Cosmology

Introduction

Cosmology: science of the universe as a whole

How did the universe evolve to what it is today?

Based on four basic facts:

The universe
- expands,
- is isotropic,
- and is homogeneous.

Isotropy and homogeneity of the universe: "*cosmological principle*".

Perhaps (for us) the most important fact is:

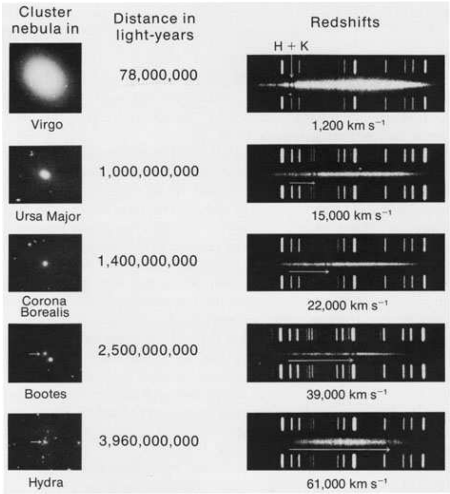
- The universe is habitable for humans.

("*anthropic principle*")

The one question cosmology does not attempt to answer is: How came the universe into being?

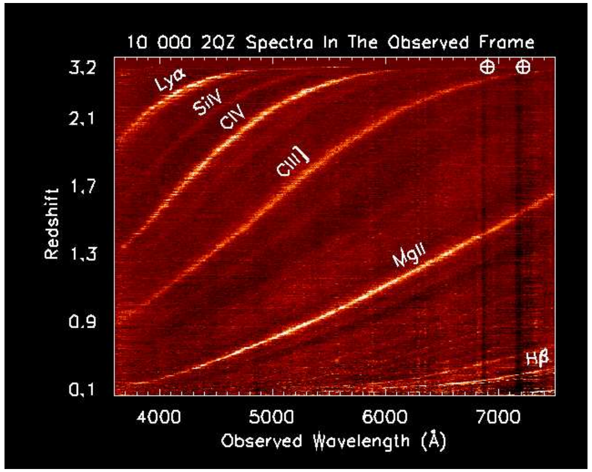
$\Longrightarrow$ Realm of theology!

Redshifts, I



Hubble: spectral lines in galaxies are more and more redshifted with increasing distance.

Redshifts, II



2dF QSO Redshift survey

Redshift:

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

interpreted as velocity:

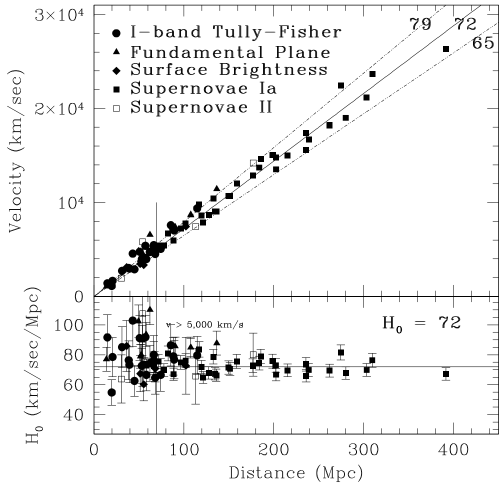
$$v = cz$$

where

$c = 300000\,\mathrm{km\,s^{-1}}$

(speed of light)

Hubble Relation, I



(Freedman, 2001, Fig.4)

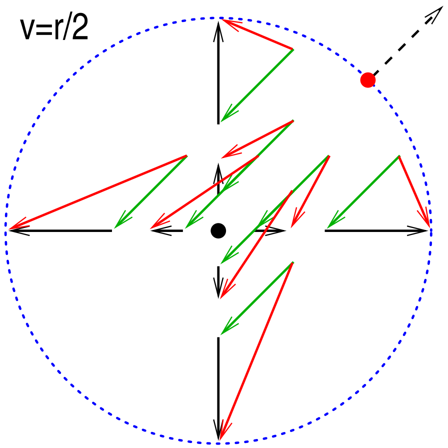
Hubble relation (1929):

The redshift of a galaxy is proportional to its distance: $v = cz = H_0 d$

where H_0: "Hubble constant".
Measurement: determine v from redshift (easy), d with standard candles (difficult)
$\Longrightarrow$ H_0 from linear regression.
Hubble Space Telescope finds

$H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$

Hubble Relation, II



The expansion law $\boldsymbol{v} = H_0\boldsymbol{r}$ is unchanged under rotation and translation: isomorphism.

Proof:

Rotation: Trivial.

