## Neutron Stars

During SN explosion:

Core of exploding star above Chandrasekhar limit $\Longrightarrow$ core collapses

Densities get so high that **neutronization** sets in:

$$\mathrm{p} + \mathrm{e}^- \longrightarrow \mathrm{n} + \nu_\mathrm{e}$$

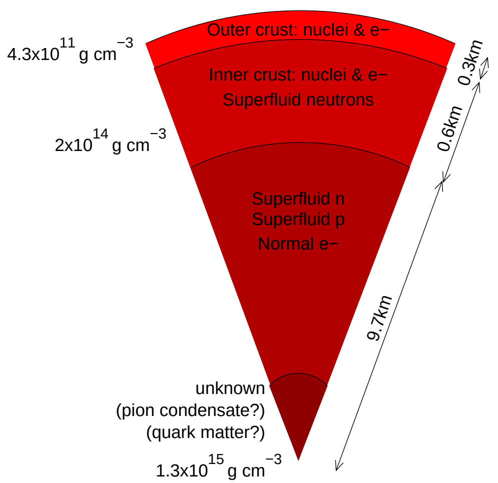
General properties:

- Pressure mainly through **degenerate neutrons** (similar to degenerate electrons for WD!).
- Typical density: $\rho \sim 10^{14}\,\mathrm{g\,cm^{-3}}$ (nuclear densities)
- Typical radius: 10...15 km (Nuremberg!)
- surface gravity $\sim 10^{11} \times$ Earth
- Detailed structure not yet fully understood,





## Neutron Stars: Structure, I




Crust: perhaps crystallized?

Atoms become elongated along $B$-field line on surface

Internal structure unclear:

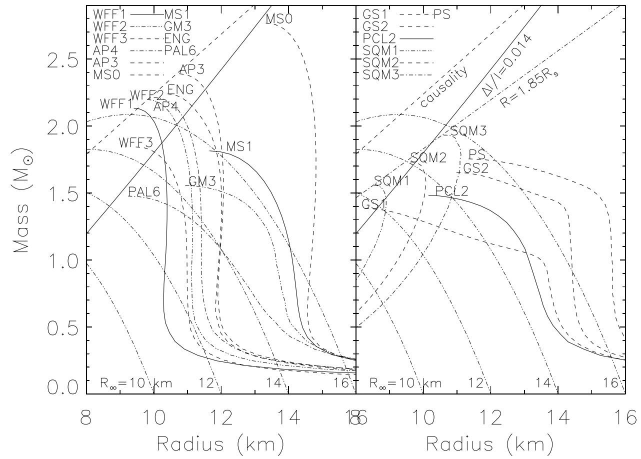
- Supraconducting matter
- Suprafluidity (i.e., fluid with no viscosity)
- central composition unknown

(after Shapiro & Teukolsky, 1983)





## Neutron Stars: Structure, II




(Lattimer & Prakash, 2001)

The structure and size of neutron stars depends strongly on the unknown properties of matter at very high densities.





## Neutron Stars: Rotation, III

During SN collapse, **angular momentum is conserved** (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

Angular momentum conservation ($J_\text{before} = J_\text{NS}$):

$$\frac{2}{5}M_\text{before}R^2_\text{before}\omega_\text{before} = \frac{2}{5}M_\text{NS}R^2_\text{NS}\omega_\text{NS}$$

or (assume $M_\text{NS} = M_\text{before}$):

$$\omega_\text{NS} = \left(\frac{R_\text{before}}{R_\text{NS}}\right)^2 \omega_\text{before} \quad \text{or} \quad P_\text{NS} = \left(\frac{R_\text{NS}}{R_\text{before}}\right)^2 P_\text{before}$$

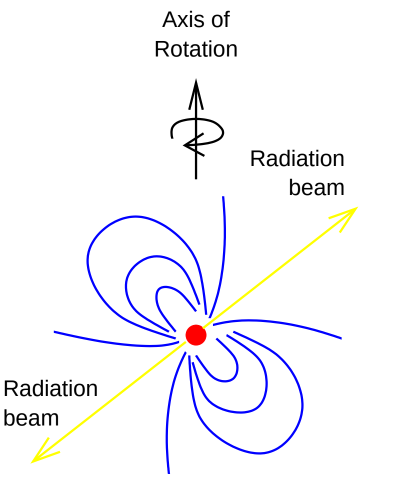
(where $P$: rotation period)

Example: $R_\text{before} = 700000\,\text{km}$ (sun), $R_\text{NS} = 15\,\text{km}$, $P_\text{Sun} = 27\,\text{d} \Longrightarrow P_\text{NS} = 0.001\,\text{s}$

**Neutron Stars are extremely fast rotators.**

close to break-up speed!

