Stellar Evolution

Now that we believe that the solar model is correct: stellar evolution

Principle:

1. Construct stellar model by solving equations of stellar structure for given radial abundances.
2. Evaluate change in elemental abundances as a function of radius based on the local fusion processes.
3. Change abundances appropriately for a time step Δt.
4. goto step 1

Characteristic Timescales

Main sequence: Hydrogen burning at the center.

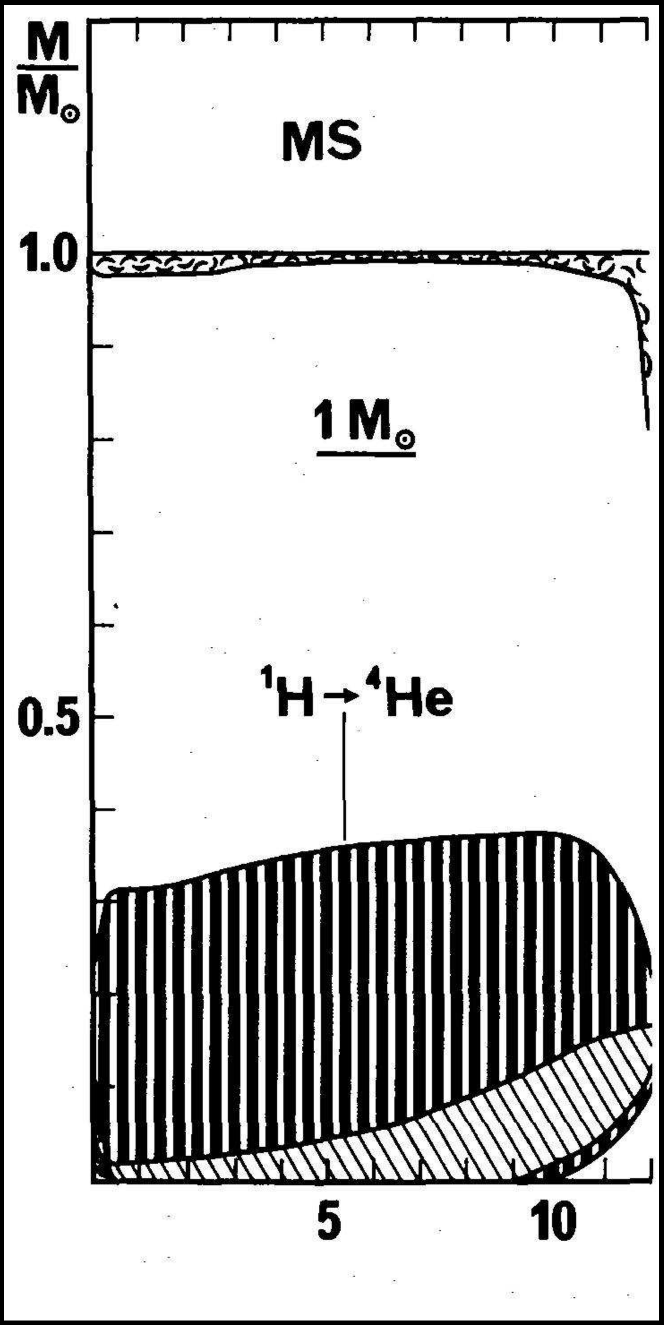
Evolution timescale dominated by the nuclear timescale $=$ timescale needed to use the fuel in the center of the star.

According to simulations, this is $\sim$10% of the available Hydrogen.
Since 0.7% of $M_{\text{core}}c^2$ converted into He, the nuclear timescale is

$$t_{\text{n}} = \frac{0.007 \cdot 0.1 M c^2}{L} = \frac{M/M_\odot}{L/L_\odot} \cdot 10^{10}\,\text{years} \tag{3.1}$$

A second important timescale is the timescale the star would need to radiate its stored thermal energy: thermal timescale.
Roughly given as

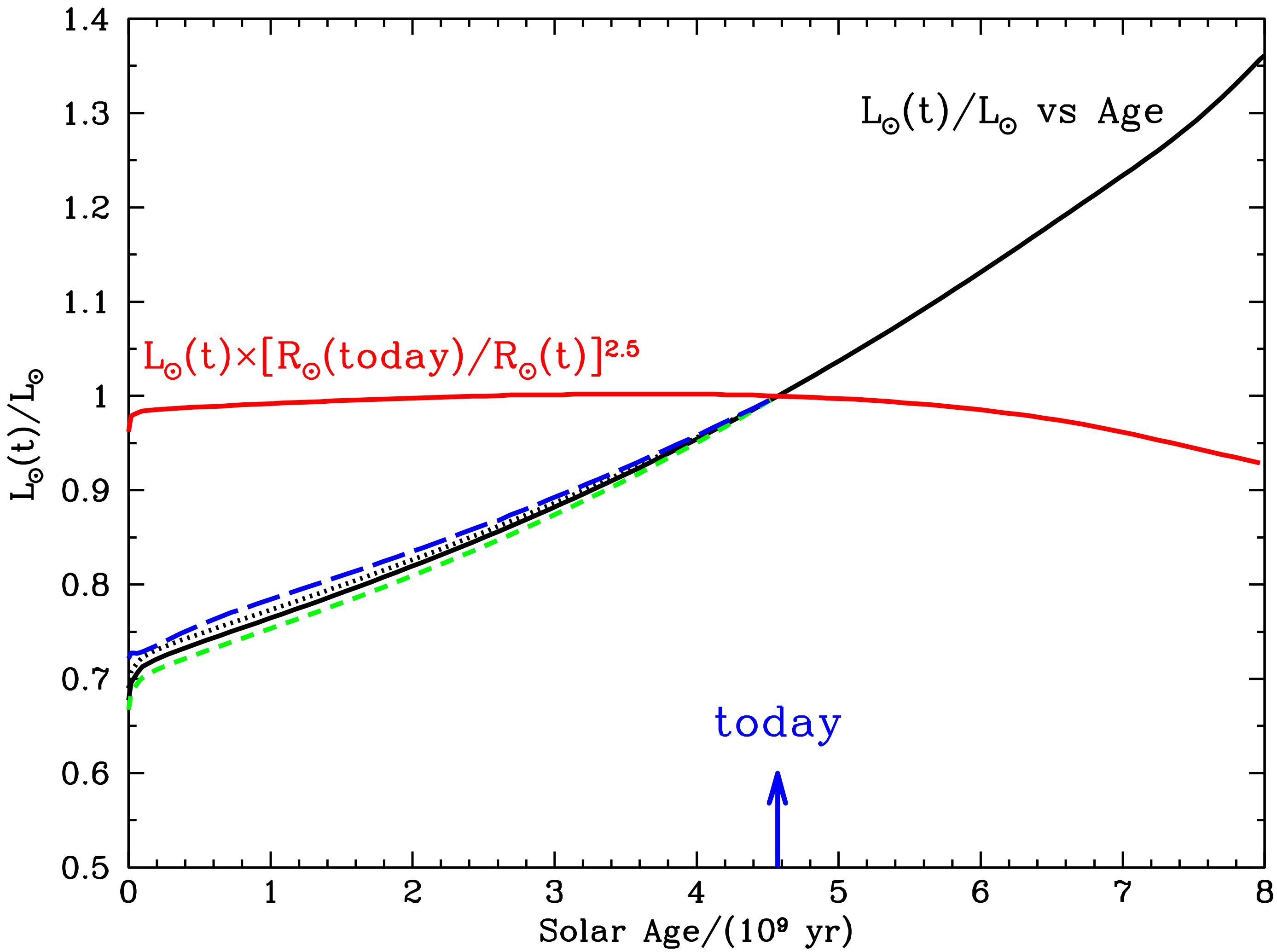
$$t_{\text{t}} = \frac{0.5 G M^2/R}{L} = \frac{(M/M_\odot)^2}{(R/R_\odot)(L/L_\odot)} \cdot 2 \times 10^7\,\text{years} \tag{3.2}$$



$t(10^9$ years$) \longrightarrow$

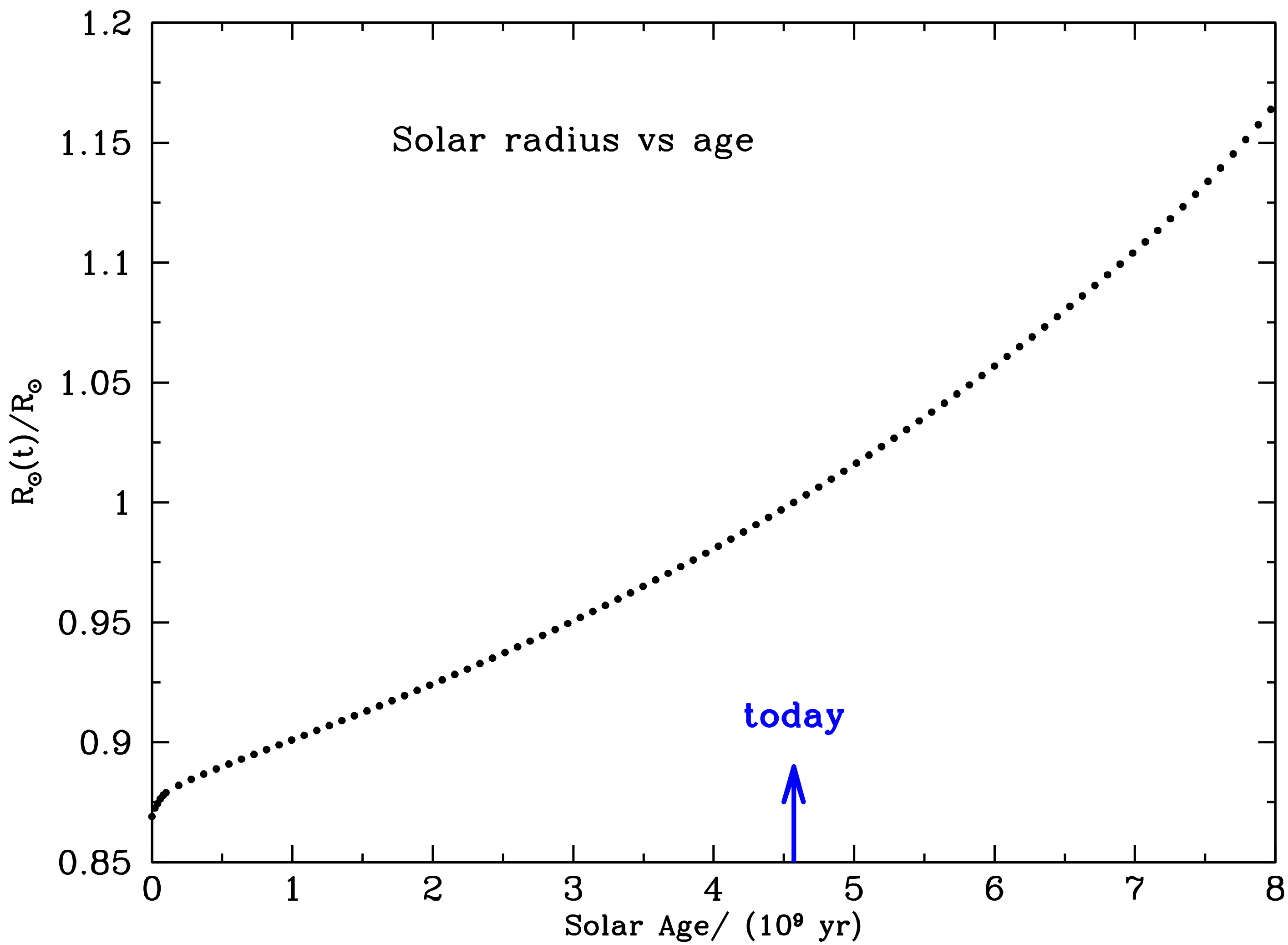
Evolution of the structure of a 1 $M_\odot$ star on the main sequence (after Maeder & Meynet, 1989).

