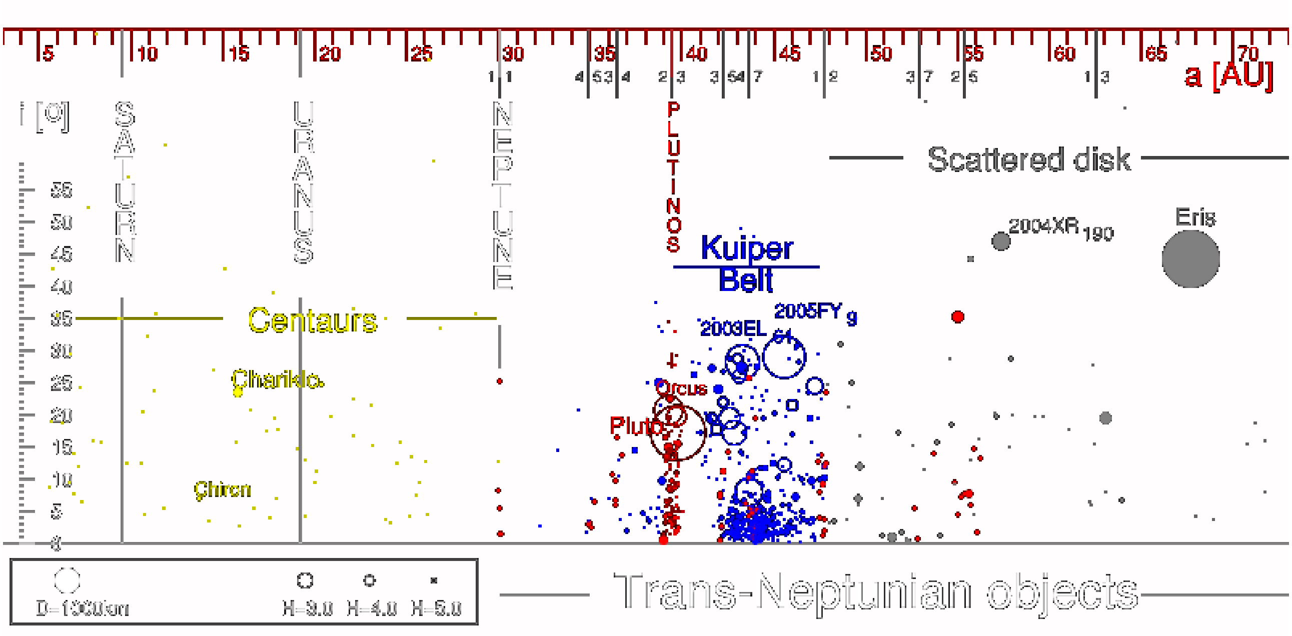
Examples:

- Halley: short-term comet
- Shoemaker-Levy: was in orbit around Jupiter; broke apart ⇒ fell onto Jupiter
- Sungrazers: loose material by sublimation processes when being to near to the Sun

Long-Period Comets: $P \geq 200$ yrs have very eccentric orbits ⇐ originate out of Oort cloud → come inwards because of interaction with bypassing stars
Short-Period Comets: $P < 200$ yrs have angular momenta like planets; mostly in plane of Solar System → come from Kuiper Belt (30 –50 AU)

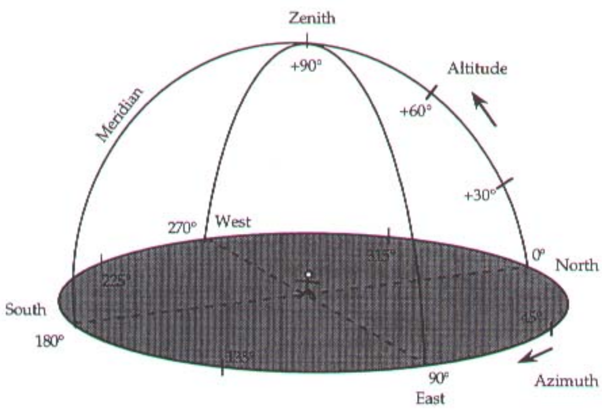
2.3. Trans-Neptunian Objects (TNOs)

Prototypes: Pluto/Charon: icy surface which is probably cratered. TNOs are further out than Neptune.