## Neutron Stars: Pulsars




"Lighthouse model" for pulsars

Another conserved observable:
magnetic flux: $\Phi = BR^2$

magnetic field after SN:

$$B_{\text{NS}} = \left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2 B_{\text{before}}$$

$\Longrightarrow$ neutron stars have strong magnetic fields (typical: $B \sim 10^6 \ldots 10^8$ T)

Radio pulsars are fast rotating (isolated) neutron stars with strong magnetic fields.





## The sounds of pulsars

- PSR 0329 – a normal pulsar ($P = 0.714519$ s)
- PSR 0833 – the Vela pulsar, a faster, younger pulsar in the Vela supernova remnant ($P = 89$ msec)
- Crab pulsar – the youngest pulsar ($P = 33$ ms)
- B1937 – one of the fastest pulsars ($P = 0.00155780644887275$ s)

See/hear
`http://www.jb.man.ac.uk/~pulsar/Education/Sounds/sounds.html`
for more examples.





## Black Holes, I

Neutron stars also have upper mass limit: Oppenheimer Volkoff limit.

Detailed mass limit unknown, causality considerations give $M \sim 3\,M_\odot$ (for "stiff equation of state" the sound speed becomes greater than speed of light at this mass)

Compact objects with mass above Oppenheimer Volkoff limit: Black Holes

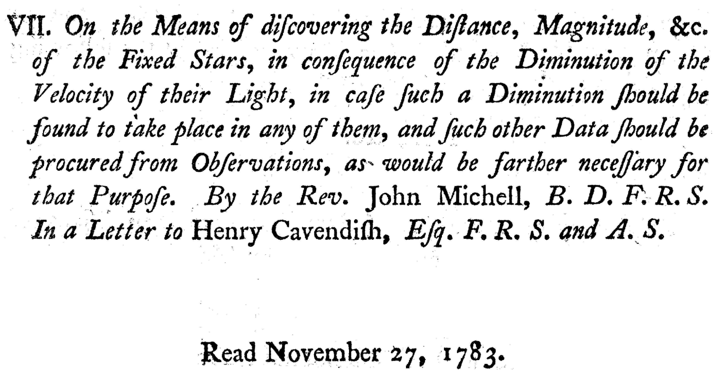
More conservative astronomers: "Black Hole Candidates".





## Black Holes, II

Rev. John Michell: *Phil. Trans. R. Soc. London*, ***74**, 35–57 (1784)*:

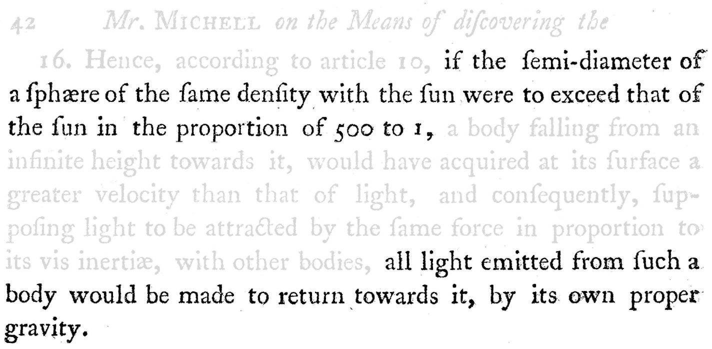


VII. *On the Means of discovering the Distance, Magnitude, &c. of the Fixed Stars, in consequence of the Diminution of the Velocity of their Light, in case such a Diminution should be found to take place in any of them, and such other Data should be procured from Observations, as would be farther necessary for that Purpose. By the Rev.* John Michell, *B. D. F. R. S. In a Letter to* Henry Cavendish, *Esq. F. R. S. and A. S.*

Read November 27, 1783.