Translation: Observations from place with position $\boldsymbol{r}'$ and velocity $\boldsymbol{v}'$: Observed distance is $\boldsymbol{r}_\circ = \boldsymbol{r} - \boldsymbol{r}'$, observed velocity is $\boldsymbol{v}_\circ = \boldsymbol{v} - \boldsymbol{v}'$. Because of the Hubble law,

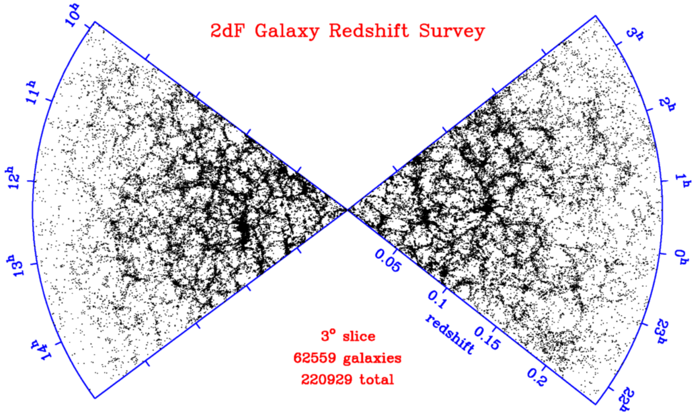
$$\boldsymbol{v}_\circ = H_0\boldsymbol{r} - H_0\boldsymbol{r}' = H_0(\boldsymbol{r} - \boldsymbol{r}') = H_0\boldsymbol{r}_\circ$$

This isomorphism is a direct consequence of the homogeneity of the universe.

Despite everything receding from us, we are not at the center of the universe $\Longrightarrow$ Copernicus principle still holds.

Copernicus principle: We are not at a special place in the universe in time or space.

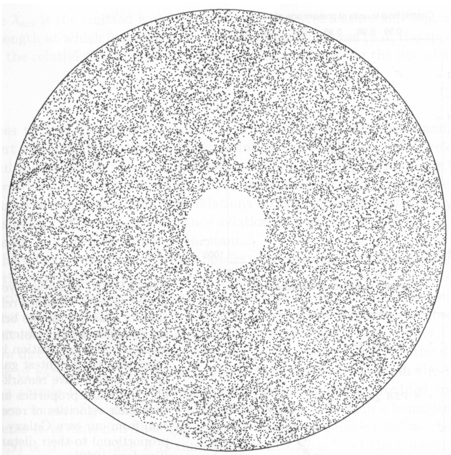
Homogeneity



2dF Survey, ~220000 galaxies total

Homogeneity: "The universe looks the same, regardless from where it is observed" (on scales $\gg$100 Mpc).

Isotropy


Peebles (1993): Distribution of 31000 radio sources on northern sky (wavelength $\lambda = 6$ cm)

Isotropy $\Longleftrightarrow$ The universe looks the same in all directions.

N.B. Homogeneity *does not* imply isotropy, and isotropy around one point does not imply homogeneity!

World Models, I

A. Einstein (1879–1955)

Albert Einstein: Presence of mass leads to curvature of space (=gravitation)

$\Longrightarrow$ General Theory of Relativity (GRT)

GRT is applicable to Universe as a whole!

World Models, VI

A. Einstein (1879–1955)

Theoretical cosmology:
Combination of

1. relativity theory
2. thermodynamics
3. quantum mechanics

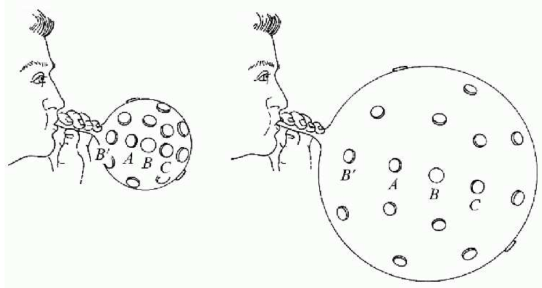
$\Longrightarrow$ complicated

Typically calculation performed in three steps:

1. Describe metric following the cosmological principle
2. Derive evolution equation from GRT
3. Use thermodynamics and quantum mechanics to obtain equation of state

. . . and then do some maths

World Models, VIII



R small R large

Misner, Thorne, Wheeler

Friedmann: Mathematical description of the Universe using normal "fixed" coordinates ("comoving coordinates"), plus scale factor R which describes evolution of the Universe.

Friedmann Equations, I

General relativistic approach: Insert metric into Einstein equation to obtain differential equation for $R(t)$:

Einstein equation:

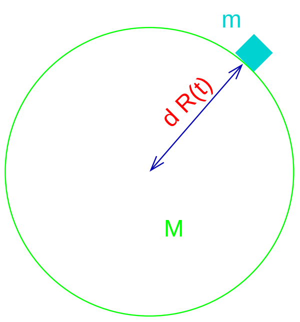
$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathscr{R} g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{9.1}$$

where

$g_{\mu\nu}$: Metric tensor ($\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$)
$R_{\mu\nu}$: Ricci tensor (function of $g_{\mu\nu}$)
$\mathscr{R}$: Ricci scalar (function of $g_{\mu\nu}$)
$G_{\mu\nu}$: Einstein tensor (function of $g_{\mu\nu}$)
$T_{\mu\nu}$: Stress-energy tensor, describing curvature of space due to fields present (matter, radiation, . . .)
Λ: Cosmological constant

$\Longrightarrow$ Messy, but doable

Friedmann Equations, II



Here, Newtonian derivation of Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (McCrea, 1937)

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \tag{9.2}$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d\,R(t)) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}m \tag{9.3}$$

Canceling $m \cdot d$ gives momentum equation:

$$\ddot{R}(t) = -\frac{4\pi G}{3}\frac{\rho_0}{R(t)^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{9.4}$$

Multiplying Eq. (9.4) with $\dot{R}$ and integrating yields the energy equation:

$$\frac{1}{2}\dot{R}(t)^2 = +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.} = +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.} \tag{9.5}$$

where the constant can only be obtained from GR.

