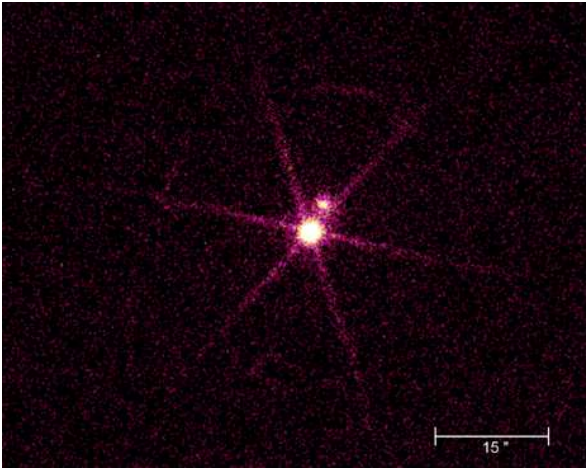




White Dwarfs

White Dwarfs:

1. End stages of evolution of stars with $M \lesssim 10 M_{\odot}$ on main sequence
2. typically $M \sim 0.6 \dots 0.8 M_{\odot}$, and always $M < 1.44 M_{\odot}$ (Chandrasekhar mass); above that: relativistic degenerate gas ($P \propto \rho^{4/3}$), can show that under these circumstances WD is not stable.
3. mainly consist of C and O
4. Radius \sim Earth
5. Typical density $\rho \sim 10^6 \text{ g cm}^{-3}$
6. interior temperature $\sim 10^7 \text{ K}$, atmosphere $\sim 10^4 \text{ K}$, slowly cooling down (observable for $\gtrsim 10^9$ years).



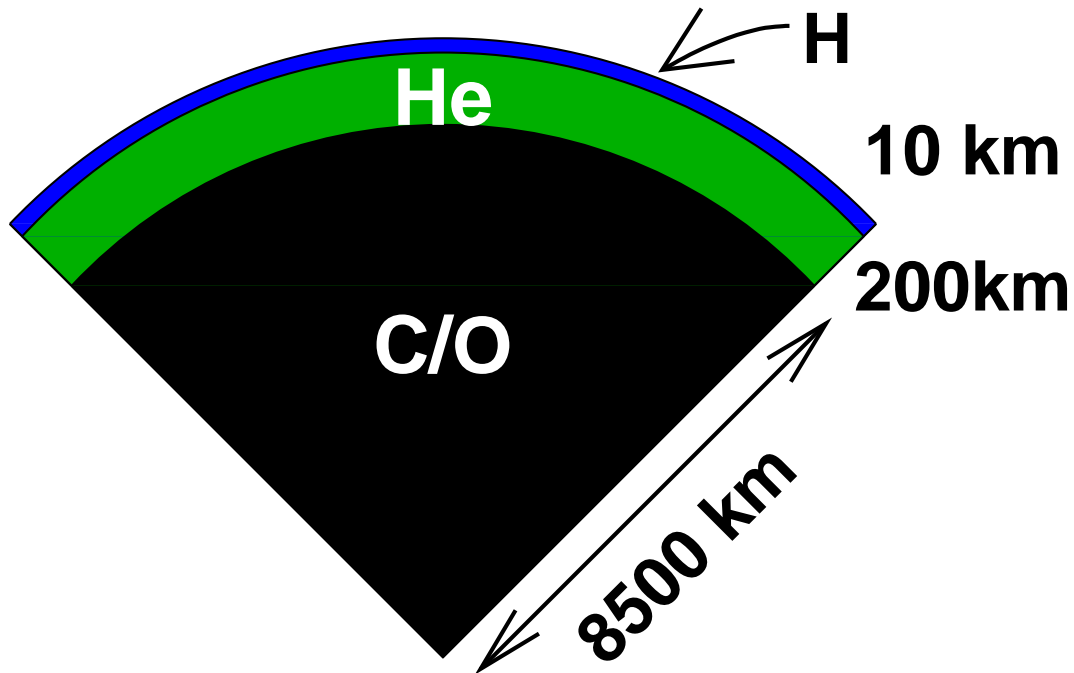
Sirius A+B: Chandra
(X-rays; WD is bright)



McDonald Observatory
(optical; WD is faint)



White Dwarfs



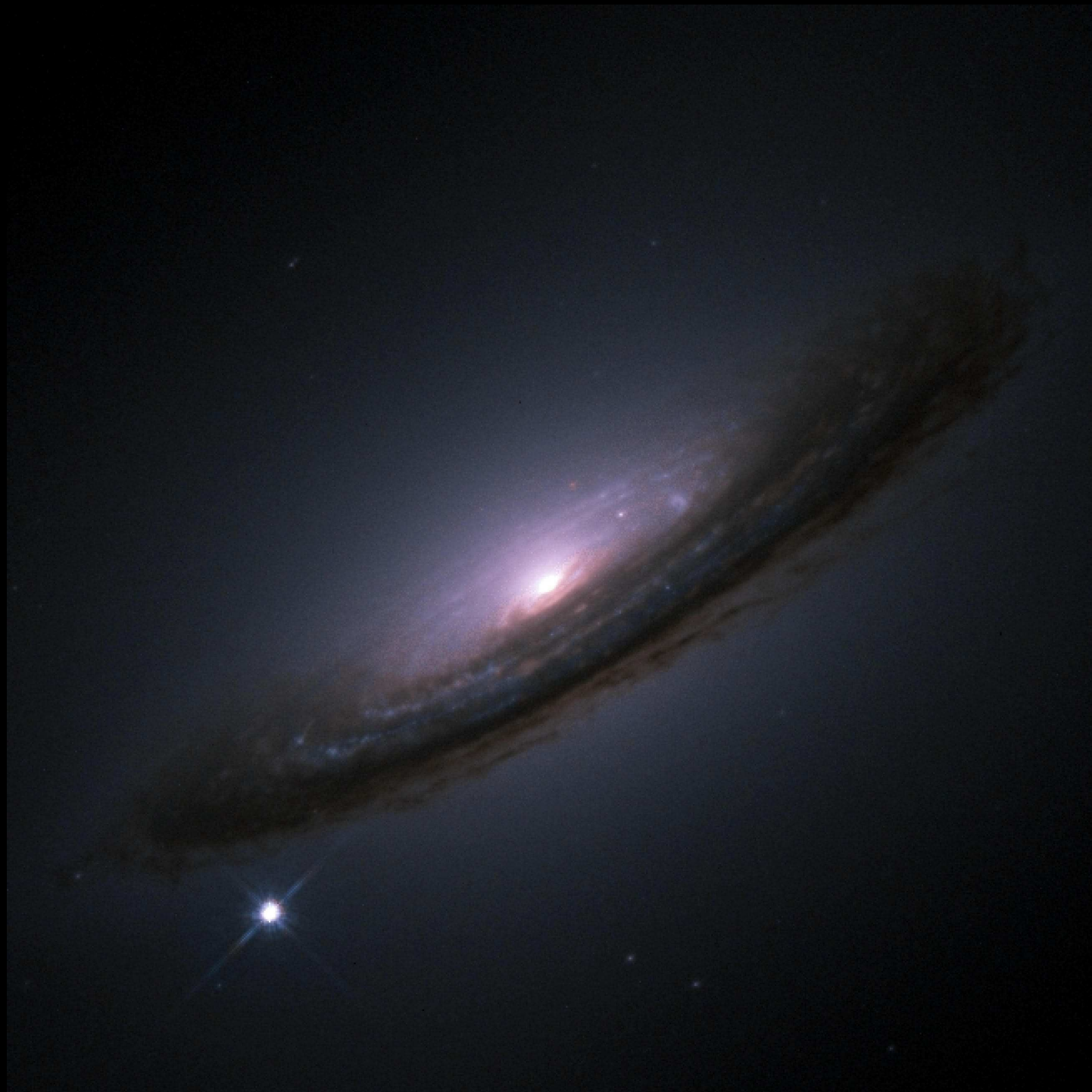
White dwarfs come in two flavors:

DA: H present in spectrum
(~80% of all WD)

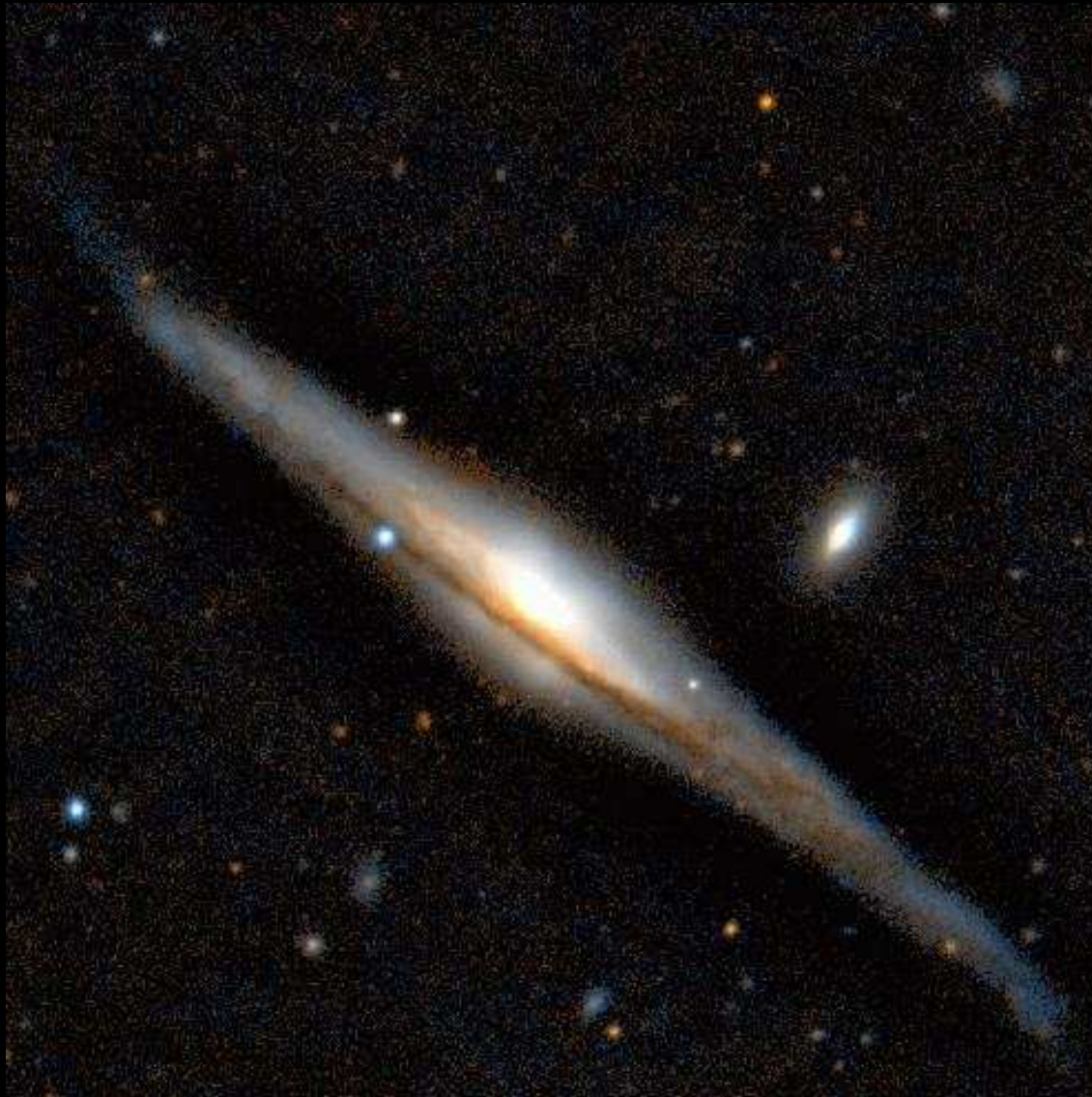
DB: He present in spectrum (~
the rest)
plus a few oddballs

Structure: gravitationally settled, so DB's really do not have any H since it would "swim on top"

⇒ layered, "onion-like" structure

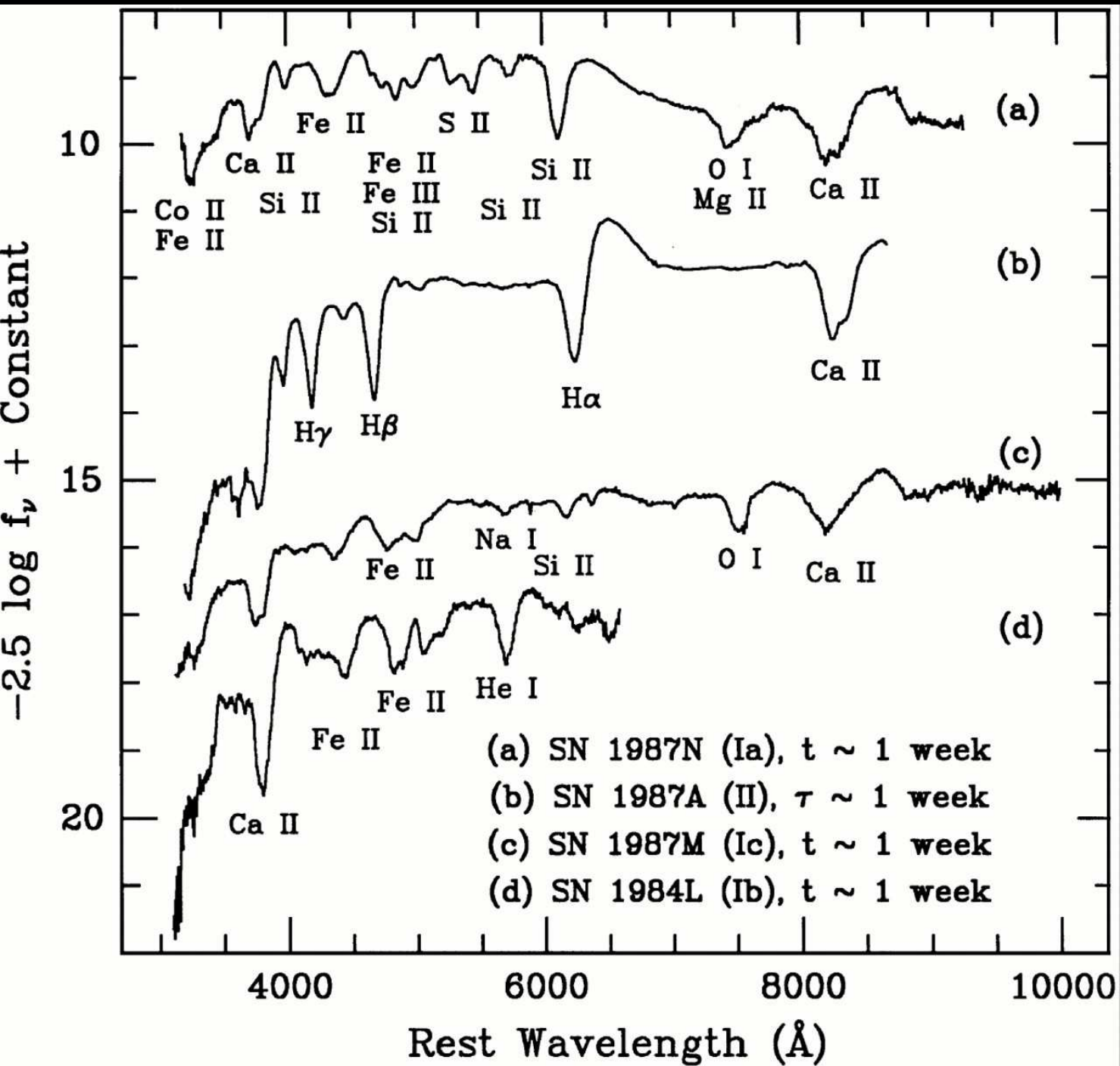


SN1994d (HST WFPC)



Type II SN2001cm in NGC5965 (2.56 m NOT, Håkon Dahle; NORDITA)

Evolution of more massive stars: fusion up to ^{56}Fe , then no energy gain
 \implies no pressure balance in centre \implies supernova explosion of type II.
energy release: 10^{46} W ($10^{20} L_{\odot}$; about 1% in light, rest in neutrinos)



Rough classification
(Minkowski, 1941):

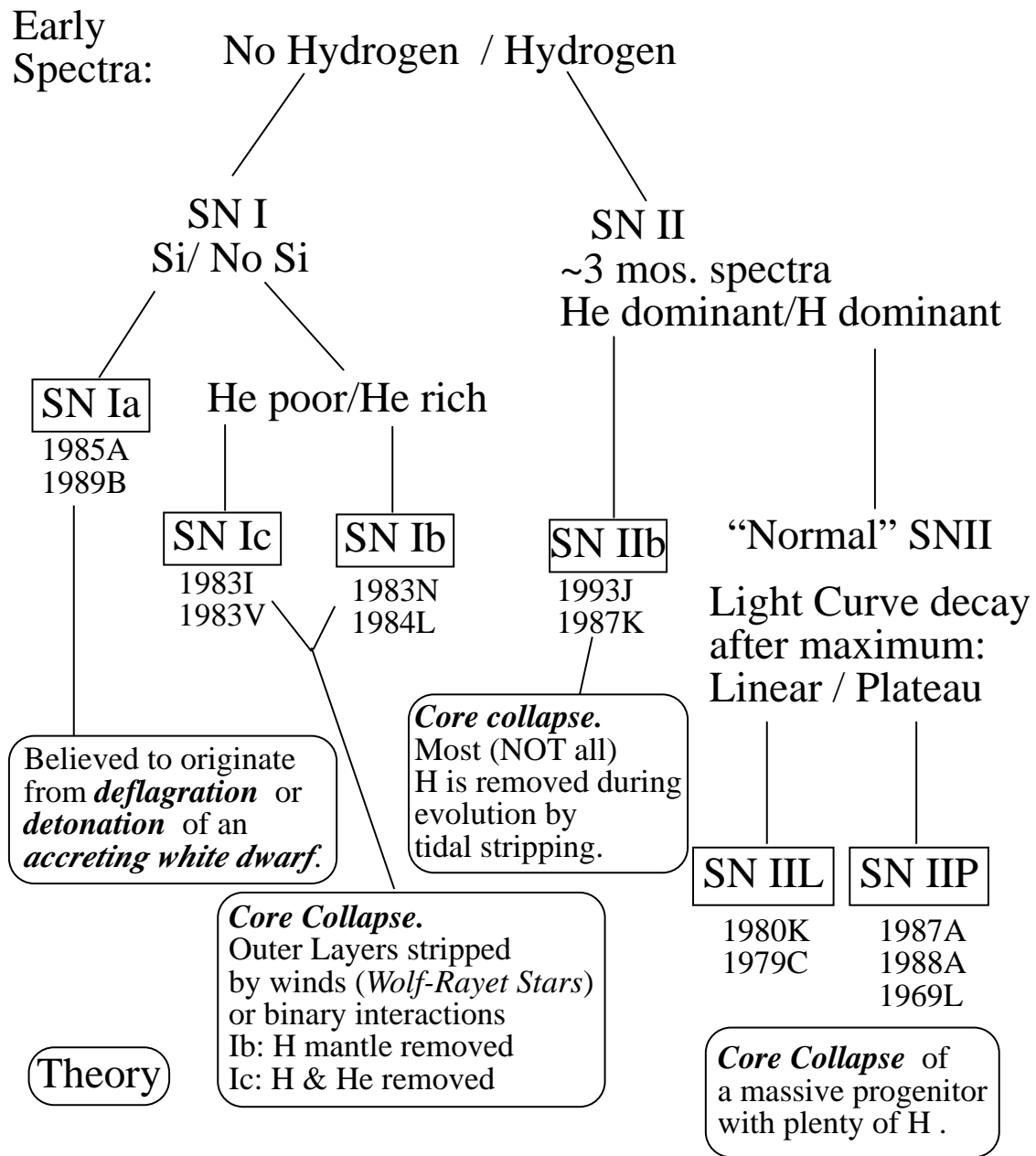
Type I: no hydrogen
in spectra;
subtypes Ia, Ib, Ic

Type II: hydrogen
present, subtypes
II-L, II-P

Note: pre 1985 subtypes Ia,
Ib had different definition
than today \implies beware when
reading older texts.