Part II. Coordinates

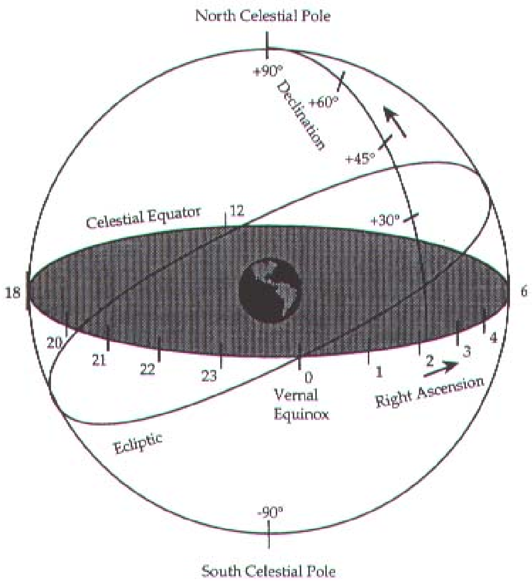
3. Horizon System



Altazimuth Coordinate System

For observing stars it's necessary to know only the position on the celestial sphere and not their real position in the universe. The easiest system is the horizontal system. The point directly above the observer is called the zenith (the one on the other side of the Earth nadir).
The reference frame for the horizon system is the tangent plane of the Earth passing through the observer. The plane is perpendicular to the line zenith-nadir.
The first coordinate in this system is the **altitude** h which is the angle from horizon towards zenith (Range:[-90°;+90°]). The zenith distance is $z = 90° - h$.
The second coordinate is the **azimuth** A which is the horizontal angle from a fixed direction. Usually S-W-N-E but can be different (Range:[0°;360°]).
Problem: depends on time of observation and position of observer

4. Equatorial System



Equatorial Coordinate System

Because of the rotation of Earth the stars seem to revolve around the celestial pole. The altitude of the pole over the horizon equals the latitude ϕ of the observer. The plane perpendicular to the rotion axis of Earth is called equatorial plane. The intersection of the equatorial plane and the celestial sphere is called **equator of the celestial sphere** or **celestial equator**. (It remains almost constant because the rotaion axis remains almost constant.)
The angle from the celestial equator to the star is called **declination** δ and is measured in degrees(Range: [-90°;90°]). The second coordinate is the **right ascension** α which is the angle from vernal equinox to the star (measured in eastern direction or counterclockwise). The vernal equinox or ascending node is the intercept point of the ecliptic (apparent path of Sun in the sky) and celestial equator. Right ascension is measured in sideral time. 24h of sideral time correspond to one rotation of the celestial sphere (1h $\hat{=}$ 15°). 0h sideral time is the moment when vernal equinox passes trough meridian.
It is usefull to define another number called the **hour angle** t which is the distance of the object from the meridian (measured clockwise). The hour angle of the vernal equinox is called **local sideral time** Θ (is measured clockwise/to the West).
It can be calculated by:

$$\Theta = t + \alpha \tag{44}$$

Sideral Time is not the same as normal time!
Another Problem: Rotational Axis of the Earth is not stable because of lunisolar precession (Earth's axis rotates around pole of ecliptic once/25800 yrs) and nutation ("wooble" due to moon and Sun around 18 yrs period).
$\Longrightarrow$ need to note epoch for coordinates (normally 1950.0, 2000.0)

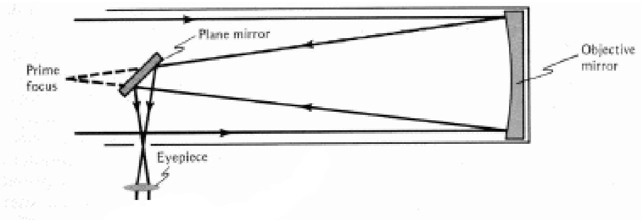
Part III.
Telescopes

5. Types

5.1. Refractors or Reflectors

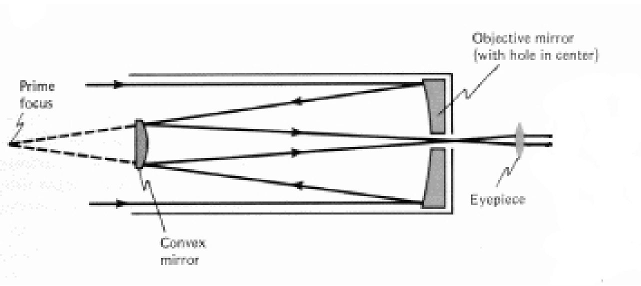
A) Lenses = Refractors: old fashioned way, because: max. diameter $\leq$ 2m due to weight $\rightarrow$ can't be supported
B) Mirrors = Reflectors: diameters up to 11m, use of parabolic mirrors (spherical aberration $\leftarrow$ would need correction)

5.2. Newtonian Telescope



Parabolic mirror; common cheaper ones; $\ominus$ large size ($\approx$ focal length)

