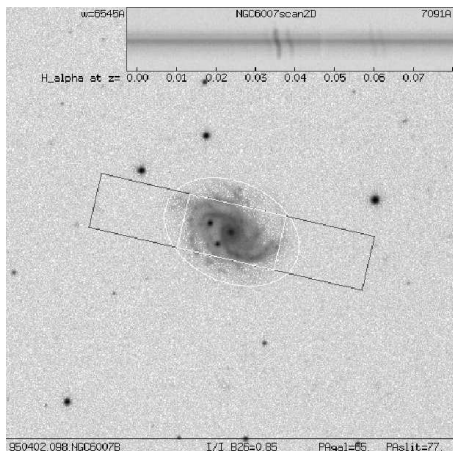




## Mass Determination, I



Spectra of galaxies: sum of all constituent spectra (mainly stars plus some contribution from nebulae).

Absorption lines show clear shift  $\Rightarrow$  Doppler effect due to motion of stars around centre:

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c} = \frac{v}{c} \sin i$$

where  $v_r$ : radial velocity,  $i$ : inclination (angle measured with respect to plane of sky).

Typical rotation speeds are a few  $100 \text{ km s}^{-1}$ .

NGC 6007 (Jansen; <http://www.astro.rug.nl/~nfgs/>)



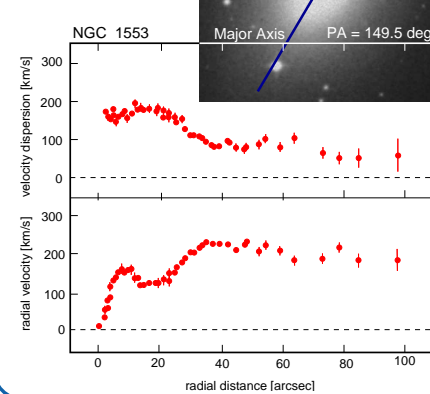
## Mass Determination, III

Spiral galaxy rotation curves are flat!

"Galaxy rotation problem", first discovered by Vera Rubin (1970)



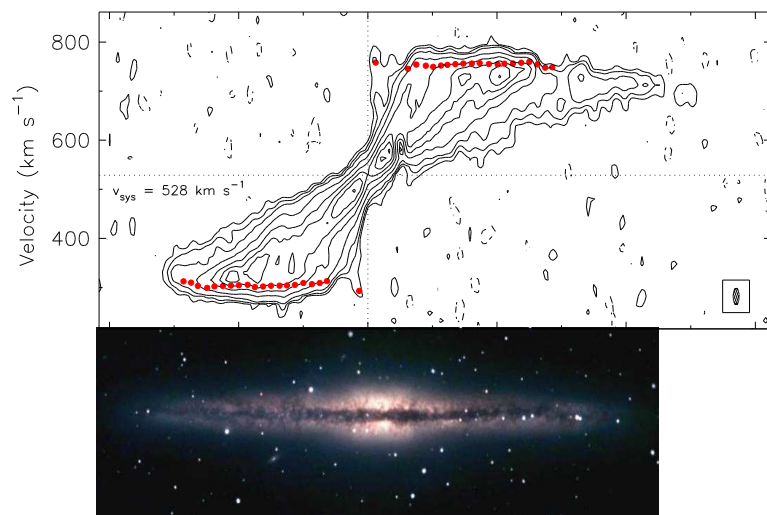
© Astron. Soc. Pacific



← NGC 1553 (S0) (after Kormendy, 1984, ApJ 286, 116)



## Mass Determination, II



NGC 891 (Swaters et al., 1997, ApJ 491, 140 / Paul LeFevre, S&T Nov. 2002)



## Rotation Curves: Interpretation

Newtonian interpretation of galaxy rotation curves:

Motion because of mass within  $r$ :

$$\frac{GM(\leq r)}{r^2} = \frac{v_{\text{rot}}^2(r)}{r}$$

such that

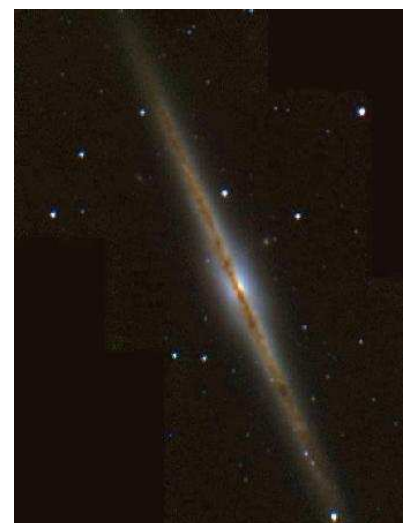
$$M(\leq r) = \frac{v_{\text{rot}}^2 r}{G}$$

therefore:

$v \sim \text{const.}$  implies  $M(\leq r) \propto r$ .