MS Evolution: Luminosity



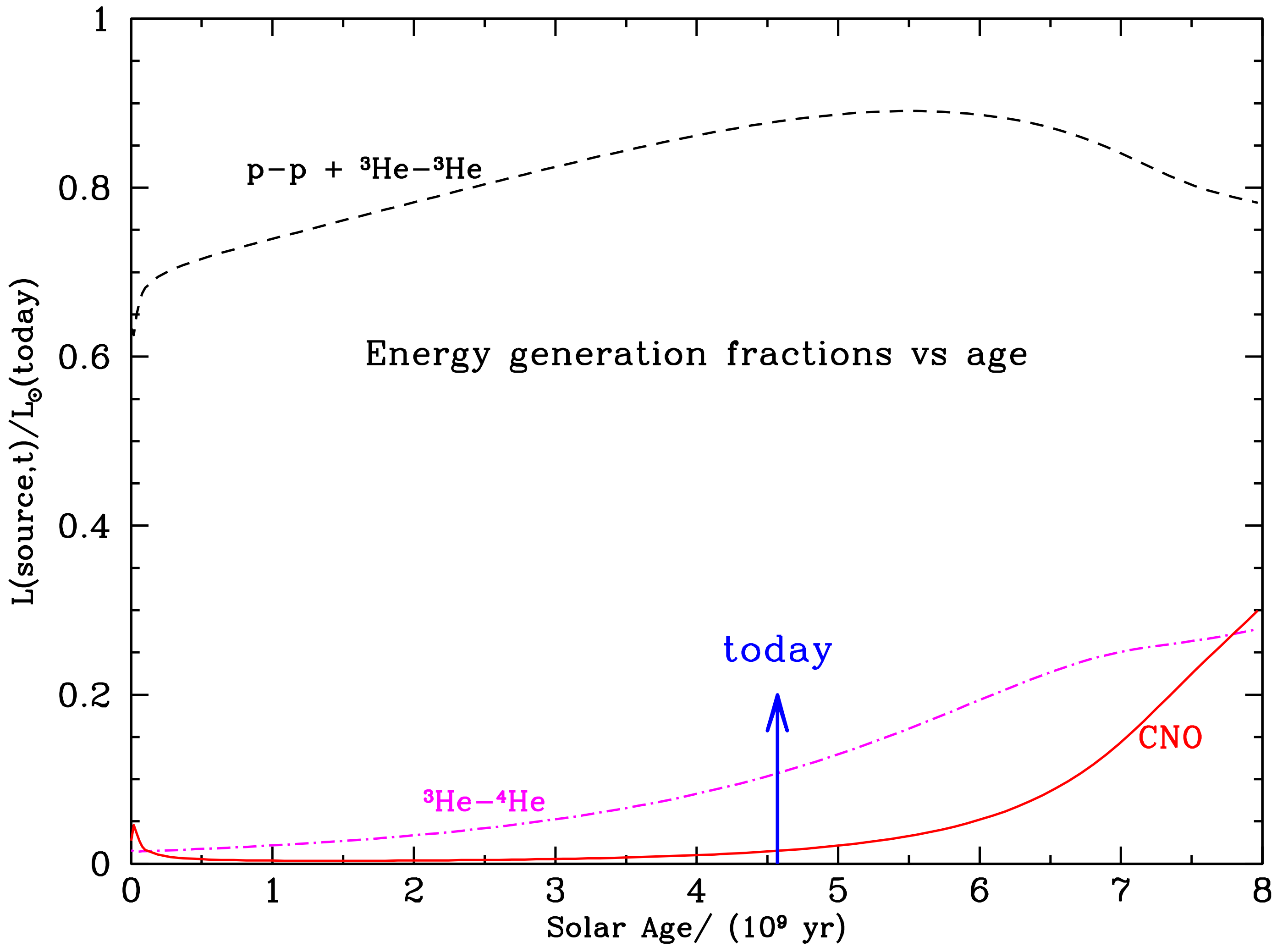
Bahcall, Pinsonneault & Basu (2001, ApJ 555, 990)

MS Evolution: Radius



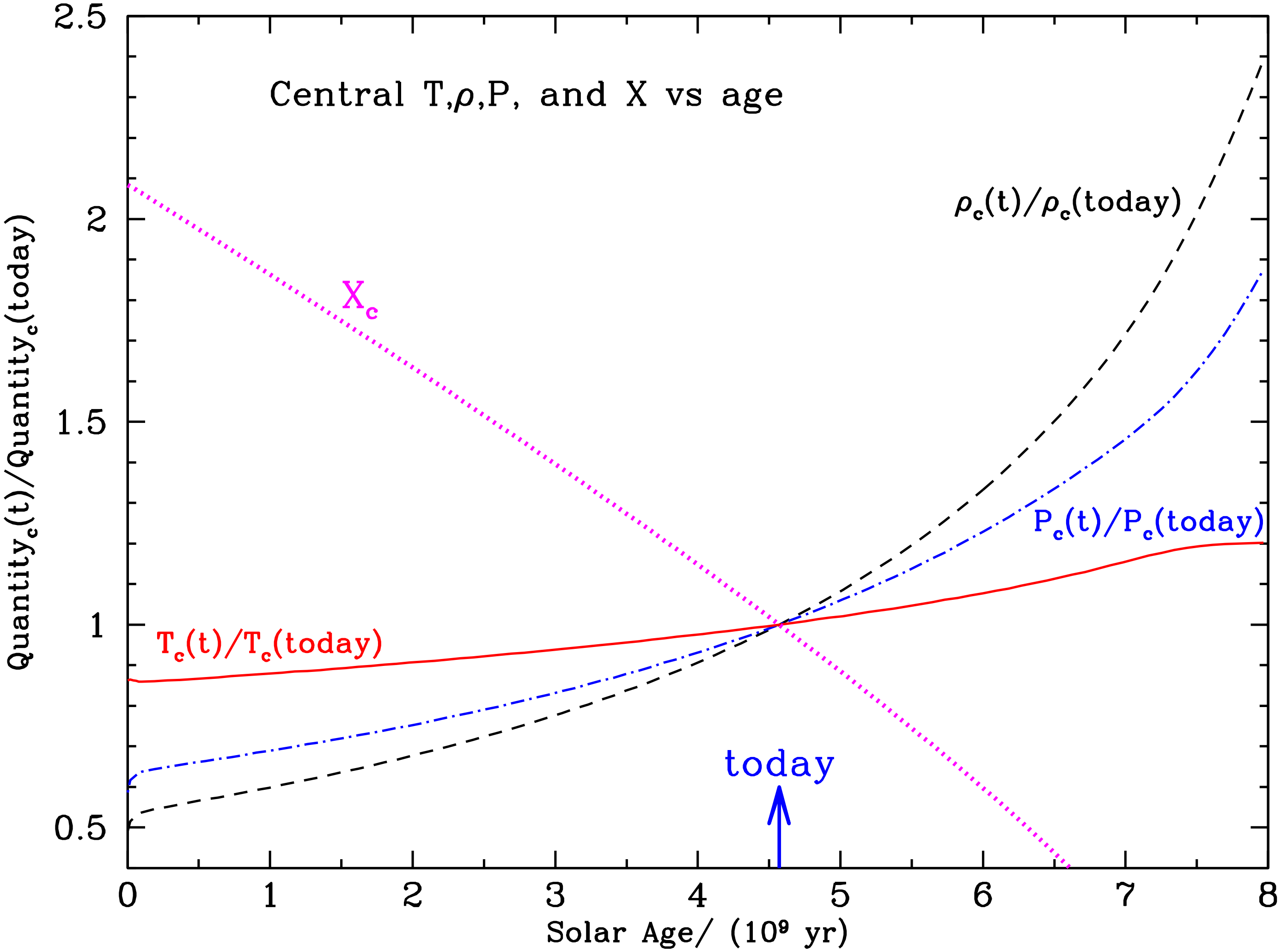
Bahcall, Pinsonneault & Basu (2001, ApJ 555, 990)

MS Evolution: Energy Generation



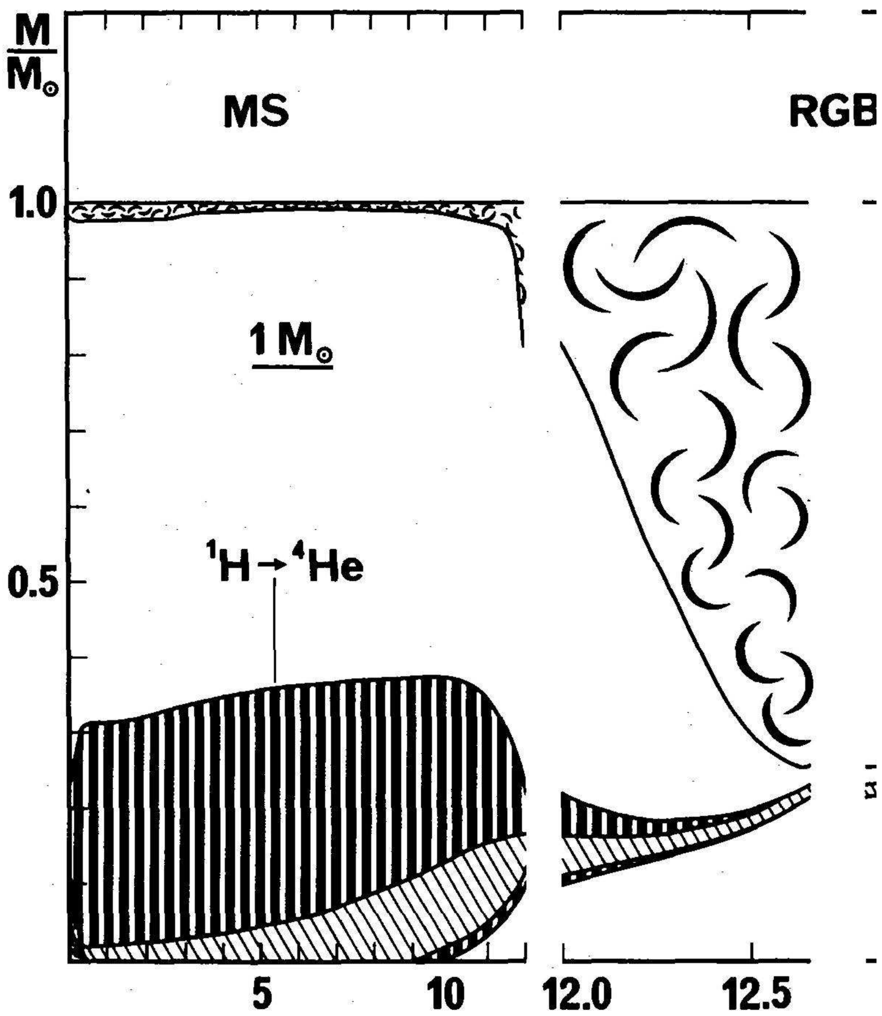
Bahcall, Pinsonneault & Basu (2001, ApJ 555, 990)

MS Evolution: Center



Bahcall, Pinsonneault & Basu (2001, ApJ 555, 990; X_c is the central H fraction)

Solar Mass Stars: Post Main Sequence, I

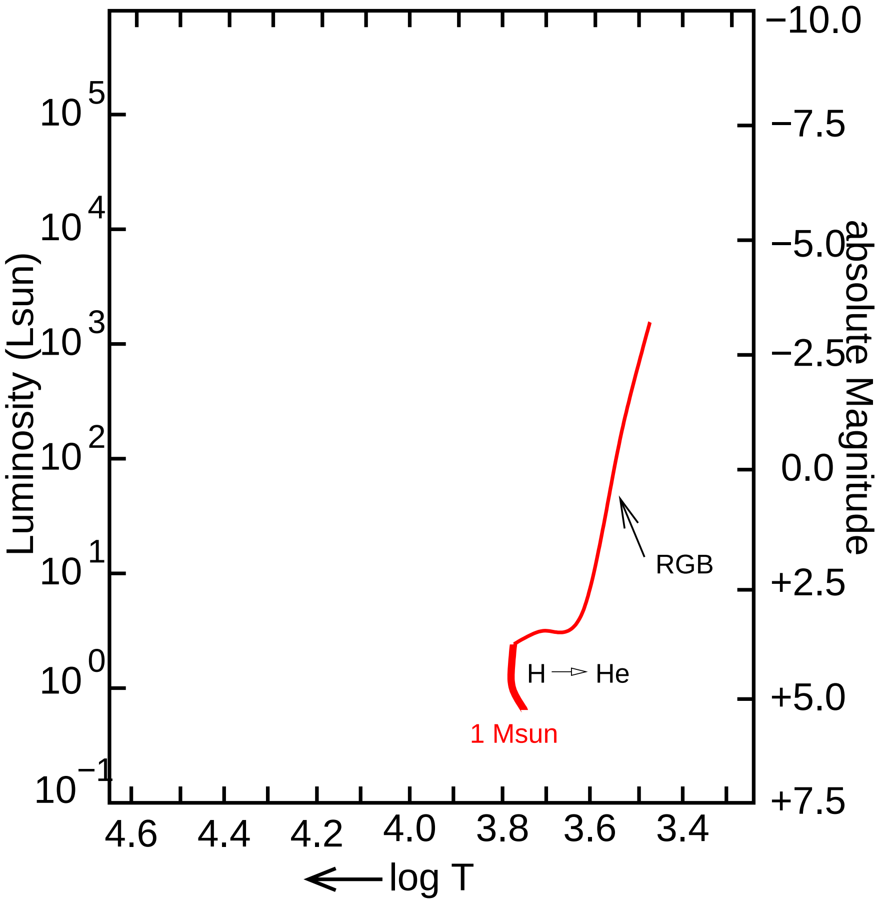


(Maeder & Meynet, 1989)

Once H is exhausted in center:
H continues to burn in a shell around the He core ("shell burning").