(Filippenko, 1997, Fig. 1); t : time after maximum light; τ : time after explosion;

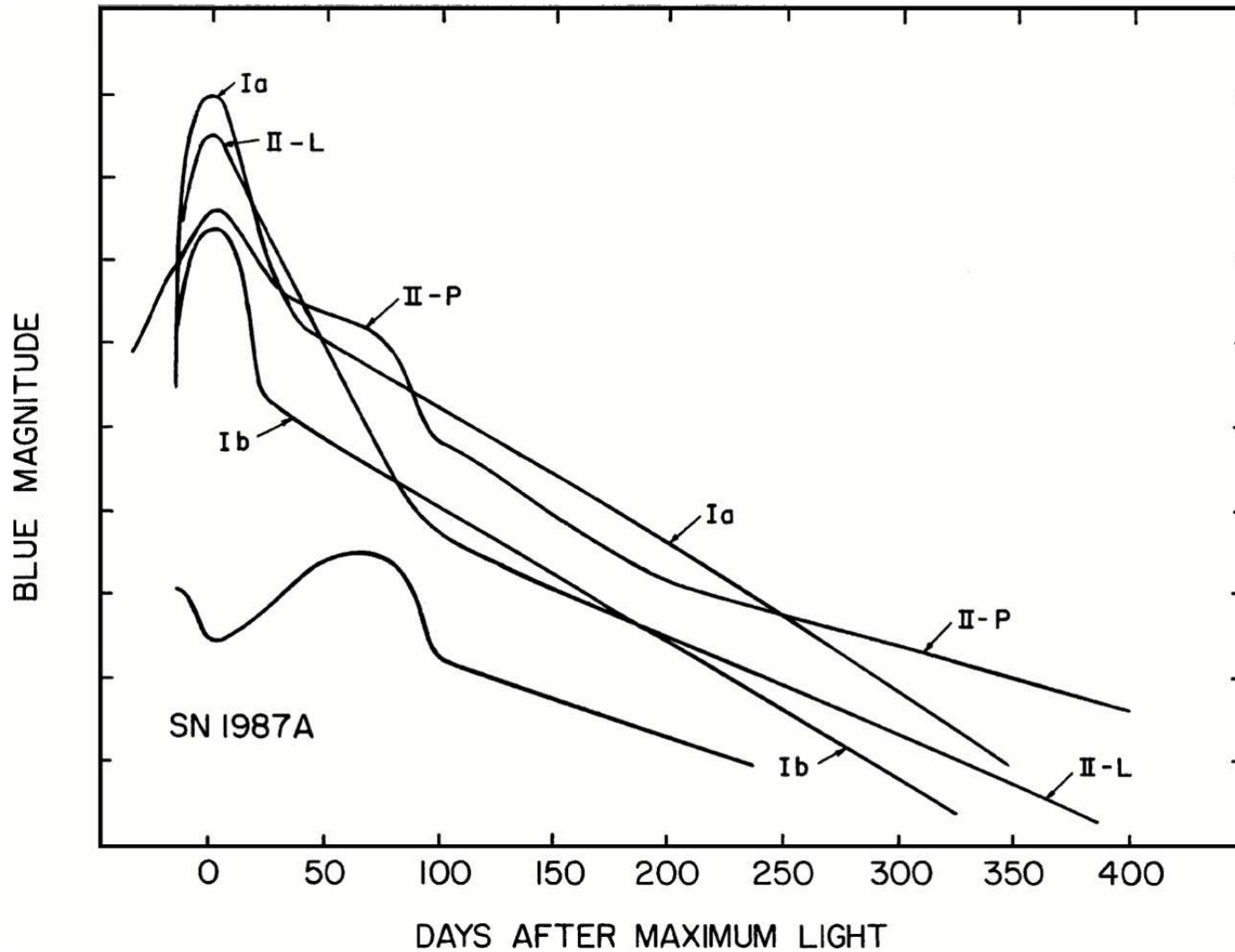
P Cyg profiles give $v \sim 10000 \text{ km s}^{-1}$



courtesy M.J. Montes



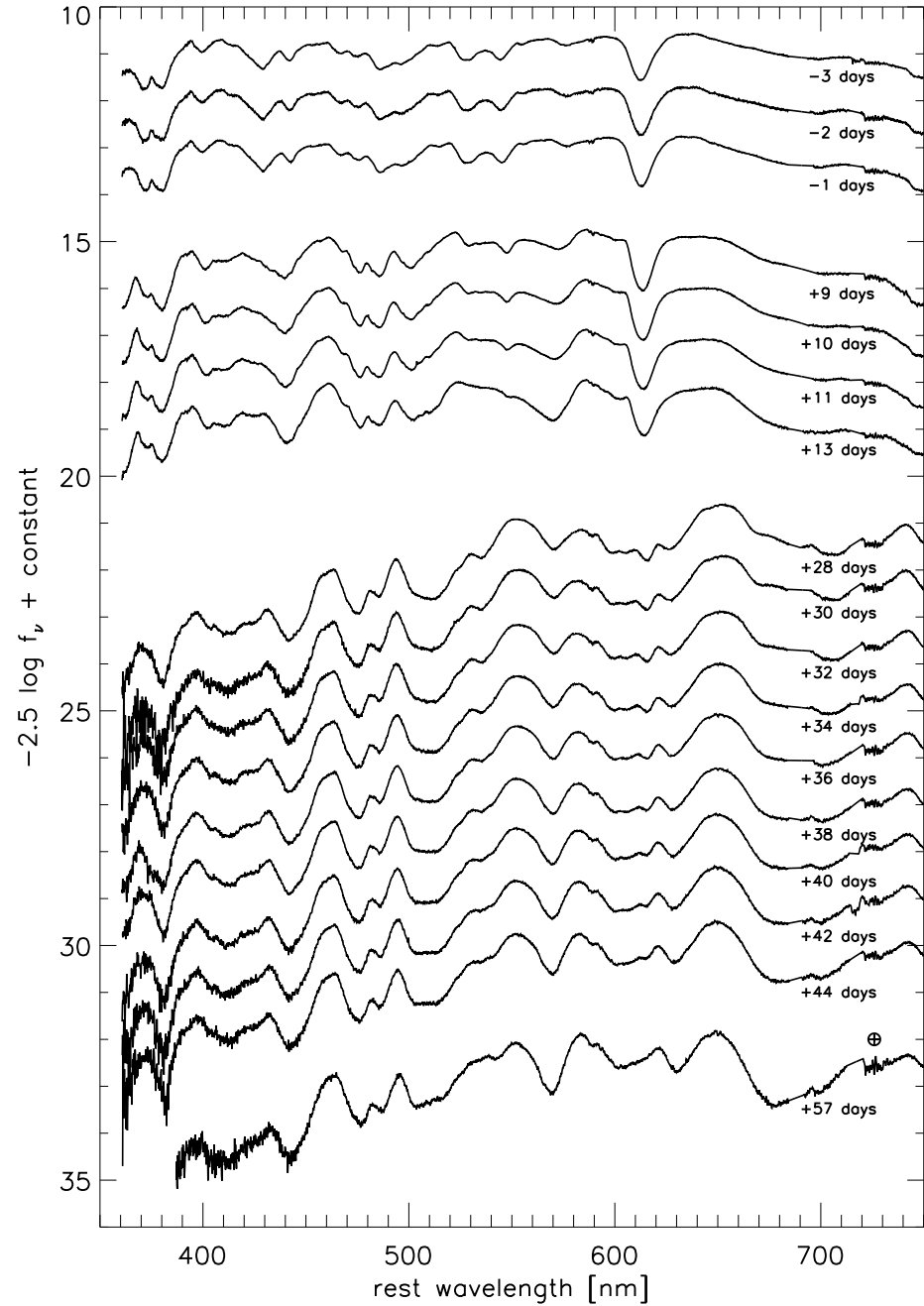
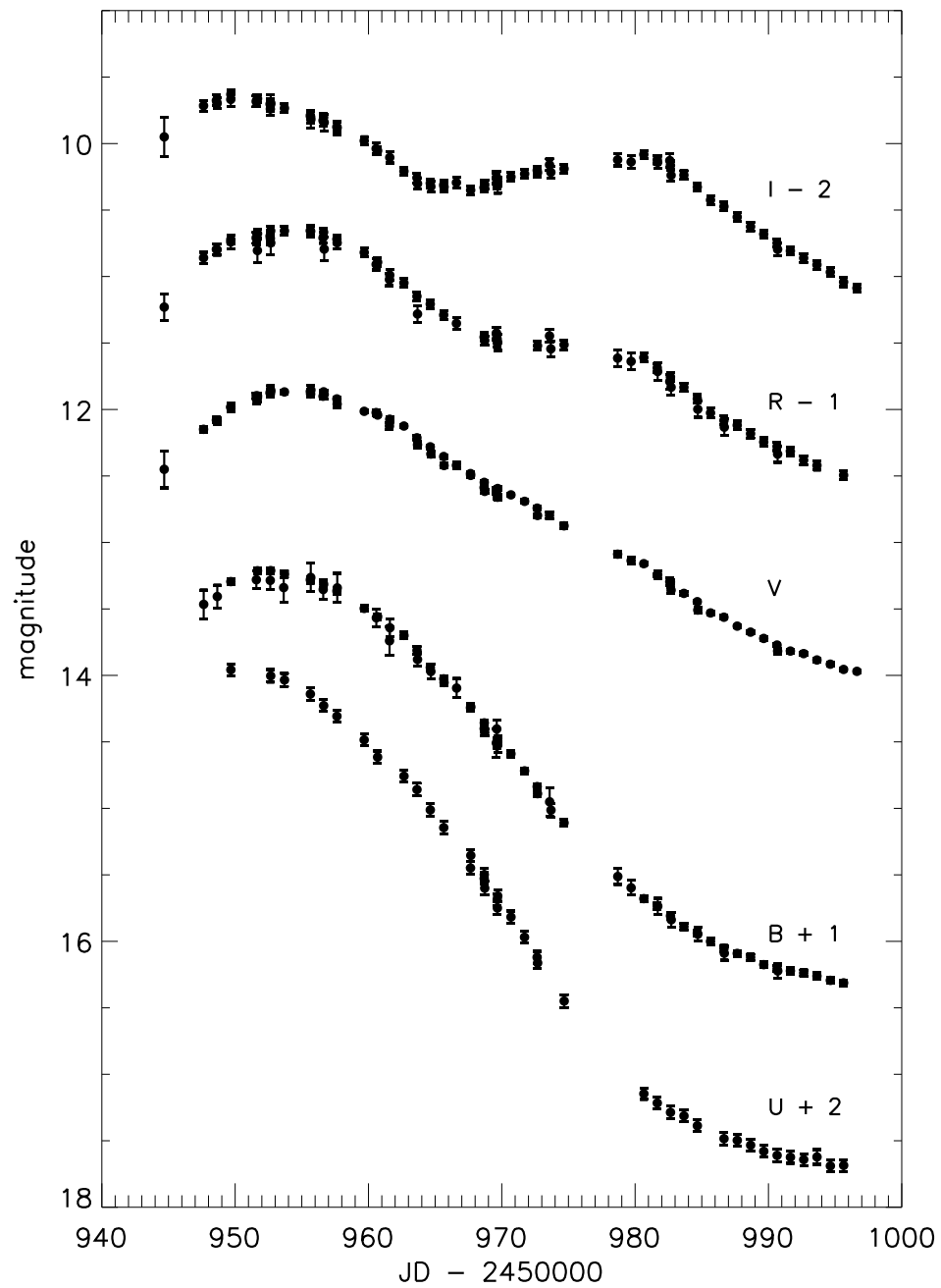
Supernovae, V



(Filippenko, 1997, Fig. 3)

Light curves of **SNe I** all very similar, **SNe II** have much more scatter.

SNe II-L (“linear”) resemble SNe I
SNe II-P (“plateau”) have const. brightness to within 1 mag for extended period of time.



(SN 1998bu in M96, Jha et al., 1999, Figs. 2 and 4)



Supernova Statistics

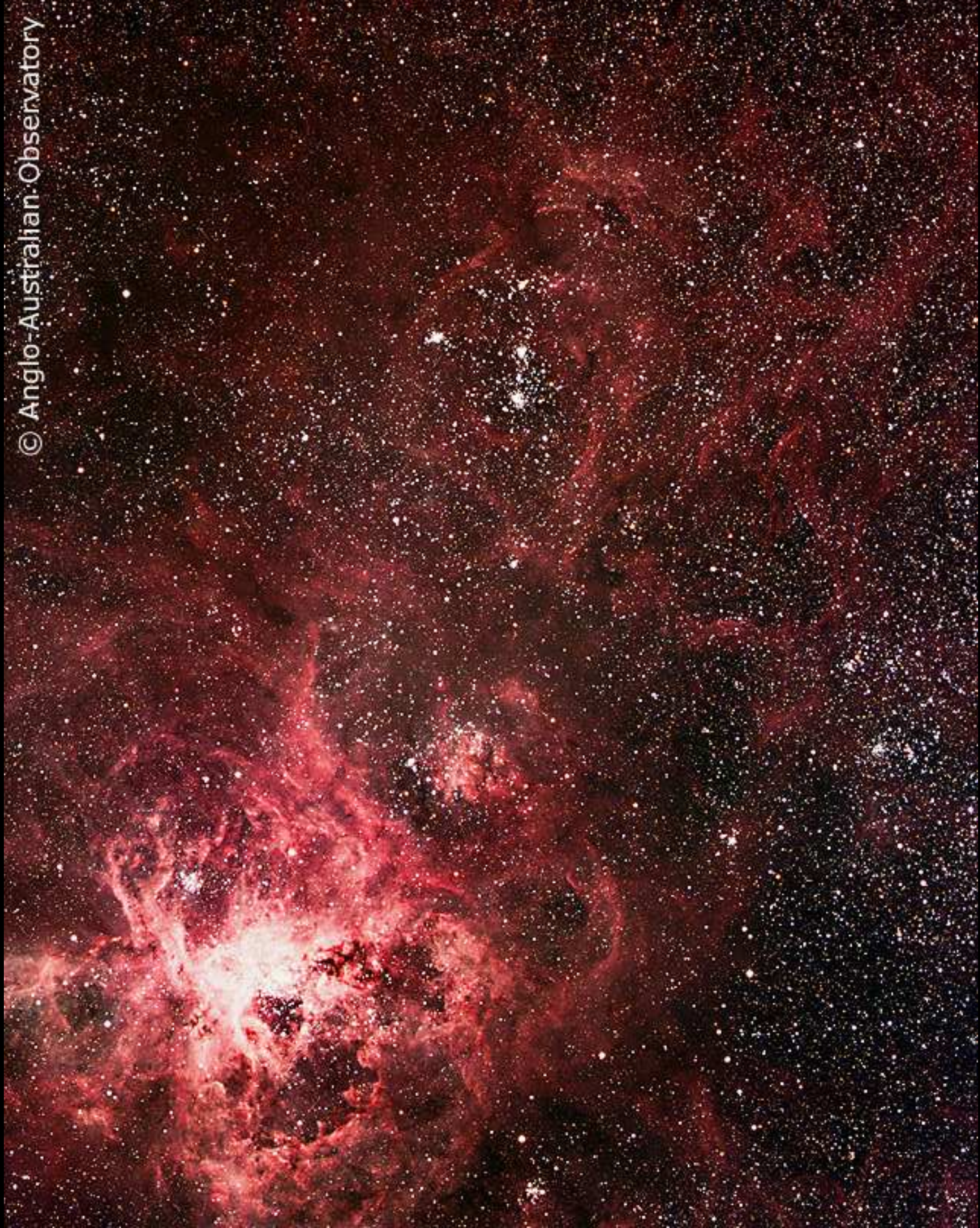
Clue on origin from supernova statistics:

- **SNe II, Ib, Ic**: never seen in elliptical galaxies, which are void of gas and have no new star formation; **generally associated with spiral arms and H II regions in spiral galaxies**, i.e., with **star forming regions**

⇒ progenitor of SNe II, Ib, Ic: **massive stars** ($\gtrsim 8 M_{\odot}$) ⇒ **“core collapse supernova”**

- **SNe Ia**: all types of galaxies, no preference for arms.

⇒ progenitor of SNe Ia: accreting **carbon-oxygen white dwarfs**, undergoing **thermonuclear runaway** (see later)



© Anglo-Australian Observatory

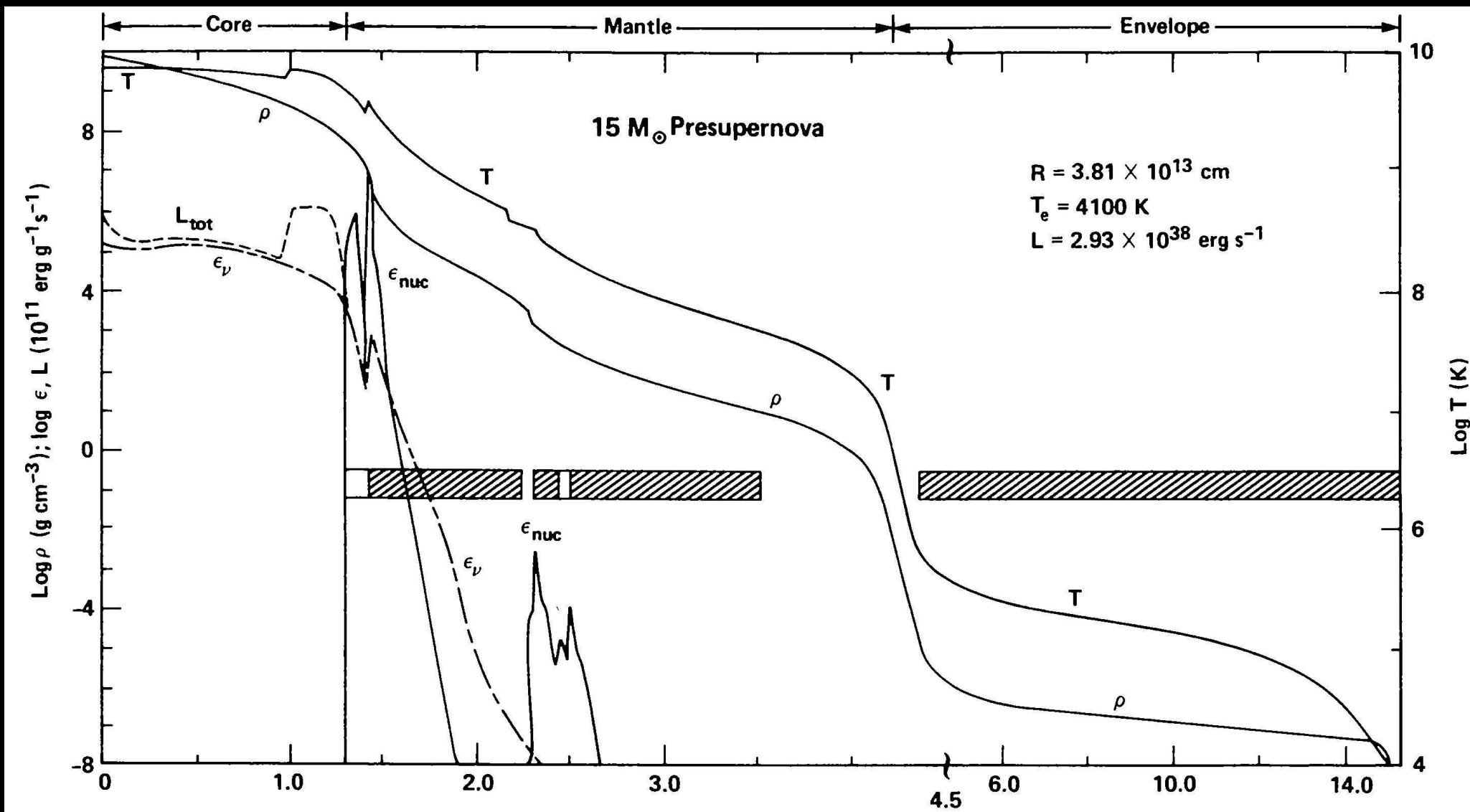




Core Collapse SNe, III

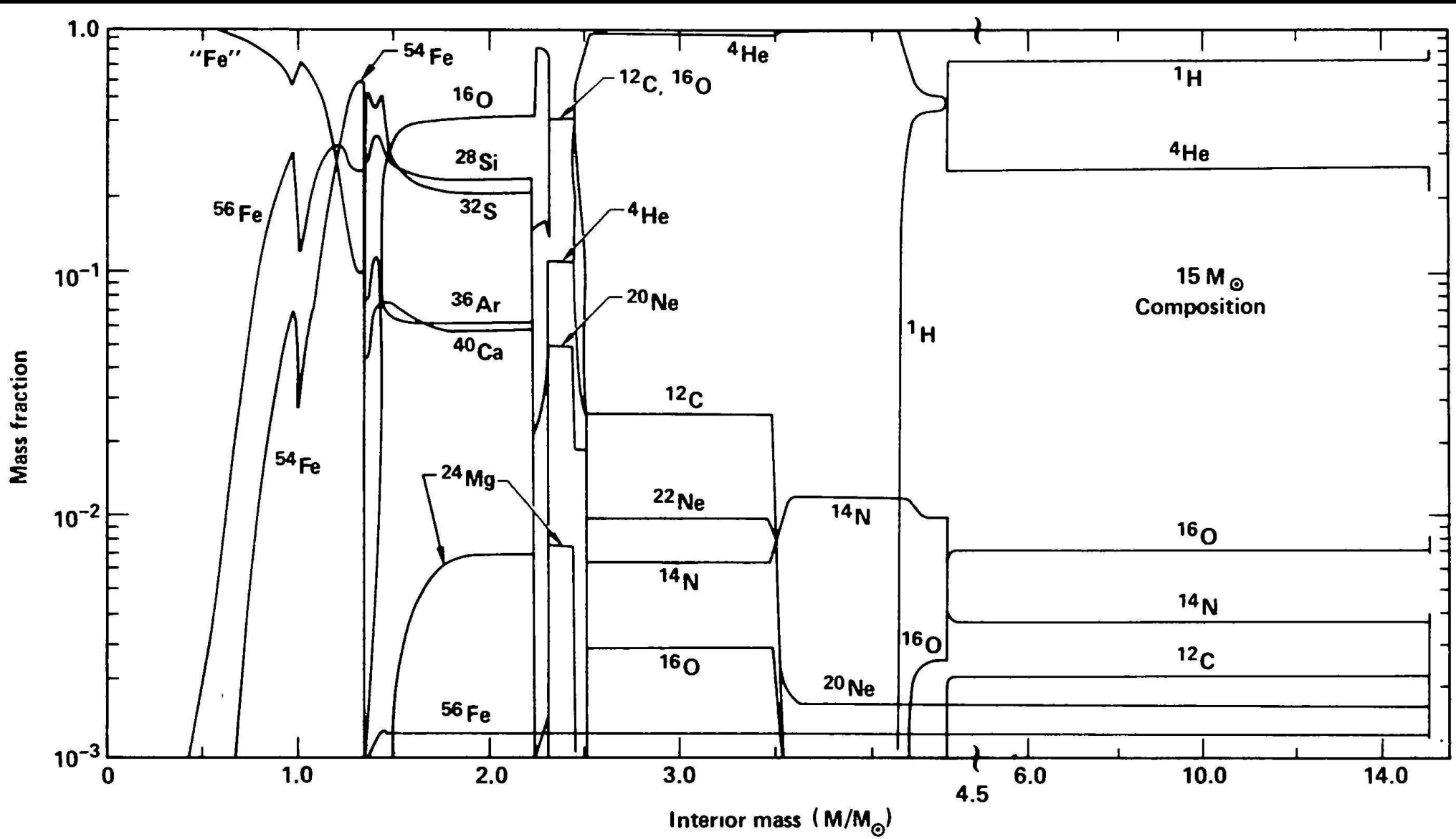
Nuclear reactions in massive ($> 8 M_{\odot}$) stars:

Reaction	above T [10^6 K]	Energy gain [MeV]
Hydrogen burning		
$4^1\text{H} \longrightarrow ^4\text{He}$	4	6.55
Helium burning		
$3^4\text{He} \longrightarrow ^8\text{Be} + ^4\text{He} \longrightarrow ^{12}\text{C}$	100	<0.61
Carbon burning		
$^{12}\text{C} + ^4\text{He} \longrightarrow ^{16}\text{O}$	600	<0.54
$2^{12}\text{C} \longrightarrow ^4\text{He} + ^{20}\text{Ne}$		
$^{20}\text{Ne} + ^4\text{He} \longrightarrow \text{n} + ^{23}\text{Mg}$		
Oxygen burning		
$2^{16}\text{O} \longrightarrow ^4\text{He} + ^{28}\text{Si}$	1000	<0.3
$2^{16}\text{O} \longrightarrow 2^4\text{He} + ^{24}\text{Mg}$		
Silicon burning		
$2^{28}\text{Si} \longrightarrow ^{56}\text{Fe}$	3000	< 0.18



Woosley & Weaver, 1988, Fig. 1

Structure of a $15 M_{\odot}$ pre-supernova star

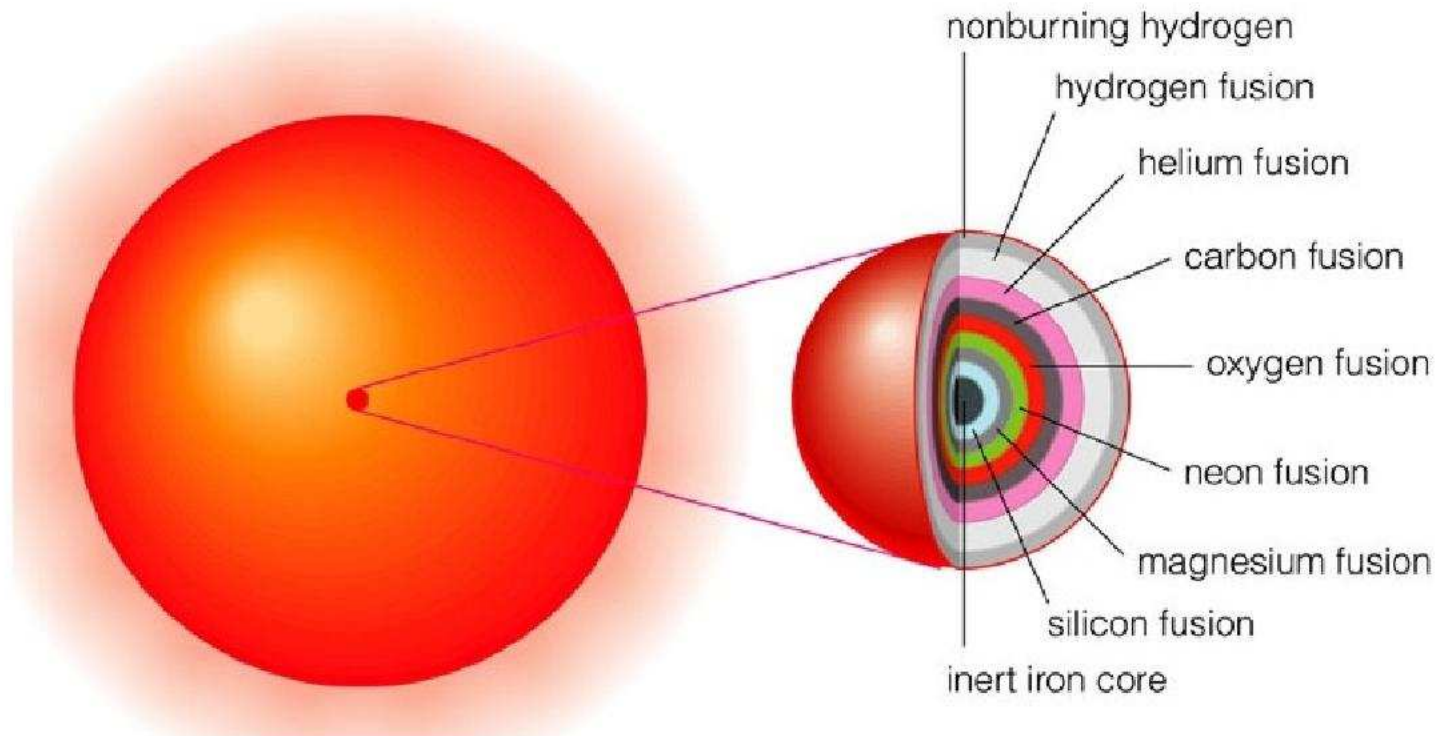


Woosley & Weaver, 1988, Fig. 1

Structure of a $15 M_{\odot}$ pre-supernova star



Core Collapse SNe, VI

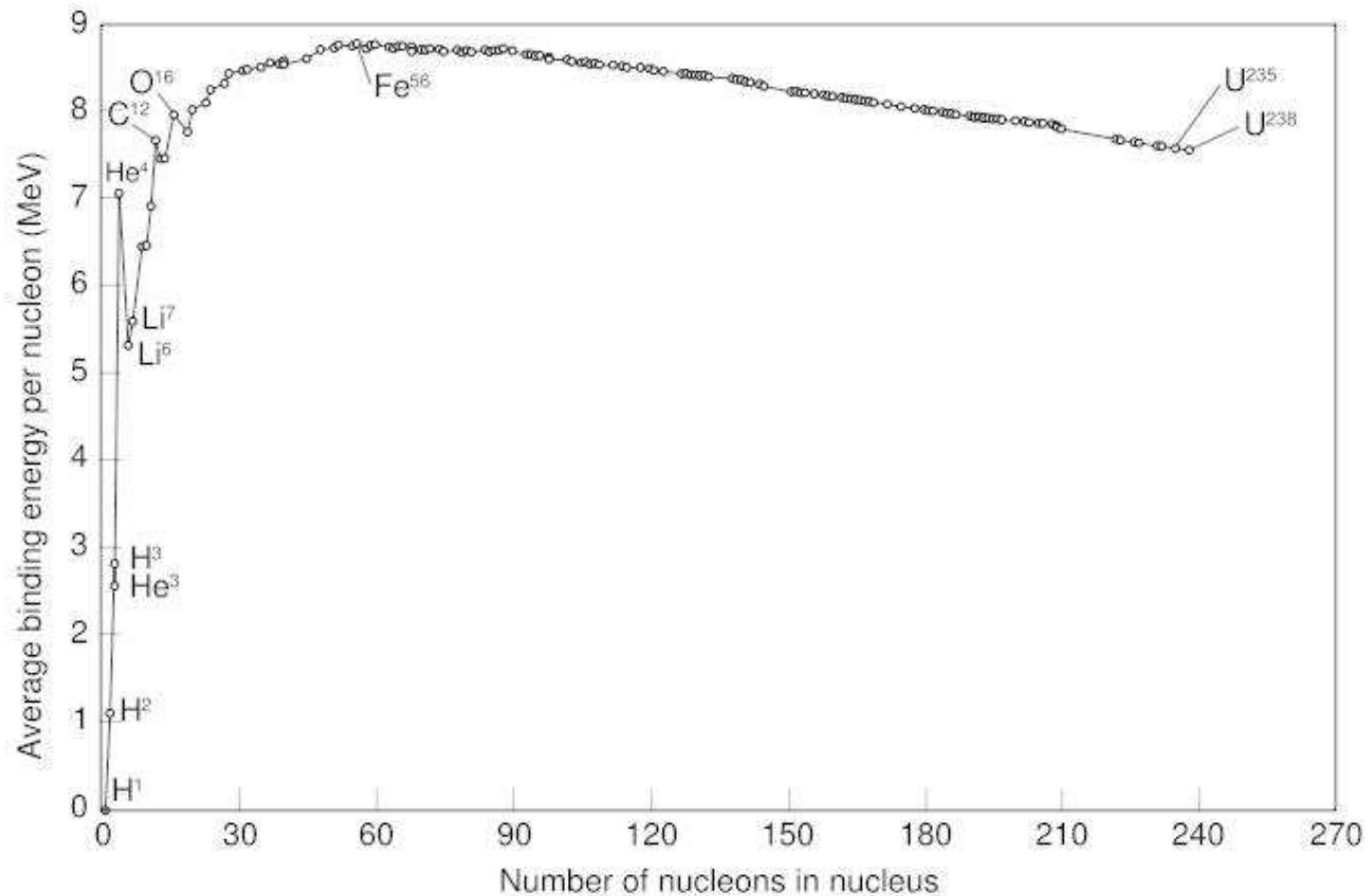


Core collapse supernovae:

- SNe II: progenitors are stars with mass $8\text{--}40 M_{\odot}$ (approximate).
- SNe Ib, Ic: more massive stars, which lost their H shell due to strong stellar winds



Core Collapse SNe, VII

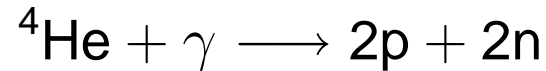
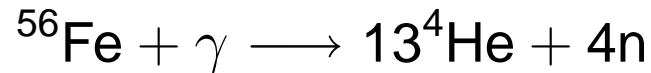


^{56}Fe is one of the most tightly bound nucleons \implies Star has a problem once ^{56}Fe reached: fusion processed become endotherm



Core Collapse SNe, VIII

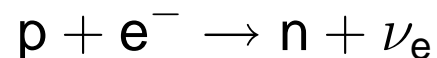
Iron core starts to shrink $\implies T$ increases $\implies {}^{56}\text{Fe}$ starts **photodisintegration**:



Typical core masses are between $1.3 M_{\odot}$ (for $10 M_{\odot}$ on ZAMS) and $2.5 M_{\odot}$ (for $50 M_{\odot}$ on ZAMS).

Until now, free electrons have degeneracy pressure and hold star

BUT: once core temperature increases to $T_c \sim 8 \times 10^9$ K and density to $\rho_c \sim 10^{10} \text{ g cm}^{-3}$: **neutronization**:



\implies **rapid energy loss** (for a $20 M_{\odot}$ star: $4.4 \times 10^{38} \text{ erg s}^{-1}$ in photons, but $3 \times 10^{45} \text{ erg s}^{-1}$ in neutrinos!) \implies **COLLAPSE**



Core Collapse SNe, IX

Once pressure support is gone:

⇒ collapse (free fall)

⇒ speeds are fast (outer core: $\sim 70000 \text{ km s}^{-1}$!)

⇒ supersonic, so outer parts don't realize what's happening

⇒ inner core compresses further through neutronization

⇒ once $\rho_c \sim 8 \times 10^{14} \text{ g cm}^{-3}$: **Neutron star forms** (degeneracy pressure of neutrons)

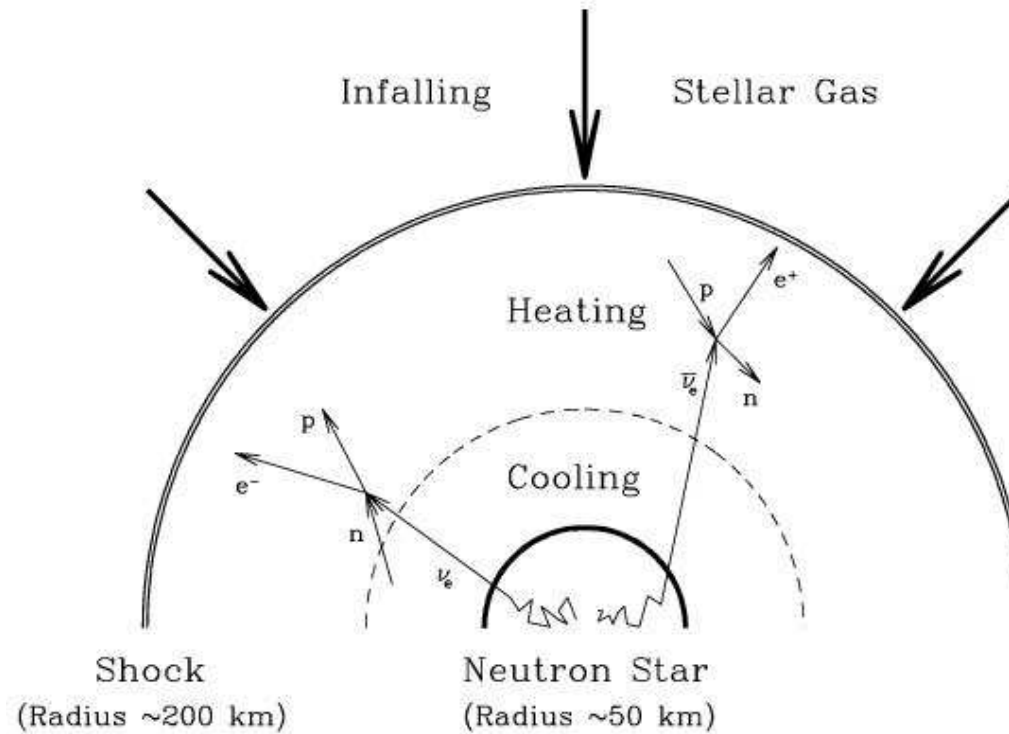
⇒ “solid surface forms”, resulting in **bounce back**

~ 20 msec for shock wave to pass through core

⇒ further photodesintegration

⇒ shock moves outwards ⇒ **explosion** (“prompt hydrodynamic explosion”)

Core Collapse SNe, X



H.-T. Janka

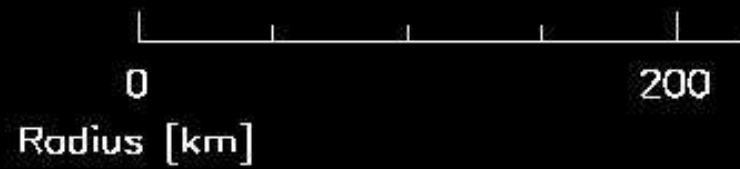
For Fe core mass of $\gtrsim 1.2 M_{\odot}$: Infalling gas damps expansion of shock \implies need sufficient **heating by neutrinos to trigger explosion!** (takes ~ 1 s \implies “delayed explosion mechanism”)

energy loss $\sim 1.7 \times 10^{51}$ ergs s⁻¹ per $0.1 M_{\odot}$ of Fe

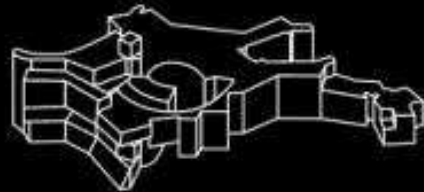
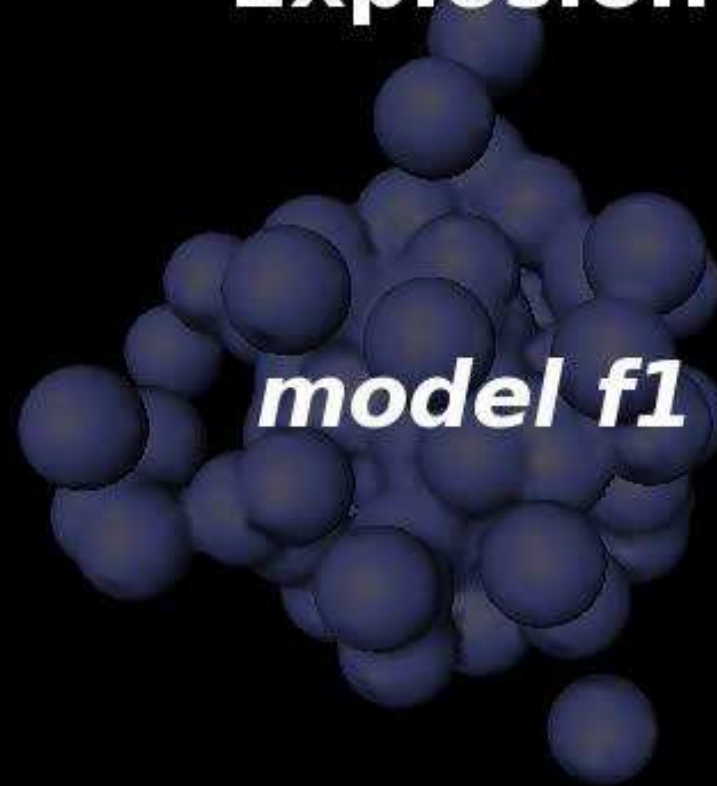
$t = 2.1 \text{ ms}$



$r_{\text{shock}} = 45 \text{ km}$



Thermonuclear Supernova Explosion

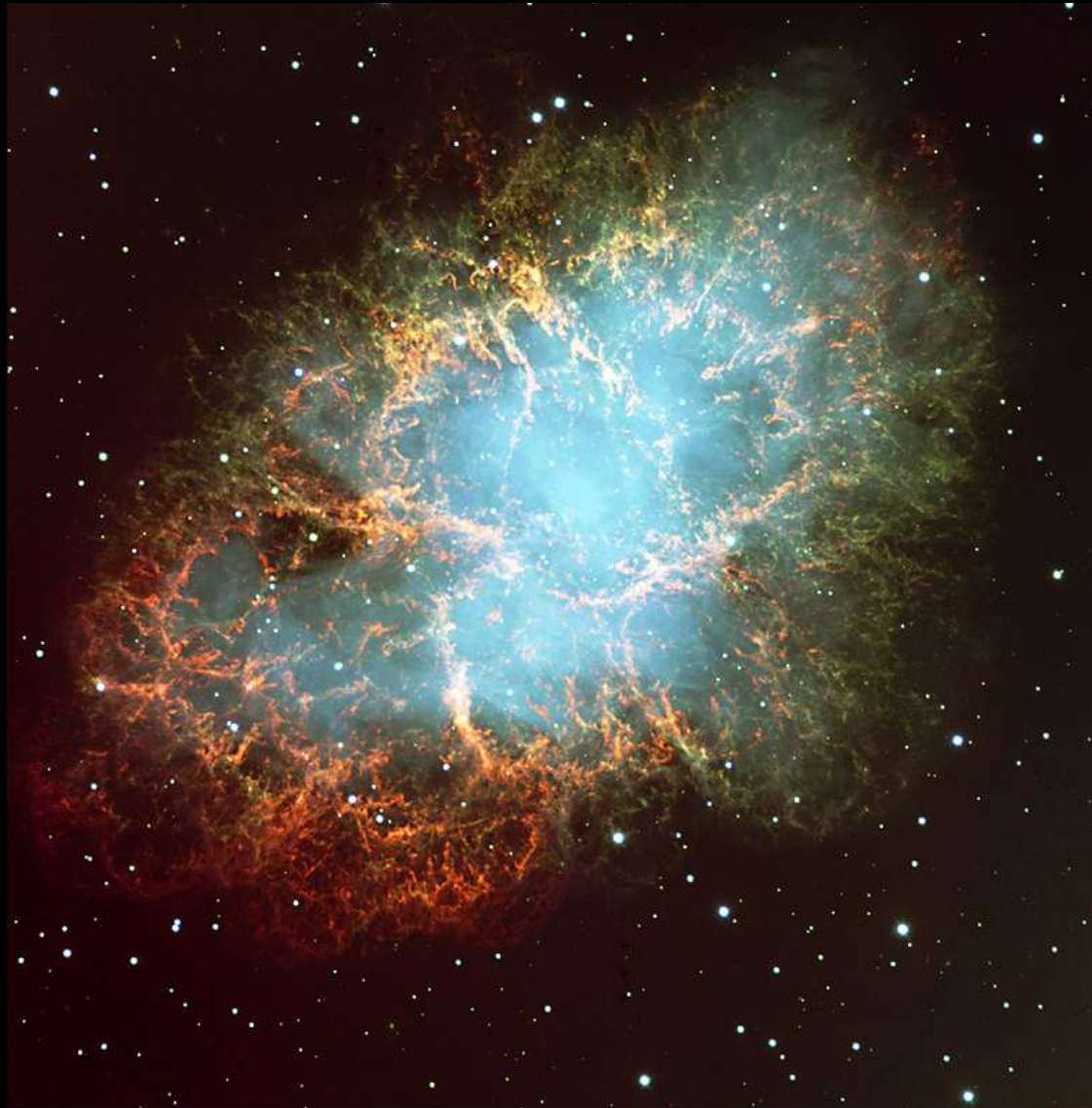


(c) Friedrich Röpke, MPA, 2004

3D-simulations show strong asymmetry of supernova explosion.