This assumption is approximately true even for nonspherical mass distributions.

NGC 891, KPNO 1.3 m  
Barentine & Esquerdo





## Rotation Curves: Interpretation

What mass distribution do we expect?

Intensity profile of disk in spiral galaxies can be well described by

$$I(r) = I_0 \exp(-r/h)$$

where  $r$ : distance from centre,  $h$ : "scale height".

Luminosity emitted within radial distance  $r_0$ :

$$L(r < r_0) = I_0 \int_0^{r_0} \exp(-r/h) 2\pi r dr = 2\pi I_0 (h^2 - \exp(-r_0/h)h(h + r_0))$$

i.e., for  $r_0 \rightarrow \infty$ :  $L(r < r_0) \rightarrow \text{const.}$

If all light comes from stars, i.e., light traces mass, then  $M/L \sim \text{const.}$ , such that  $M(< r) \sim \text{const.}$  outside a certain radius and  $v \propto r^{-1/2} \Rightarrow$  not what is observed!

Canonical interpretation: a large fraction of gravitating material does not emit light  $\Rightarrow$  spiral galaxies have large and massive halos made of dark matter, resulting in  $M/L \sim 30$ .

Galaxies: Masses

5



## Dark Matter: Nonbaryonic

Nonbaryonic dark matter:

*Requirements:*

- gravitating
- no or very weak other interaction with baryons (=“us”)

$\Rightarrow$  Grab-box of elementary particle physics:

### 1. Neutrinos with non-zero mass

**Pro:** It exists, mass limits are a few eV, need only  $(m_\nu c^2) \sim 10 \text{ eV}$

**Contra:**  $\nu$  are relativistic ( $v \sim c$ ), this has implications for galaxy formation that make it unlikely that they form a major part of dark matter.

### 2. Axions ( $mc^2 \sim 10^{-5} \dots -2 \text{ eV}$ ) and WIMPs (weakly interacting massive particles; masses $mc^2 \sim \text{GeV}$ )

**Pro:** help with cosmology as well

**Contra:** We do not know they exist... (but they might soon be detectable)

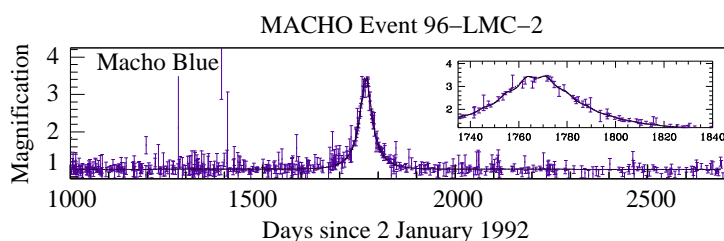
$\Rightarrow$  Jury is still out, question on origin of flat rotation curves is still open.

Mass: Interpretation

2



## Dark Matter: MACHOS



after Alcock et al. (2001, Fig. 2)

MACHOS (Massive Compact Halo Objects): White dwarfs in the galaxy's halo

**Pro:**

1. very low luminosity objects  $\Rightarrow$  very difficult to detect
2. detected by **microlensing** towards SMC and LMC (see figure)  $\Rightarrow$  MW halo consists of 50% white dwarfs

**Contra:**

1. possible "self-lensing" (by stars in MW or SMC/LMC; confirmed for a few cases)
2. inferred white dwarf formation rate too high ( $100 \text{ year}^{-1} \text{ Mpc}^{-3}$  instead of  $< 1$  as previously assumed)

Mass: Interpretation

1



## MOND

Modified Newtonian Dynamics (Milgrom, 1983ff.; MOND): Alternative to Dark Matter

Reviews: Sanders & McGaugh, 2002, Ann. Rev. Astron. Astrophys. 40, 263; Milgrom, 2001, astro-ph/0112069

*Idea:* Modify Newton's Laws:

Acceleration on particle in gravitational field:

$$a = \frac{GM}{r^2} \cdot \frac{1}{\mu(a/a_0)} \quad \text{with} \quad \mu(x) \rightarrow \begin{cases} 1 & \text{for } x \rightarrow \infty \\ x & \text{for } x \rightarrow 0 \end{cases}$$

i.e., for accelerations  $a \ll a_0$ ,  $a \rightarrow \sqrt{GMa_0/r^2}$ , giving circular motion in the limit of small accelerations:

$$\sqrt{\frac{GM(\leq r)a_0}{r^2}} = \frac{v^2}{r} \quad \Rightarrow \quad M(\leq r) = \frac{v^4}{Ga_0}$$

and therefore independent of  $r$ !