## Black Holes, III

Rev. John Michell: *Phil. Trans. R. Soc. London*, ***74***, *35–57 (1784)*:


42 Mr. MICHELL *on the Means of diſcovering the*

16. Hence, according to article 10, if the ſemi-diameter of a ſphære of the ſame denſity with the ſun were to exceed that of the ſun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its ſurface a greater velocity than that of light, and conſequently, ſuppoſing light to be attracted by the ſame force in proportion to its vis inertiæ, with other bodies, all light emitted from ſuch a body would be made to return towards it, by its own proper gravity.





## Black Holes, IV

In more modern usage (but still Newtonian!):

*Total energy of a mass $m$:*

$$E = E_{\text{pot}} + E_{\text{kin}} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

Mass $m$ is unbound if $E > 0$, i.e., for

$$v \geq v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

Black Hole: Body of mass $M$ and radius $R$ for which $v_{\text{escape}} > c$, where $c$ is the speed of light.

This is the case if

$$R \leq R_{\text{S}} = \frac{2GM}{c^2} \sim 3\,\text{km}\frac{M}{M_\odot}$$

the Schwarzschild Radius.





## Einstein, I

Albert Einstein (1879–1955)

Special Relativity (1905):

- Speed of light has the same value in *all* frames of reference
- Observer with constant velocity measure the same physical laws

From these axioms follows:

$\Longrightarrow$ Space and time are relative ("4D-space-time")

$\Longrightarrow$ $E = mc^2$ ("Mass and Energy are equivalent")



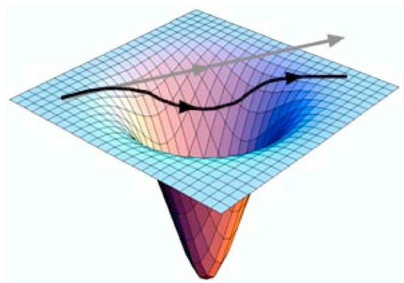


## Einstein, III

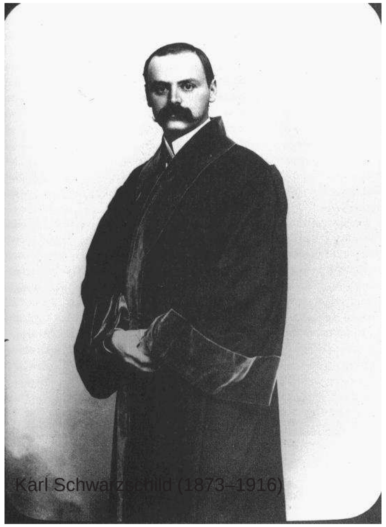
Albert Einstein (1879–1955)

General relativity (1916):

- Mass curves space ("Metric")
- Light moves through curved space



## post-Einstein, I



Karl Schwarzschild (1873–1916)

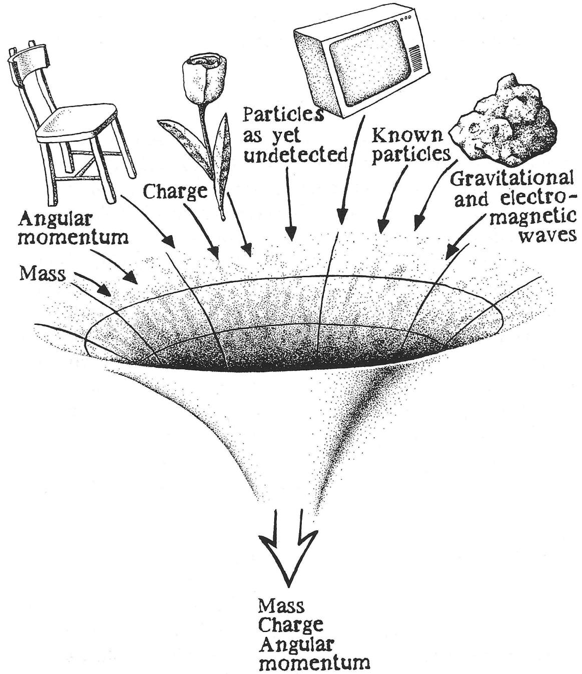
Directly after publication of GRT:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2$$

(Schwarzschild Metric).