5.3. Cassegrain Telescope



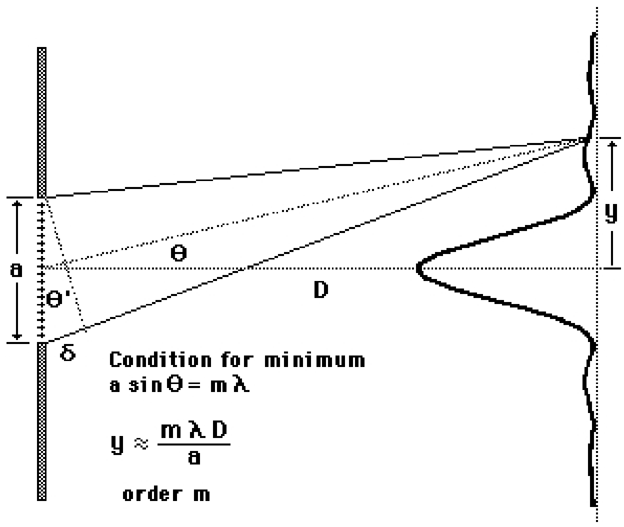
"Folded optical path" $\Rightarrow$ smaller=shorter $\Rightarrow$ Telescope of choice today

5.4. Schmidt Telescope

spherical mirror; mostly for: wide angle; needs corrector plate

6. Resolution

Interference occurs due to light waves:



At a telescope rings → bright maximum in the middle called Airy disk!
⇒ Rayleigh criterion: maximum of diffraction pattern of one source must fall into minimum of diffraction pattern of other source. Therefore the diffraction limited resolution is:

$$\boxed{\alpha = \frac{1.220\,\lambda}{d} \qquad \alpha_{\text{opt}} = \frac{12''}{D/1\text{cm}}} \tag{45}$$

where α is the minimum angle. Results are better if telescope is bigger or wavelengths are shorter!
In reality: 3× better is achievable

7. Adaptive Optics

Adaptive Optics means correcting the atmospheric turbulences by adapting the mirror with small "rubbers" behind it. Because of turbulences it's not possible to see stars with less than $\theta \geq 0.3''$ (stars → disks). So if telescope diameter is bigger than 40cm resolution doesn't increase automatically. Adaptive Optics improves astronomical seeing but works on Earth only in IR (→ optical and UV observations better in space). For knowing how to change you need a guide star or an artificial laser.

8. Active Optics

For large mirrors it's necessary to have active optics which corrects the mirror into perfect shape. This needs to be done to keep focus properly.

Part IV. Stars

9. Observations

9.1. Distances and Proper motion

Parallax measurement (several times over year): parallax angle (small-angle approximination):

$$p = \frac{1AU}{d} \tag{46}$$

The parsec (pc) is the distance at which 1 AU subtends 1".

$\boxed{1 \text{ pc} \approx 3.26 \text{ ly}}$
If p is known in arcsecs then distance:

$$d = \frac{1}{p} \tag{47}$$

best parallax measurements:
Hipparcos satellite: 120 000 objects in mas errors ($B -$ V,V $-$ J)
Tycho catalogue: 10^6 stars with 20 –30mas precision (2 band photometry)
Direct distant measures $\approx$ 1kpc
Accuracy $\approx$ 0.01" Earth, 1 mas space
But: Gaia will bring better results

9.2. Brightness and Luminosity

9.2.1. Luminosity

Def.: Luminosity: The total energy emitted by a star per second is called its luminosity.

$$L_{\odot} = 3.9 \cdot 10^{26}\text{W} \tag{48}$$

Assumption: isotropic radiation
$\Rightarrow$ Flux (inverse square law):

$$F = \frac{L}{4\pi\, r^2} \tag{49}$$

Star fluxes are very small $\approx 10^{-8}$

9.2.2. Magnitudes

$\boxed{\text{A brightness difference of 5 magnitudes corresponds to a ratio of 100 in detected flux.}}$
Two stars have magnitudes m_1 and m_2:

$$\frac{f_1}{f_2} = 100^{(m_2 - m_1)/5} \tag{50}$$

$$\Rightarrow \log\left(\frac{f_1}{f_2}\right) = \frac{m_2 - m_1}{5} \log_{10} 100 = \frac{2}{5}(m_2 - m_1) \tag{51}$$

or

$$\boxed{m_2 - m_1 = 2.5 \log_{10}\left(\frac{f_1}{f_2}\right) = -2.5 \log_{10}\left(\frac{f_2}{f_1}\right)} \tag{52}$$

LARGER MAG = FAINTER STAR!!!
$\Rightarrow$ Inverse square law links different distances to magnitudes:

$$\frac{F}{f} = \frac{L/(4\pi\, D^2)}{L/(4\pi\, d^2)} = \left(\frac{d}{D}\right)^2 \tag{53}$$

$\Rightarrow$ Absolute magnitude M (distance= 10pc):

$$\Rightarrow \boxed{m - M = 2.5 \log_{10}\left(\frac{F}{f}\right) = 2.5 \log_{10}\left(\frac{d}{10pc}\right)^2 = 5 \log_{10} d - 5} \tag{54}$$