MOND can explain the flat rotational curves (by construction!).

Mass: Interpretation

3



## MOND

Fits of rotational curves give

$$a_0 = 1.2 \times 10^{-8} \text{ cm s}^{-2}$$

and  $M/L \sim 1$ , so not bad!

**BUT:**

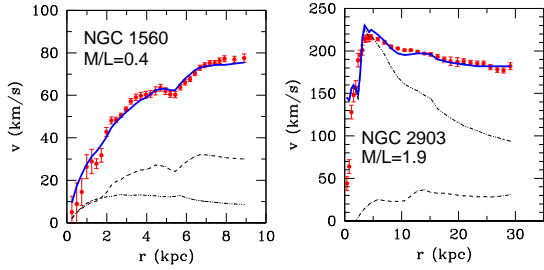
- where is the physics behind  $a_0$ ?

- violation of the strong equivalence principle

("outcome of any physical experiment is independent of where and when in the universe it is performed, and it is independent on whether the experimental apparatus is free falling or stationary")

⇒ At the moment MOND does not seem to be a viable alternative to other theories of dark matter.

... but it shows that even today people are not afraid to attack Newton's laws, and this is good for progress of physics as a whole



after Sanders & McGaugh (2002)

Mass: Interpretation

4



## The Astronomical Distance Ladder



## Introduction

Distances are required to determine properties such as the luminosity or the size of an astronomical object.

Only *direct* method:

1. Trigonometric parallax

Most other methods based on "standard candles", i.e., use known absolute magnitude of an object to derive distance via distance modulus.

2. Main Sequence Fitting
3. Variable stars: RR Lyrae and Cepheids
4. Type Ia Supernovae
5. Tully-Fisher for spiral galaxies
6.  $D_n$ - $\sigma$  for ellipticals
7. Brightest Cluster Galaxies

For the farthest objects, can also use expansion of universe:

8. Hubble's law

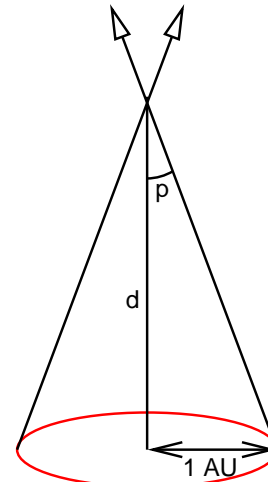
Methods are calibrated using distances from the previous step of the distance ladder.

Distance Ladder

5



## Trigonometric Parallax



Motion of Earth around Sun ⇒ Parallax  
Produces apparent motion of star; projected on sky see angular motion, opening angle

$$\tan p \sim p = \frac{r_{\text{Earth}}}{d} = \frac{1 \text{ AU}}{d}$$

$p$  is called the trigonometric parallax.

Note: requires several at several positions of the Earth

Measurement difficult:  $\pi \lesssim 0.76''$  ( $\alpha$  Cen).

Define unit for distance:

Parsec: Distance where 1 AU has  $p = 1''$ .

$$1 \text{ pc} = 206265 \text{ AU} = 3.086 \times 10^{16} \text{ m} = 3.26 \text{ ly}$$

Direct Methods

1



## Trigonometric Parallax

Best measurements to date: Hipparcos satellite (1989–1993)

- systematic error of position:  $\sim 0.1$  mas
- effective distance limit: 1 kpc
- standard error of proper motion:  $\sim 1$  mas/yr
- photometry
- magnitude limit: 12
- complete to mag: 7.3–9.0

Results available at <http://astro.estec.esa.nl/Hipparcos/>:

**Hipparcos catalogue:** 120000 objects with milliarcsecond precision.

**Tycho catalogue:**  $10^6$  stars with 20–30 mas precision, two-band photometry

Direct Methods

2



## Standard Candles, I

Assuming isotropic emission, the flux measured at distance  $d$  from object with luminosity  $L$  is given by the “inverse square law”,

$$f(d) = \frac{L}{4\pi d^2}$$

note that  $f$  is a function of the  $d$ .