Friedmann Equations, III

Problems with the Newtonian derivation:

1. Cloud is implicitly assumed to have $r_{\text{cloud}} < \infty$ (for $r_{\text{cloud}} \longrightarrow \infty$ the force is undefined)
   $\Longrightarrow$ violates cosmological principle.
2. Particles move *through* space
   $\Longrightarrow$ $v > c$ possible
   $\Longrightarrow$ violates SRT.

Why do we get correct result?

GRT $\longrightarrow$ Newton for small scales and mass densities; since universe is isotropic $\Longrightarrow$ scale invariance on Mpc scales $\Longrightarrow$ Newton sufficient (classical limit of GR).

(In fact, point 1 above *does* hold in GR: Birkhoff's theorem).

Friedmann Equations, IV

The exact GR derivation of Friedmanns equation gives:

$$\begin{aligned}\ddot{R} &= -\frac{4\pi G}{3}R\left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right] \\ \dot{R}^2 &= +\frac{8\pi G\rho}{3}R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right]\end{aligned} \tag{9.6}$$

Notes:

1. For $k = 0$: Eq. (9.6) $\longrightarrow$ Eq. (9.5).
2. k determines the curvature of space:
   - $k > 0$: open universe (infinite volume)
   - $k = 0$: flat universe
   - $k < 0$: closed universe (finite volume)
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. There is energy associated with the vacuum, parameterized by the parameter Λ.

Hubble's Law

The variation of $R(t)$ implies Hubble's Law:

Small scales $\Longrightarrow$ Euclidean geometry

Proper distance between two observers with comoving distance d:

$$D(t) = d \cdot R(t) \tag{9.7}$$

Expansion $\Longrightarrow$ D changes:

$$\frac{\Delta D}{\Delta t} = \frac{R(t+\Delta t)d - R(t)d}{\Delta t} \quad \text{and for} \lim_{\Delta t \to 0} \quad v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}\,d = \frac{\dot{R}}{R}D =: H\,D \tag{9.8}$$

$\Longrightarrow$ Identify local Hubble "constant" as

$$H = H(t) = \frac{\dot{R}(t)}{R(t)} \tag{9.9}$$

$\Longrightarrow$ Hubble "constant" is time-dependent! $\Longrightarrow$ "Hubble parameter"

Critical Density

Looking at the energy equation for $\Lambda = 0$,

$$\dot{R}^2 = +\frac{8\pi G\rho}{3}R^2 - kc^2 \qquad (9.10)$$

we find that the evolution of the Hubble parameter is:

$$\left(\frac{\dot{R}}{R}\right)^2 = H(t)^2 = \frac{8\pi G\rho(t)}{3} - \frac{kc^2}{R^2} \qquad (9.11)$$

and therefore

$$k \cdot \frac{c^2}{R(t)^2 H(t)^2} = \frac{8\pi G}{3H(t)^2}\rho(t) - 1 = \frac{\rho(t)}{\rho_{\text{crit}}} - 1 = \Omega - 1 \qquad (9.12)$$

where Ω is called the critical density:

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} \quad \text{where} \quad \rho_{\text{crit}} = \frac{3H^2}{8\pi G} \qquad (9.13)$$

currently: $\rho_{\text{crit}} \sim 1.67 \times 10^{-24}\,\text{g}\,\text{cm}^{-3}$ (3...10 H-Atoms m^{-3}).

Ω describes the curvature of the universe:

$\Omega > 1 \Longrightarrow k > 0$: closed | $\Omega = 1 \Longrightarrow k = 0$: flat | $\Omega < 1 \Longrightarrow k < 0$: open

Critical Density

World Model: Evolution of R as a function of time

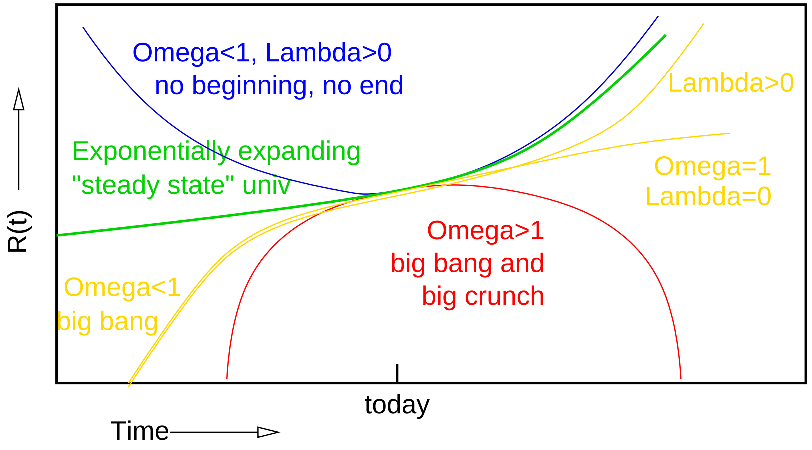
Solution of Friedmann equations depends on boundary conditions:

1. Value of H as measured today (H is time dependent!)
2. Density Parameter of universe

Note: total Ω is sum of:

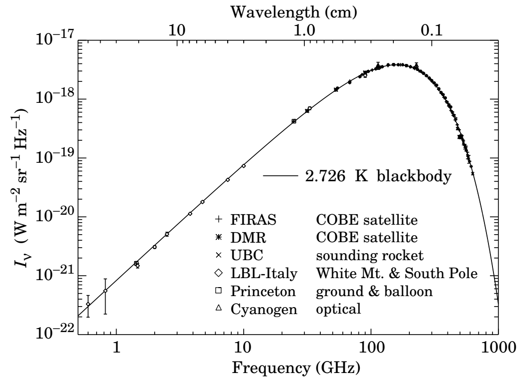
1. Ω_{m}: Matter, i.e., everything that leads to gravitative effects
   Ω_{m} in baryonic matter is $\lesssim$3%, but note there might be "nonbaryonic dark matter" as well!
2. $\Omega_\Lambda = \Lambda c^2/3H^2$: contribution caused by vacuum energy density Λ
   (Λ is often called "dark energy" for PR reasons)

Critical Density



Many different kinds of world models are possible, behaviour of universe depends on Ω und Λ.

3K CMB



Penzias & Wilson (1965): "Measurement of Excess Antenna Temperature at 4080 Mc/s"
$\Longrightarrow$ Cosmic Microwave Background radiation (CMB)

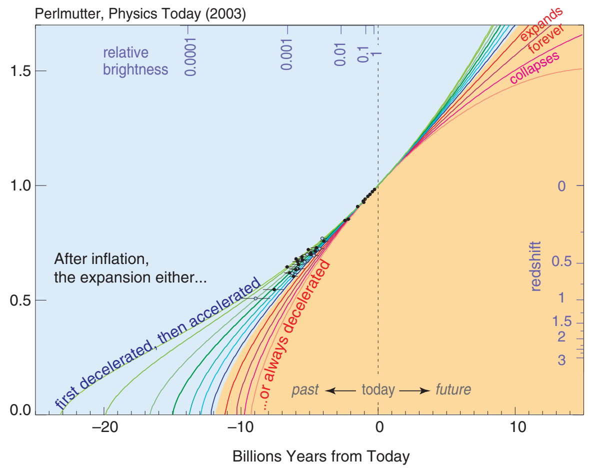
(Smoot et al., 1997, Fig. 1)

CMB spectrum is blackbody with temperature $T_{\text{CMB}} = 2.728 \pm 0.004\,\text{K}$.

Extrapolating CMB temperature back in time (see homework) shows:

Universe started with a hot big bang, has since cooled down.

World Models



Note: Extrapolation backwards gives age of universe as *roughly* $1/H_0$!

for $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 2.3 \times 10^{-18}\,\mathrm{s^{-1}}$, giving an age of 13.6 Gyr.

History of the universe, I

$R(t)$	t since BB	T[K] [K]	ρ_{matter} [g cm^{-3}]	Major Events
	10^{-42}	10^{30}		Planck era, “begin of physics”
	$10^{-40\ldots-30}$	10^{25}		Inflation (IMPLIES $\Omega = 1$)
10^{-13}	$\sim10^{-5}$ s	$\sim10^{13}$	$\sim10^{9}$	generation of p-p$^-$, and baryon anti-baryon pairs from radiation background
3×10^{-9}	1 min	10^{10}	0.03	generation of e$^-$-e$^+$ pairs out of radiation background
10^{-9}	10 min	3×10^{9}	10^{-3}	nucleosynthesis
$10^{-4}\ldots10^{-3}$	$10^{6\ldots7}$ yr	$10^{3\ldots4}$	$10^{-21\ldots-18}$	End of radiation dominated epoch
7×10^{-4}	380000 yr	4000	10^{-20}	Hydrogen recombines, decoupling of matter and radiation
	200×10^{6} yr			first stars formed
1	13.7×10^{9} yr	3	10^{-30}	now

History of the universe, II

BB works remarkably well in explaining the observed universe.

There are, however, quite big problems with the classical BB theories:

Horizon problem: CMB looks too isotropic $\Longrightarrow$ Why?

Flatness problem: Density close to BB was very close to $\Omega = 1$ (deviation $\sim 10^{-16}$ during nucleosynthesis) $\Longrightarrow$ Why?

Hidden relics problem: There are no observed magnetic monopoles, although predicted by GUT, neither gravitinos and other exotic particles $\Longrightarrow$ Why?

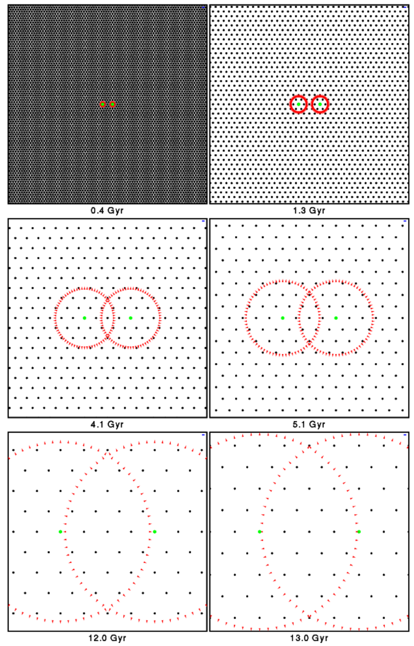
Vacuum energy problem: Energy density of vacuum is 10^{120} times smaller than predicted $\Longrightarrow$ Why?