$m - M \mathrel{\hat{=}}$ distance modulus;
(Sun: -26.7 full moon: -12.6 naked eye limit: $+6.0$ best achievable: $+30$)

9.3. Temperature and Spectrum

9.3.1. Planck's Radiation Law

approximately: thermodynamic equilibrium → Max Planck's Blackbody Radiation:

$$\boxed{F_\lambda = \frac{2\,\frac{h\,c^2}{\lambda^5}}{e^{\frac{h\,c}{\lambda\,k\,t}}-1}} \tag{55}$$

(F_λ is the energy emitted per second and wavelength interval)
Stefan-Boltzmann Law: Power emitted per 1m^2 surface of blackbody

$$\boxed{P = \sigma\,T^4} \tag{56}$$

(hotter body → higher luminosity)
Wien's displacement law: maximum blackbody radiation

$$\boxed{\lambda_{max}\,T = 2.898 \cdot 10^{-3}\,m \cdot\, K} \tag{57}$$

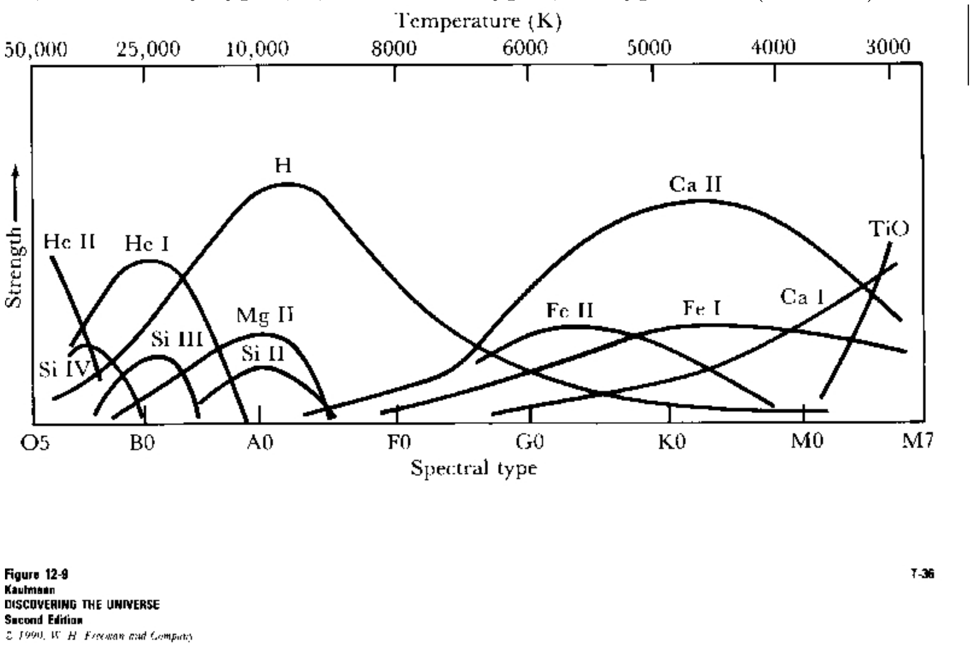
(hotter → peak at shorter wavelength)

9.3.2. Spectroscopy

blackbody → atmosphere: absorbing → line spectrum (Sun: Fraunhofer lines):
Spectral types are a temperature sequence!
Spectral classes:

O B A F G K M

30,000 K "early type", 3,000 K "late type", subtypes 0...9 (sun: G2)



L, T - Stars: Brown dwarfs
L-dwarfs:

- T = 1200K-2500K
- low mass
- some haven't fusion

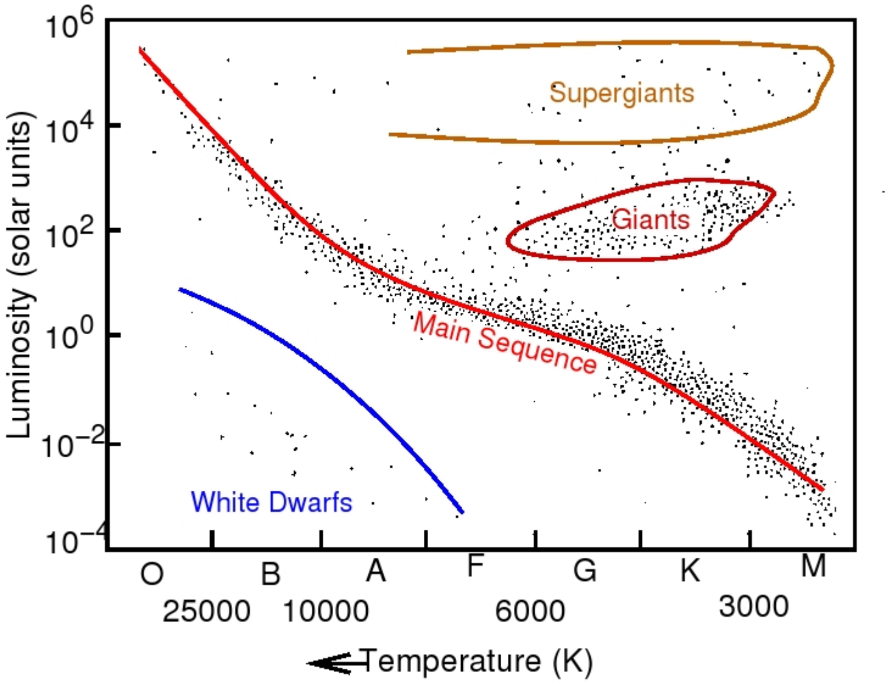
- peak in IR
- optical: prominent lines from metal hybrides and alkalimetals