Remember that the magnitude is defined through comparing two fluxes,

$$m_2 - m_1 = 2.5 \log_{10}(f_1/f_2) = -2.5 \log_{10}(f_2/f_1)$$

To allow the comparison of sources at different distances, define

$$\text{absolute magnitude } M = \text{magnitude if star were at distance 10 pc}$$

Because of this

$$M - m = -2.5 \log_{10}(f(10 \text{ pc})/f(d)) = -2.5 \log_{10}\left(\frac{L/(4\pi(10 \text{ pc})^2)}{L/(4\pi d^2)}\right) = -2.5 \log_{10}\left(\frac{d}{10 \text{ pc}}\right)^2$$

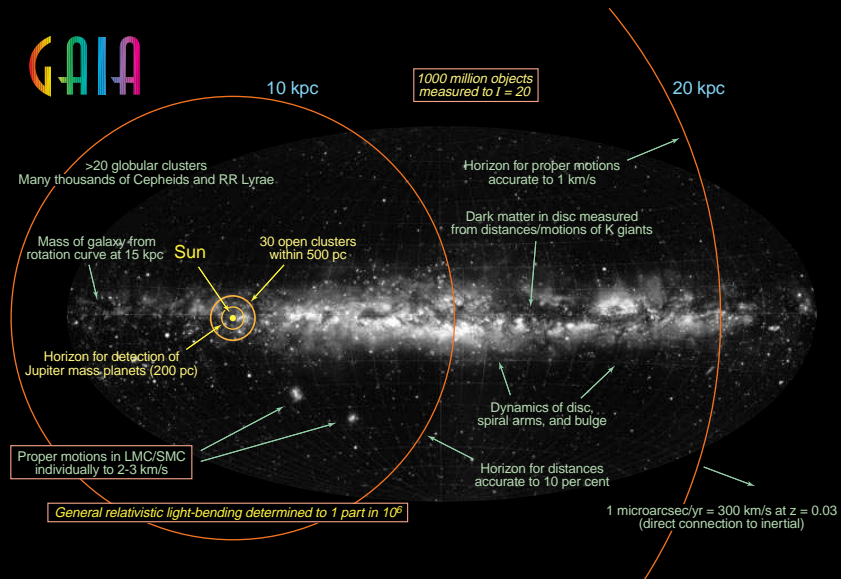
The difference  $m - M$  is called the distance modulus,

$$m - M = 5 \log_{10}\left(\frac{d}{10 \text{ pc}}\right)$$

Indirect Methods

1

Plans for the future: (ESA mission, launch 2010, observations 2011–2016):



GAIA:  $\sim 4 \mu\text{arcsec}$  precision, 4 color to  $V = 20$  mag,  $10^9$  objects.



## Standard Candles, IV

To obtain distance, use standard candles

Standard candles are defined to be objects for which their absolute magnitude is known.

Requirements:

- physics of standard candle well understood (i.e., need to know *why* object has certain luminosity).
- absolute magnitude of standard candle needs to be calibrated, e.g., by measuring its distance by other means (this is a *big problem*)

To determine distance to astronomical object:

1. find standard candle(s) in object,
2. measure their  $m$
3. determine  $m - M$  from known  $M$  of standard candle
4. compute distance  $d$

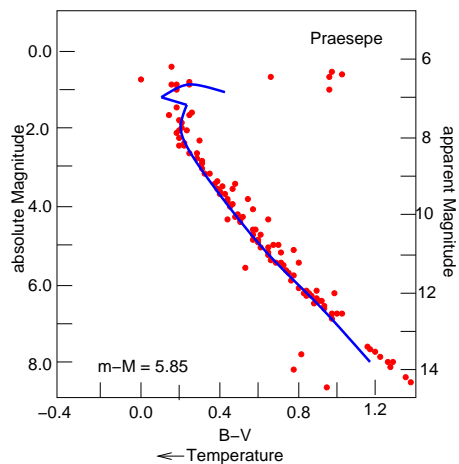
Often, distances are given in terms of  $m - M$ , and not in pc, so last step is not always performed.

Indirect Methods

4



## Main Sequence Fitting



MS fitting applied to Praesepe  
(after Vandenberg & Bridges 1984)

Clusters: if Main Sequence in Hertzsprung Russell Diagram determinable:

Shift observed HRD until main sequence agrees with location of MS measured for stars in solar vicinity  $\Rightarrow$  distance modulus.

Currently: distances to  $\sim 200$  open clusters known

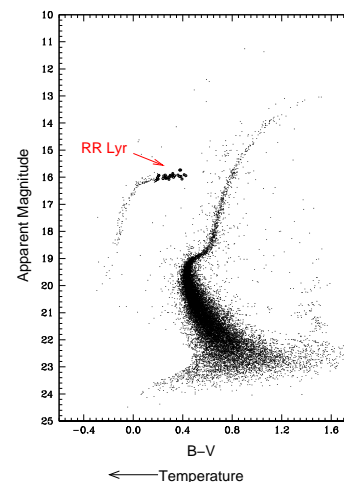
Distance limit  $\sim 7$  kpc.

Indirect Methods

6



## RR Lyrae, I



HRD of Globular Cluster M2  
(after Lee et al., 1999, Fig. 2)

RR Lyrae variables:

- Variability ( $P \sim 0.2 \dots 1$  d)
- Mainly temperature change
- RR Lyr gap clearly observable in globular cluster HRD

Absolute magnitude of RR Lyr gap:

$M_V = 0.6$ ,  $M_B = 0.8$ , i.e.,  $L_{RR} \sim 50 L_{\odot}$ .