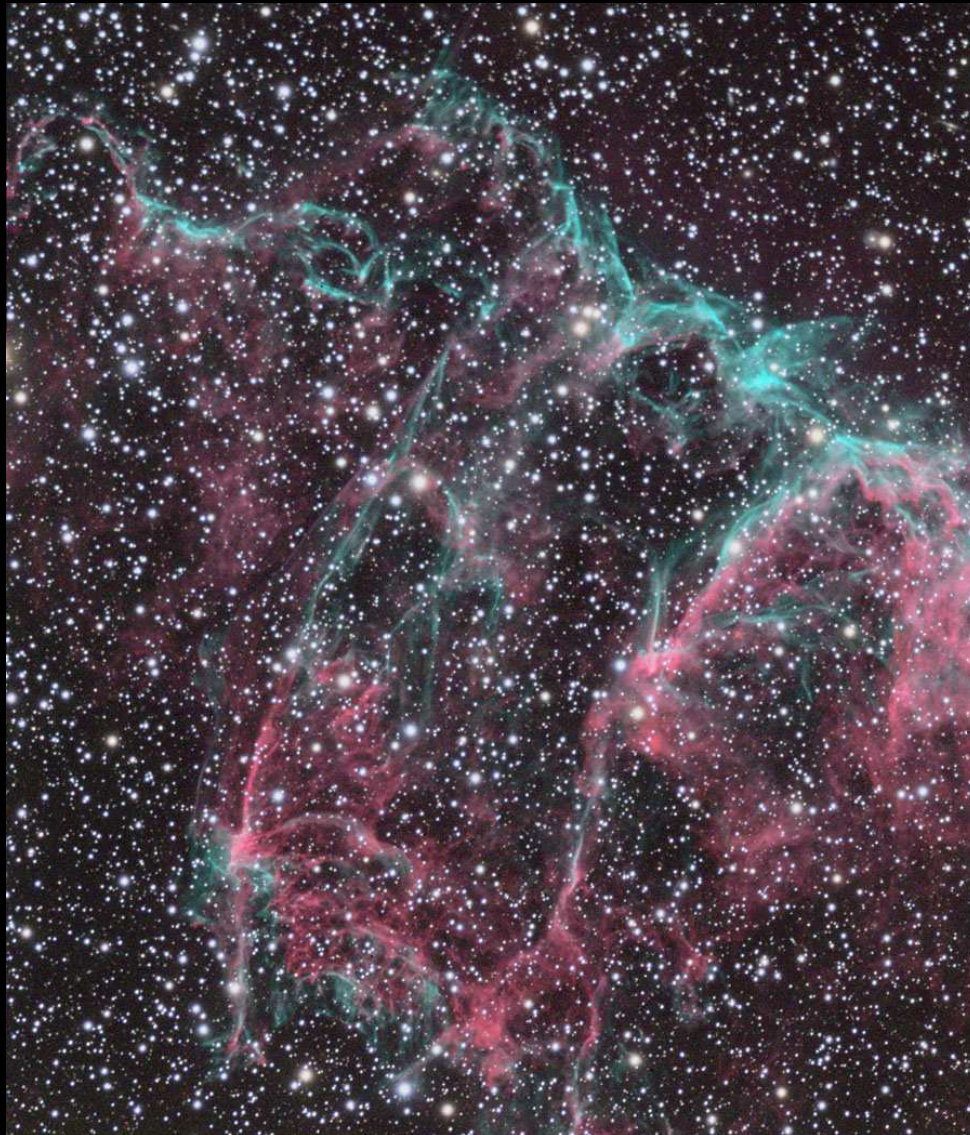


CXC



(ESO VLT/FORS 2)

Crab nebula: young remnant of SN of 1054, observed light due to **synchrotron radiation** (radiation emitted by electrons accelerated in magnetic field)



5000–10000 year old IC 1340/Veil Nebula/Cygnus Loop (©Loke Kun Tan)

Older supernova remnants: “wispy structure” due to interaction with interstellar medium, radiation (line emission) mainly caused by heating due to shocks.

The progenitor of a Type Ia supernova



Two normal stars are in a binary pair.



The more massive star becomes a giant...



...which spills gas onto the secondary star, causing it to expand and become engulfed.



The secondary, lighter star and the core of the giant star spiral inward within a common envelope.



The common envelope is ejected, while the separation between the core and the secondary star decreases.



The remaining core of the giant collapses and becomes a white dwarf.



The aging companion star starts swelling, spilling gas onto the white dwarf.



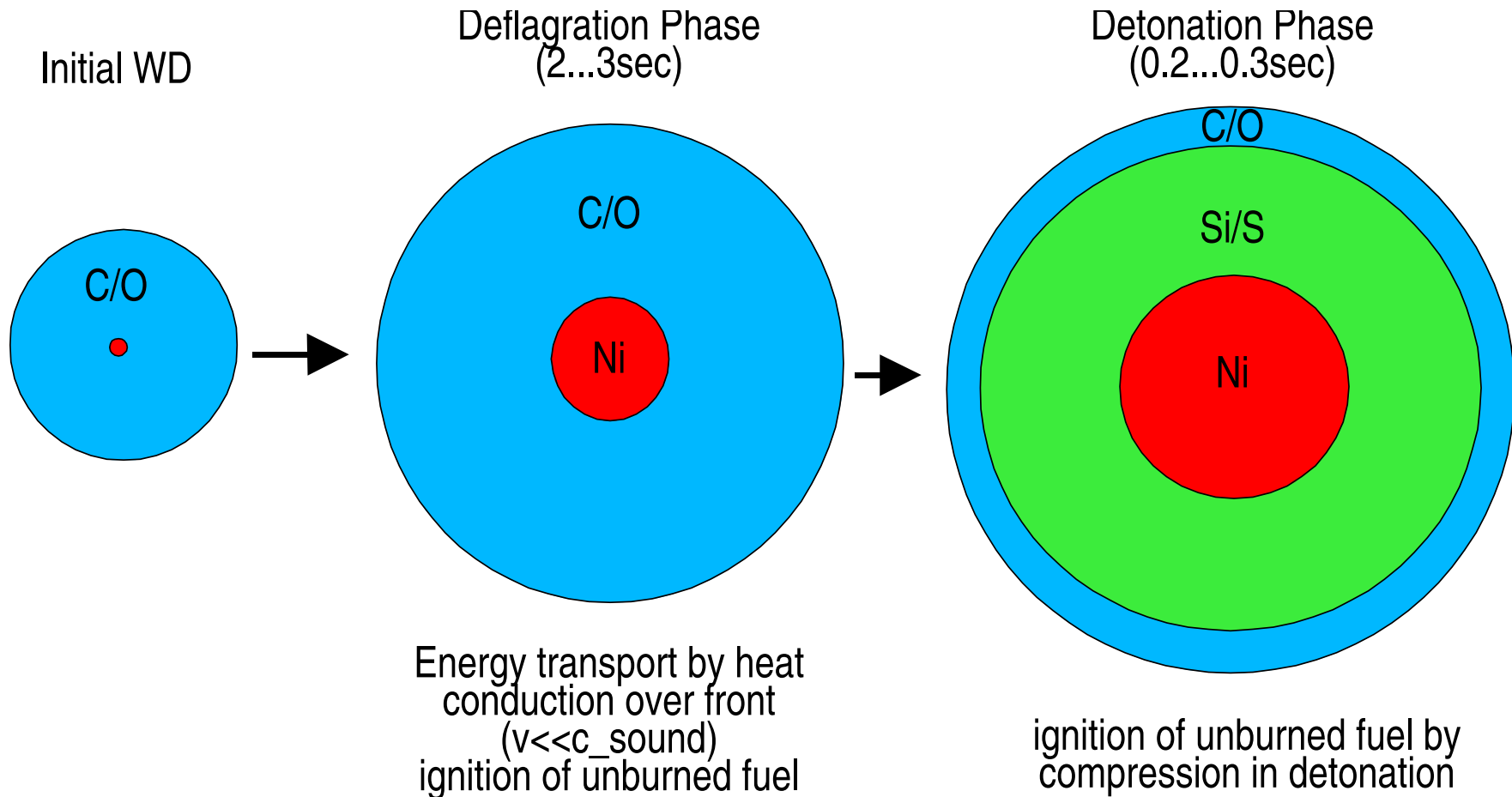
The white dwarf's mass increases until it reaches a critical mass and explodes...



...causing the companion star to be ejected away.



Supernova Mechanisms – Ia



after P. Höflich

SNe Ia: **thermonuclear runaway** in **carbon-oxygen white dwarfs** (See later for details)



Neutron Stars

During SN explosion:

Core of exploding star above Chandrasekhar limit \implies core collapses

Densities get so high that neutronization sets in:

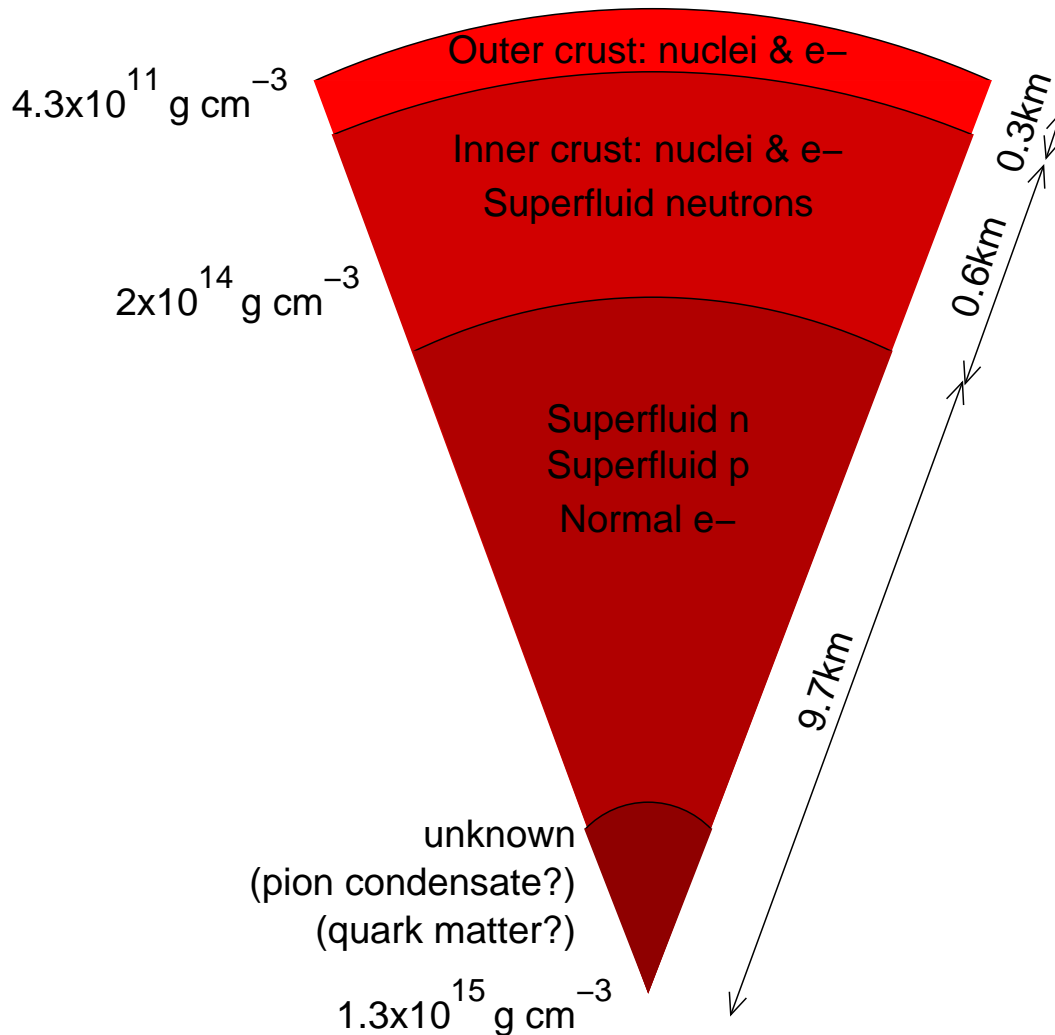


General properties:

- Pressure mainly through degenerate neutrons (similar to degenerate electrons for WD!).
- Typical density: $\rho \sim 10^{14} \text{ g cm}^{-3}$ (nuclear densities)
- Typical radius: 10... 15 km (Nuremberg!)
- surface gravity $\sim 10^{11} \times \text{Earth}$
- Detailed structure not yet fully understood



Neutron Stars: Structure, I



Crust: perhaps crystallized?

Atoms become elongated along B -field line on surface

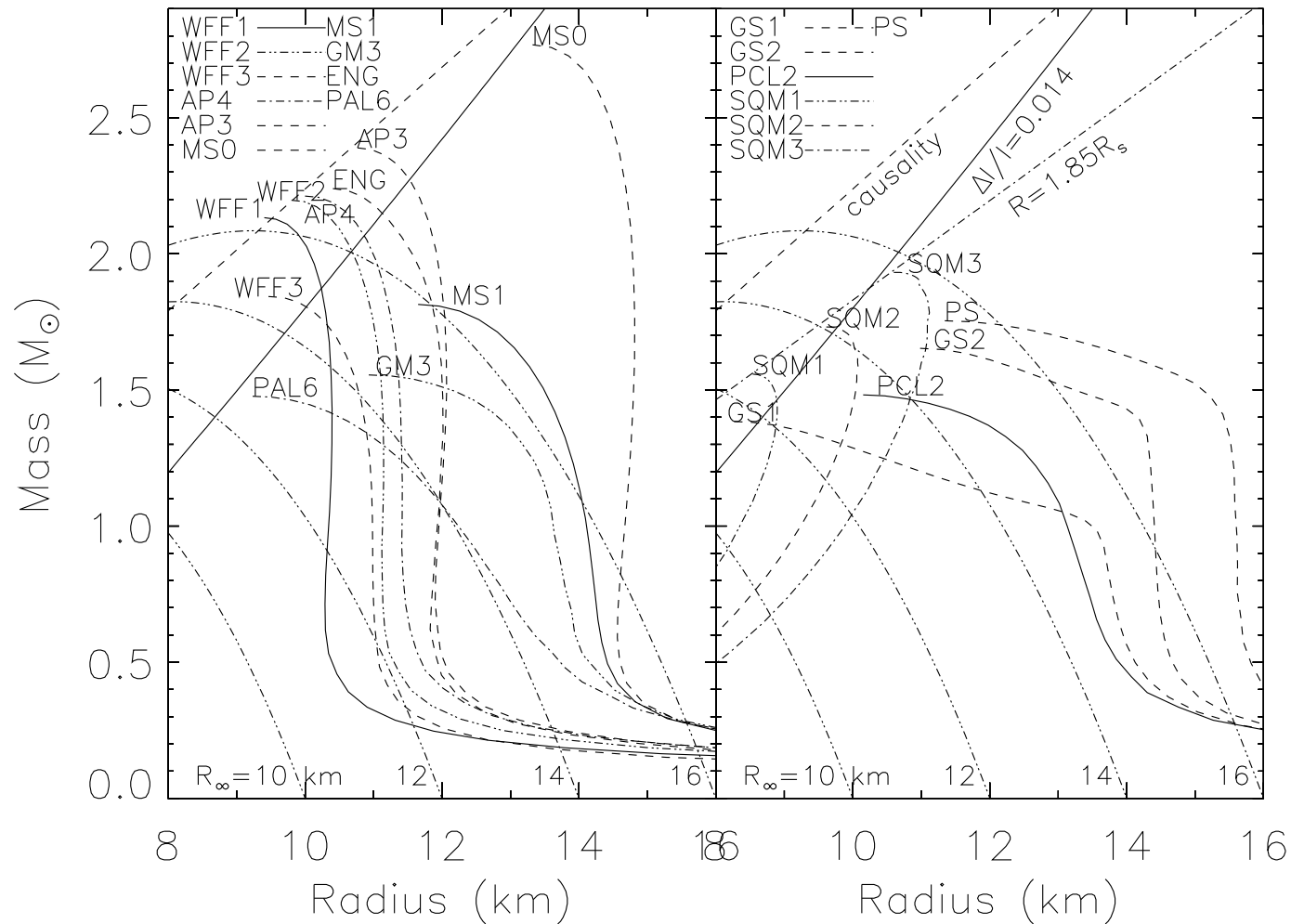
Internal structure unclear:

- **Supraconducting** matter
- **Suprafluidity** (i.e., fluid with no viscosity)
- **central composition unknown**

(after Shapiro & Teukolsky, 1983)



Neutron Stars: Structure, II



(Lattimer & Prakash, 2001)

The structure and size of neutron stars depends strongly on the unknown properties of matter at very high densities.



Neutron Stars: Rotation, I

During SN collapse, **angular momentum is conserved** (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$



Neutron Stars: Rotation, II

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Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

Angular momentum conservation ($J_{\text{before}} = J_{\text{NS}}$):

$$\frac{2}{5}M_{\text{before}}R_{\text{before}}^2\omega_{\text{before}} = \frac{2}{5}M_{\text{NS}}R_{\text{NS}}^2\omega_{\text{NS}}$$

or

$$\omega_{\text{NS}} = \left(\frac{M_{\text{before}}}{M_{\text{NS}}}\right) \left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2 \omega_{\text{before}} \quad \text{or} \quad P_{\text{NS}} \sim \left(\frac{R_{\text{NS}}}{R_{\text{before}}}\right)^2 P_{\text{before}}$$

(where P : rotation period)



Neutron Stars: Rotation, III

During SN collapse, **angular momentum is conserved** (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

Angular momentum conservation ($J_{\text{before}} = J_{\text{NS}}$):

$$\frac{2}{5}M_{\text{before}}R_{\text{before}}^2\omega_{\text{before}} = \frac{2}{5}M_{\text{NS}}R_{\text{NS}}^2\omega_{\text{NS}}$$

or

$$\omega_{\text{NS}} = \left(\frac{M_{\text{before}}}{M_{\text{NS}}}\right) \left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2 \omega_{\text{before}} \quad \text{or} \quad P_{\text{NS}} \sim \left(\frac{R_{\text{NS}}}{R_{\text{before}}}\right)^2 P_{\text{before}}$$

(where P : rotation period)

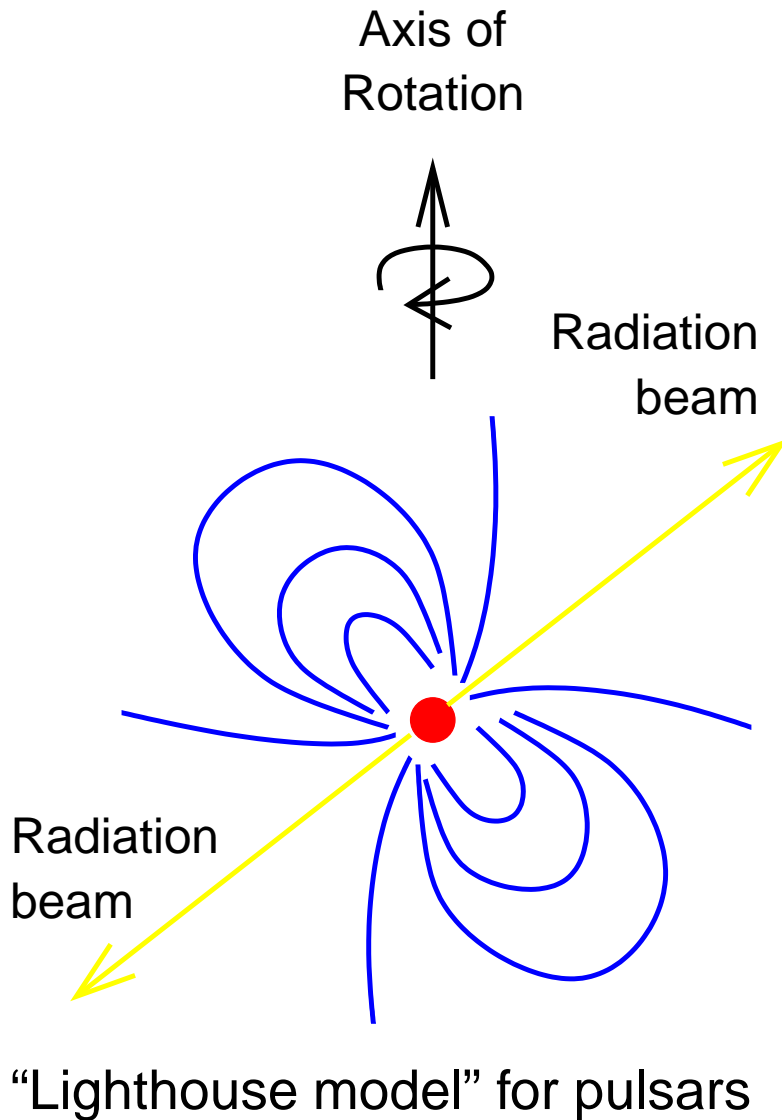
Example: $R_{\text{before}} = 700000 \text{ km (sun)}$, $R_{\text{NS}} = 15 \text{ km}$, $P_{\text{Sun}} = 27 \text{ d} \implies P_{\text{NS}} = 0.001 \text{ s}$

Neutron Stars are extremely fast rotators.

close to break-up speed!