Describes "shape of space" in vicinity of mass $M$.



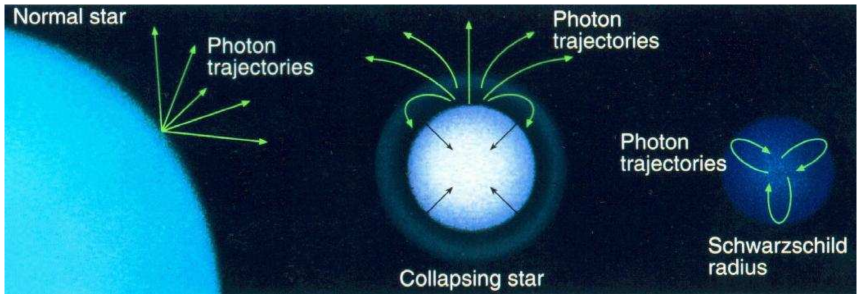



Black holes are very simple physical objects, determined by

- Mass
- (Charge)
- Angular momentum



## post-Einstein, II




$R > R_S$ $R \sim R_S$ $R < R_S$

Behavior of light is determined from location of emission, in dependence from the Schwarzschild Radius:

$$R_S = \frac{2GM}{c^2} \sim 3\,\text{km}\,\frac{M}{M_\odot}$$

Same value as in Newtonian derivation!

J.N. Imamura





## Black Holes: Accretion, III

Astrophysical energy sources:

1. Nuclear fusion

Reactions à la

$$4\text{p} \longrightarrow {}^4\text{He} + \Delta E_\text{nuc}$$

Energy released:

**Fusion produces $\sim 6 \times 10^{11}\,\text{J}\,\text{g}^{-1}$**

(i.e., $\Delta E_\text{nuc} \sim 0.007 m_\text{p} c^2$)

2. Gravitation

Accretion of mass $m$ from $\infty$ to $R_S$ on black hole with mass $M$ gives

$$\Delta E_\text{acc} = \frac{GMm}{R_S} \quad \text{where} \quad R_S = \frac{2GM}{c^2}$$

**Accretion produces $\sim 10^{13}\,\text{J}\,\text{g}^{-1}$**

(i.e., $\Delta E_\text{acc} \sim 0.1\, m_\text{p} c^2$)

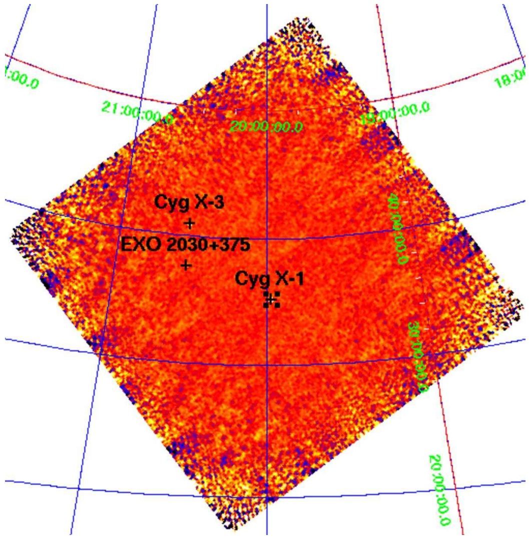
$\Longrightarrow$ Accretion of material is the most efficient astrophysical energy source.

. . . thus accreting objects are the most luminous in the whole universe.