Works out to LMC ( $d \sim 50$  kpc) and other dwarf galaxies of local group, mainly used for globular clusters and local group.

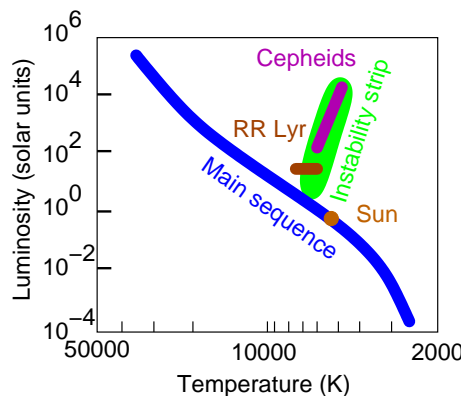
Example: M5: gap at  $m = 16$  mag  $\Rightarrow m - M = 15.4$  mag  $\Rightarrow d = 12$  kpc.

Variable Stars

2



## Introduction



Instability strip in the Hertzsprung-Russell Diagram

Certain regions of HRD: stars prone to instability:

Ionisation of Helium: transparency of outer parts of star changes

$\Rightarrow$  size of star changes

$\Rightarrow$  surface temperature and luminosity variations

Most important variables of this kind:

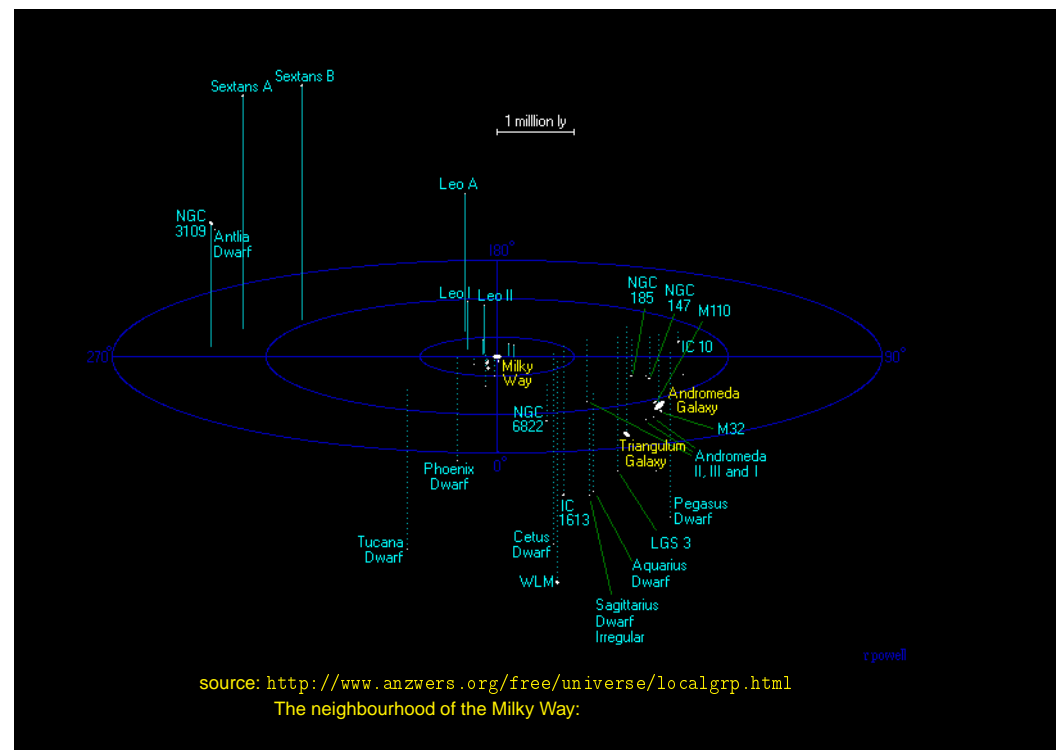
1. RR Lyr variables

mainly in globular clusters: lower metallicity of clusters ("population II") allows stars to enter instability strip