For stars with $M \lesssim 1\,M_\odot$: Star reacts by expanding convective hull until it is almost fully convective.

Solar Mass Stars: Post Main Sequence, II



(after Iben, 1991)

Once H is exhausted in center: H continues to burn in a shell around the He core ("shell burning").

For stars with $M \lesssim 1\,M_\odot$: Star reacts by expanding convective hull until it is almost fully convective.

$\Longrightarrow$ luminosity increases, temperature decreases

$\Longrightarrow$ motion in HRD horizontally towards the right, then upwards to higher L: red giant stage.

Solar Mass Stars: Post Main Sequence, III

Reminder: stars are in hydrostatic equilibrium: inwards gravitational pressure balanced by outwards gas pressure

Since the gas pressure is $P = nkT$: energy source needed to heat gas (=fusion).

This is a problem for the core during the red giant stage, as virtually no fusion ongoing

$\Longrightarrow$ Core gets compressed

$\Longrightarrow$ ρ and T increase

BUT:

collapse cannot continue indefinitely!

$\Longrightarrow$ once ρ has increased appreciably, there must be a point where quantum mechanical effects become important.

Different ways to write the equation of state of an ideal gas

Among the more confusing subjects of thermodynamics are the many different ways in which the ideal gas equation can be written.

The one I prefer for astronomy is

$$P = nkT$$

where

- P: Pressure (measured in $N\,\mathrm{m}^{-2}$)
- n: particle density (i.e., number of particles per cubic meter, unit: m^{-3})
- $k = 1.38066 \times 10^{-23}\,\mathrm{J\,K^{-1}}$: Boltzmann constant
- T: Temperature (measured in Kelvins)

This equation has the advantage that it counts all particles individually (thus using n). If you know the mass of the gas particles, m_{gas} then another way of writing the ideal gas equation is

$$P = \frac{n m_{\mathrm{gas}}}{m_{\mathrm{gas}}} kT = \rho kT \frac{1}{m_{\mathrm{gas}}}$$

illustrating that for an ideal gas, $P \propto \rho$, where ρ is the mass density.

Another way to write the ideal gas equation is in terms of the total number of gas molecules, $N = nV$, where V is the volume. The ideal gas equation then is

$$P = \frac{N}{V} kT \quad \Longleftrightarrow PV = NkT \quad \Longleftrightarrow \frac{PV}{T} = Nk$$

This version has the problem, however, that the number of gar molecules is typically rather large (there are 6×10^{23} molecules in a volume of 22.4 liters of gas, this number of particles is called one *mole*). Because working with smaller numbers is generally thought a good idea, chemists prefer to work with moles. Per definition, the unit of particle number here is the Avogadro number $N_{\mathrm{A}} = 6.0221 \times 10^{23}$. So, if you want to work with moles, then the above equation becomes

$$PV = \frac{N}{N_{\mathrm{A}}} AkT = N_{\mathrm{mol}} RT$$

where

- N_{mol}: the number of moles of the gas in the volume V,
- $R = N_{\mathrm{A}} k 8.3145\,\mathrm{J\,mol^{-1}\,K^{-1}}$: the universal gas constant

To summarise, each of these equations has its own uses, and which one you want to use, really depends on the circumstances of the problem you are solving. For your future life as physicists, try to remember one of them, and then understand how you get from this one to the others, instead of memorising all four ones. This approach will need less memory and lead to a better understanding of what is really going on behind the scenes.

QM interlude, I

Quantum mechanics: One of the stranger phenomena in QM is the Pauli exclusion principle:

> For particles such as electrons ("Fermions"), at least one of their quantum numbers must be different.

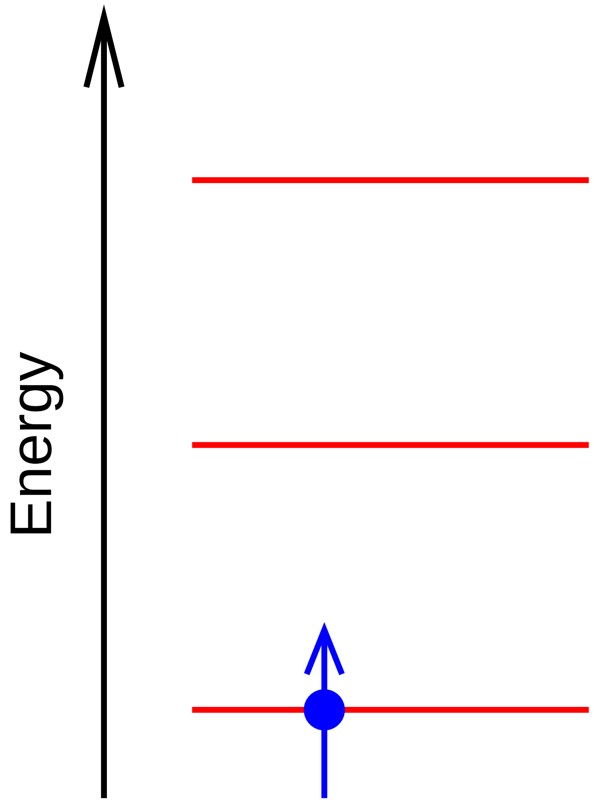
Quantum numbers are, e.g.,

- position (x, y, z),
- momentum (mv_x, mv_y, mv_z),
- angular momentum,
- spin (s)

All of these numbers are "quantized", i.e., can only have discrete values (e.g., spin: $+1/2$, $-1/2$).

In a typical gas, this is not a problem ("phase space is (almost) empty") once it becomes dense $\Longrightarrow$ exclusion principle kicks in.