Expansion problem: The universe expands $\Longrightarrow$ Why?

Baryogenesis: There is virtually no antimatter in the universe $\Longrightarrow$ Why?

Structure formation: Standard BB theory produces no explanation for lumpiness of universe.

Inflation attempts to answer all of these questions.



courtesy E. Wright.
Expansion of horizon in an expanding universe.

History of the universe, IV

Use the Friedmann equation with a cosmological constant:

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{9.14}$$

where $a = R(t)/R_0$

Basic assumption of inflationary cosmology:

During the big bang there was a phase where Λ dominated the Friedmann equation.

$$H(t) = \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}} = \text{const.} \tag{9.15}$$

since $\Lambda = \text{const.}$ (probably...).

Solution of Eq. (9.15):

$$a \propto e^{Ht} \tag{9.16}$$

History of the universe, V

When did inflation happen?

Typical assumption: Inflation = phase transition of a scalar field ("inflaton") associated with Grand Unifying Theories.

Therefore the assumptions:

- temperature $kT_{\text{GUT}} = 10^{15}$ GeV, when $1/H \sim 10^{-34}$ sec ($t_{\text{start}} \sim 10^{-34}$ s).
- inflation lasted for 100 Hubble times, i.e., for $\Delta T = 10^{-32}$ s.

With Eq. (9.16):

Inflation: Expansion by factor $e^{100} \sim 10^{43}$.

...corresponding to a volume expansion by factor $\sim 10^{130} \Longrightarrow$ solves hidden relics problem!

Evolution of the Universe

Extrapolating backwards, universe is asymptotically flat

$\Longrightarrow$ physics of early universe ~independent of later evolution

What is *very* dependent on H and Ω is later evolution, i.e., formation of structure and evolution of universe to what it is today

Modern Cosmology: Determination of H_0, Ω and Λ from observations and comparison with theory

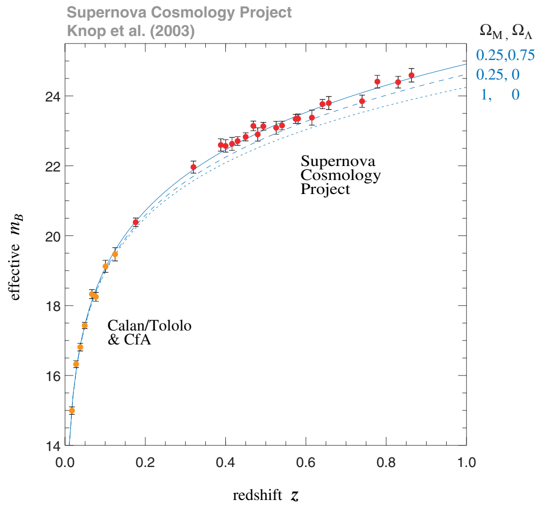
H_0: value of Hubble parameter today

In the following: Examples for new measurements to determine Ω and Λ:

- Supernova observations and
- Cosmic Microwave Background (WMAP).

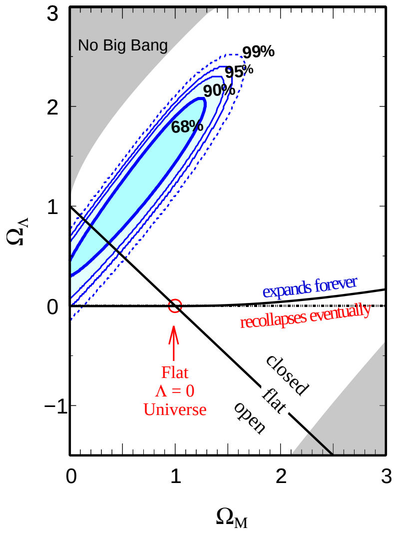
General hope: confirmation that $\Omega_m + \Omega_\Lambda = 1$ as predicted by theory of inflation (this implies a *flat* universe).

Supernovae



Supernova observations are well explained by models with $\Omega_m = 0.25$ and $\Omega_\Lambda = 0.75$.

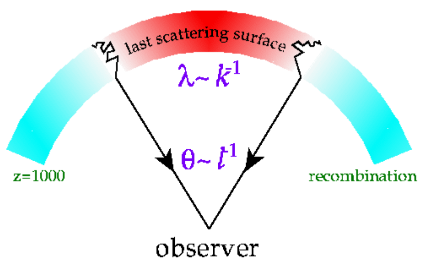
Supernovae



Supernova observations are well explained by models with $\Omega_m = 0.25$ and $\Omega_\Lambda = 0.75$.

$\Omega_\Lambda = 0$ is *excluded* by data!