Neutron Stars: Pulsars



Another conserved observable:
magnetic flux: $\Phi = BR^2$

magnetic field after SN:

$$B_{\text{NS}} = \left(\frac{R_{\text{before}}}{R_{\text{NS}}} \right)^2 B_{\text{before}}$$

⇒ **neutron stars have strong magnetic fields** (typical: $B \sim 10^6 \dots 10^8 \text{ T}$)

Radio pulsars are fast rotating (isolated) neutron stars with strong magnetic fields.



The sounds of pulsars

- PSR 0329 – a normal pulsar ($P = 0.714519$ s)
- PSR 0833 – the Vela pulsar, a faster, younger pulsar in the Vela supernova remnant ($P = 89$ msec)
- Crab pulsar – the youngest pulsar ($P = 33$ ms)
- B1937 – one of the fastest pulsars ($P = 0.00155780644887275$ s)

See/hear

<http://www.jb.man.ac.uk/~pulsar/Education/Sounds/sounds.html>
for more examples.



Black Holes, I

Neutron stars also have upper mass limit: **Oppenheimer Volkoff limit**.

Detailed mass limit unknown, causality considerations give $M \sim 3 M_{\odot}$ (for “stiff equation of state” the sound speed becomes greater than speed of light at this mass)

Compact objects with mass above Oppenheimer Volkoff limit: **Black Holes**

More conservative astronomers: “Black Hole Candidates”.



Black Holes, II

Rev. John Michell: *Phil. Trans. R. Soc. London*, **74**, 35–57 (1784):

VII. On the Means of discovering the Distance, Magnitude, &c. of the Fixed Stars, in consequence of the Diminution of the Velocity of their Light, in case such a Diminution should be found to take place in any of them, and such other Data should be procured from Observations, as would be farther necessary for that Purpose. By the Rev. John Michell, B. D. F. R. S. In a Letter to Henry Cavendish, Esq. F. R. S. and A. S.

Read November 27, 1783.



Black Holes, III

Rev. John Michell: *Phil. Trans. R. Soc. London*, **74**, 35–57 (1784):

42 *Mr. MICHELL on the Means of discovering the*

16. Hence, according to article 10, if the semi-diameter of a sphere of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiae, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.



Black Holes, IV

In more modern usage (but still Newtonian!):

Total energy of a mass m :

$$E = E_{\text{pot}} + E_{\text{kin}} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

Mass m is unbound if $E > 0$, i.e., for

$$v \geq v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

Black Hole: Body of mass M and radius R for which $v_{\text{escape}} > c$, where c is the speed of light.

This is the case if

$$R \leq R_{\text{S}} = \frac{2GM}{c^2} \sim 3 \text{ km} \frac{M}{M_{\odot}}$$

the **Schwarzschild Radius**.



Einstein, I



Albert Einstein (1879–1955)

Special Relativity (1905):

- Speed of light has the same value in *all* frames of reference
- Observer with constant velocity measure the same physical laws

From these axioms follows:

⇒ Space and time are relative
("4D-space-time")

⇒ $E = mc^2$
("Mass and Energy are equivalent")



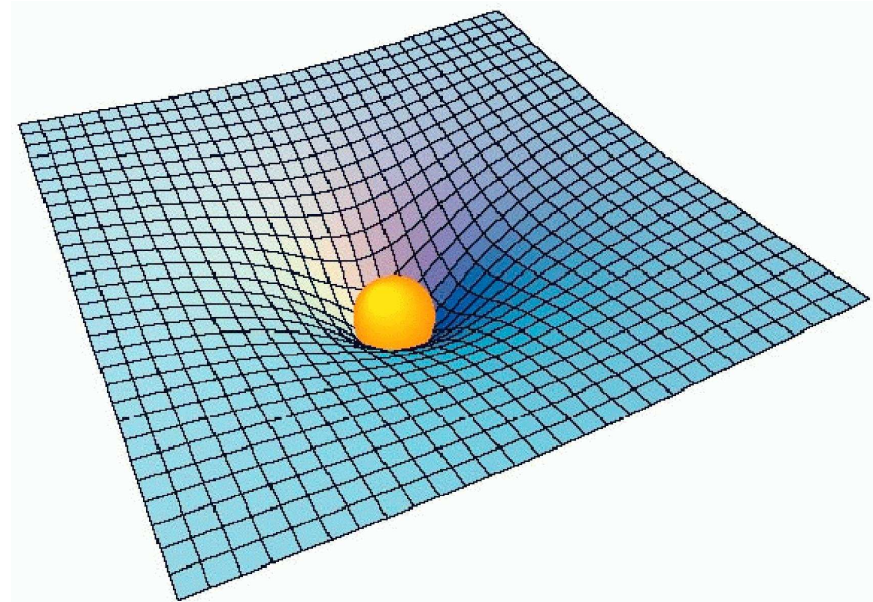
Einstein, II



Albert Einstein (1879–1955)

General relativity (1916):

- Mass curves space (“Metric”)





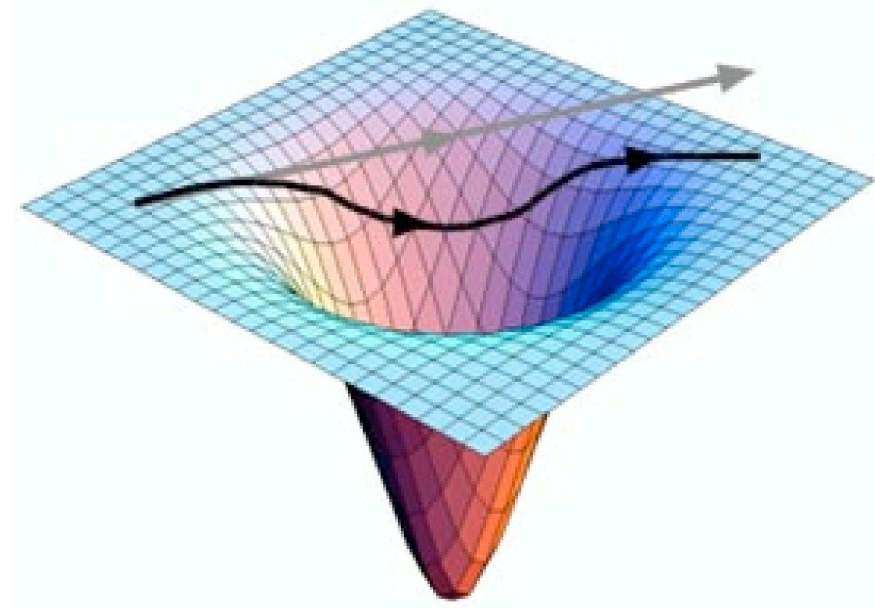
Einstein, III



Albert Einstein (1879–1955)

General relativity (1916):

- Mass curves space (“Metric”)
- Light moves through curved space





post-Einstein, I



Karl Schwarzschild (1873–1916)

Directly after publication of GRT:

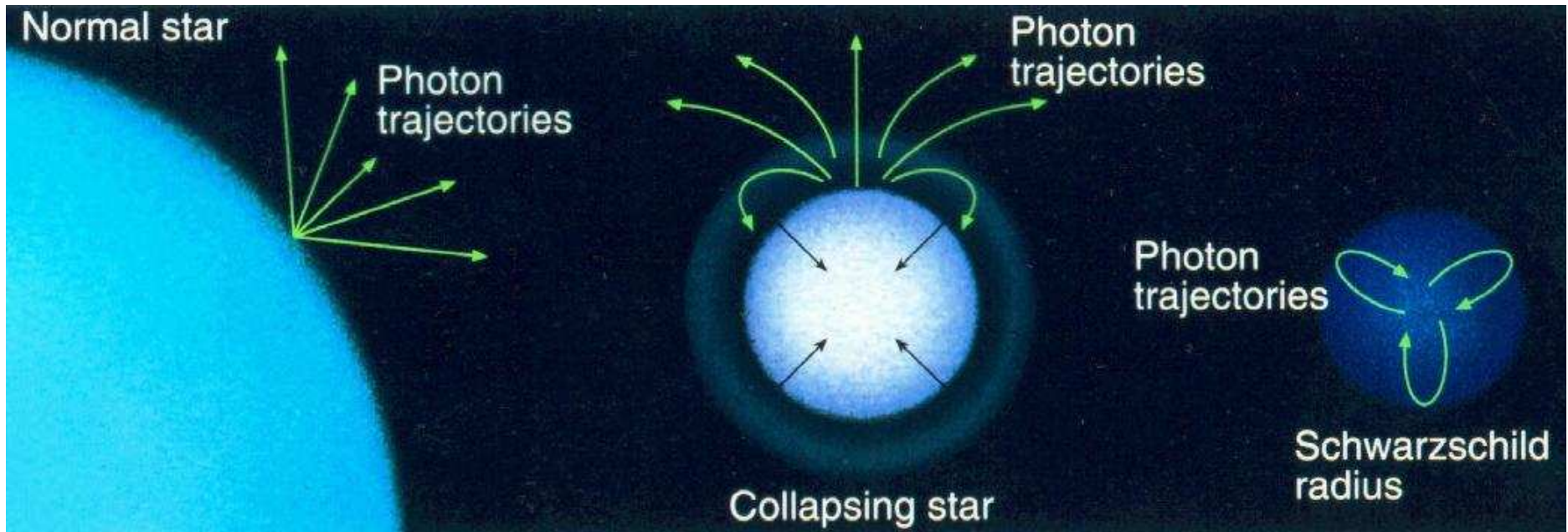
$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2$$

(Schwarzschild Metric).

Describes “shape of space” in vicinity of mass M .



post-Einstein, II



$$R > R_S$$

$$R \sim R_S$$

$$R < R_S$$

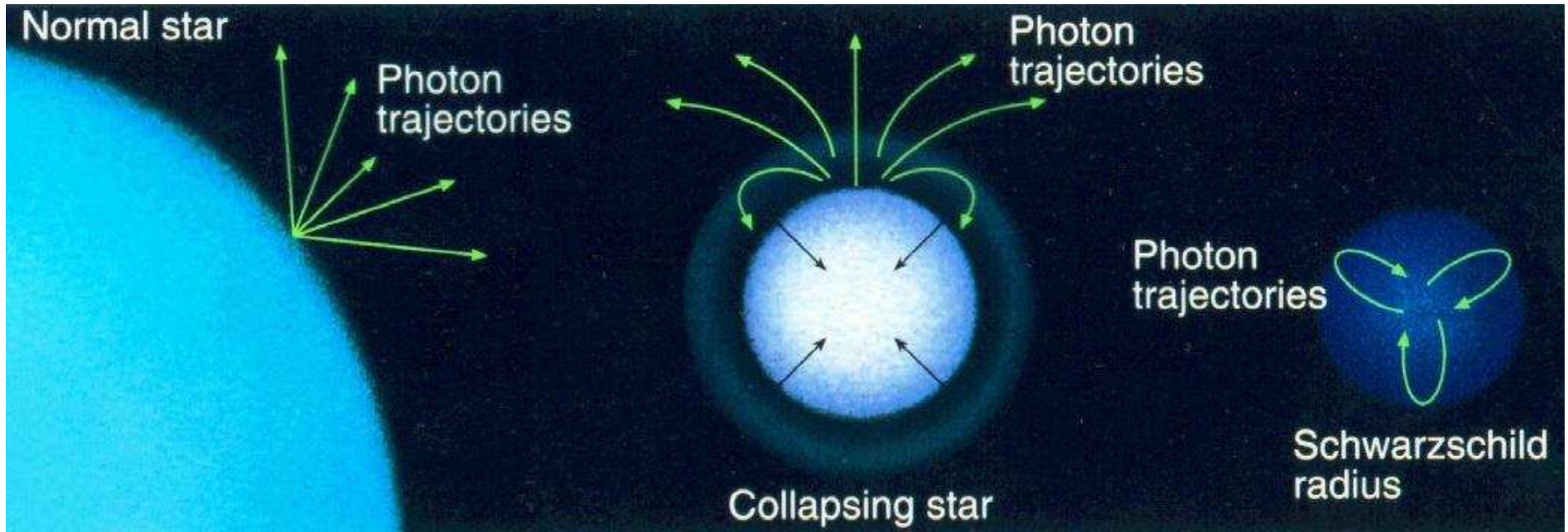
Behavior of light is determined from location of emission, in dependence from the **Schwarzschild Radius**:

$$R_S = \frac{2GM}{c^2} \sim 3 \text{ km} \frac{M}{M_\odot}$$

Same value as in Newtonian derivation!



post-Einstein, III



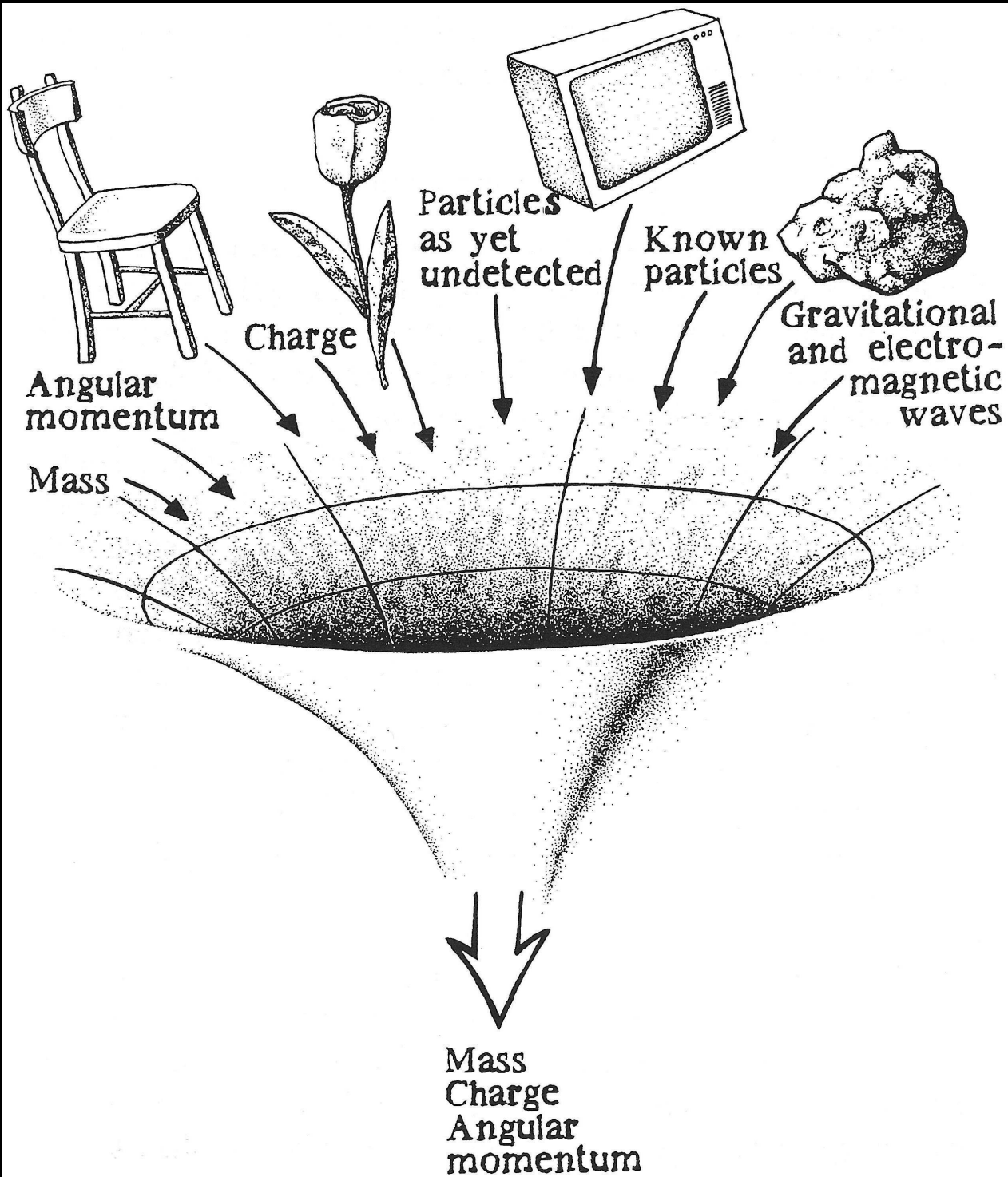
$$R > R_s$$

$$R \sim R_s$$

$$R < R_s$$

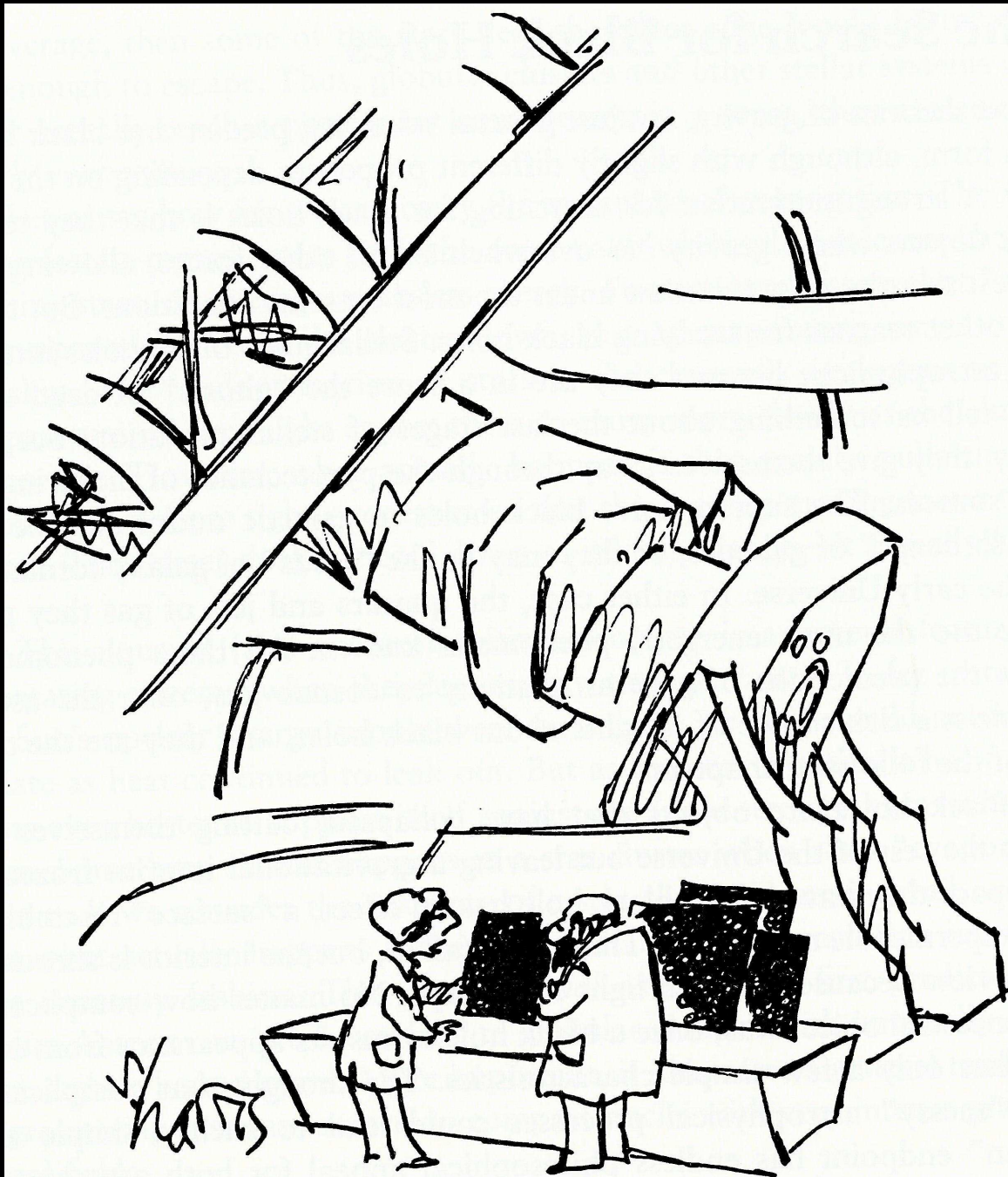
Black hole in GRT: Bodies smaller than their Schwarzschild radius.