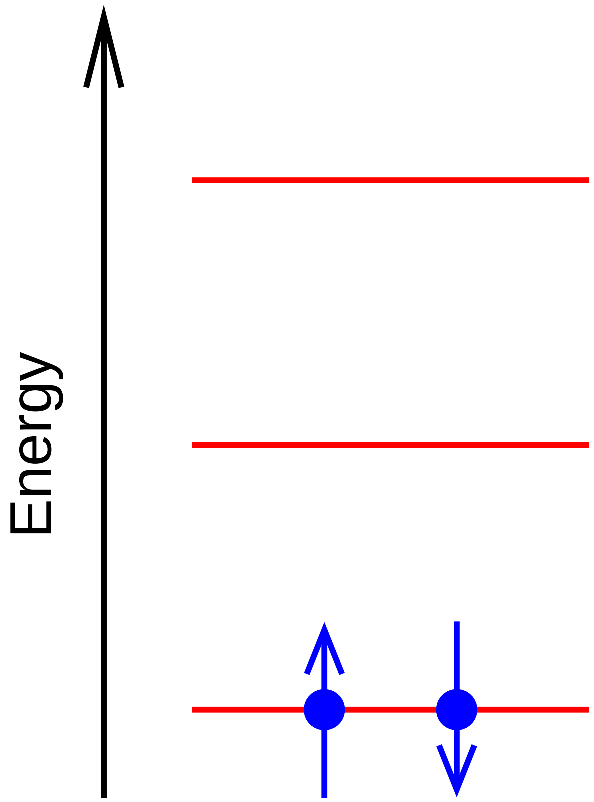
QM interlude, II



Effect of high density on electron energy

Energy of electrons at the same position in space

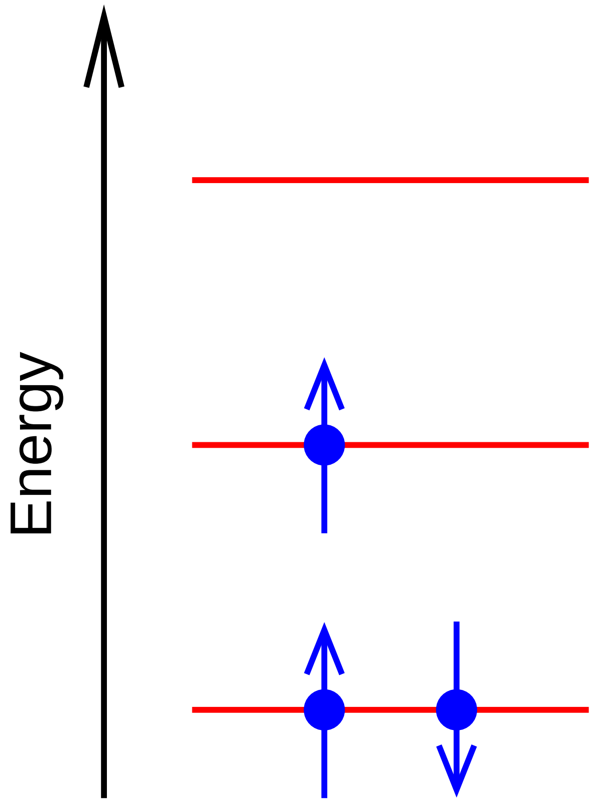
QM interlude, III



Energy of electrons at the same position in space

Effect of high density on electron energy

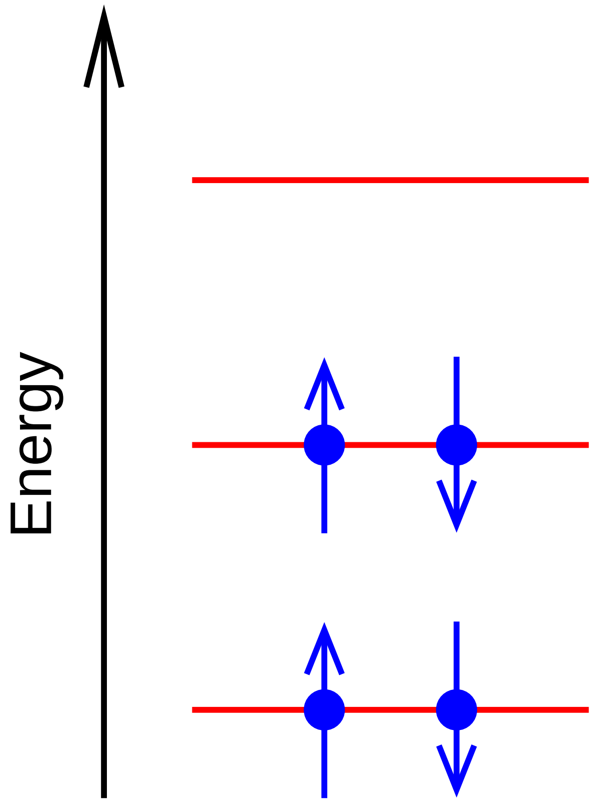
QM interlude, IV



Energy of electrons at the same position in space

Effect of high density on electron energy

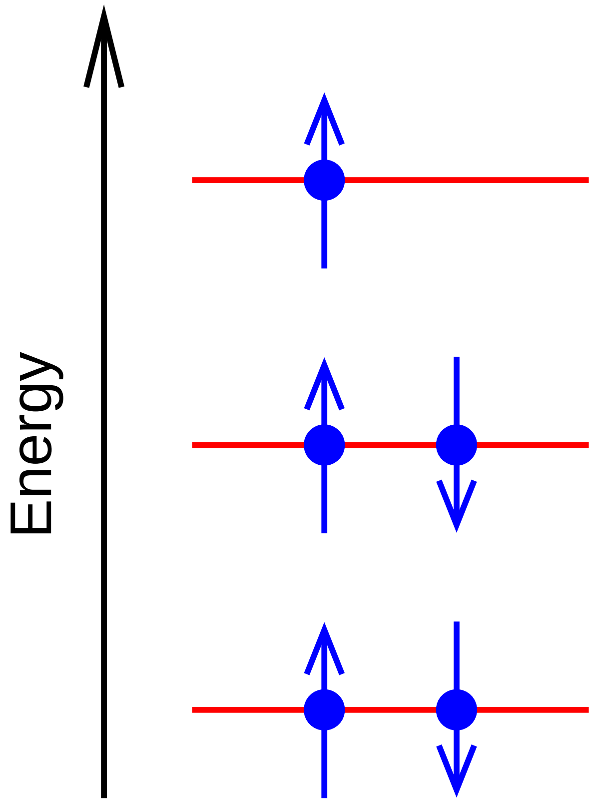
QM interlude, V



Energy of electrons at the same position in space

Effect of high density on electron energy

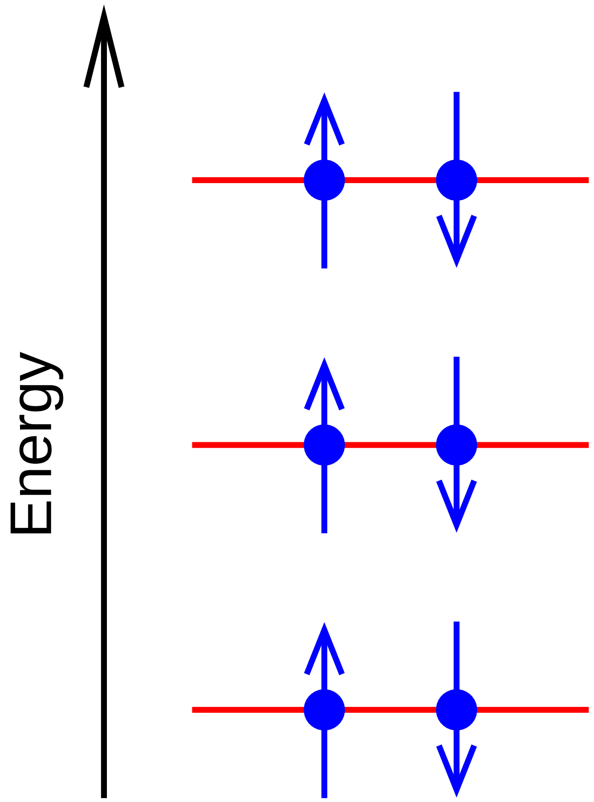
QM interlude, VI



Effect of high density on electron energy

Energy of electrons at the same position in space

QM interlude, VII

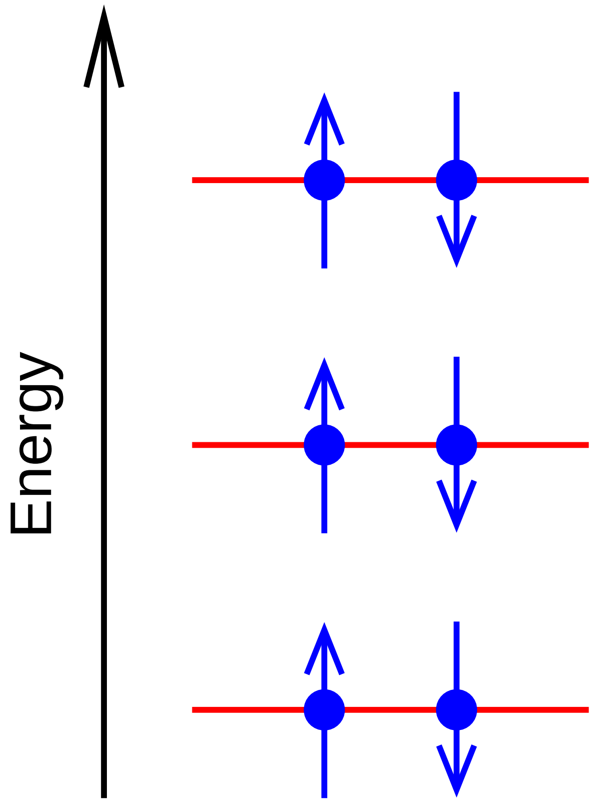


Energy of electrons at the same position in space

Effect of high density on electron energy:

In degenerate electron gases, electrons have much higher energies than in thermal gas.

QM interlude, VIII



Energy of electrons at the same position in space

Effect of high density on electron energy:

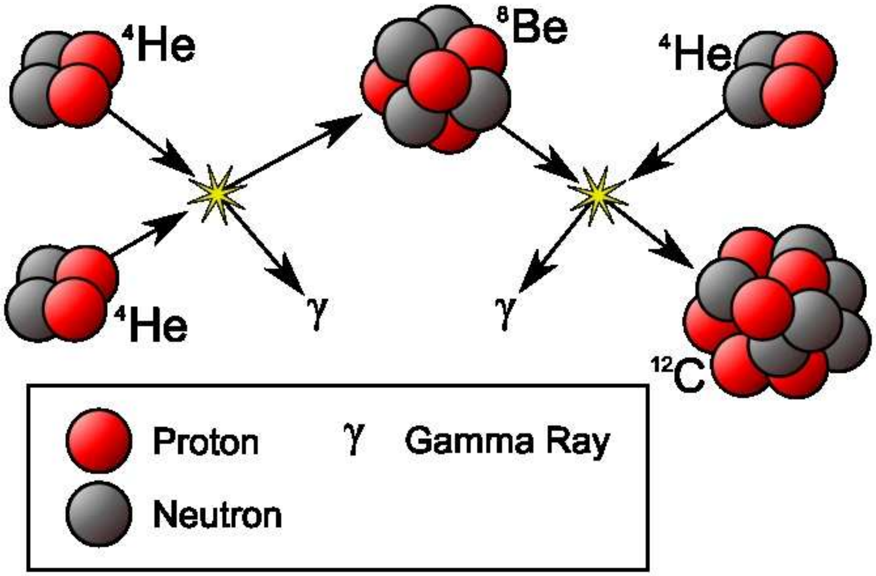
In degenerate electron gases, electrons have much higher energies than in thermal gas.

Interaction of electrons results in degeneracy pressure:

$$P = \frac{\hbar^2}{m_\mathrm{e}} n_\mathrm{e}^{5/3} \propto \rho^{5/3}$$

Note: The degeneracy pressure is independent of the temperature!

Solar Mass Stars: Post Main Sequence, I



In the degenerate core, once T_{core} ~100 × 10^6 K: Triple alpha process starts:

$$^{4}\text{He} + {}^{4}\text{He} \leftrightarrow {}^{8}\text{Be}$$
$$^{8}\text{Be} + {}^{4}\text{He} \rightarrow {}^{12}\text{C}$$

Since ^{8}Be has a half life of only 2.6×10^{-16} s: effectively this can only work if 3 α-particles collide.

But core is degenerate:

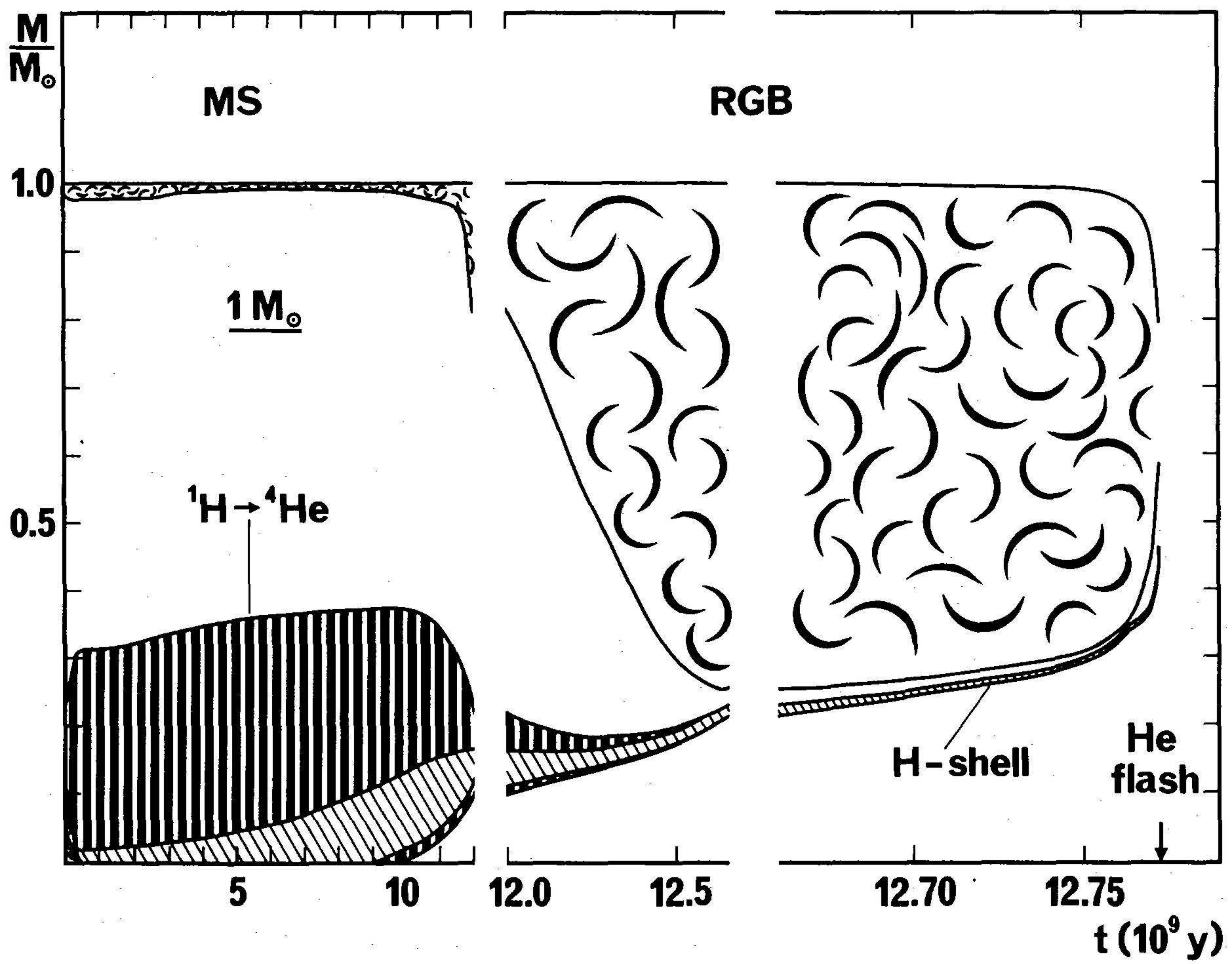
$\Longrightarrow$ High thermal conductivity of electrons

