



# *The Astronomical Distance Ladder*



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2. **Main Sequence Fitting**
3. Variable stars: **RR Lyrae** and **Cepheids**
4. **Type Ia Supernovae**
5. **Tully-Fisher for spiral galaxies**
6.  **$D_n$ - $\sigma$  for ellipticals**
7. **Brightest Cluster Galaxies**



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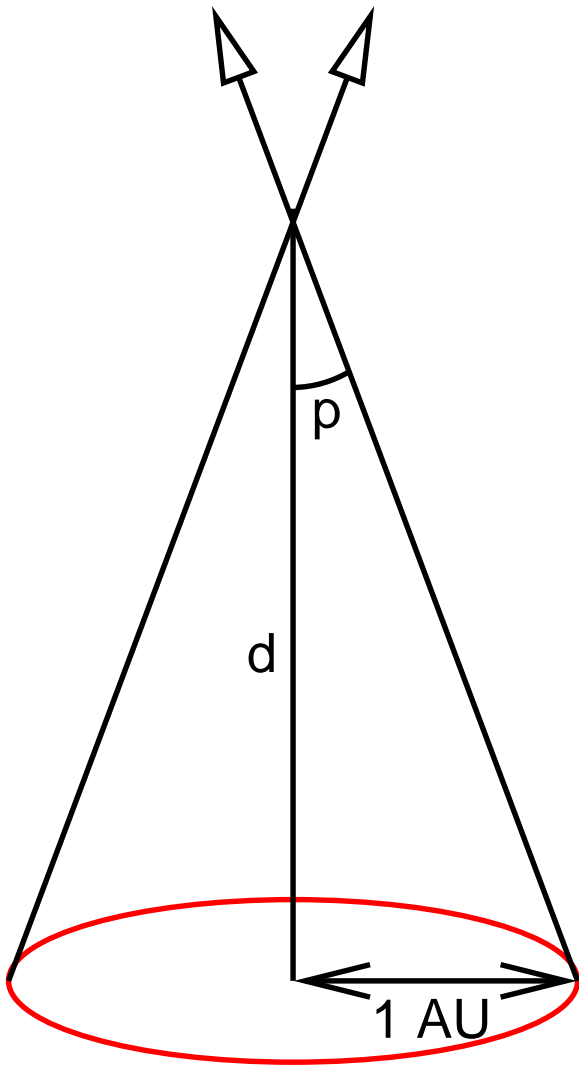
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Methods are calibrated using distances from the previous step of the **distance ladder**.



## Trigonometric Parallax



Motion of Earth around Sun  $\implies$  **Parallax**

Produces apparent motion of star; projected on sky see angular motion, opening angle

$$\tan p \sim p = \frac{r_{\text{Earth}}}{d} = \frac{1 \text{ AU}}{d}$$

$p$  is called the **trigonometric parallax**.

Note: requires several at several positions of the Earth

Measurement difficult:  $\pi \lesssim 0.76''$  ( $\alpha$  Cen).

Define unit for distance:

**Parsec**: Distance where 1 AU has  $p = 1''$ .

$$1 \text{ pc} = 206265 \text{ AU} = 3.086 \times 10^{16} \text{ m} = 3.26 \text{ ly}$$



## Trigonometric Parallax

Best measurements to date: **Hipparcos satellite** (1989–1993)

- systematic error of position:  $\sim 0.1$  mas
- effective **distance limit: 1 kpc**
- standard error of proper motion:  $\sim 1$  mas/yr
- photometry
- **magnitude limit: 12**
- complete to mag: 7.3–9.0