J.N. Imamura



Black holes are very simple physical objects, determined by

- Mass
- (Charge)
- Angular momentum



"It's black, and it looks like a hole. I'd say it's a black hole."



Black Holes: Accretion, I

Astrophysical energy sources:

1. Nuclear fusion

Reactions à la



Energy released:

Fusion produces $\sim 6 \times 10^{11} \text{ J g}^{-1}$

(i.e., $\Delta E_{\text{nuc}} \sim 0.007 m_p c^2$)



Black Holes: Accretion, II

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1. Nuclear fusion

Reactions à la



Energy released:

Fusion produces $\sim 6 \times 10^{11} \text{ J g}^{-1}$

(i.e., $\Delta E_{\text{nuc}} \sim 0.007 m_p c^2$)

2. Gravitation

Accretion of mass m from ∞ to R_S on black hole with mass M gives

$$\Delta E_{\text{acc}} = \frac{GMm}{R_S} \text{ where } R_S = \frac{2GM}{c^2}$$

Accretion produces $\sim 10^{13} \text{ J g}^{-1}$

(i.e., $\Delta E_{\text{acc}} \sim 0.1 m_p c^2$)



Black Holes: Accretion, III

Astrophysical energy sources:

1. Nuclear fusion

Reactions à la



Energy released:

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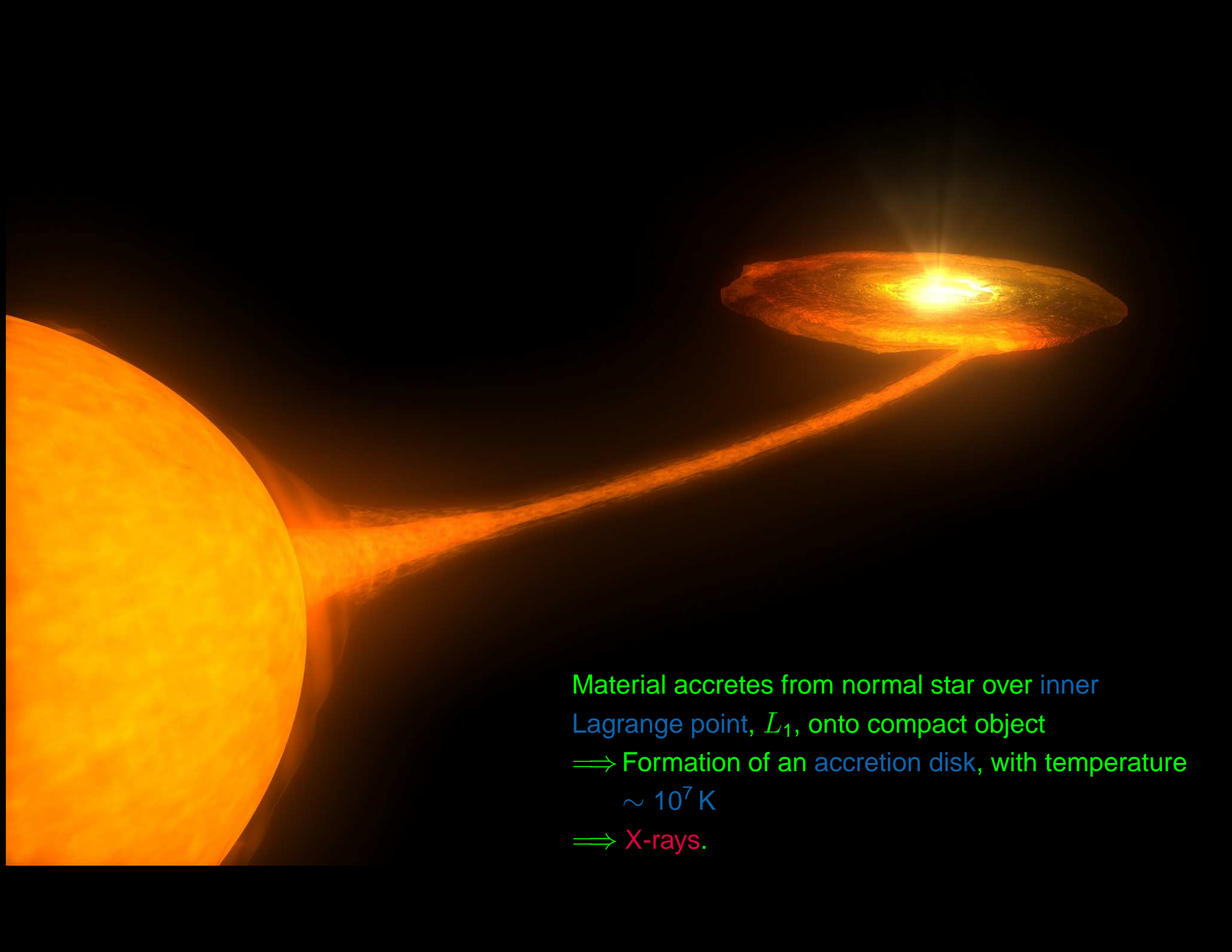
Accretion produces $\sim 10^{13} \text{ J g}^{-1}$

(i.e., $\Delta E_{\text{acc}} \sim 0.1 m_p c^2$)

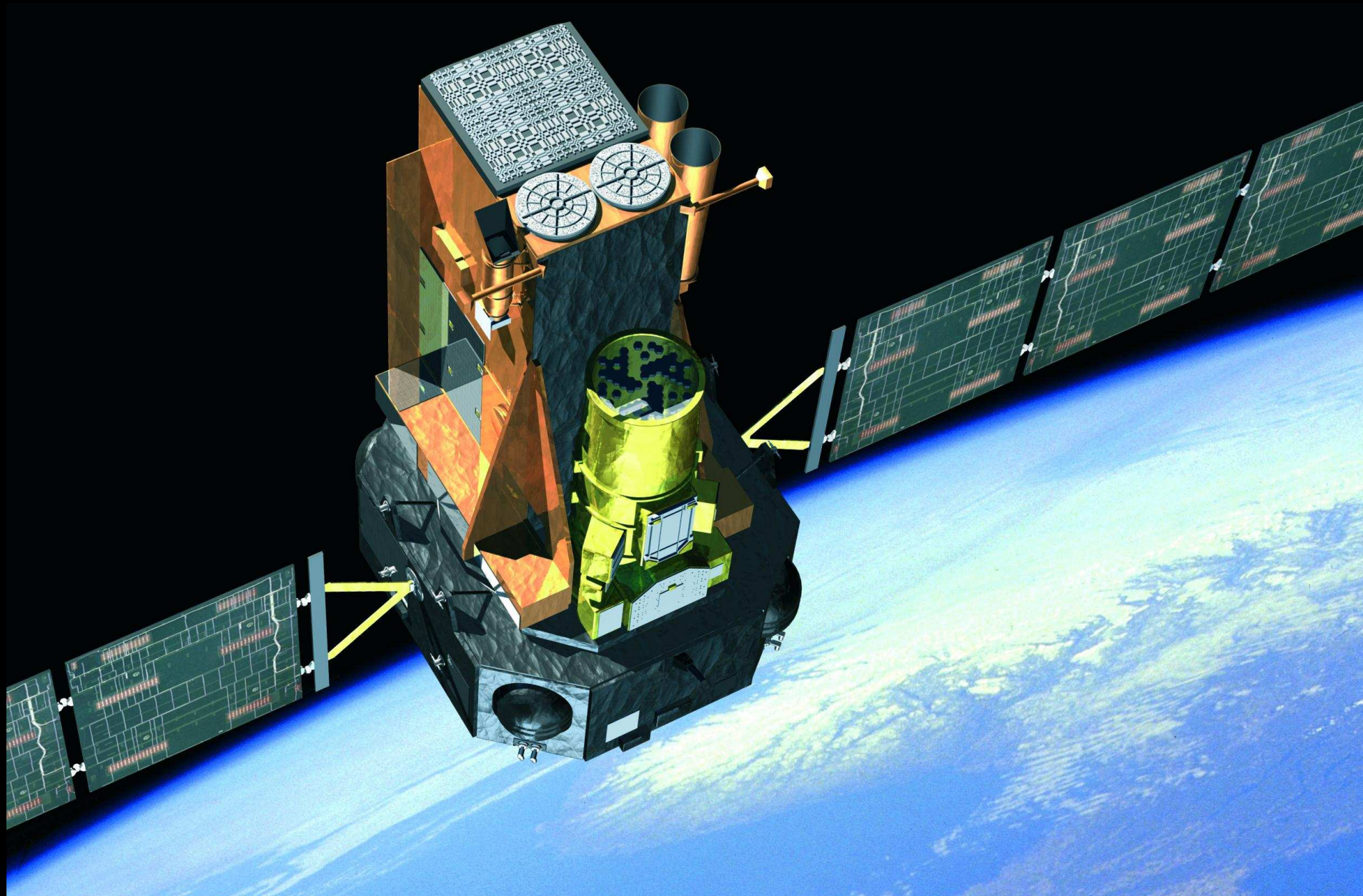
⇒ Accretion of material is the **most efficient** astrophysical energy source.

... thus accreting objects are the most luminous in the whole universe.

Note: energy gets radiated away from *outside* the Schwarzschild radius!



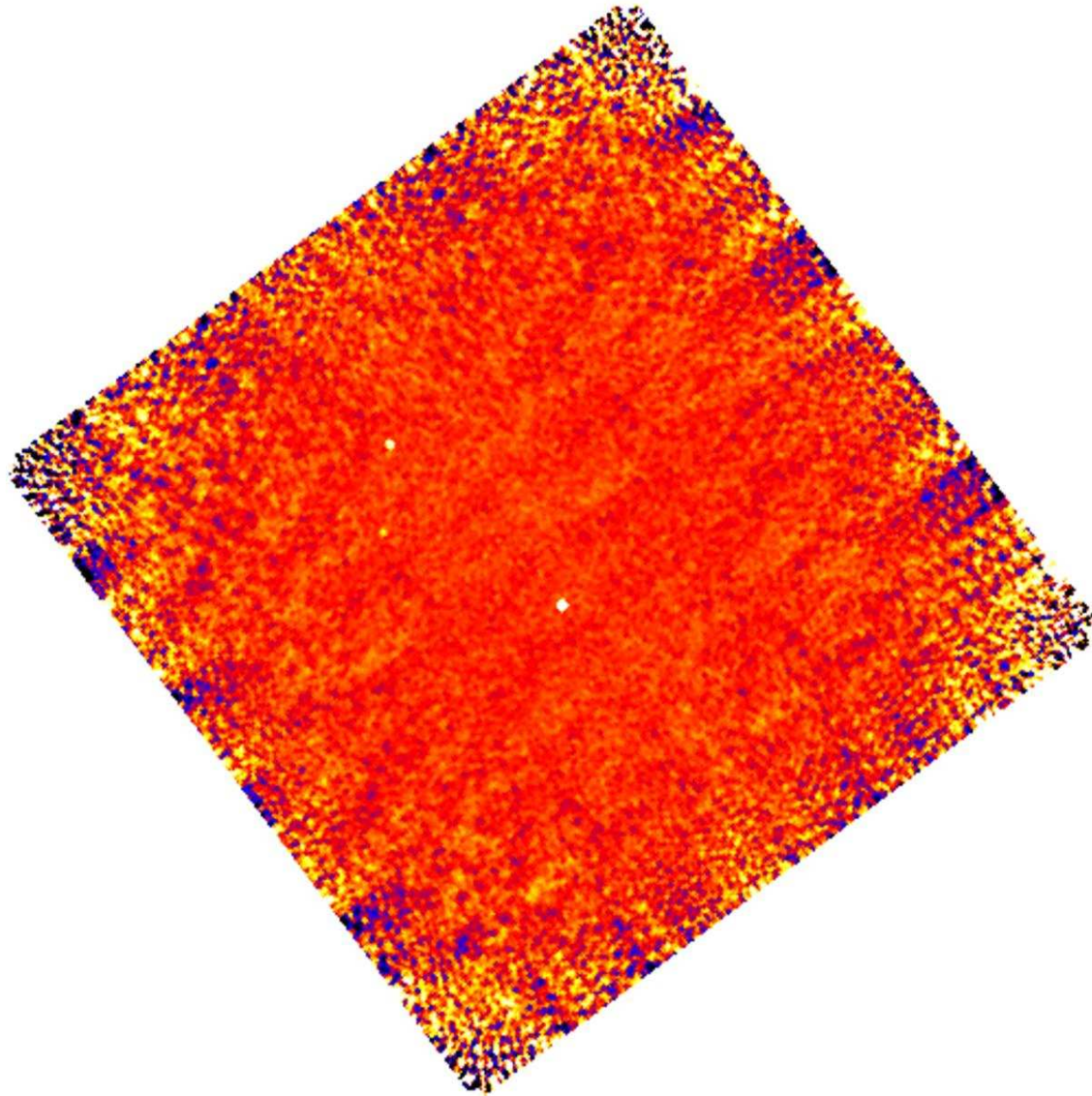
Material accretes from normal star over inner
Lagrange point, L_1 , onto compact object
⇒ Formation of an accretion disk, with temperature
 $\sim 10^7$ K
⇒ X-rays.