2.  $\delta$  Cepheids

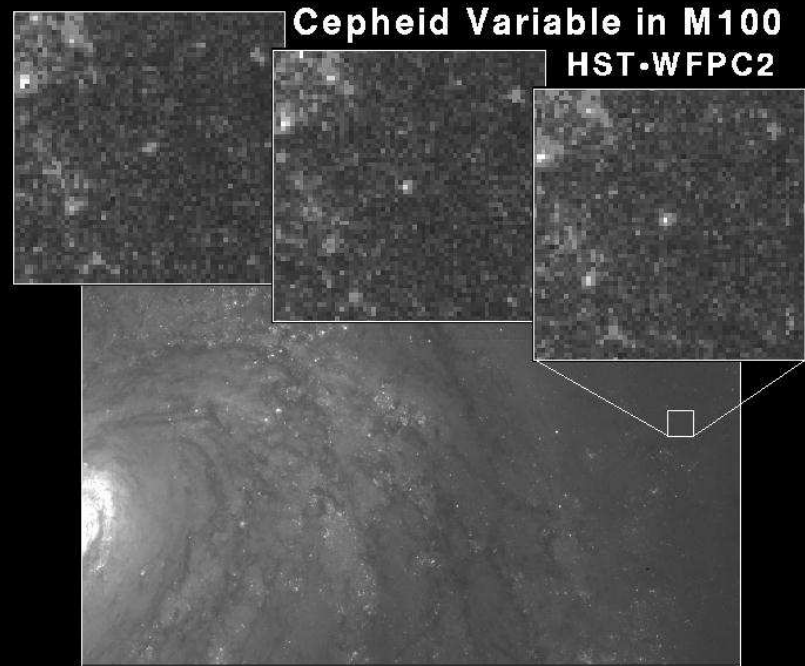
Variable Stars

1



source: <http://www.answers.org/free/universe/localgrp.html>  
The neighbourhood of the Milky Way:

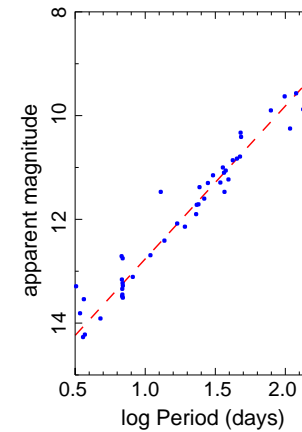
rpowell



Cepheid Variable in M100  
HST-WFPC2



Cepheids



Henrietta Leavitt(1907):

Cepheids have a period luminosity relationship:  $M \propto -\log P$

Low luminosity Cepheids have lower period

Observations find:

$$M = -2.76 \log P - 1.40$$

( $P$  in days)

Calibrated from observing Large Magellanic Cloud Cepheids (see figure), and determining LMC distance from other means (MS fitting, RR Lyr,...) to find absolute magnitudes...

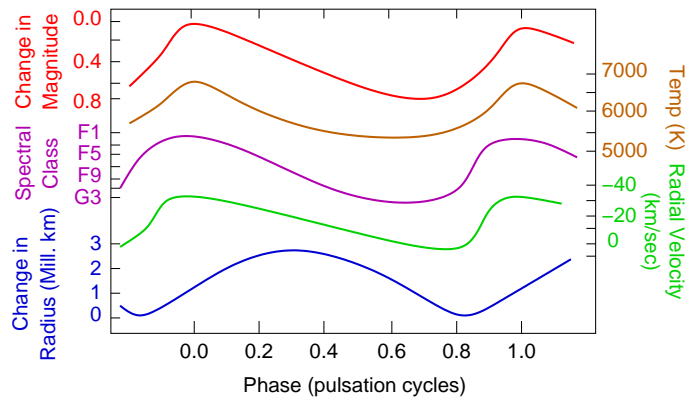
With HST: works out to Virgo cluster

( $d = 18.5$  Mpc).

Period-Luminosity relation for the LMC Cepheids after Mould et al. (2000, Fig. 2)



Cepheids



after <http://csep10.phys.utk.edu/astr162/lect/index.html>

Cepheids: Luminous stars ( $L \sim 1000 L_{\odot}$ ) in instability strip with large luminosity amplitude variation,  $P \sim 2 \dots 150$  d (easily measurable).

The origin of the Period-Luminosity relationship is in the Helium ionisation instability discussed before. The details of this are rather messy, however, it is easy to see that a Period-Luminosity relationship as that observed for the Cepheids is a simple consequence of the fact that the pulsating star is not disrupted by its oscillation.