$\Longrightarrow$ core has uniform temperature

$\Longrightarrow$ 3α onset is rapid

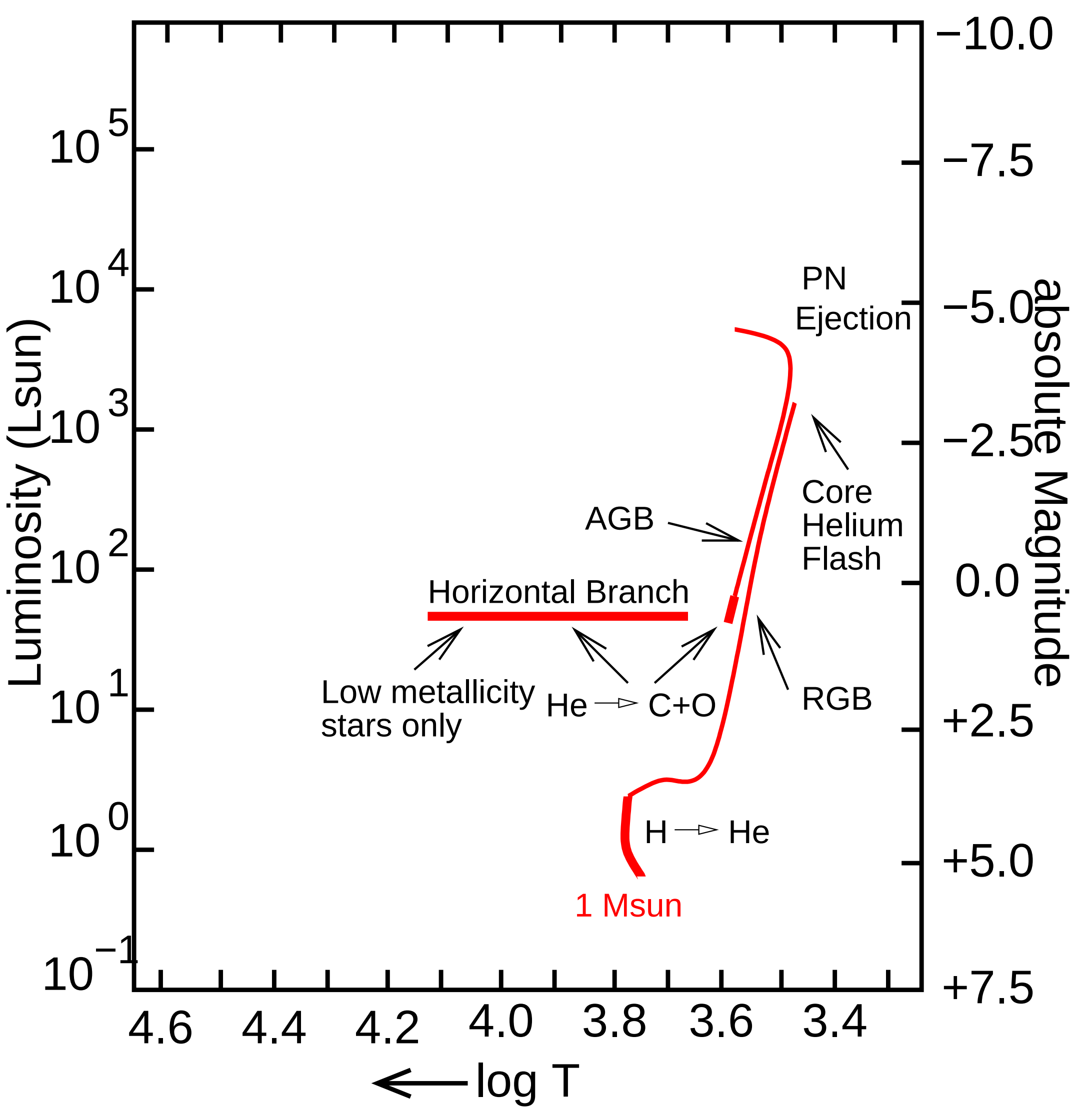
$\Longrightarrow$ He flash

Not seen on surface (“buffered” by convective envelope).



Evolution of the structure of a 1 $M_\odot$ star to the Helium flash (Maeder & Meynet, 1989).

Solar Mass Stars: Post Main Sequence, III



After the He flash star has He burning in core and H shell burning

$\Longrightarrow$ starts to expand again

$\Longrightarrow$ "asymptotic giant branch"

Instable He fusion processes ("thermal pulses") lead to ejection of outer layers ($\sim$50% of total mass!)

Effect of He core being unable to transport energy away quickly enough.

$\Longrightarrow$ inner (hotter) parts of star become visible.

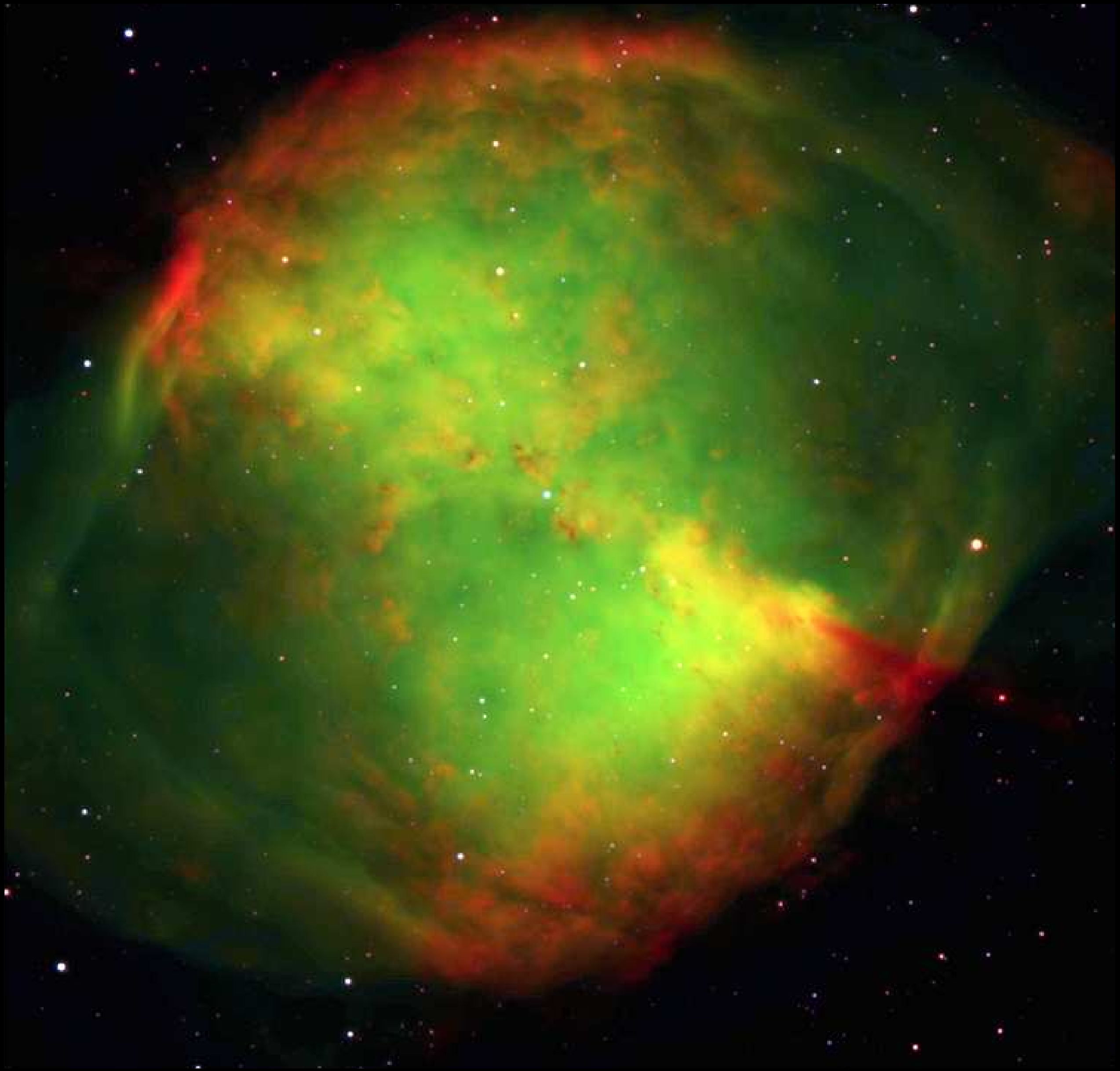
after Iben, 1991

Abell 39 (WIYN, AURA, NOAO, NSF)

planetary nebulae: material ejected during AGB phase, photoionized once remaining core of former star has shed enough mass to emit UV photons.

Ring Nebula (HST/STScI/NASA)

planetary nebulae: material ejected during AGB phase, photoionized once remaining core of former star has shed enough mass to emit UV photons.

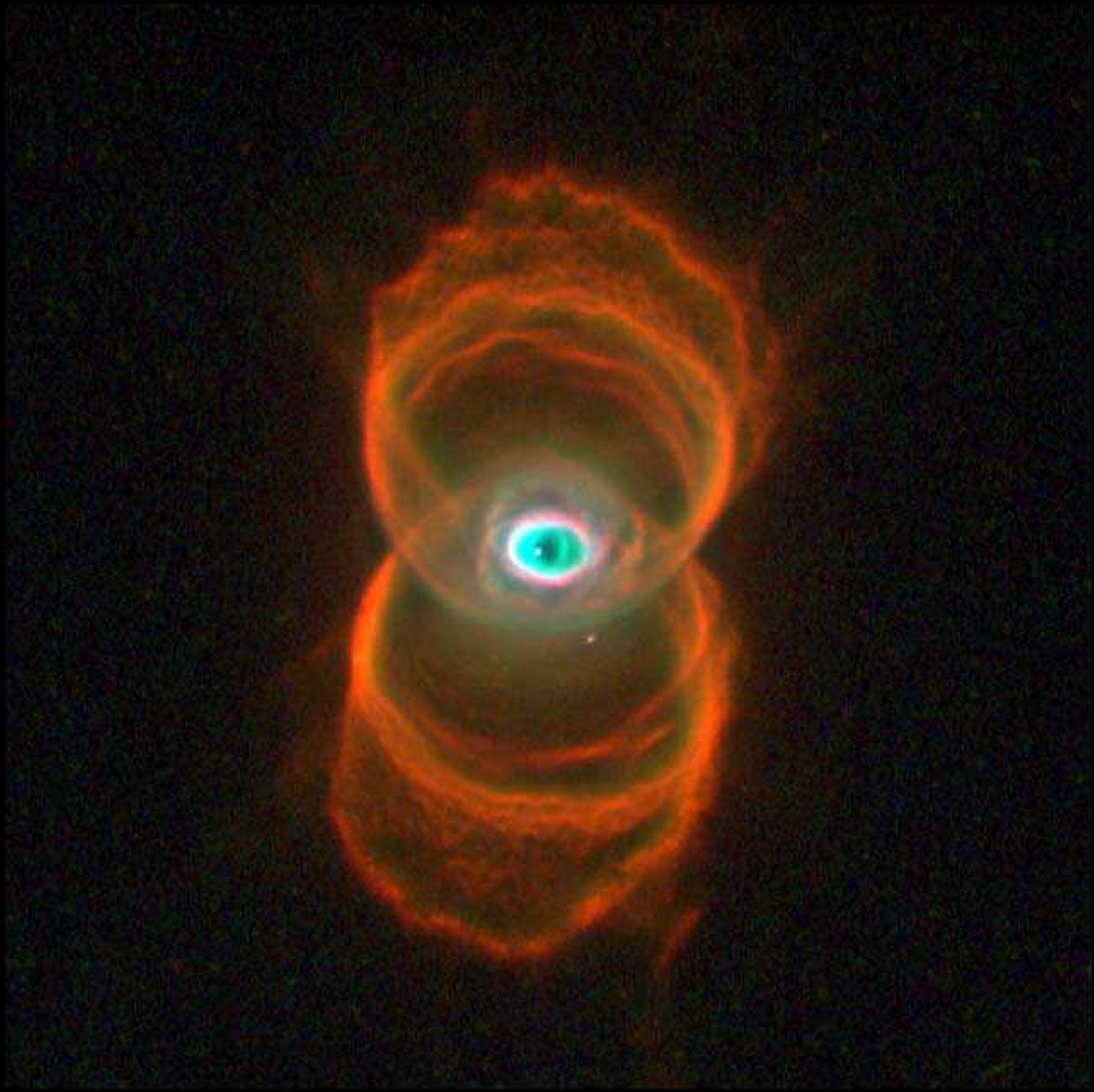


NGC 6853/M27 (“Dumbbell Nebula”; ESO VLT/FORS)

planetary nebulae: material ejected during AGB phase, photoionized once remaining core of former star has shed enough mass to emit UV photons.

IC4406 (ESO VLT)

planetary nebulae: material ejected during AGB phase, photoionized once remaining core of former star has shed enough mass to emit UV photons.



Hourglass Nebula (HST/Sahai/Trauger)

planetary nebulae: material ejected during AGB phase, photoionized once remaining core of former star has shed enough mass to emit UV photons.

Stellar Evolution: Massive Stars

Structure on the Main Sequence: Simulations show existence of two regimes:

lower main sequence : stars have structure similar to Sun:

- energy generation: pp-chain ($\epsilon \propto T^5$)
- inner radiative core
- convective hull

upper main sequence : for central temperatures of 18×10^6 K (1.5 $M_\odot$ stars): pp-chain and CNO-cycle produce equal amounts of energy. Above that: CNO dominates.