T-dwarfs:

- $T \approx 100$ K
- strong molecule lines such as methane

9.4. Hertzsprung Russell Diagram (HRD)

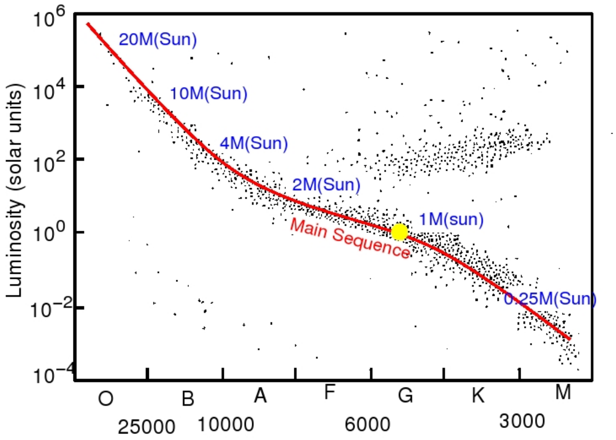
Stellar temperature (or color index or spectral class) vs. stellar luminosity (or absolute magnitude)

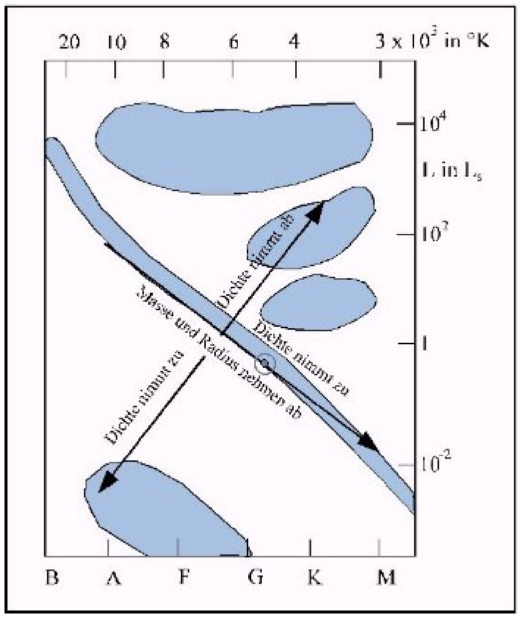


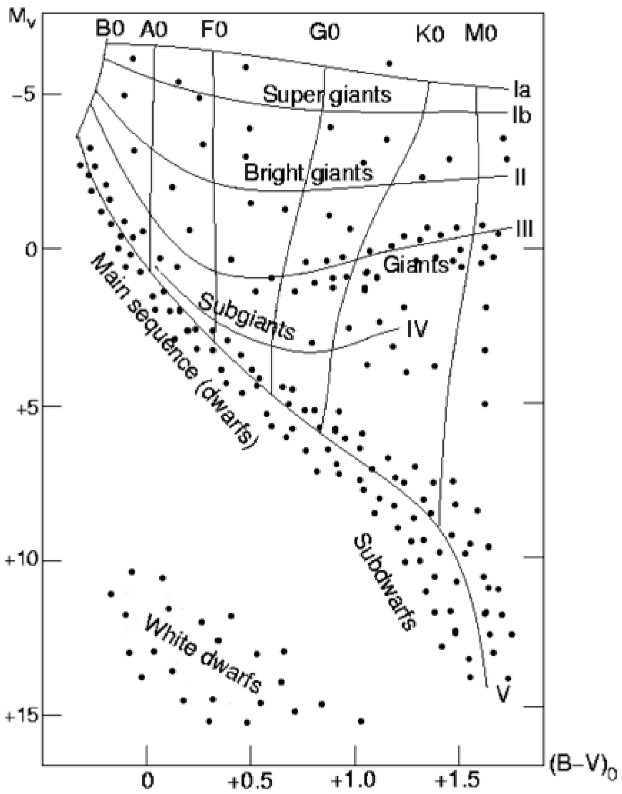
- most on main sequence (called "dwarfs")
- luminosity: $L = 4\pi R^2 \sigma T^4 \propto R^2 T^4$ → cold but luminous stars → "Giants"
- Hot, underluminous stars → are small → "white dwarfs"