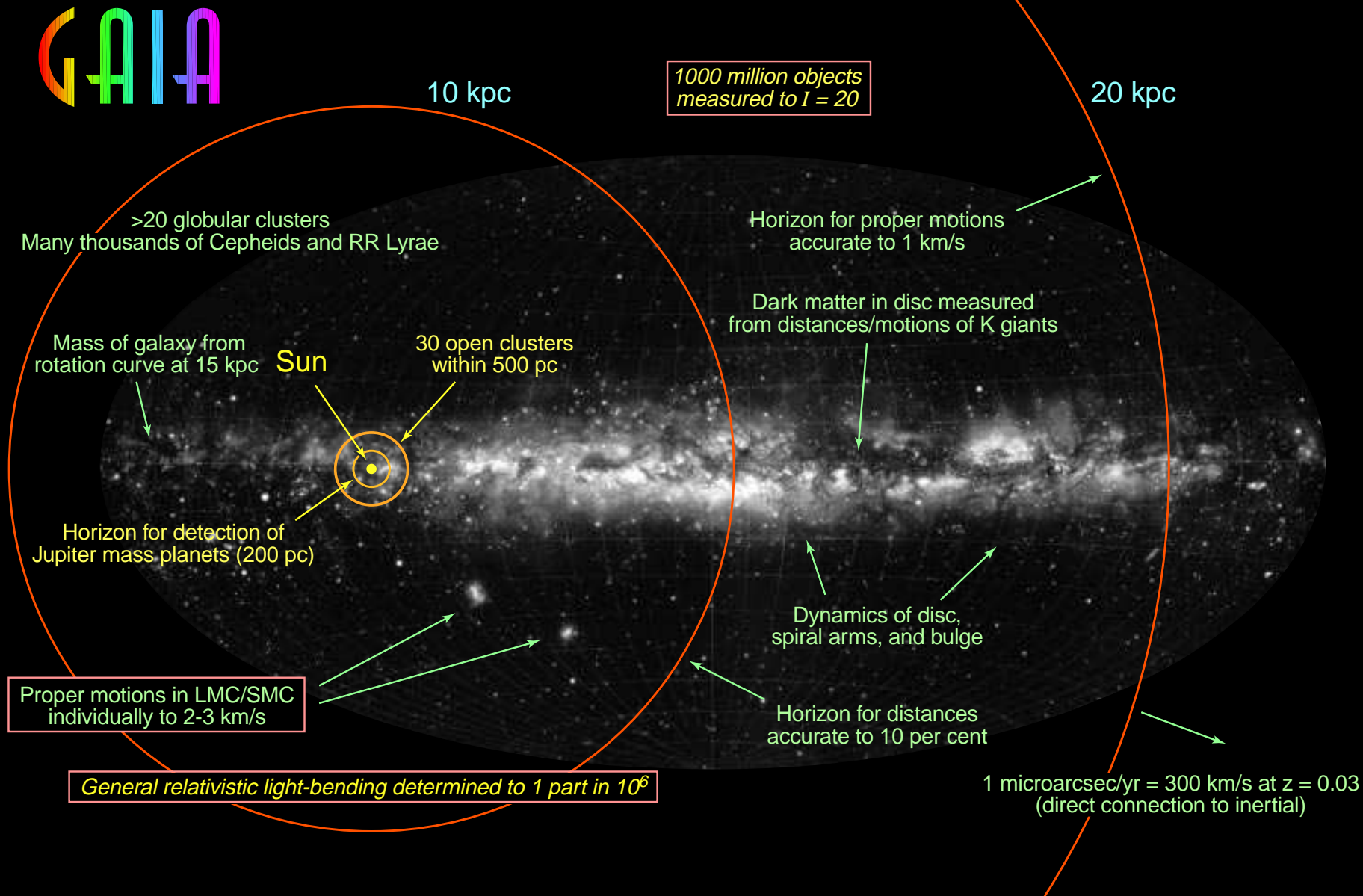
Results available at <http://astro.estec.esa.nl/Hipparcos/>:

**Hipparcos catalogue**: 120000 objects with milliarcsecond precision.

**Tycho catalogue**:  $10^6$  stars with 20–30 mas precision, two-band photometry



# Plans for the future: GAIA (ESA mission, launch 2010, observations 2011–2016):



GAIA:  $\sim 4 \mu\text{arcsec}$  precision, 4 color to  $V = 20$  mag,  $10^9$  objects.



## Standard Candles, I

Assuming **isotropic emission**, the **flux** measured at distance  $d$  from object with **luminosity**  $L$  is given by the “**inverse square law**”,

$$f(d) = \frac{L}{4\pi d^2}$$

note that  $f$  is a function of the  $d$ .