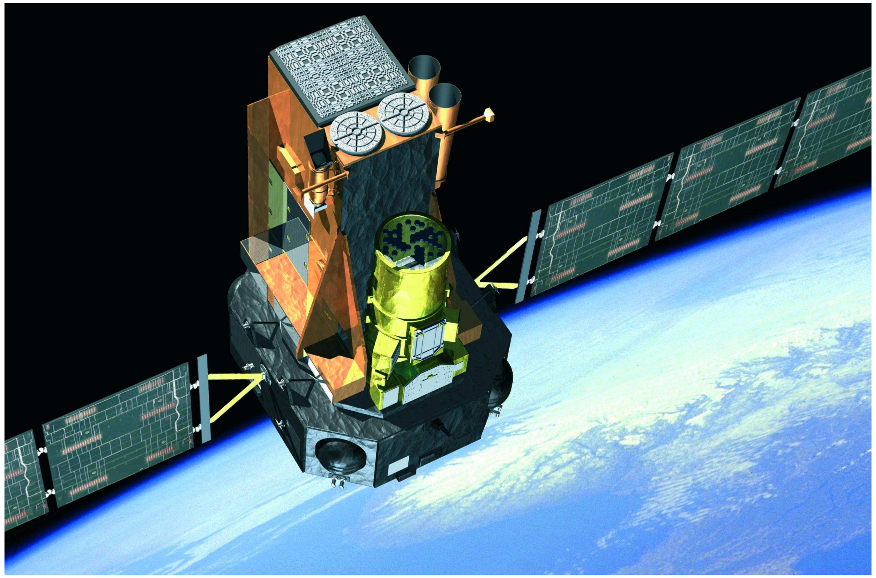
*Note:* energy gets radiated away from *outside* the Schwarzschild radius!



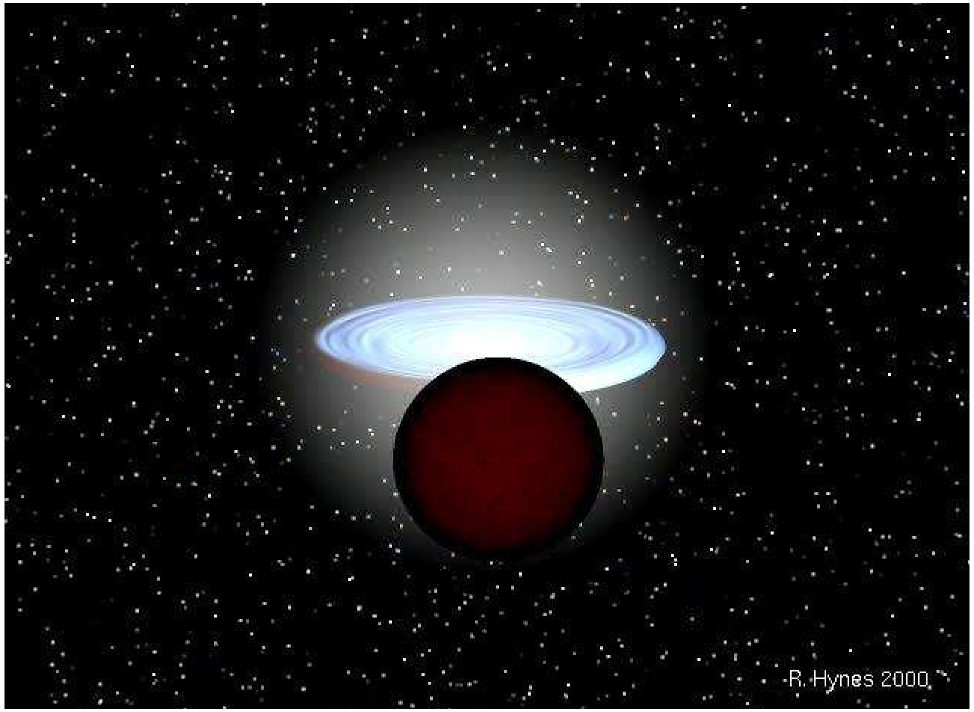

## Black Holes: Accretion, VII



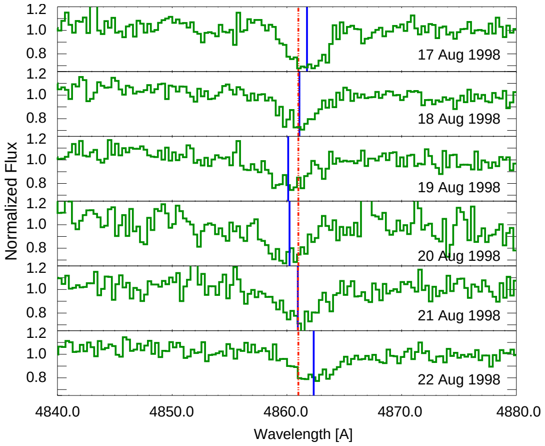


*INTEGRAL*



## Mass determination, II




Motion of H$\beta$ linie in HDE 226868/Cyg X-1 (Pottschmidt, Wilms)