CMB, I



courtesy Wayne Hu

After Big Bang: universe dense ("foggy"), photons efficiently scatter off electrons $\Longrightarrow$ coupling of radiation and matter

Universe cools down: recombination of protons and electrons into hydrogen

$\Longrightarrow$ no free electrons

$\Longrightarrow$ scattering far less efficient

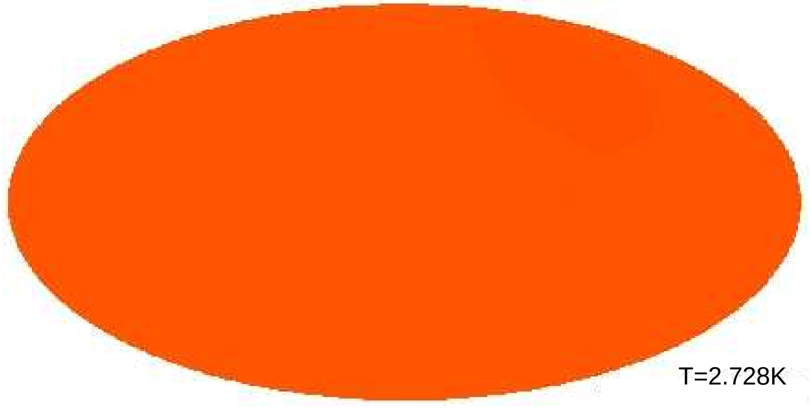
$\Longrightarrow$ Photons: "free streaming"

Photons escaping from overdense regions loose energy (gravitational red shift)

$\Longrightarrow$ Observable as temperature fluctuation (Sachs Wolfe Effect)

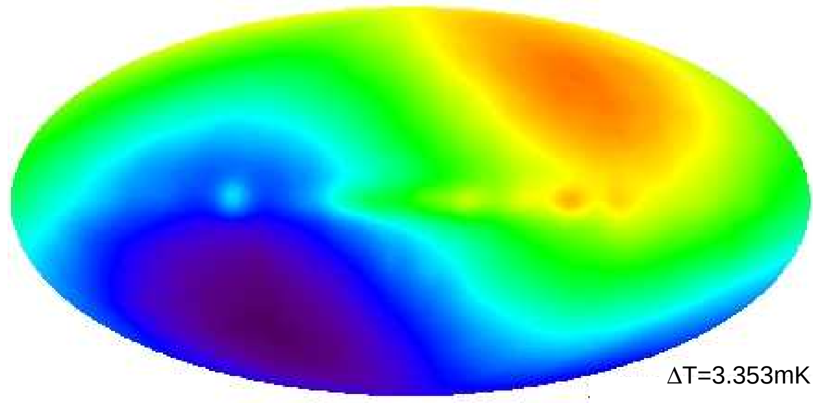
CMB Fluctuations $\sim$ Gravitational potential at $z \sim 1100$ $\Longrightarrow$ structures

CMB, II



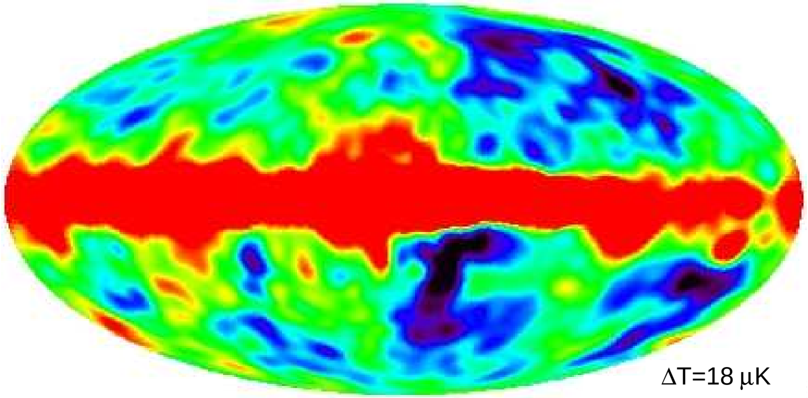
COBE (1992): First map of 3 K CMB
$T = 2.728\,\mathrm{K}$

CMB, III

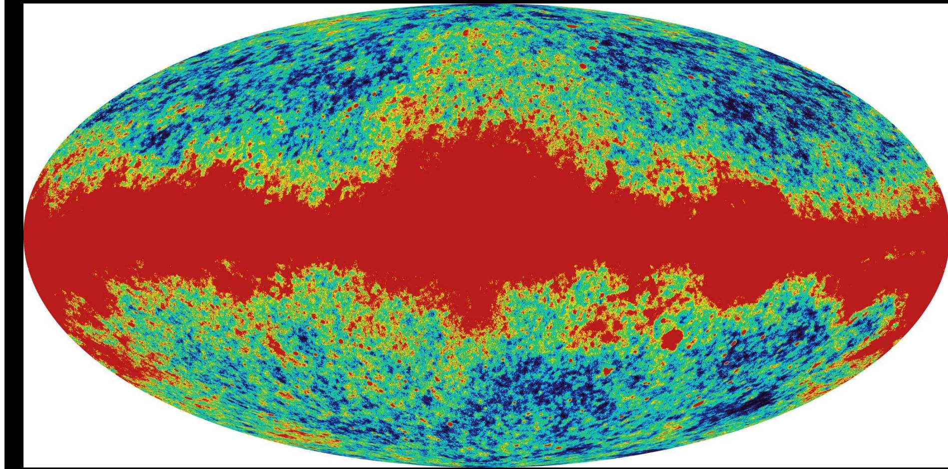


Overlaid: Dipole anisotropy caused by motion of the solar system
Temperature fluctuation: $\Delta T/T \sim 10^{-4}$