Mass-luminosity relationship + HRD: Main Sequence is a Mass Sequence!
(M-dwarf $\approx 0.25\ M_\odot$, G $\approx$ Sun, O,B $\approx M \geq 20\ M_\odot$)
Morgan-Keenan classes: luminosity classes (Sun: G2 V)
Filter: UBV transparent for:
U → ultraviolett
B → blue
V → yellow, green

What to do? Measure with each one → take luminosity → compare the ratios → find out peak area → estimate surface temperature!
So whats the use? Find out exact position in HRD: lines + peak → magnitude/luminosity → distance!







→ If you know: luminosity (apparent brightness + spectrum → spectral class → HRD → luminosity) and apparent brightness/magnitude (measurement) → distance: $d = 10^{(m-M+5)/5}$ → not exact.
in global clusters all stars have same age $\Longrightarrow$ HRD points out stellar evolution.

9.5. Masses

50 –80% of all stars in solar neighborhood are part of multiple systems

- apparent binaries: ("optical double") just seem to belong together
- visual binaries: bound sytem that can be imaged (e.g. Mizar) → motion can be imaged (periods ≈ 1 –100yrs)

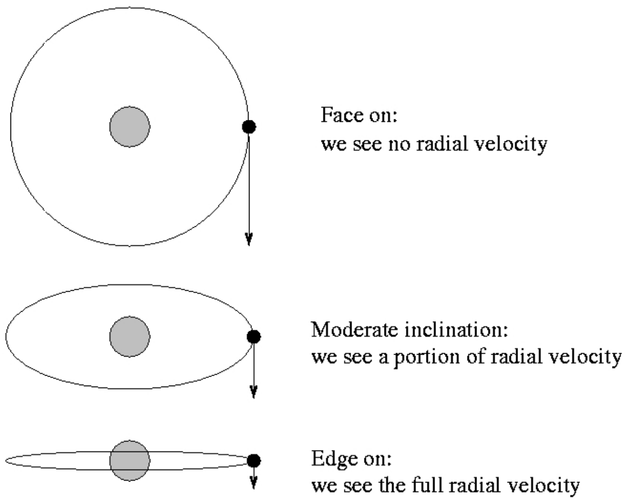
- spectroscopic binaries or spectrum binaries: bound sytem, cannot resolve image into stars, but: Doppler effect in stellar spectrum (short periods: hrs, months) → wooble around CMS (cause CMS is straight line)

9.5.1. Visual Binaries

→ Stellar masses: Kepler 3

$$\frac{a^3}{P^2} = \frac{G}{4\pi^2}(m_1 + m_2) \tag{58}$$

(Observational Parameters: P- directly measurable, a- measurable from image if and only if distance to binary+inclination are known)
Inclination:



simpliest case: real major axis; if not: $a_{\mathrm{obs}} = a_{\mathrm{real}} \cos i$
To figure out individual masses:

$$\frac{M_1}{M_2} = \frac{a_2}{a_1} \tag{59}$$

(a are the semimajor axis around CMS)

9.5.2. Photometric Binaries

Special case of spectroscopic binaries; Describable by Roche potential (potentials of two stars + rotating system → Coriolis force)
Isosurfaces: only very near to star spherical, elsewhere not → stellar magnitude changes in orbit
Eclipsing binaries: photometric binaries where the orbital plane is perpendicular to the celestial plane

9.5.3. Spectroscopic Binaries

Only possible to measure radial velocity in line of sight!
For circular orbit, angle θ on orbit:

$$\theta = \omega\, t \text{ with } \omega = \frac{2\pi}{P} \tag{60}$$

observed radial velocity:

$$v_{\mathrm{r}} = v \cdot \cos(\omega\, t) \tag{61}$$

from observed $v_{\mathrm{r}}(t) \;\to\; v \cdot \sin i$ ("velocity amplitude").
Motion of star visible through Doppler shift in stellar spectrum:

$$\frac{\Delta\lambda}{\lambda} = \frac{v_{mboxr}}{c} = \frac{v}{c}\sin i\, \cos(\omega\, t) \tag{62}$$

(for stars no relativistic Doppler effect needed)