- energy generation: CNO-cycle ($\epsilon \propto T^{17}$)
- inner convective core since energy generation from CNO cycle strongly peaked towards center.
- outer radiative hull

Stellar Evolution: Massive Stars

Evolution on MS similar, however, faster than for low mass stars.

More massive stars ($\gtrsim 1.5\,M_\odot$) reach threshold temperature for 3α before reaching degeneracy

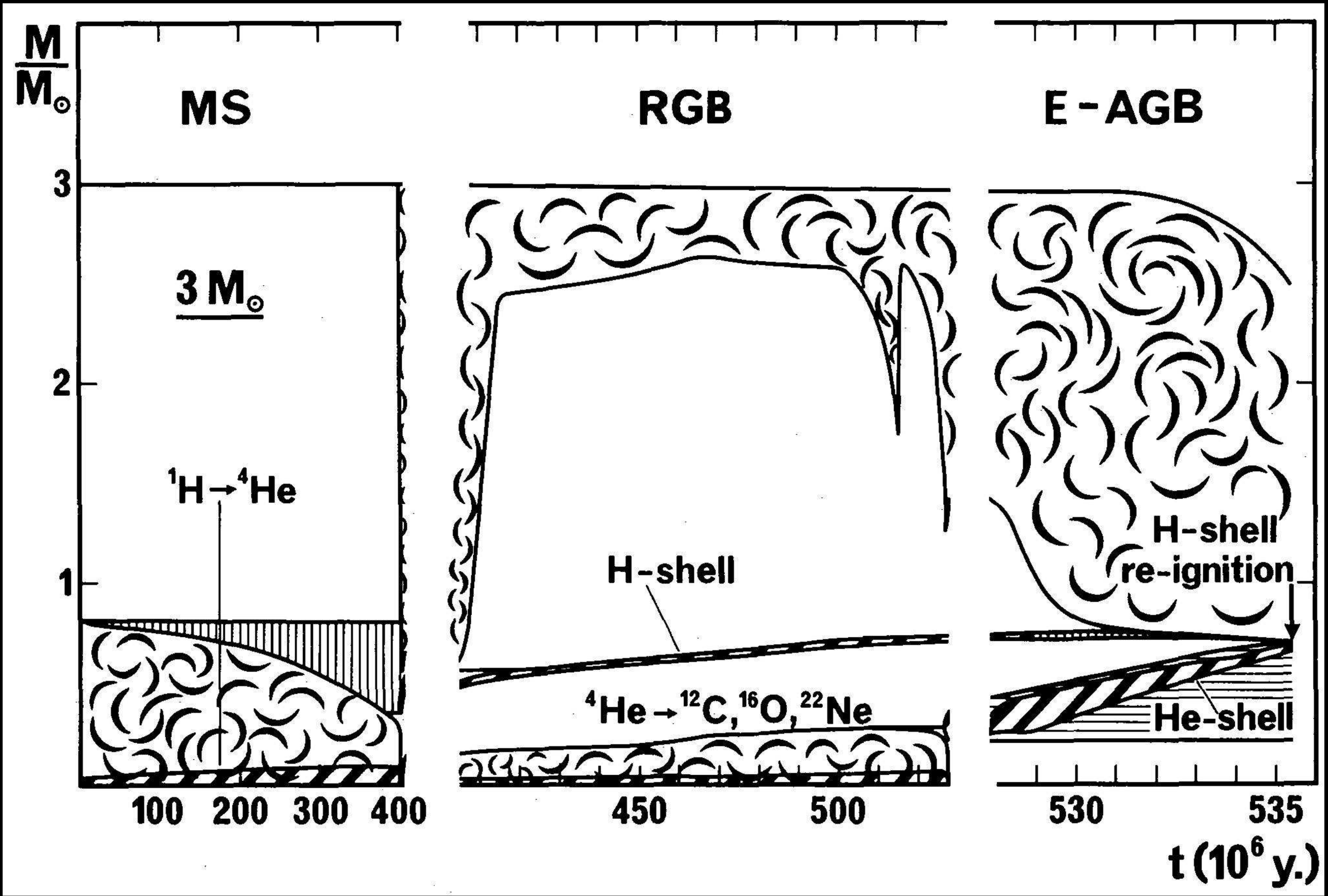
$\Longrightarrow$ He just starts to burn.

In these objects, higher order fusion processes can kick in (but are energetically unimportant): alpha reactions

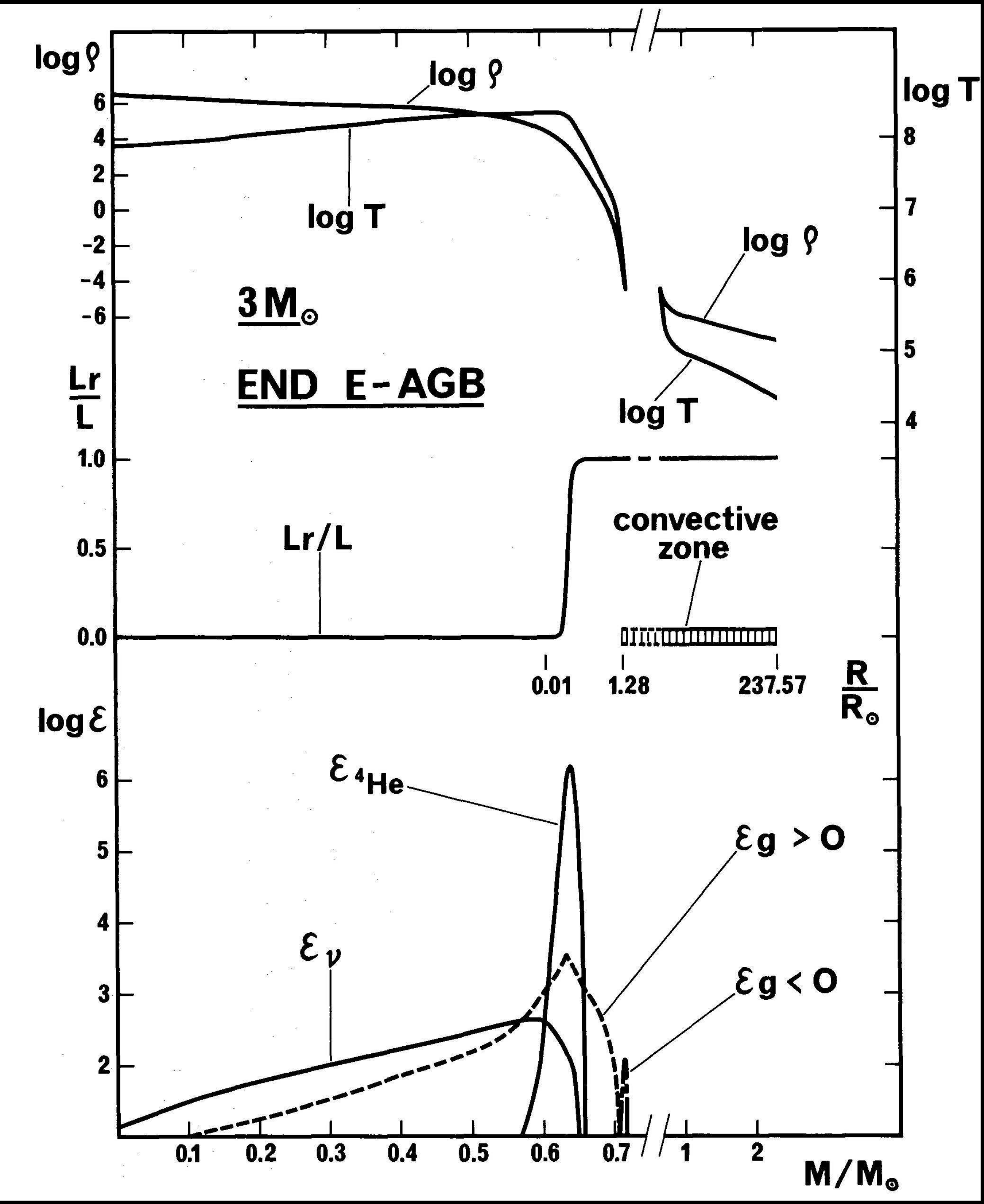
$$^{12}\mathrm{C} + {}^{4}\mathrm{He} \longrightarrow {}^{16}\mathrm{O} + \gamma$$
$$^{16}\mathrm{O} + {}^{4}\mathrm{He} \longrightarrow {}^{20}\mathrm{Ne} + \gamma$$
$$^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \longrightarrow {}^{24}\mathrm{Mg} + \gamma$$

Outer layers continue H shell burning.

During evolution of star on red giant branch: convective hull moves deeper into core, can mix fusion products into outer layers.

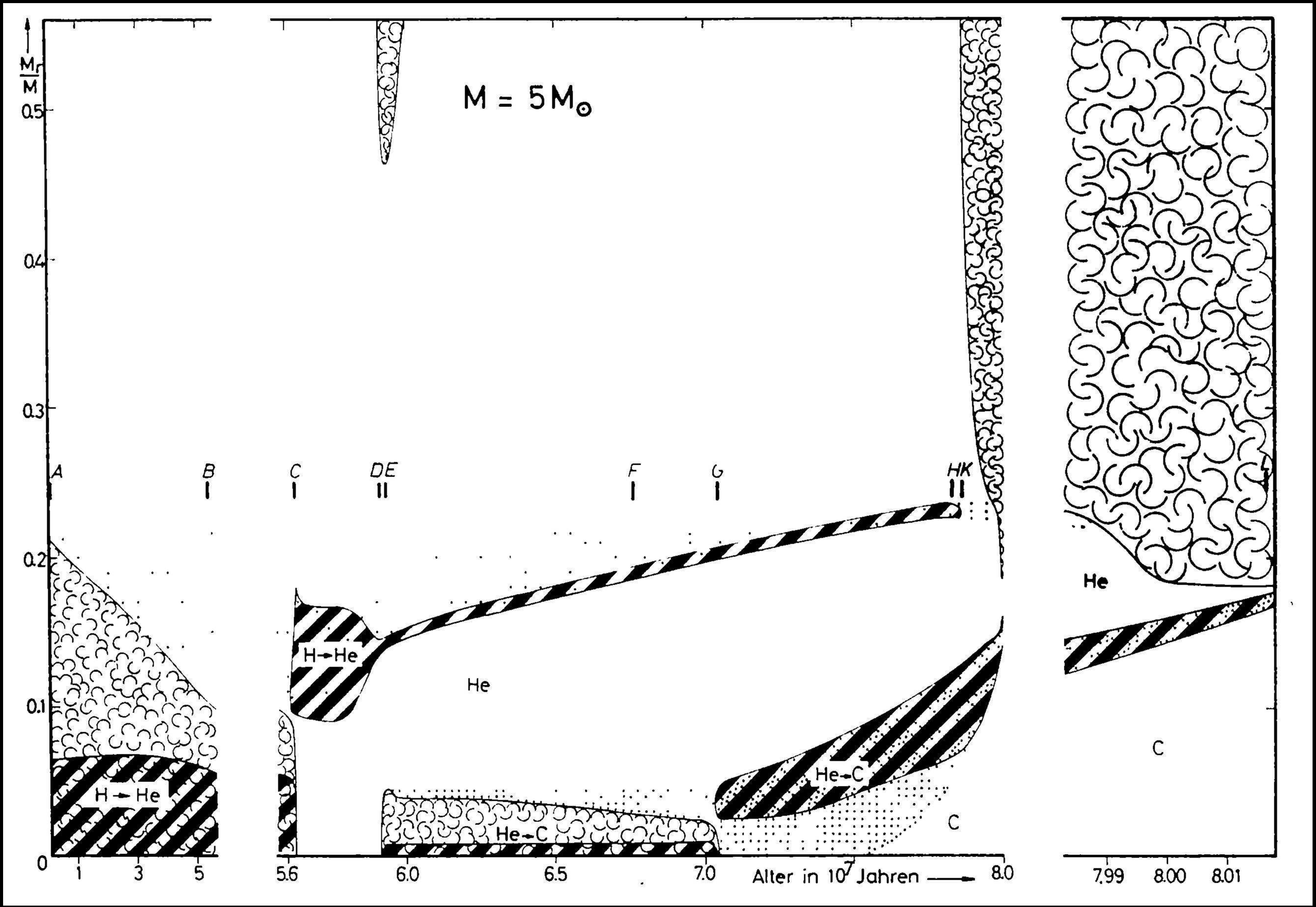


Evolution of the structure of a 3 $M_{\odot}$ star to the early Asymptotic Giant Branch (Maeder & Meynet, 1989).