INTEGRAL

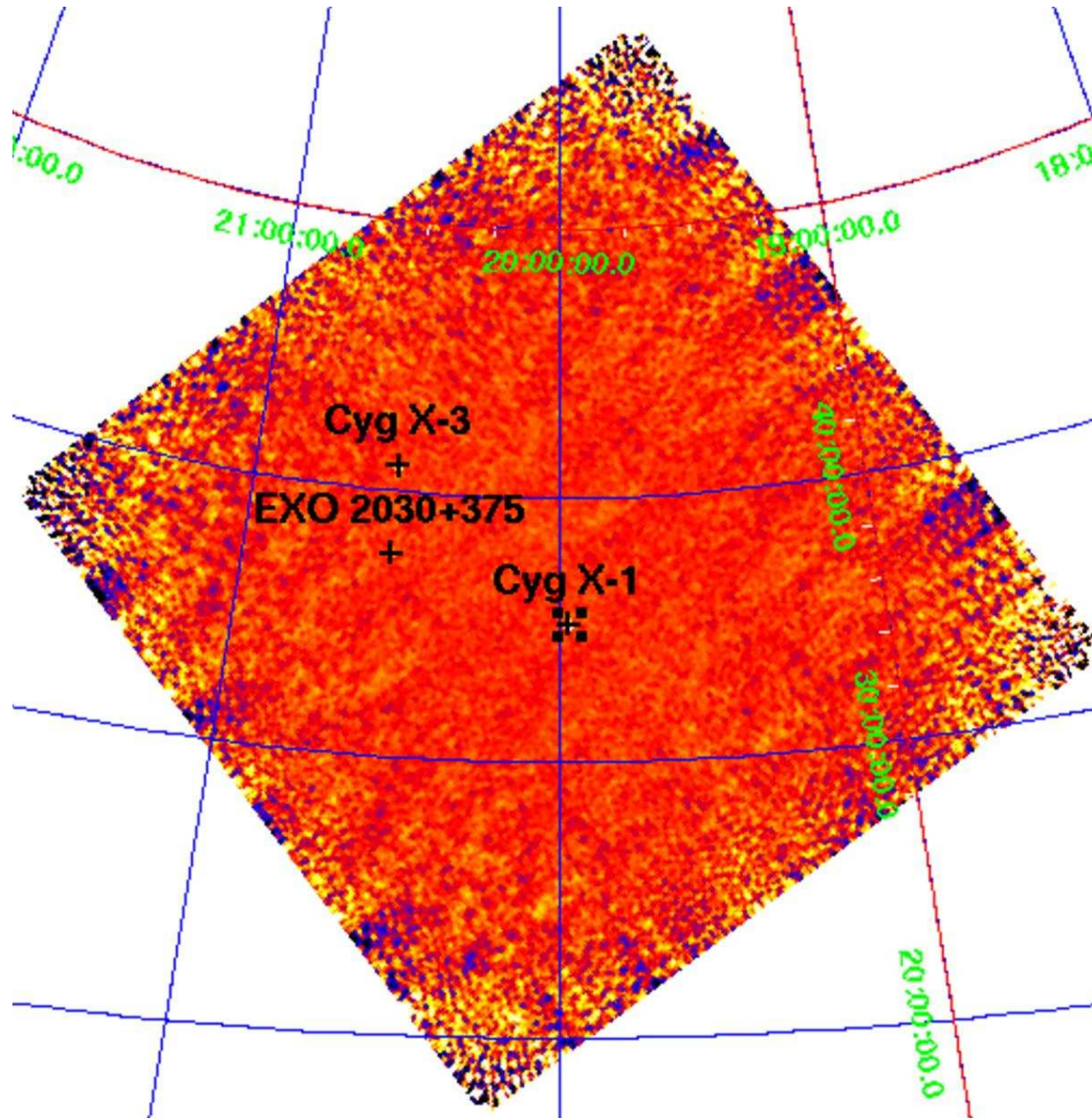


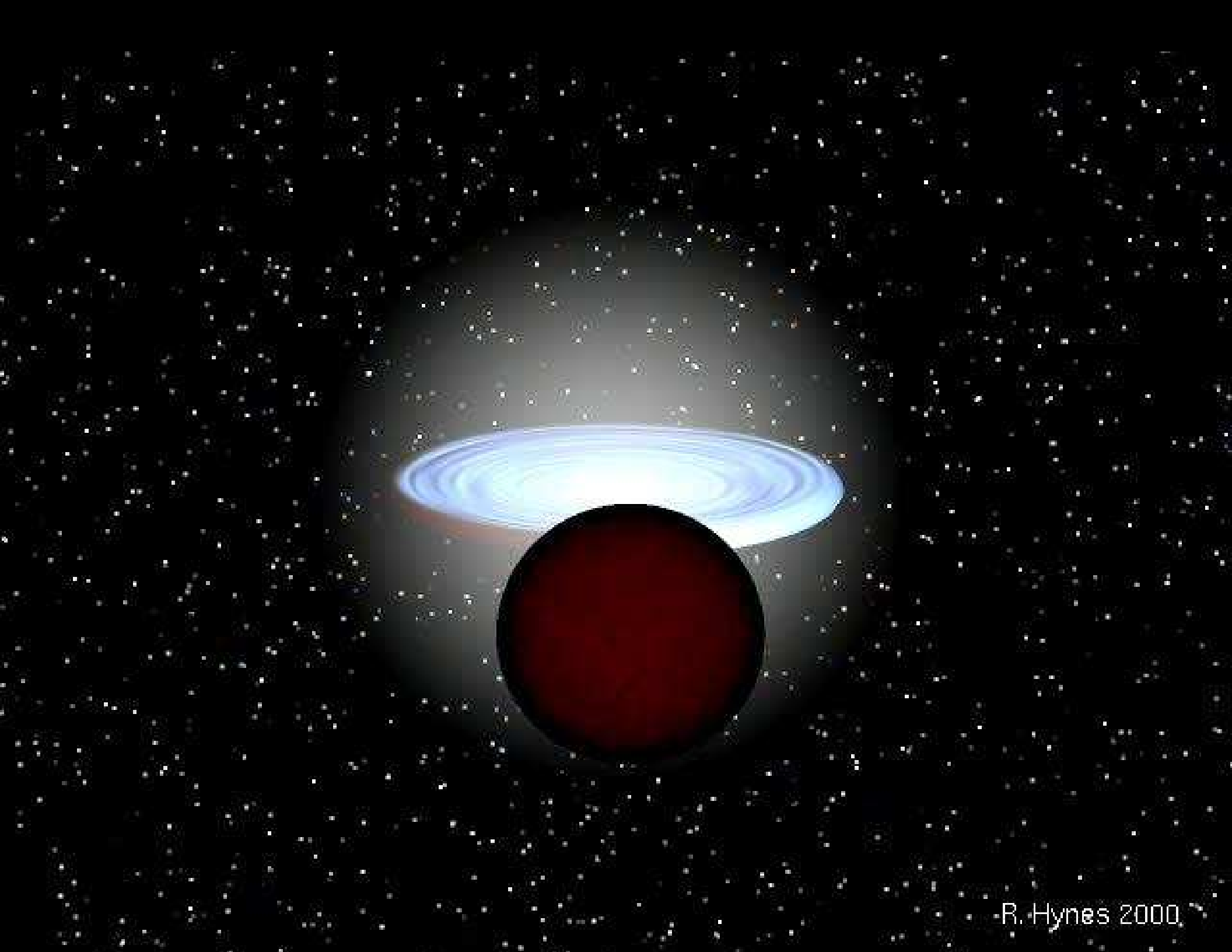
Black Holes: Accretion, VI





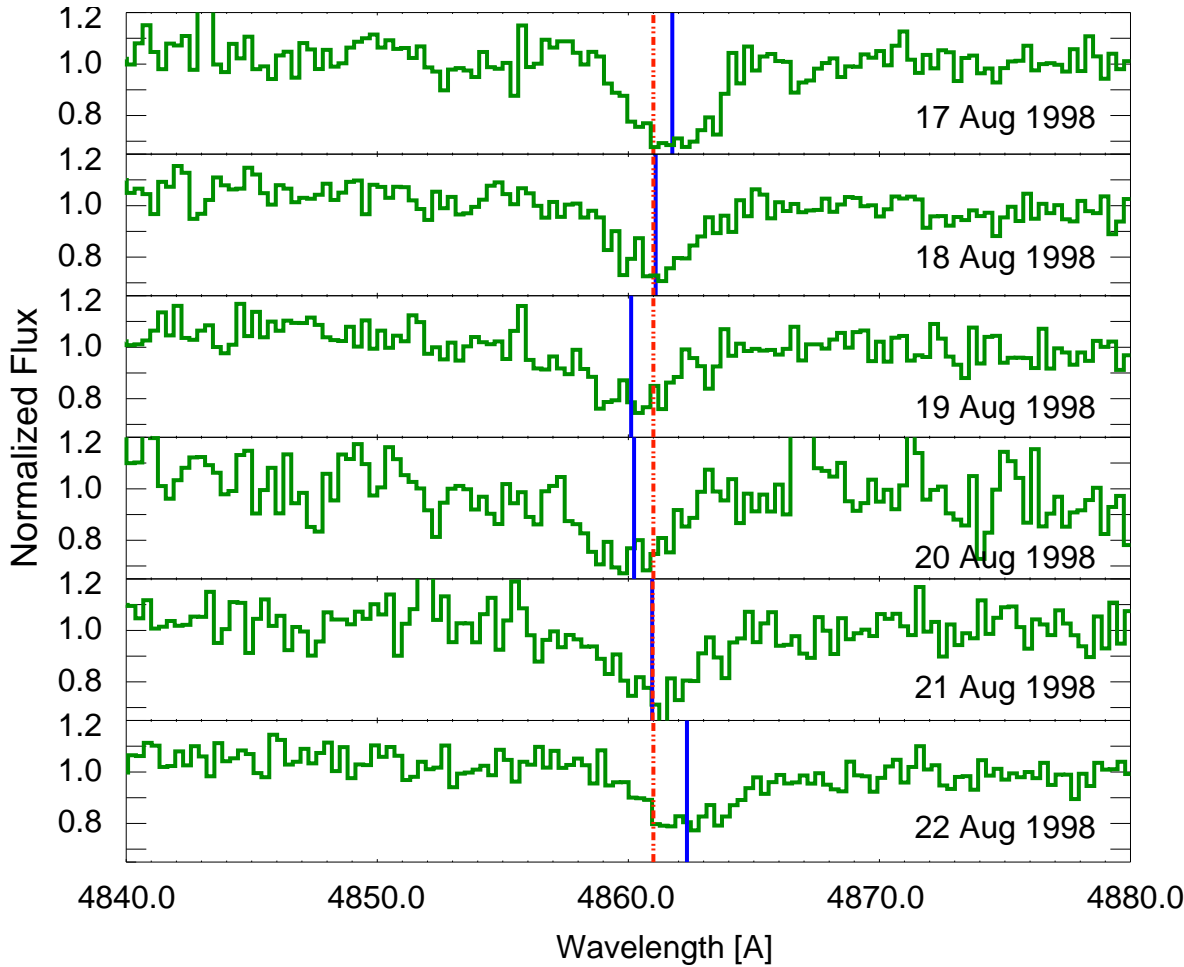
Black Holes: Accretion, VII







Mass determination, II



Motion of H β line in HDE 226868/Cyg X-1
(Pottschmidt, Wilms)

Reminder: In **binary systems**: Mass of compact object from 3rd Kepler:

$$\frac{a^3}{P^2} = \frac{G(M_1 + M_2)}{4\pi^2}$$

(a : semi-major axis, P : orbital period, $M_{1,2}$: Mass).

Derive from this: **Mass function**

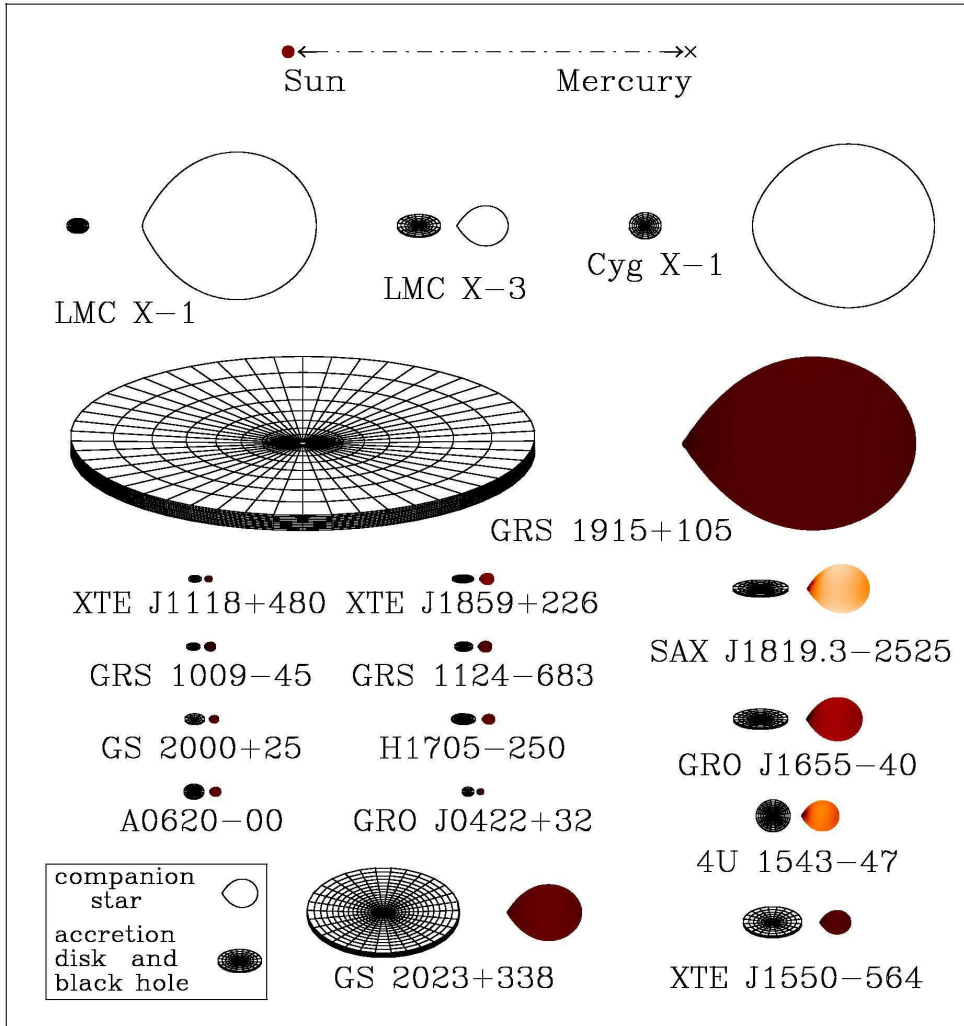
$$MF = \frac{M_2^3 \sin^3 i}{(1 + (M_1/M_2))^2} = \frac{K_1^2 P}{2\pi G}$$

MF is **lower limit** for M_2 .

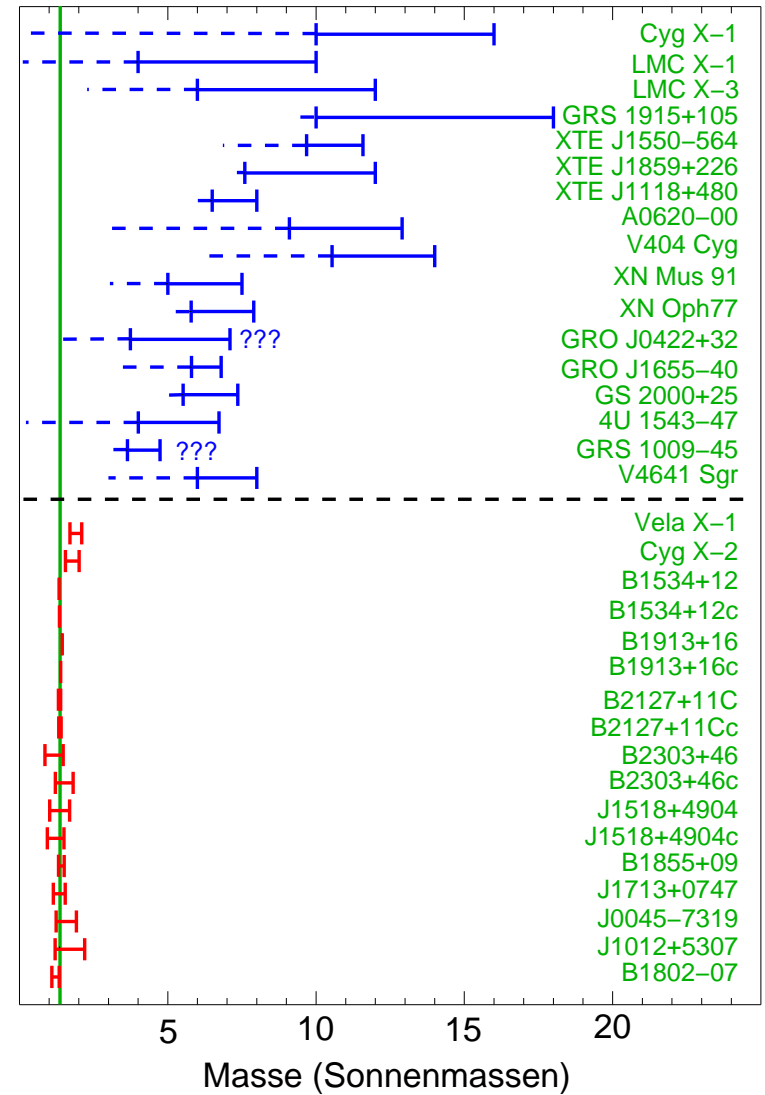
(K_2 : velocity amplitude,
 P : period)



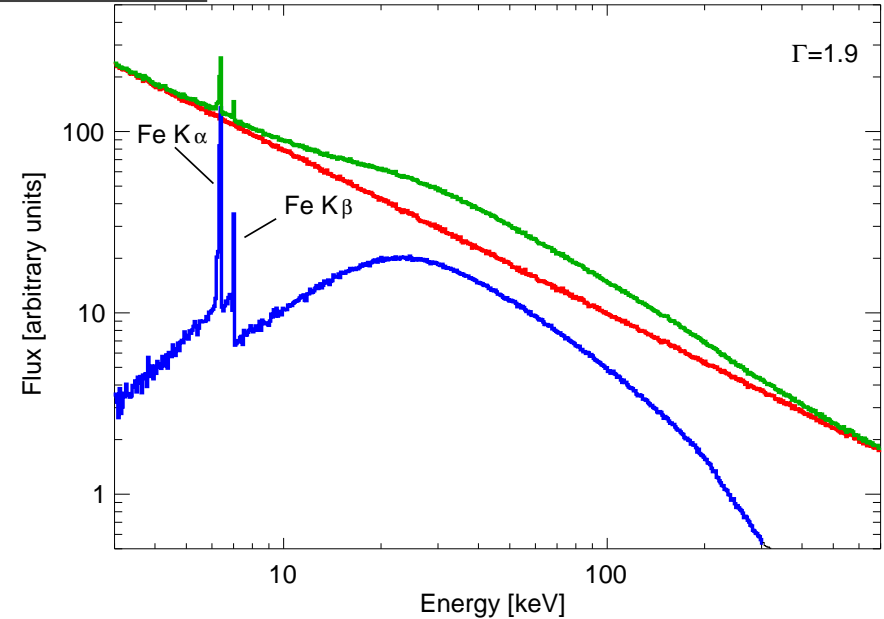
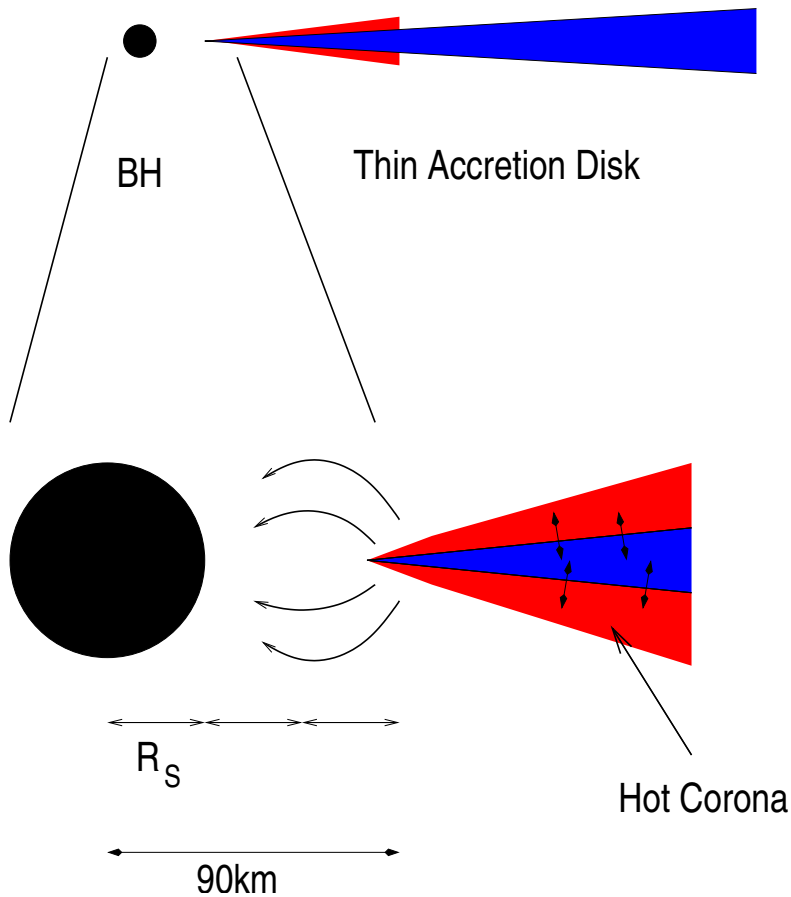
Mass determination, III



Orosz, 2003, priv. comm.



Relativistic Lines

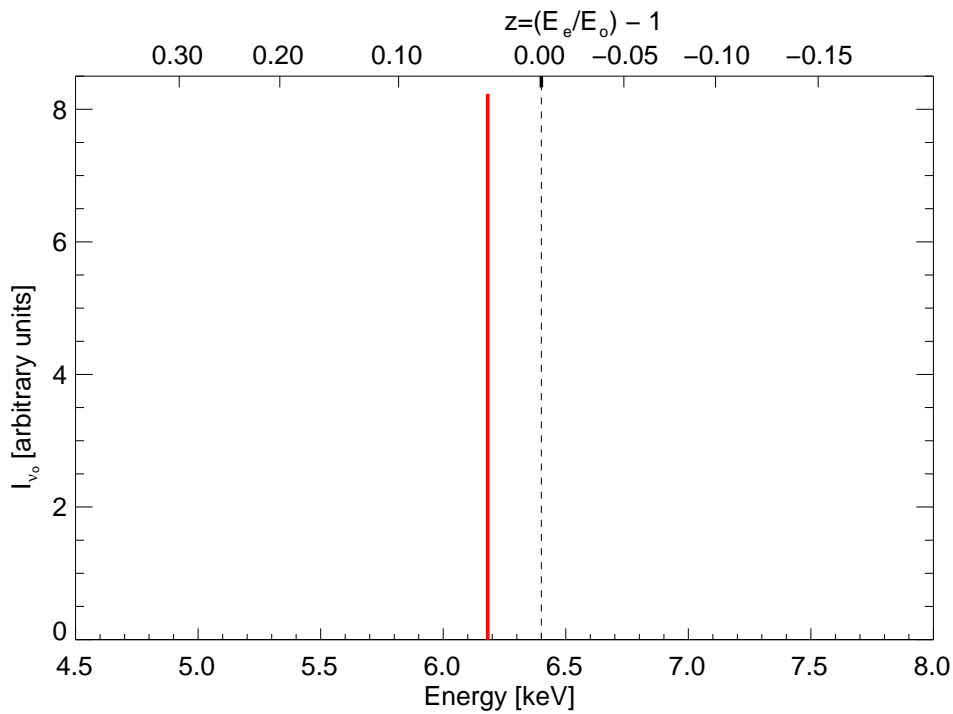
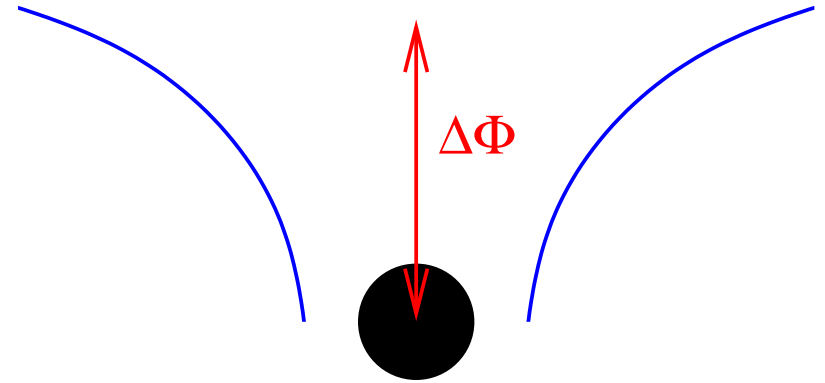


Black Hole X-Ray Spectrum:

- **Comptonization** of soft X-rays from accretion disk in **hot corona** ($T \sim 10^8$ K): **power law continuum**.
- **Thomson scattering** of power law photons in disk: **Compton Reflection Hump**
- **Photoabsorption** of power law photons in disk: **fluorescent Fe $K\alpha$ Line** at ~ 6.4 keV



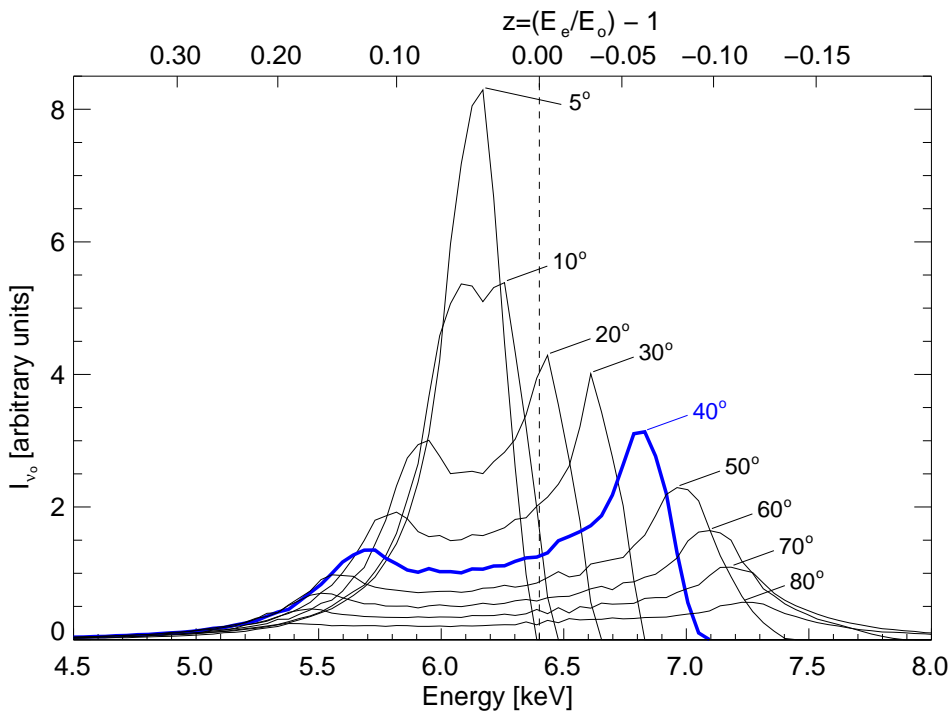
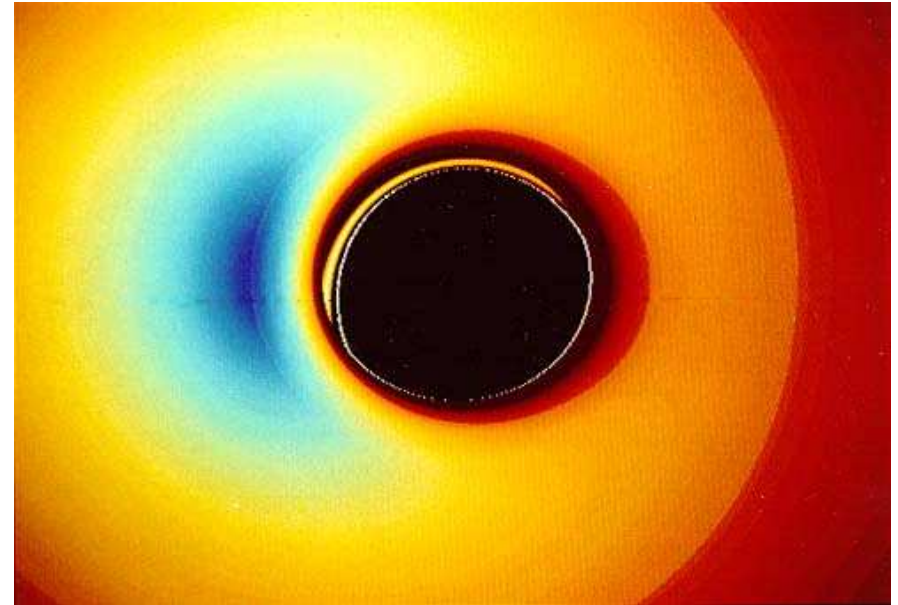
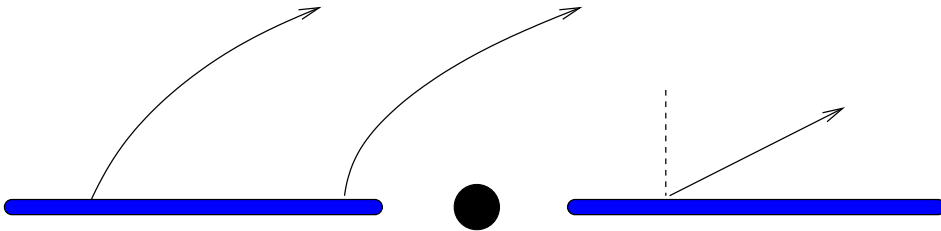
Relativistic Lines