9.5.4. Mass function

Kepler 3:

$$\frac{G}{4\pi^2}(M_1 + M_2) = \frac{R^3}{P^2} \tag{63}$$

Assumption: observation of lines of star 1 only

$$\rightarrow \text{ CMS: } M_1\, r_1 = M_2\, r_2 \tag{64}$$

$$R = r_1 + r_2 = r_1(1 + \frac{r_2}{r_1}) = r_1(1 + \frac{M_1}{M_2}) \tag{65}$$

Assumption: circular

$$v_1 = \frac{2\pi\, r_1}{P} \tag{66}$$

But: inclination unknown, we only observe radial component, i.e.:

$$v_{\text{obs}} = v_1\, \sin i \tag{67}$$

$$\iff r_1 = \frac{P}{2\pi}\, v_1 = \frac{P\, v_{\text{obs}}}{2\pi\, \sin i} \tag{68}$$

$$\implies R = r_1(1 + \frac{M_1}{M_2}) = \frac{P\, v_{\text{obs}}}{2\pi\, \sin i}\,(1 + \frac{M_1}{M_2}) \tag{69}$$

⇒ Insert into Kepler 3:

$$\frac{G}{4\pi^2}(M_1 + M_2) = \frac{1}{P^2}\frac{P^3}{\sin^3 i}(1 + \frac{M_1}{M_2}) \tag{70}$$

$$\Rightarrow \boxed{\frac{M_2^3}{(M_1 + M_2)^2}\, \sin^3 i = \underbrace{\frac{P\, v_{\text{obs}}^3}{2\pi\, G}}_{\text{Observables}}} =: f_{\text{M}} \tag{71}$$

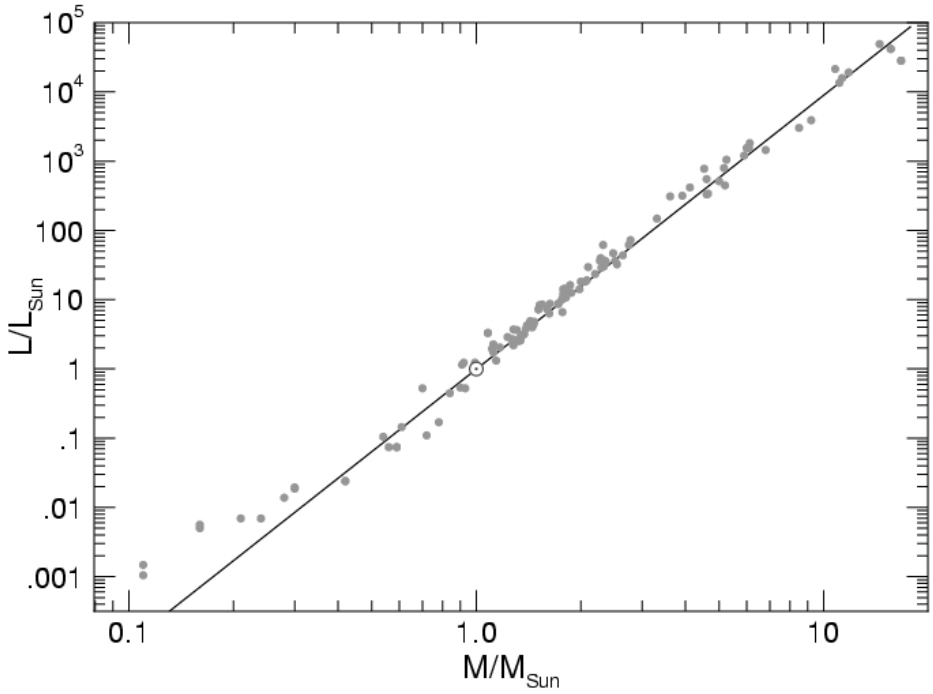
(mass function)

⇒ $\mathrm{f_M}$ is lower limit for M_2
⇒ application:

9.5.5. Mass-Luminosity Relation

- apparent magnitude m and distance ⇒ luminosity
- mass from binary stars

⇒ mass-luminosity relationship



Emperical results:

$$\frac{L}{L_\odot} = \begin{cases} 0.23\left(\frac{M}{M_\odot}\right)^{2.4} & (M < 0.43 M_\odot) \\ (\frac{M}{M_\odot})^{4.0} & (M \geq 0.43 M_\odot) \end{cases} \tag{72}$$

⇒ more massive stars ⇒ lot higher L (factor 2 in M ⇒ factor 8 in L)
⇒ More massive stars live much shorter!

10. Exoplanets

10.1. General Stuff

2 ways to detect Extrasolar Planets:
A) direct method = direct imaging
B) indirect method:

- gravitational interaction with star in radial velocity
- gravitational interaction with star in motion of star
- influence of planet on light from behind planet (gravitational lensing)

10.2. Direct Imaging

Need good telescope! (Resolution power + contrast)

10.2.1. Contrast

Energy passing through 1m^2 per second at distance r ("flux") when assuming isotropic spread:

$$F = \frac{L}{4\,\pi\,r^2} \tag{73}$$

at Earth the luminosity is 10^{10} times weaker than at the Sun and in IR 10^7 (for Jupiter: 10^9 times)
⇒ It's necessary to get contrasts of 1:10^9 ⇒ for Solar System planets ⇒ not possible today.