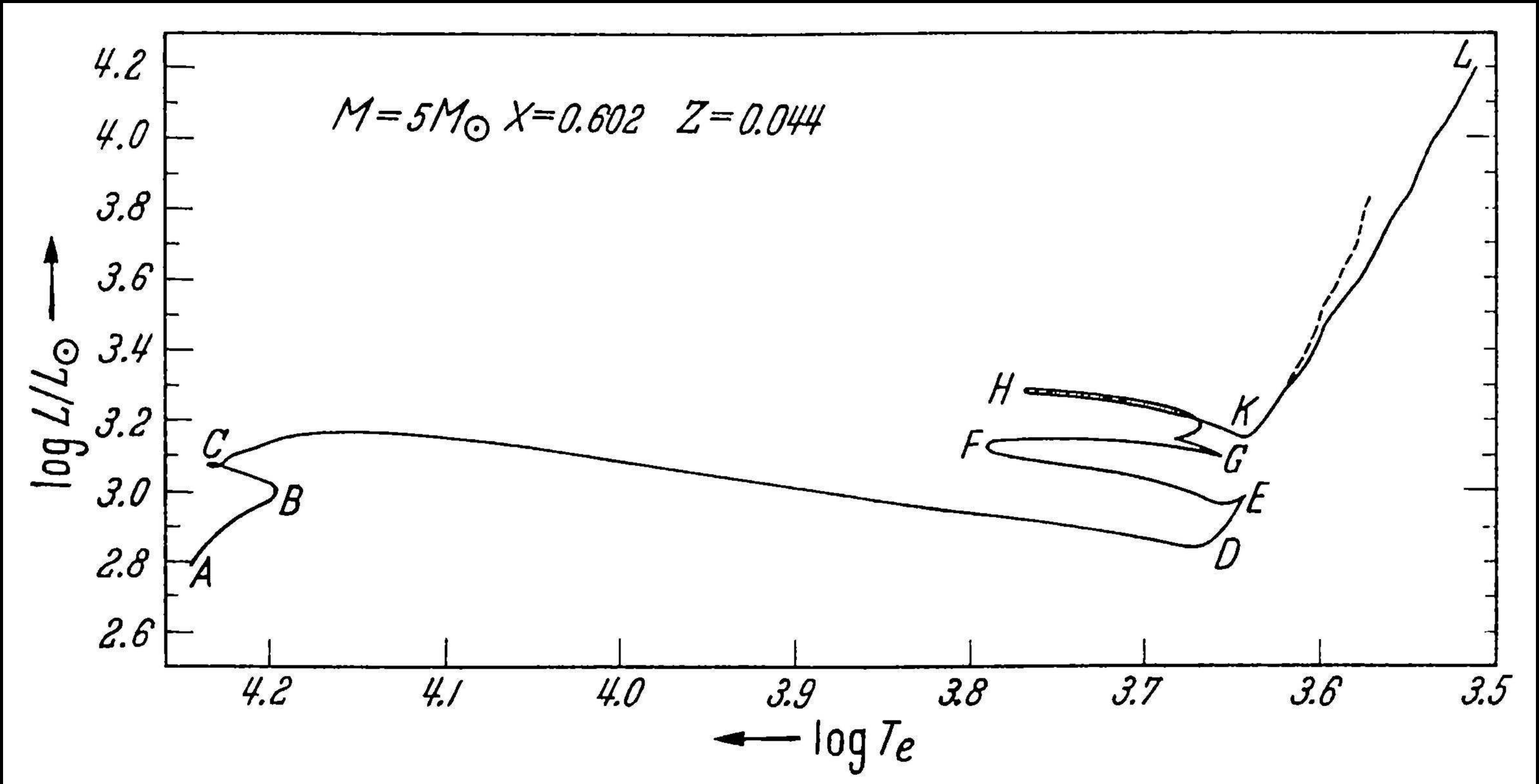


Internal structure of a 3 $M_\odot$ star which has reached the early Asymptotic Giant Branch.

Maeder & Meynet, 1989

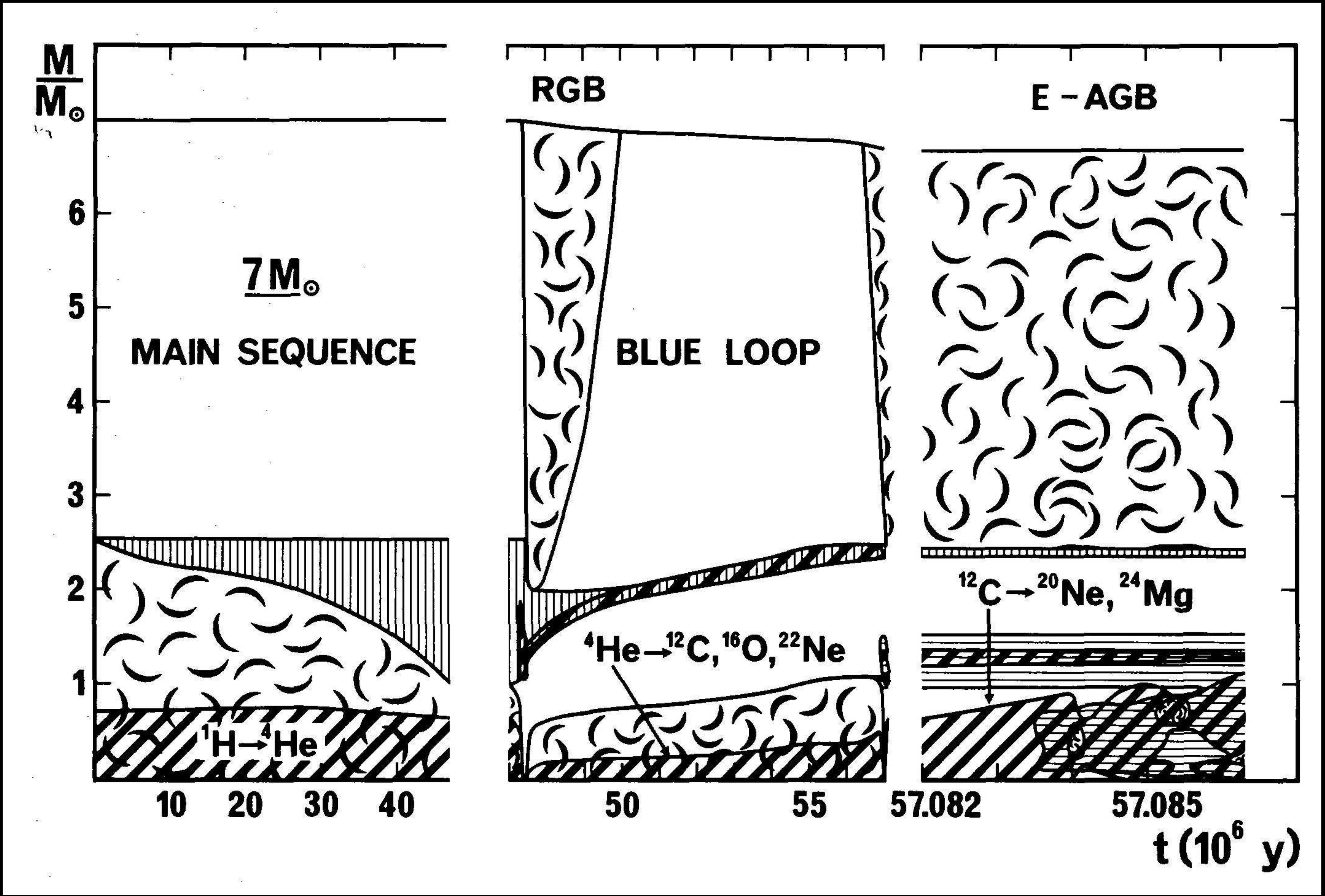


Kippenhahn et al. (1965): Evolution of the internal structure of a 5 $M_\odot$ star.

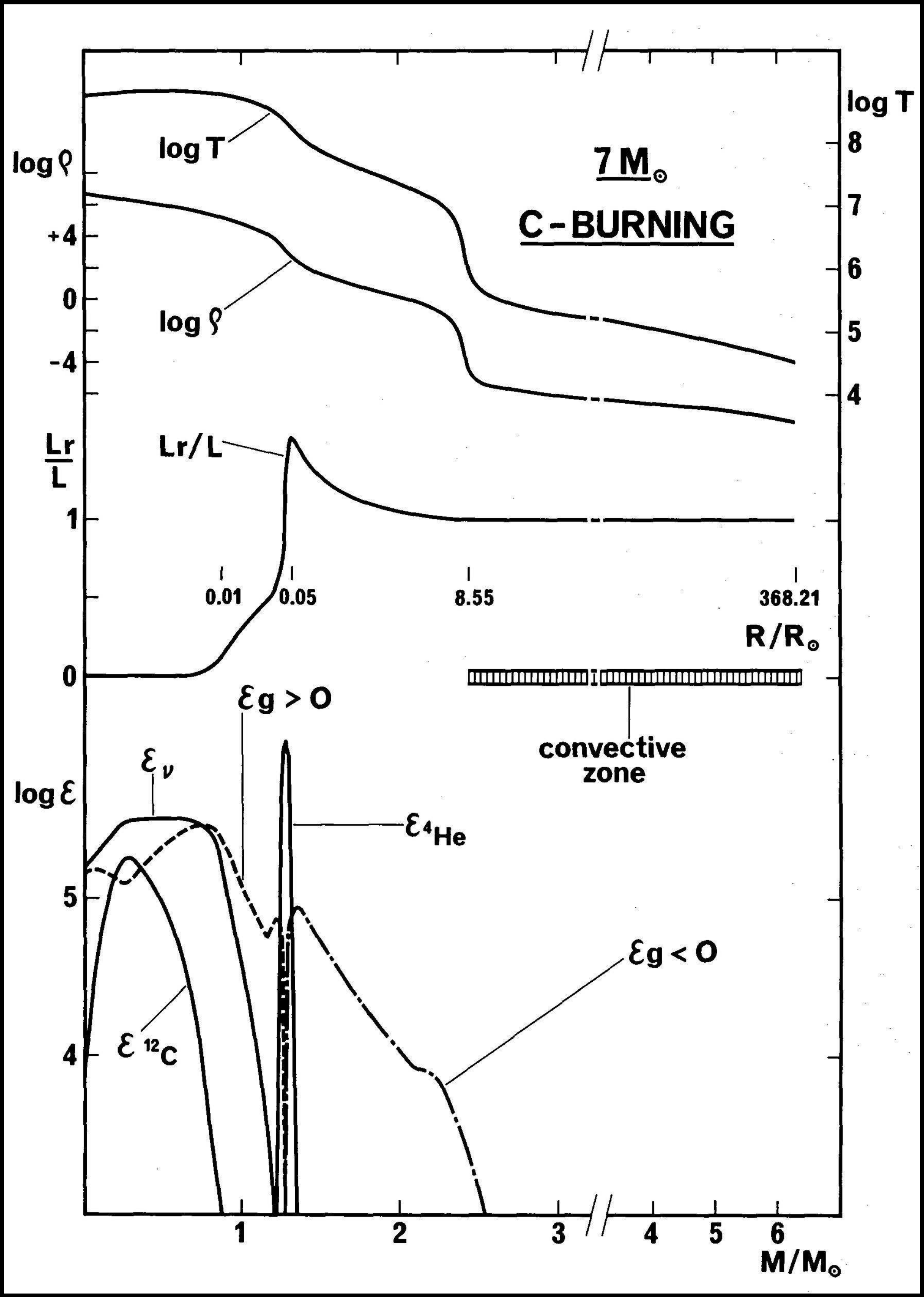


Kippenhahn et al. (1965)

Evolution of a 5 $M_{\odot}$ star in the HRD.