Reminder: In binary systems: Mass of compact object from 3rd Kepler:

$$\frac{a^3}{P^2} = \frac{G(M_1 + M_2)}{4\pi^2}$$

($a$: semi-major axis, $P$: orbital period, $M_{1,2}$: Mass).

Derive from this: Mass function

$$\mathrm{MF} = \frac{M_2^3 \sin^3 i}{(1 + (M_1/M_2))^2} = \frac{K_1^2 P}{2\pi G}$$

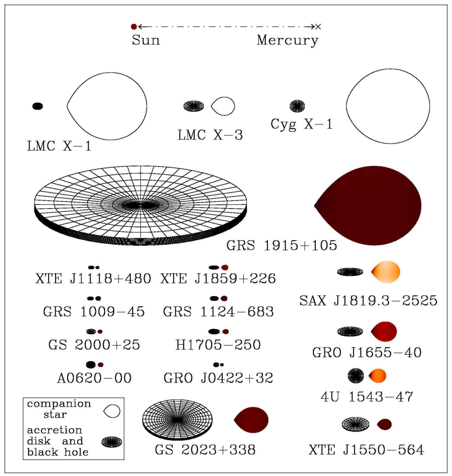
MF is lower limit for $M_2$.

($K_2$ : velocity amplitude, $P$: period)

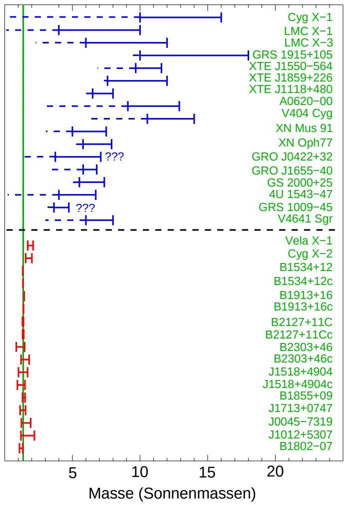




## Mass determination, III



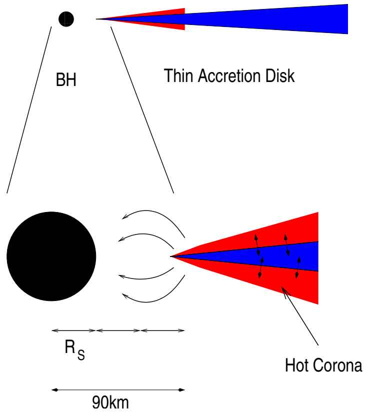

Orosz, 2003, priv. comm.

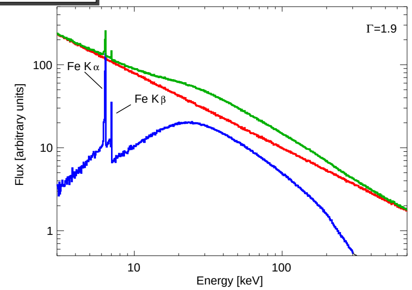







## Relativistic Lines



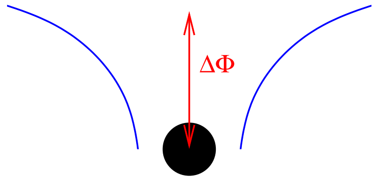

Black Hole X-Ray Spectrum:

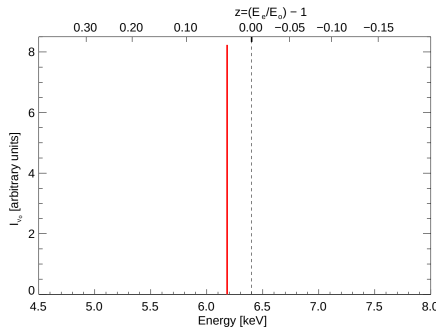
- Comptonization of soft X-rays from accretion disk in hot corona ($T \sim 10^8$ K): power law continuum.
- Thomson scattering of power law photons in disk: Compton Reflection Hump
- Photoabsorption of power law photons in disk: fluorescent Fe K$\alpha$ Line at ~6.4 keV





## Relativistic Lines

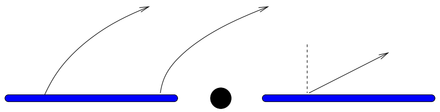


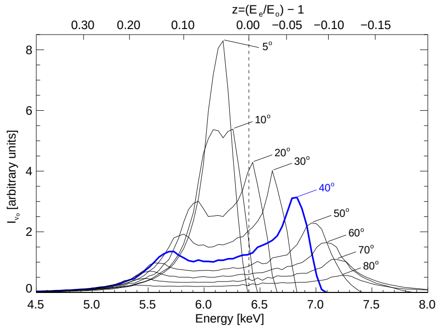



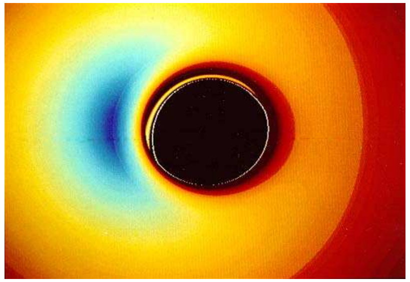

Total observed line profile affected by

- grav. Redshift

## Relativistic Lines





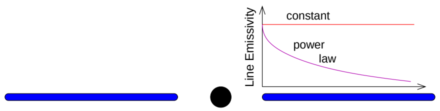
Total observed line profile affected by

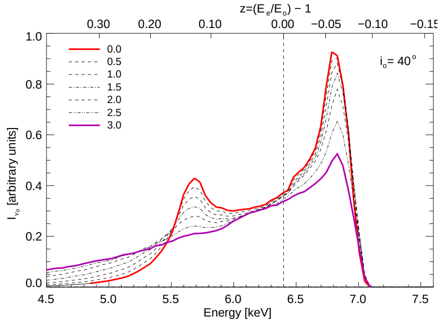
- grav. Redshift
- Light bending
- rel. Doppler shift

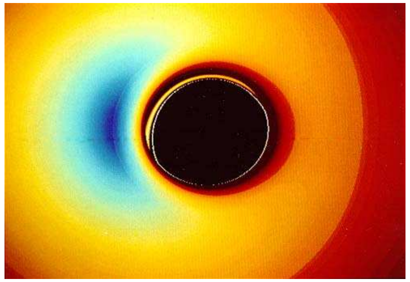




## Relativistic Lines



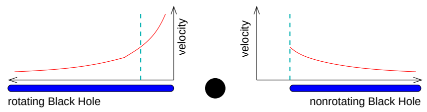
Total observed line profile affected by

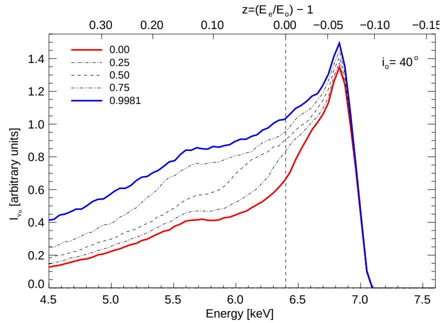
- grav. Redshift
- Light bending
- rel. Doppler shift
- emissivity profile