10.2.2. Angular Separation

$$\tan\theta = \frac{r}{d} \tag{74}$$

where r is distance between planet and star and d is the distance to the star.
⇒ because of small-angle approximination:

$$\theta \approx \frac{r}{d} \tag{75}$$

Typical distance to stars: $d \approx 100$ ly; Typical distances in planetary systems: $r \approx 1$ AU ⇒ θ=0.03"
⇒ resolving power of telescope:

$$\alpha = \frac{12''}{D/1\text{cm}} \tag{76}$$

⇒ 0.03" $\hat{=}$ 4m=D → works
BUT NO: Atmosphere → resolution ≈ 0.5" ("seeing") → only from space possible
⟹ NO DIRECT IMAGING FROM EARTH (only in IR: with adaptive optics, but only dim stars because of contrast = it's possible to display regions close to stars but no good resolution)

10.3. Radial Velocity Measurements

2-body problem: Assumption circular orbit (CMS):

$$\frac{m_1}{r_2} = \frac{m_2}{r_1} \tag{77}$$

velocity of star due to action of planet:

$$v_1 = \frac{2\pi\, r_1}{P} = \frac{2\pi\, m_2}{P\, m_1}\, r_2 \tag{78}$$

→ with Jupiter $v_1 = 13.1\,\mathrm{m\,s^{-1}} \approx 50\,\mathrm{km\,h^{-1}}$
→ need to be able to measure star velocities with better than $13\,\mathrm{m\,s^{-1}}$
→ works with spectroscopic methods via Doppler effect:

$$\boxed{\frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{v}{c}} \tag{79}$$

→ good spectrographs needed → $\frac{\Delta\lambda}{\lambda} = 4 \cdot 10^{-8}$ → only doable with "tricks" (periodic variations of the effect have to be taken into account + long-term observations + good spectrograph → some chance of finding anything)

10.4. Results

About 550 exoplanets found

1. Mass: $M > M_{\text{Jupiter}}$ for most ones → selection effect (more change in v)
2. Semimajor axis: small (P = years) → selection effect (e.g. "hot Jupiters" like comets) because of short observation times
3. Eccentricities: most in eccentric orbits → different from Solar System → ????
4. Habitable Zone: one planet found orbiting in the habitable zone around the red dwarf star Gliese 581 ($a = 0.073AU$, $e = 0.16$)

11. Star formation

Stars are born in "Giant Molecular Clouds" (diameter: 50 –1000pc, molecular gas: H_2,CO, ...,very cool (10 –20K), densities: $n \approx 10^6 - 10^{10}\,\frac{1}{\mathrm{cm}^3}$)
→ Collapse triggered by collision of clouds or supernovae; idea: instable cloud grav > thermal pressure:

$$\Longrightarrow\; R > R_\mathrm{J} = \sqrt{\frac{15\,k\,T}{8\pi\,G\,\mathrm{m_p}\,\rho}} \approx \sqrt{\frac{k\,T}{G\,\mathrm{m_p}\,\rho}} \qquad \text{Jeans Radius} \tag{80}$$

⇒ Problem: Masses too large!
Reality: ISM has B-fields ⇒ particle motion ⊥ to B-field difficult ⇒ stops gas collapsing
Good thing because after Jeans stuff too much star formation! ⇒ more difficult theory inculding fields needed
Process of protostar development:

1. Stellar mass cores form because of fragmentation along B-fields
2. Material collapses inwards until material in center has enough pressure and heat (T >10^6K) to start fusion (process is called "inside out collapse" because at start density in middle was higher than around and therefore rose much faster in the middle) Around Protostar an accretion disk forms!
3. because of stellar wind "bipolar outflow" ⇒ radio lobes ⇒ O, B stars start hydrogen burning (Orion: Trapezium) ⇒ UV light ionises gas ⇒ winds push gas outwards ⇒ around newly formed stars empty space ⇒ compression of other gas ⇒ triggers more star formation ⇒ reflection nebula + open cluster is formed
   ⟹ **Stars usually don't come to be alone!**
4. Star now part of zero age main sequence (ZAMS) (still has circumstellar disk) ⇒ sometimes disks make collimated outflows (jets) ⟸ Herbing Haro Objects
   Protostar → ZAMS a few 10^6 years

The main sequence is state of fusion of hydrogen into helium.
(≈ 10^9 years for Sun)
Star is in "hydrostatic equilibrium".