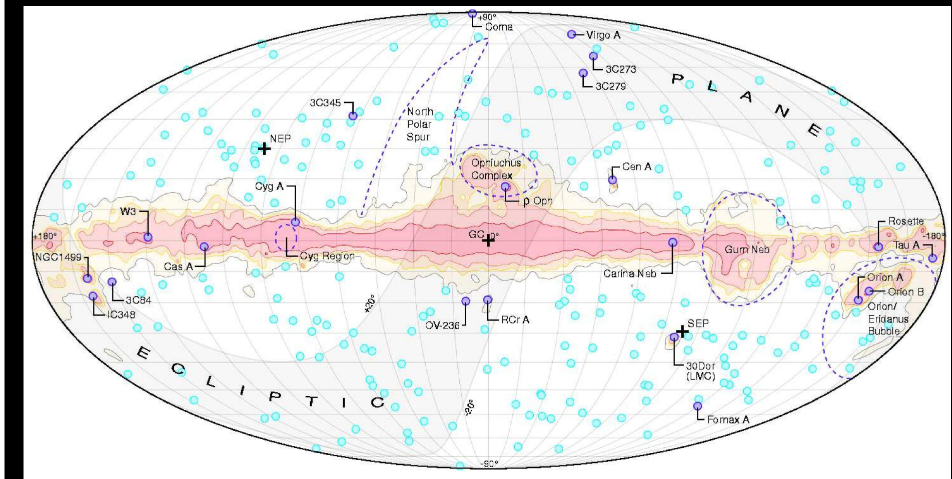
CMB, IV



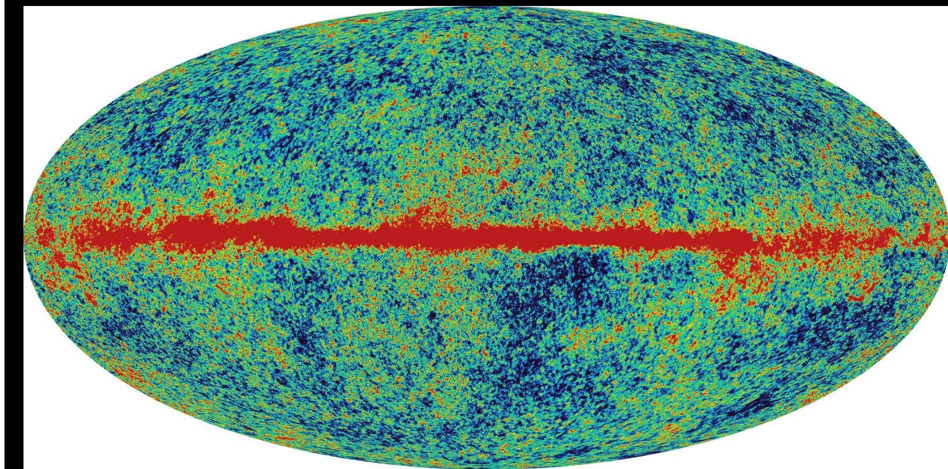
At level of $\Delta T/T \sim 10^{-5}$: Deviations from isotropy due to structure formation



WMAP, K-Band, $\lambda = 13\,\mathrm{mm}$, $\nu = 22.8\,\mathrm{GHz}$, resolution 0.83°

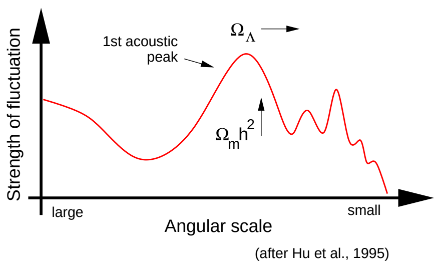


Foreground features



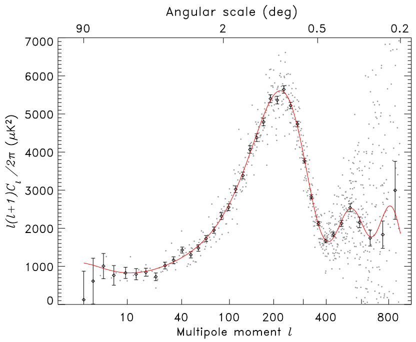
WMAP, W-Band, $\lambda = 3.2\,\mathrm{mm}$, $\nu = 93.5\,\mathrm{GHz}$, resolution 0.21°

Results



Power spectrum of CMB depends on

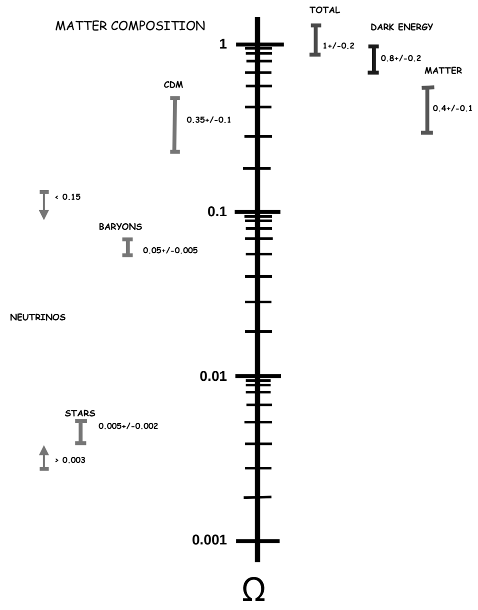
$\Omega_m \quad H_0 \quad \Omega_\Lambda$