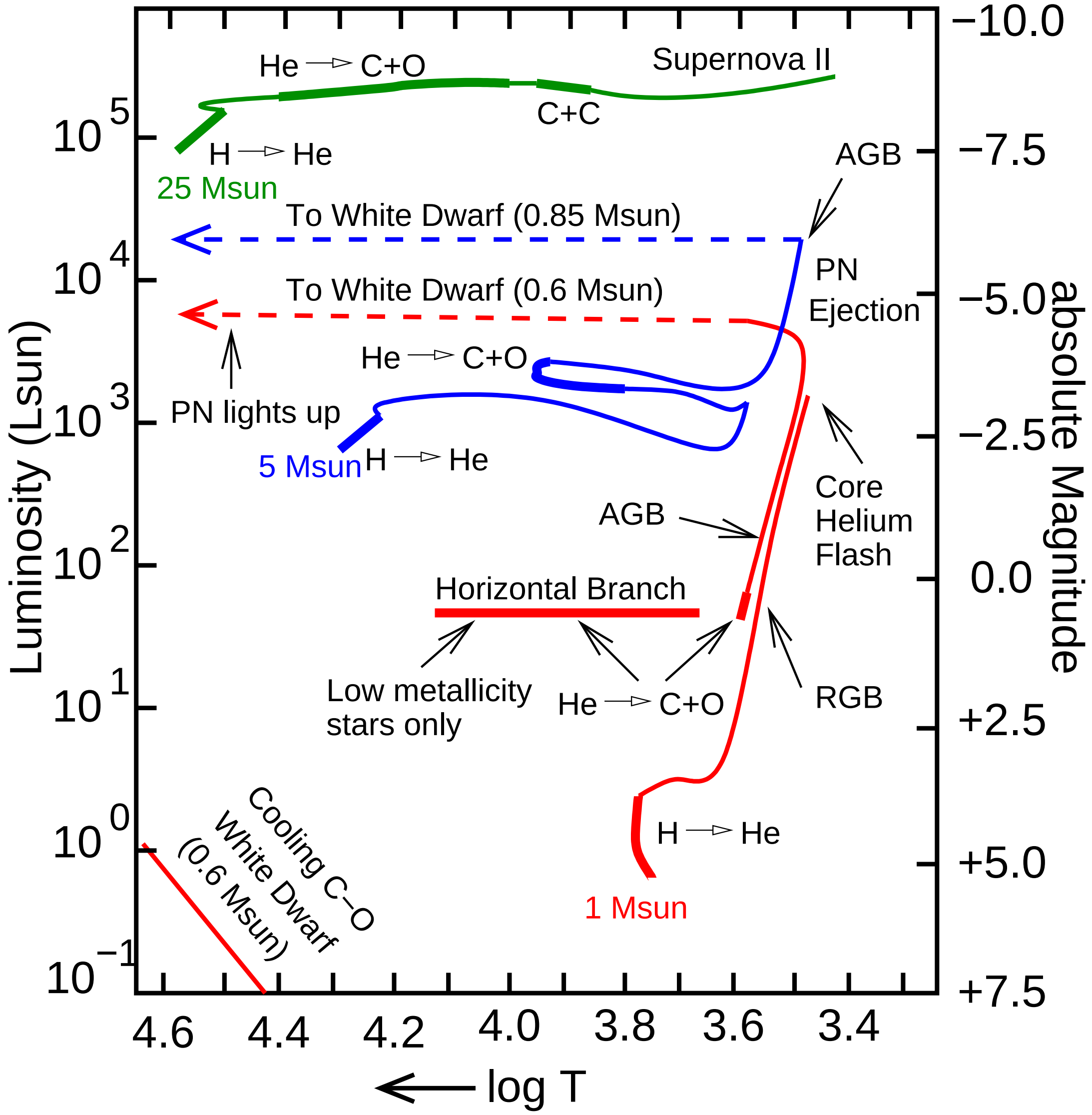
Evolution of the structure of a 7 $M_\odot$ star to the carbon burning phase (Maeder & Meynet, 1989).



Internal structure of a 7 $M_{\odot}$ star which just starts its carbon burning phase.

Maeder & Meynet, 1989

Stellar Evolution: Massive Stars



after Iben, 1991

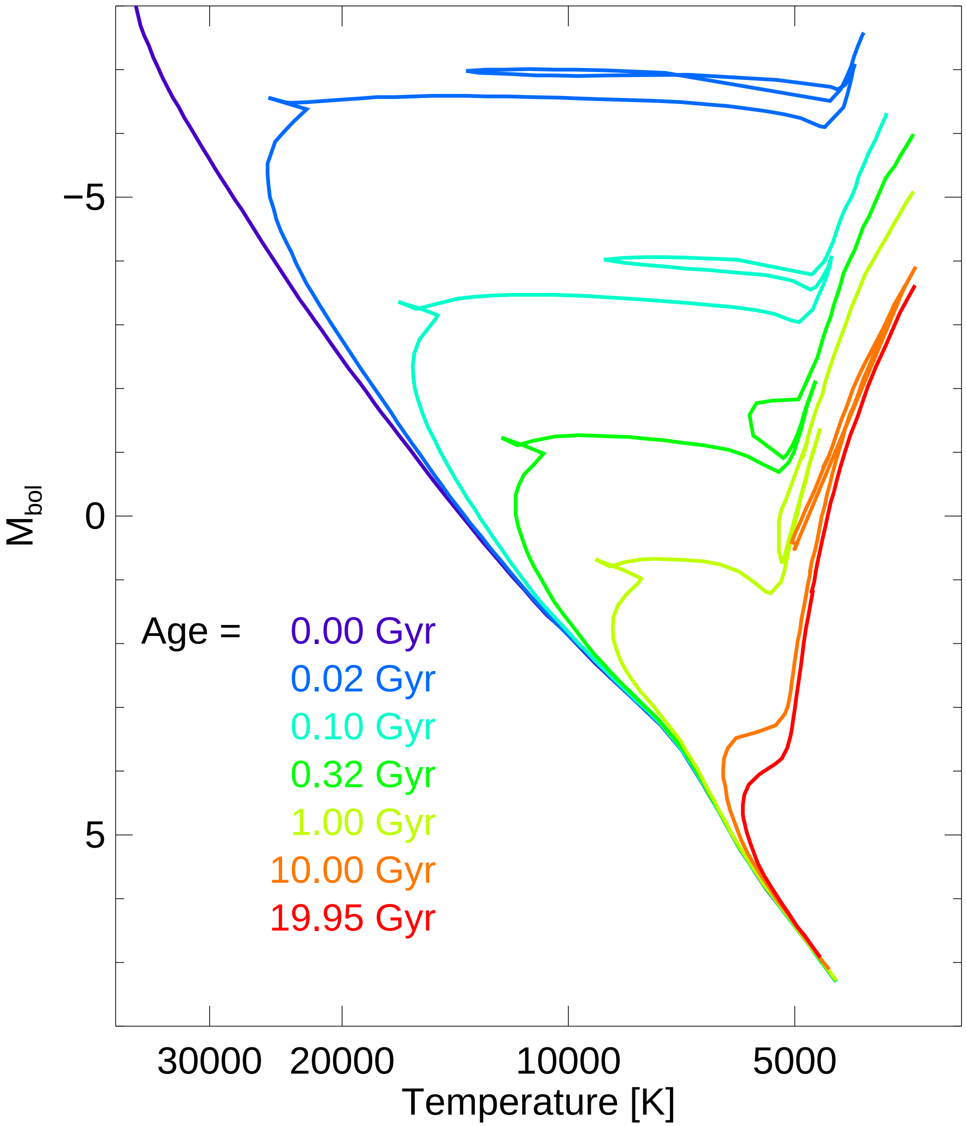
Evolution of stars in the HRD from main sequence to death

Typical timescales (units of 10^6 yr; Schaller et al. 1992):

	$1\,M_\odot$	$5\,M_\odot$	$25\,M_\odot$
H→He	10000	94	6.4
He→C		12	0.6
C+C			0.01
PN	$\lesssim 0.01$	$\lesssim 0.01$	N/A
WD	∞	∞	N/A

Post-H-burning burning: need higher core temperatures (Coulomb barrier!), less energy release $\Longrightarrow$ last much shorter than hydrogen burning.

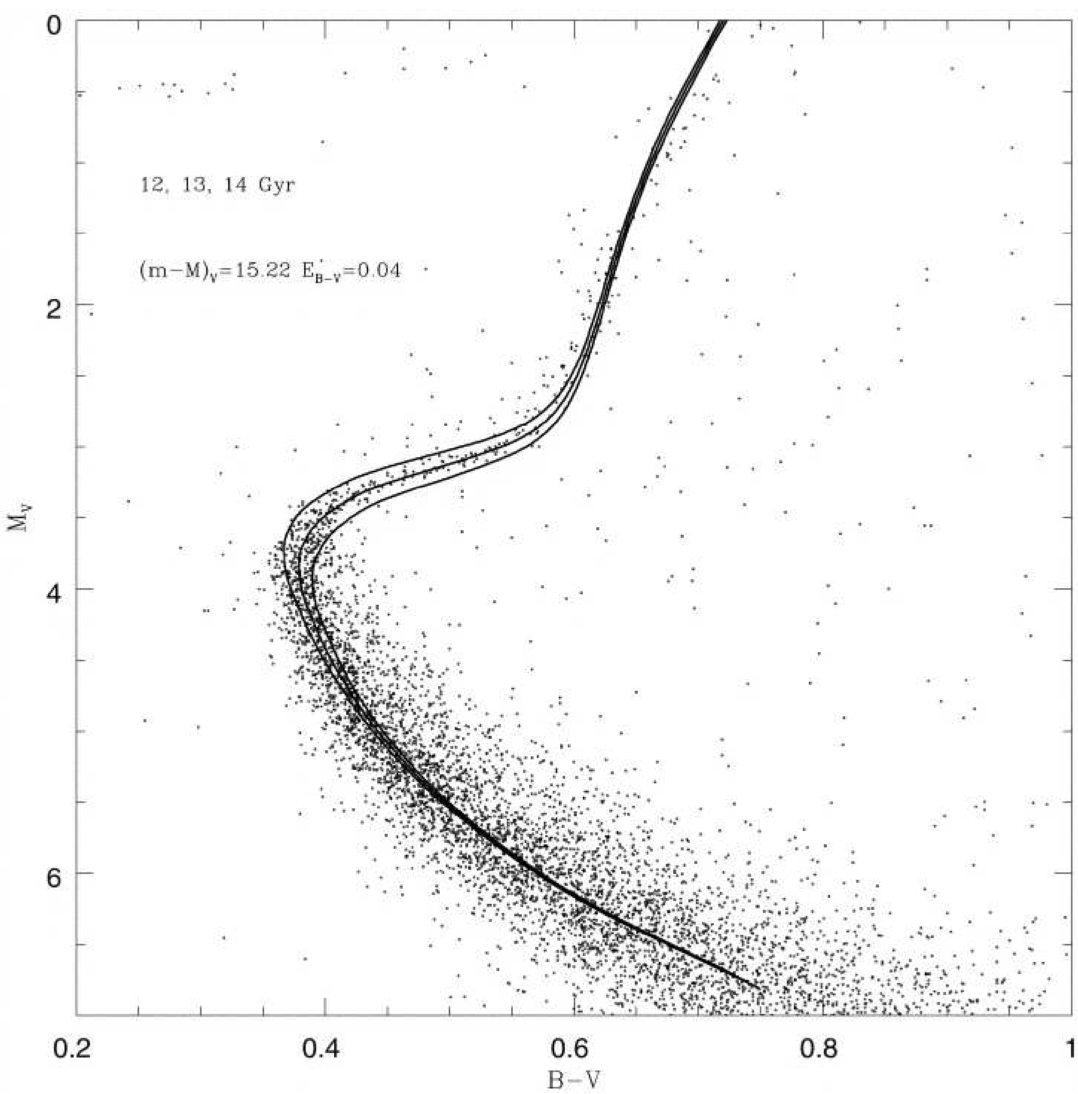
Stellar Evolution: Massive Stars



after Bertelli et al. (1994)

Predicted evolution of HRD from globular clusters can reproduce their HRDs, allows age determination

Stellar Evolution: Massive Stars



(M68; Straniero et al., 1997; Fig. 11)

Predicted evolution of HRD from globular clusters can reproduce their HRDs, allows age determination

Result: $\sim$12. . . 13 billion years

$\Longrightarrow$ GCs are oldest objects in the universe!

End-Stages of Stellar Evolution