For the outer parts of the star to remain bound, the kinetic energy of the pulsating outer parts of the stars has to remain smaller than their binding energy:

$$\frac{1}{2}mv^2 \lesssim \frac{GMm}{R}$$

But we know that for the velocity

$$v < \frac{2R}{P}$$

where  $P$  is the period of the star and  $R$  its radius at maximum extension (we observe the star to expand to a radius  $R$  once every  $P$  seconds, so the maximum distance the expanding material can go during that time is  $2R$ ). Inserting  $v$  into the above equation gives

$$\frac{14R^2}{2P^2} \lesssim \frac{GM}{R} \iff P^2 \gtrsim \frac{2R^3}{GM} = \frac{2}{G} \frac{1}{M/R^3}$$

If we assume that the pulsation is close to the break-up speed, and noting that  $M/R^3$  is proportional to the average density of the star, then it is easy to see that

$$P \propto (G\rho)^{-1/2}$$

In the homework for this week you are asked to convince yourself that  $(G\rho)^{-1/2}$  has the dimension of a period, i.e., for all gas balls oscillating close to the break up speed, we expect that  $P \propto \rho^{-1/2}$ . To obtain the period luminosity relationship, you need to remember that the emissivity per square-metre of the surface of a star with temperature  $T$  is  $\sigma T^4$  (per the Stefan-Boltzmann law), while the surface of the star is proportional to  $R^2$ . Therefore, the luminosity of the star is  $L \propto R^2 T^4$ .

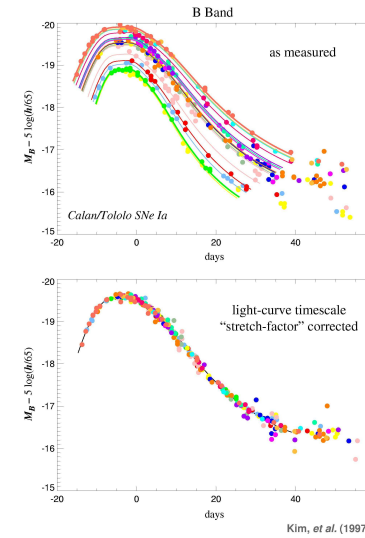
This week's homework asks you to use  $L \propto R^2 T^4$  and  $P \propto \rho^{-1/2}$  to show that from these the absolute magnitude of a pulsating star is related to the period through

$$\log P \propto -m$$

as observed for Cepheids.



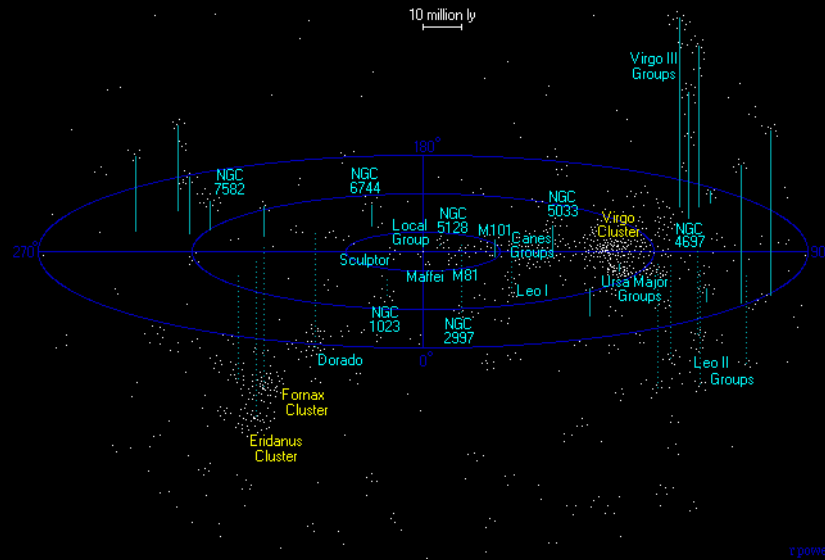
## Supernovae



After correction of systematic effects and time dilatation (expansion of the universe, see later):  
SN Ia lightcurves all look the same  
⇒ standard candle

Supernovae

2



The universe out to the Virgo Cluster

source: <http://www.answers.org/free/universe/virgo.html>



## Supernovae

SN Ia = Explosion of CO white dwarf when pushed over Chandrasekhar limit ( $1.4 M_{\odot}$ ) (via accretion?).

⇒ Always similar process

⇒ Very characteristic light curve: fast rise, rapid fall, exponential decay ("FRED") with half-time of 60 d.

60 d time scale from radioactive decay  $\text{Ni}^{56} \rightarrow \text{Co}^{56} \rightarrow \text{Fe}^{56}$   
(“self calibration” of lightcurve if same amount of  $\text{Ni}^{56}$  produced everywhere)

*Calibration:* SNe Ia in nearby galaxies where Cepheid distances known.