WMAP best fit parameters (assuming $\Omega = 1$, $H_0 =: h \cdot 100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$):

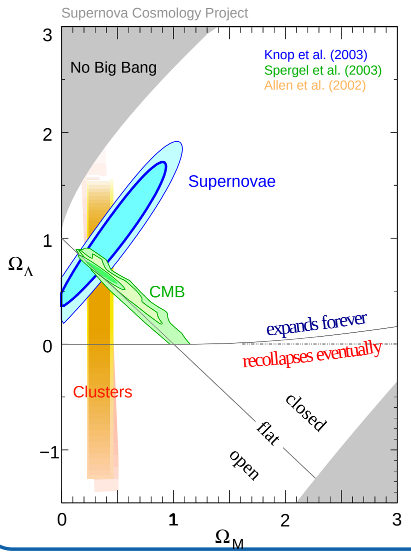
$$h = 0.72 \pm 0.05$$
$$\Omega_m h^2 = 0.14 \pm 0.02$$

MATTER / ENERGY in the UNIVERSE



M.J. Turner

Results



Confidence regions for Ω_Λ and Ω_m.

dark: 68% confidence, outer region: 90%

$$\Omega = 1.02 \pm 0.02$$
$$\Omega_m = 0.14 \ldots 0.3$$
$$H_0 = 72 \pm 5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

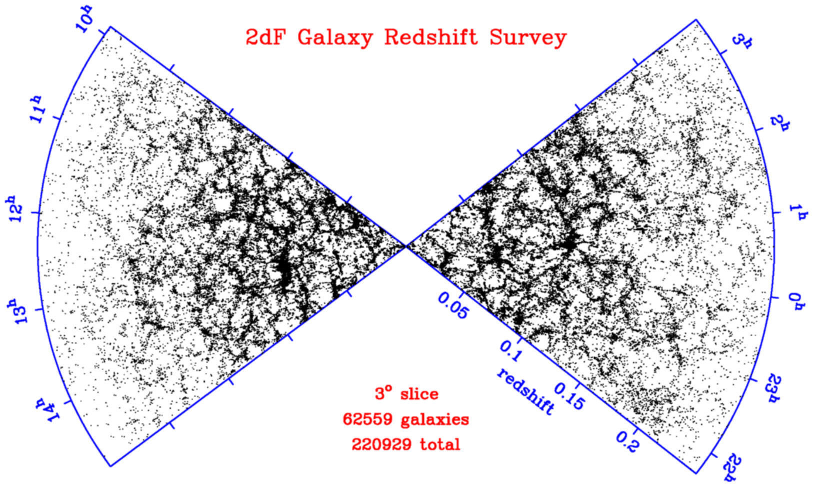
leading to an age of the universe of 13.7 billion years.

This means:

~70% of the universe is due to "dark energy"

. . . and what this is: we have no clue

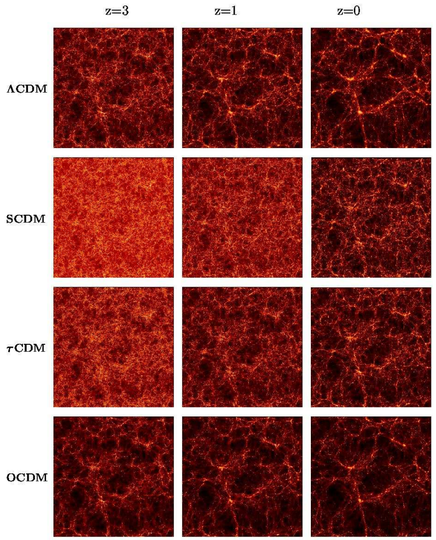
Large Scale Structures



2dF Survey, ~220000 galaxies total ⟹ structures

Hubble Ultra Deep Field (11 days exposure!)

Theoretical Structure Formation



Virgo collaboration

We can use theories for nature of Λ and measured values of H_0 and Ω to predict how galaxies evolve in the universe.

Theoretical Structure Formation

"Structure formation": How to form density perturbations in an initially approximately smooth universe. Perturbations then grow, forming structures (galaxies, galaxy clusters)

To understand formation of structures, need to study evolution of universe with dark matter:

Hot Dark Matter: relativistic particles (e.g., neutrinos): moving with $v \sim c$. Fast particles

$\Longrightarrow$ smears out small density perturbations

$\Longrightarrow$ "top down structure formation"

Not what is observed

(observed: galaxies were there first, clusters are still forming)

Cold Dark Matter: slow particles, condense first, forming potential wells while matter still coupled to radiation.

Once radiation decouples from matter (when universe is cold enough), matter falls in gravity wells.

$\Longrightarrow$ "bottom up structure formation"

Closer to what is observed

Best models: combination of CDM and Λ