Total observed line profile affected by

- grav. Redshift

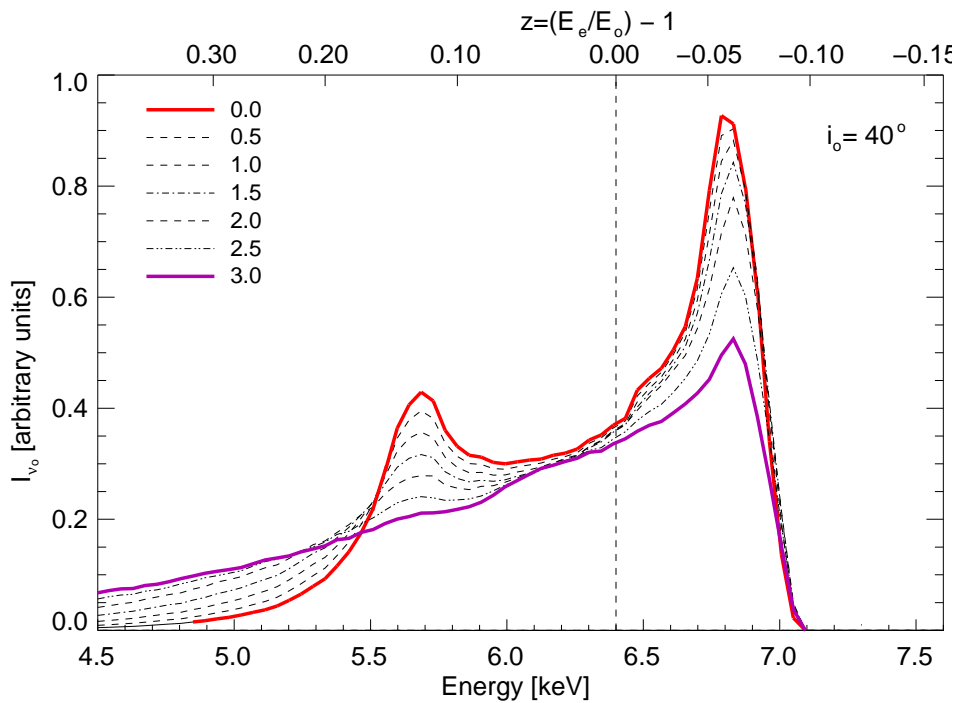
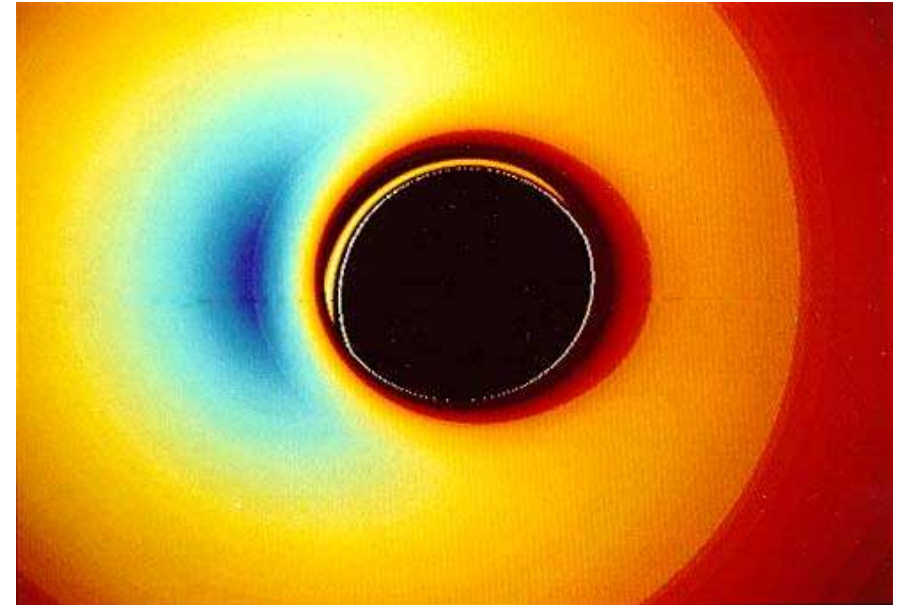
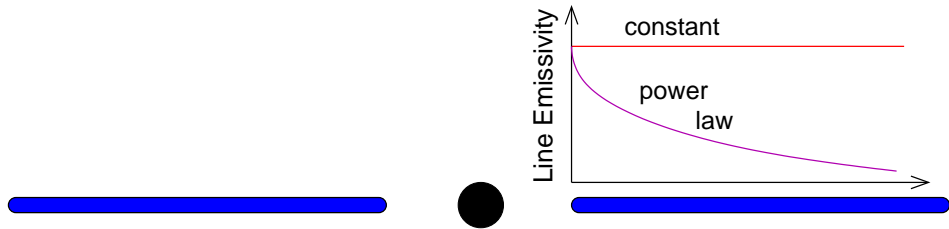
Relativistic Lines



Total observed line profile affected by

- grav. Redshift
- Light bending
- rel. Doppler shift

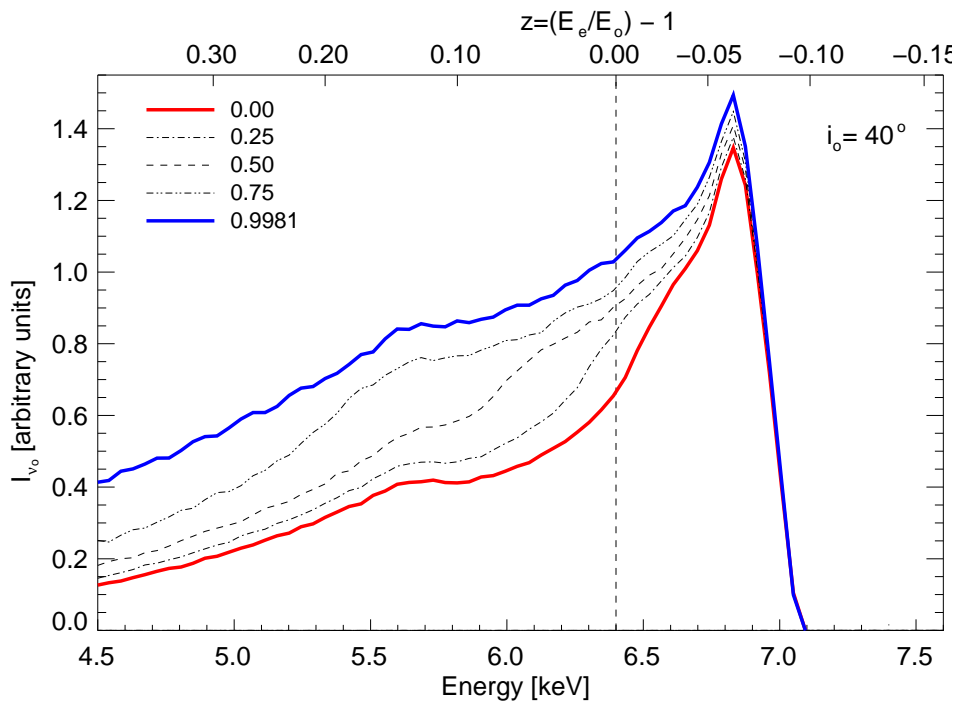
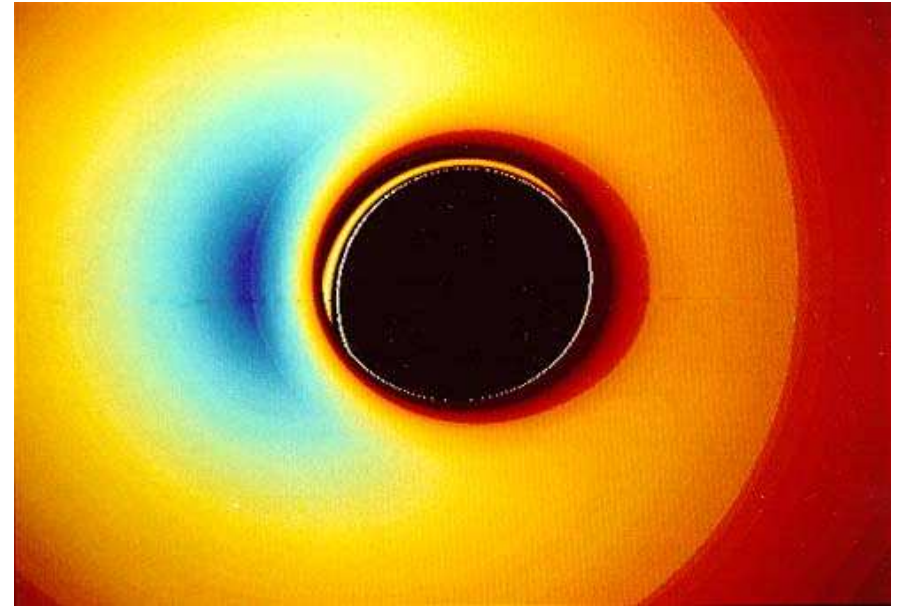
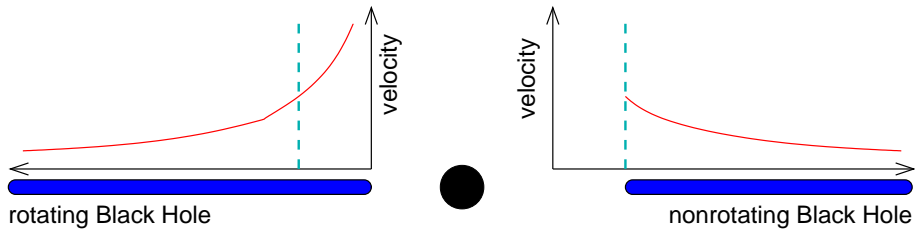
Relativistic Lines



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Relativistic Lines



Total observed line profile affected by

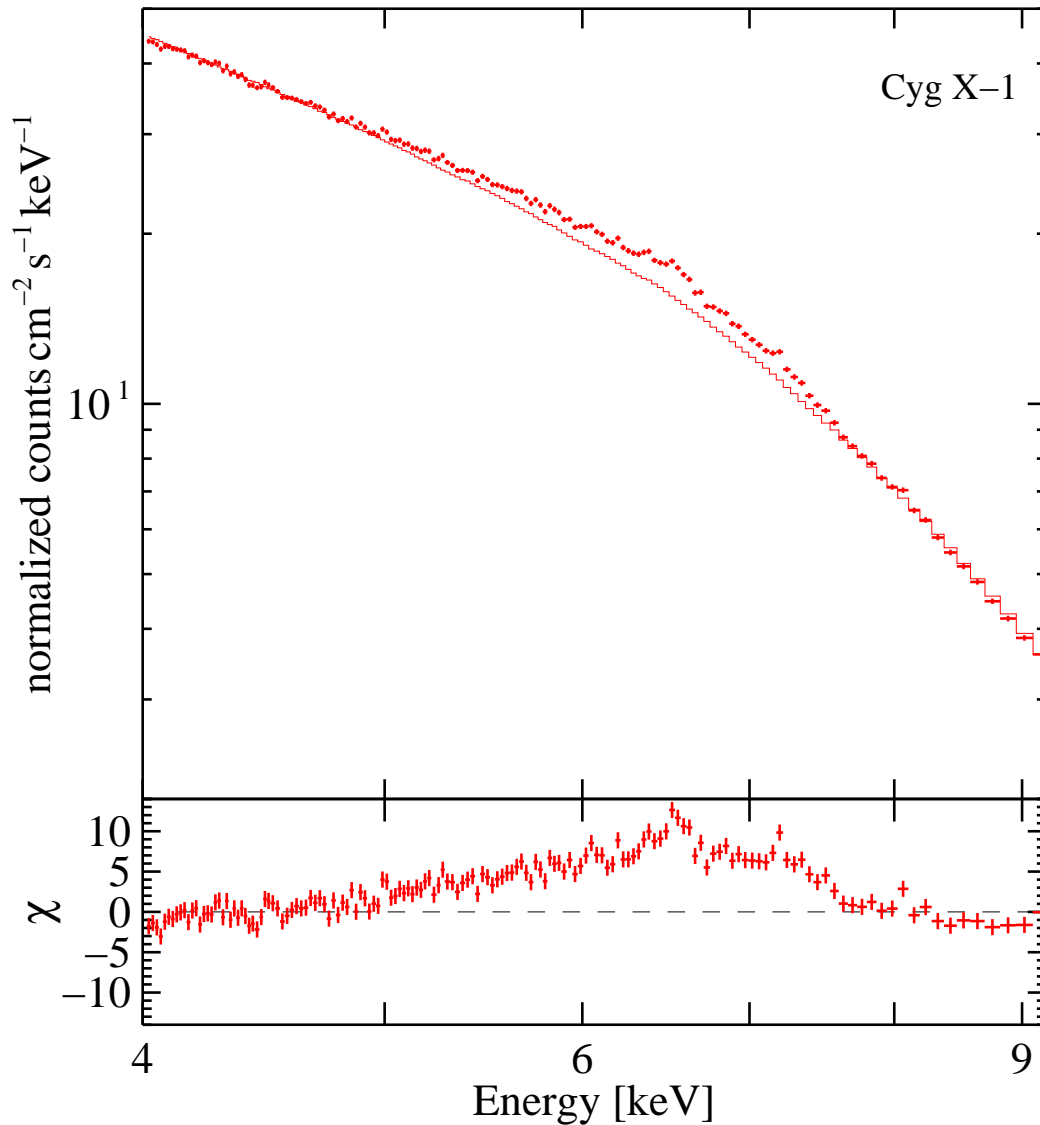
- grav. Redshift
- Light bending
- rel. Doppler shift
- emissivity profile
- spin of black hole



XMM-Newton



Relativistic Lines

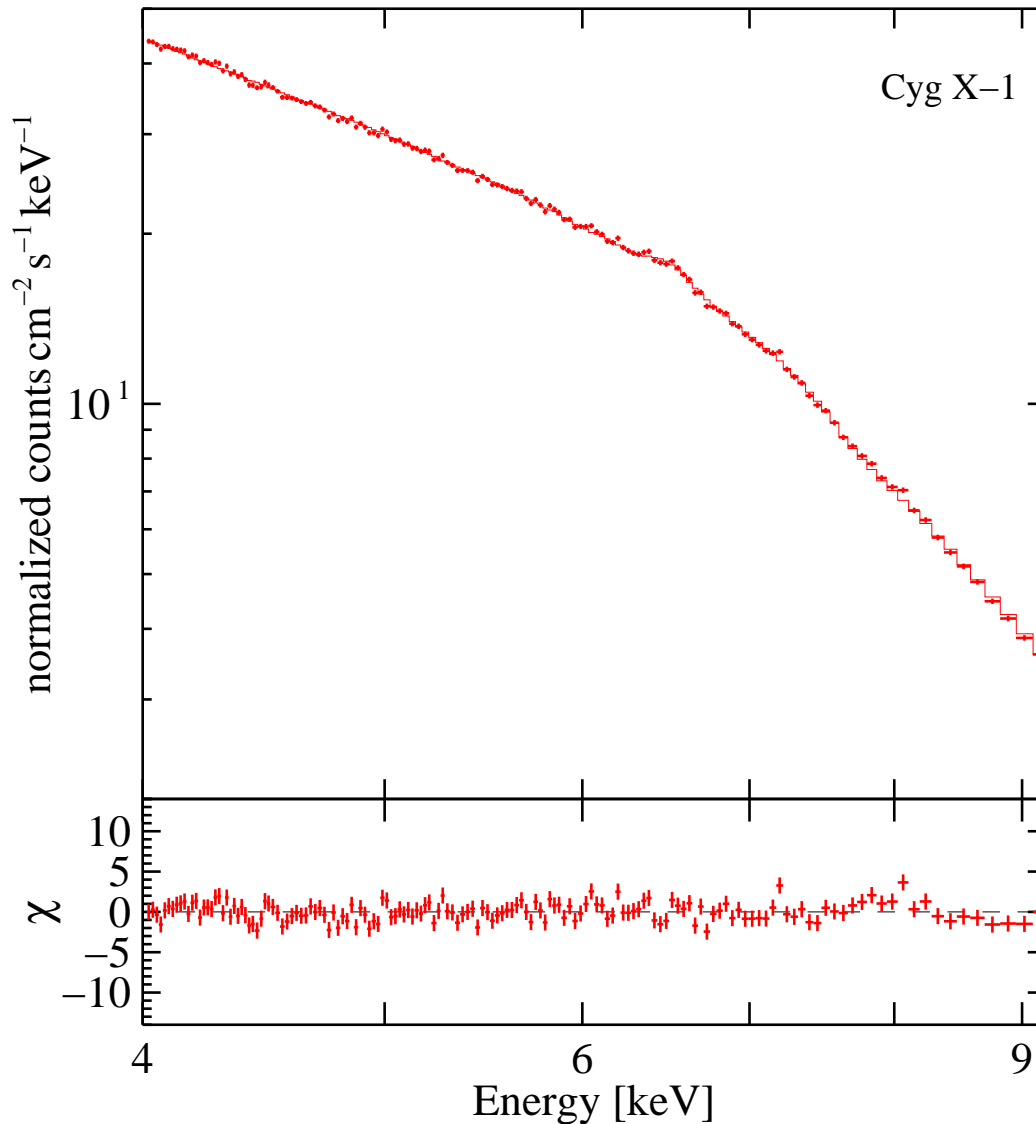


XMM-Newton Observation of
Cyg X-1: Power-law fit to
 $E \leq 5 \text{ keV}$ and $E \geq 8 \text{ keV}$:
strong residuals in Fe $K\alpha$ region

Wilms et al. (2006)



Relativistic Lines



Wilms et al. (2006)

4–9 keV spectrum: well explained ($\chi_{\text{red}}^2 = 1.3$) with:

- Power law

$$\Gamma = 1.90 \pm 0.01$$

- narrow line

$$E = 6.52 \pm 0.02 \text{ keV},$$

$$\sigma = 80 \pm 35 \text{ eV},$$

$$\text{EW} = 14 \text{ eV}$$

- relativistic line (Kerr)

$$E = 6.76 \pm 0.1 \text{ keV},$$

emissivity $\propto r^{-4.3 \pm 0.1}$ (strongly centrally peaked!).

Parameters similar (but not equal) to *Chandra* intermediate state observations (Miller et al., 2002)



Summary

Stars end their lives as one of three kinds of **compact objects**:

White Dwarf: $R \sim R_{\text{Earth}}$, $\rho \sim 10^{5\dots6} \text{ g cm}^{-3}$

$M < 1.44 M_{\odot}$ (**Chandrasekhar Limit**)

Equilibrium between gravitation and pressure of degenerate electrons



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Black Hole: Above OV-Limit no stable configuration known

\implies star collapses

\implies **Black Hole**

$M \gtrsim 4 M_{\odot}$

Event horizon at $R_S = 2GM/c^2 = 3(M/M_{\odot}) \text{ km}$ (**Schwarzschild radius**)

Filippenko, A. V., 1997, *Ann. Rev. Astron. Astrophys.*, 35, 309

Jha, S., et al., 1999, *ApJS*, 125, 73

Lattimer, J. M., & Prakash, M., 2001, *ApJ*, 550, 426

Shapiro, S. L., & Teukolsky, S. A., 1983, *Black Holes, White Dwarfs, and Neutron Stars*, (New York: Wiley)