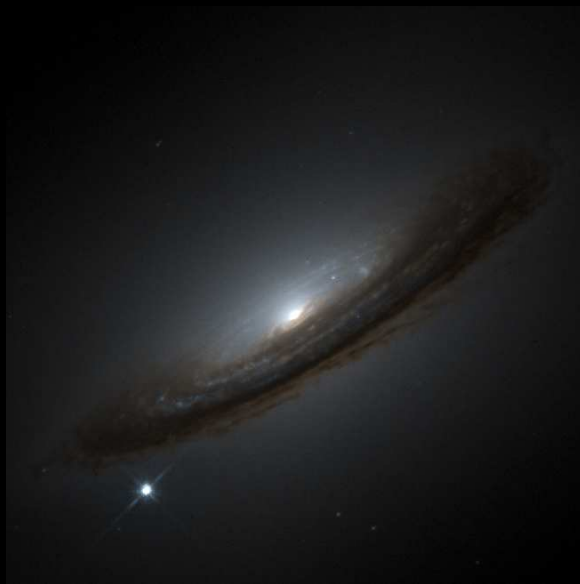
At maximum light:

$$M_B = -18.33 \pm 0.11 \iff L \sim 10^{9 \dots 10} L_{\odot}$$

Observable out to  $\gtrsim 1$  Gpc ⇒ covers almost the whole universe. . .

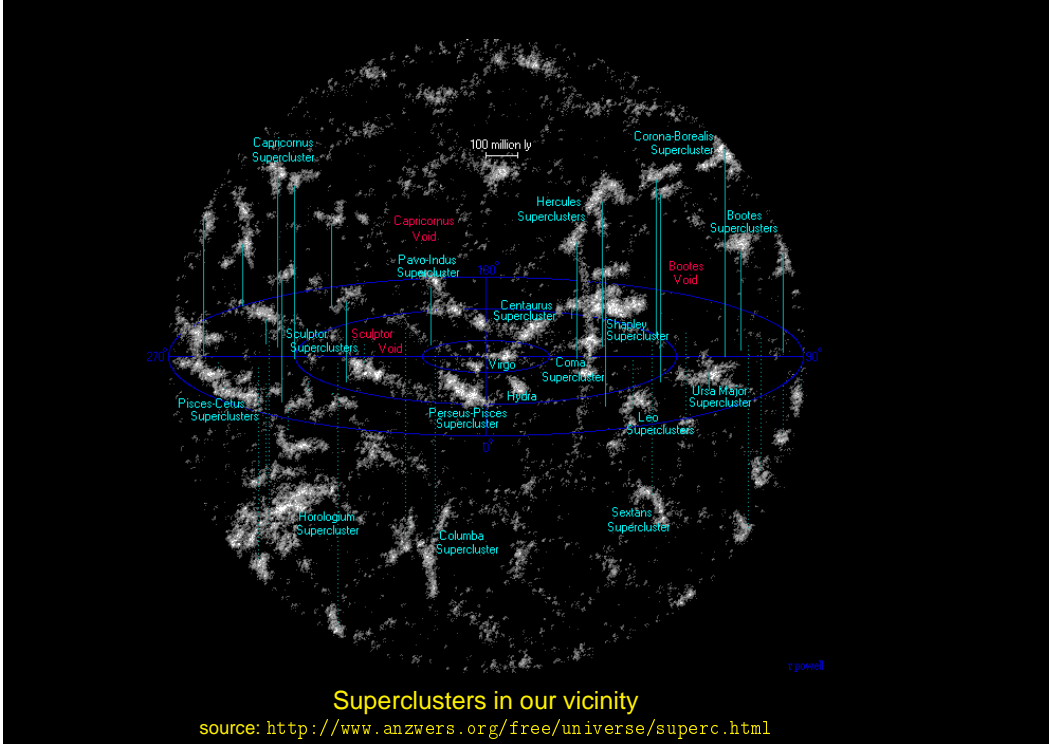
Supernovae

3



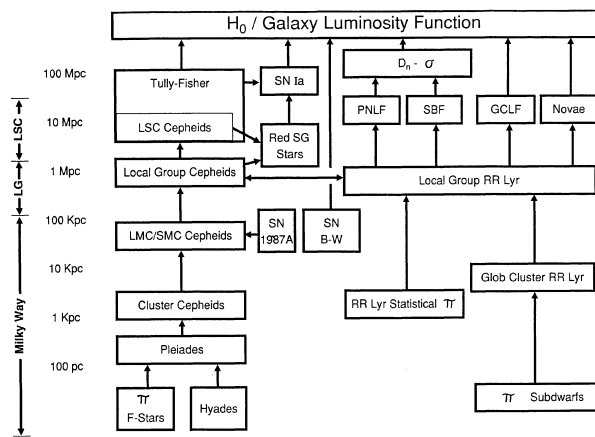
SN1994d (HST WFPC)

Supernovae have luminosities comparable to whole galaxies:  
 $\sim 10^{51}$  erg/s in light,  $100 \times$  more in neutrinos.



7-20

### Summary: Distance Ladder



Pathways to Extragalactic Distances

Jacoby (1992, Fig. 1)