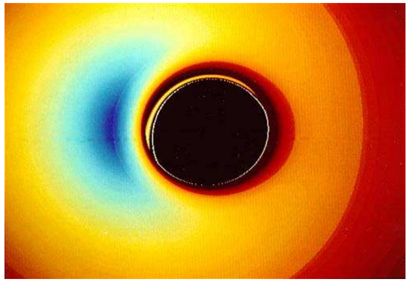




## Relativistic Lines



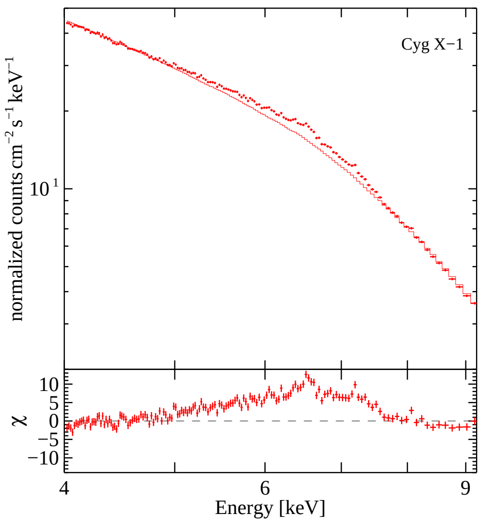
Total observed line profile affected by

- grav. Redshift
- Light bending
- rel. Doppler shift
- emissivity profile
- spin of black hole



*XMM-Newton*

## Relativistic Lines



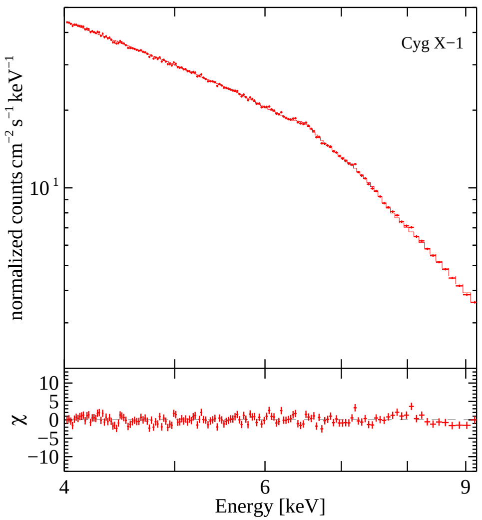

*XMM-Newton* Observation of Cyg X-1: Power-law fit to $E \leq 5$ keV and $E \geq 8$ keV: strong residuals in Fe K$\alpha$ region

Wilms et al. (2006)





## Relativistic Lines




4–9 keV spectrum: well explained ($\chi^2_{\text{red}} = 1.3$) with:

- Power law
  $\Gamma = 1.90 \pm 0.01$
- narrow line
  $E = 6.52 \pm 0.02$ keV,
  $\sigma = 80 \pm 35$ eV,
  EW=14 eV
- relativistic line (Kerr)
  $E = 6.76 \pm 0.1$ keV,
  emissivity $\propto r^{-4.3\pm0.1}$ (strongly centrally peaked!).

Parameters similar (but not equal) to *Chandra* intermediate state observations (Miller et al., 2002)

Wilms et al. (2006)





## Summary

Stars end their lifes as one of three kinds of compact objects:

**White Dwarf:** $R \sim R_{\text{Earth}}$, $\rho \sim 10^{5\ldots6}$ g cm$^{-3}$
$M < 1.44\,M_\odot$ (Chandrasekhar Limit)
Equilibrium between gravitation and pressure of degenerate electrons

**Neutron Star:** $R \sim 10$ km, $\rho \sim 10^{13}\ldots10^{16}$ g cm$^{-3}$
$1.44\,M_\odot < M \lesssim 3\ldots4\,M_\odot$ (Oppenheimer-Volkoff Limit)
Density implies inv. $\beta$-decay ($p + e^- \rightarrow n$), i.e., star has high neutron content

**Black Hole:** Above OV-Limit no stable configuration known
$\Longrightarrow$ star collapses
$\Longrightarrow$ Black Hole
$M \gtrsim 4\,M_\odot$
Event horizon at $R_S = 2GM/c^2 = 3(M/M_\odot)$ km (Schwarzschild radius)



4–54

Filippenko, A. V., 1997, Ann. Rev. Astron. Astrophys., 35, 309
Jha, S., et al., 1999, ApJS, 125, 73
Lattimer, J. M., & Prakash, M., 2001, ApJ, 550, 426
Shapiro, S. L., & Teukolsky, S. A., 1983, Black Holes, White Dwarfs, and Neutron Stars, (New York: Wiley)