Remember that the **magnitude** is defined through comparing two fluxes,

$$m_2 - m_1 = 2.5 \log_{10}(f_1/f_2) = -2.5 \log_{10}(f_2/f_1)$$

To allow the comparison of sources at different distances, define

absolute magnitude  $M$  = magnitude if star were at distance 10 pc

Because of this

$$M - m = -2.5 \log_{10}(f(10 \text{ pc})/f(d)) = -2.5 \log_{10}\left(\frac{L/(4\pi(10 \text{ pc})^2)}{L/(4\pi d^2)}\right) = -2.5 \log_{10}\left(\frac{d}{10 \text{ pc}}\right)^2$$

The difference  $m - M$  is called the **distance modulus**,

$$m - M = 5 \log_{10}\left(\frac{d}{10 \text{ pc}}\right)$$



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Requirements:

- **physics of standard candle well understood** (i.e., need to know *why* object has certain luminosity).
- **absolute magnitude of standard candle needs to be calibrated**, e.g., by measuring its distance by other means (this is a *big problem*)



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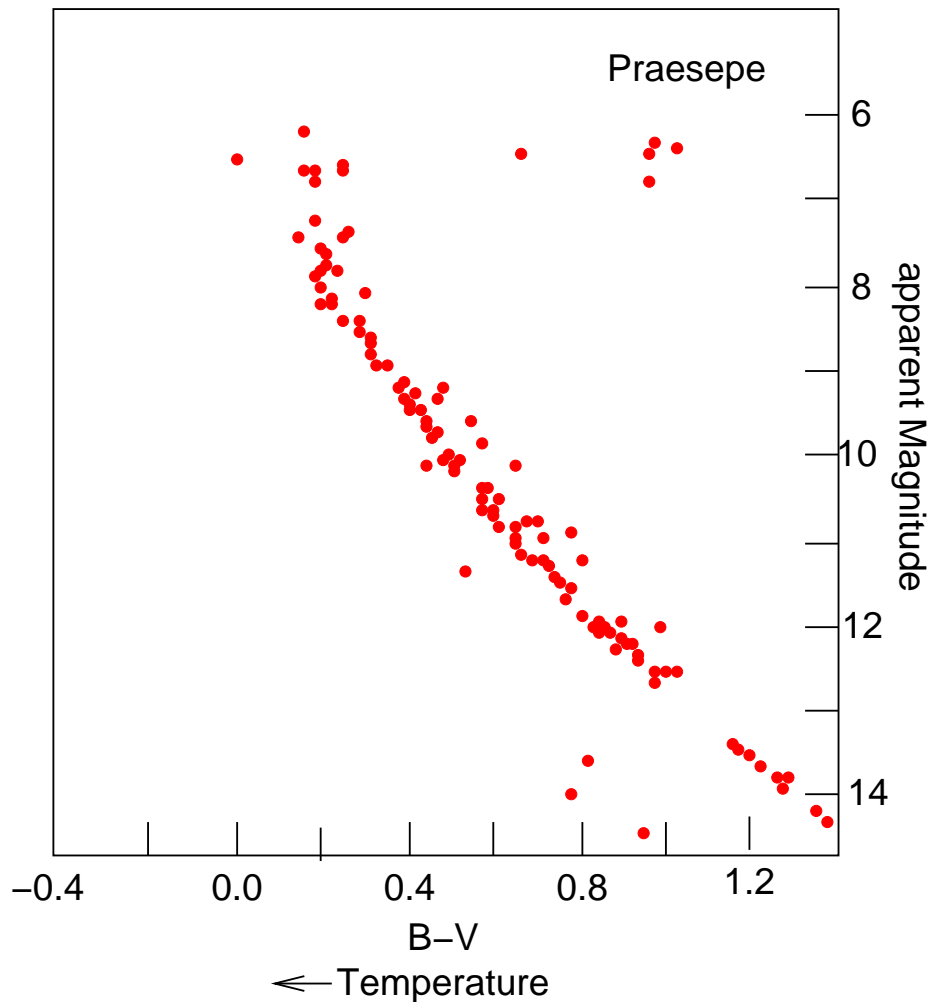
To determine distance to astronomical object:

1. **find standard candle(s)** in object,
2. **measure** their  $m$
3. **determine**  $m - M$  from known  $M$  of standard candle
4. **compute** distance  $d$

Often, distances are given in terms of  $m - M$ , and not in pc, so last step is not always performed.



## Main Sequence Fitting



MS fitting applied to Praesepe  
(after Vandenberg & Bridges 1984)

*Clusters:* if **Main Sequence** in Hertzsprung Russell Diagram determinable:

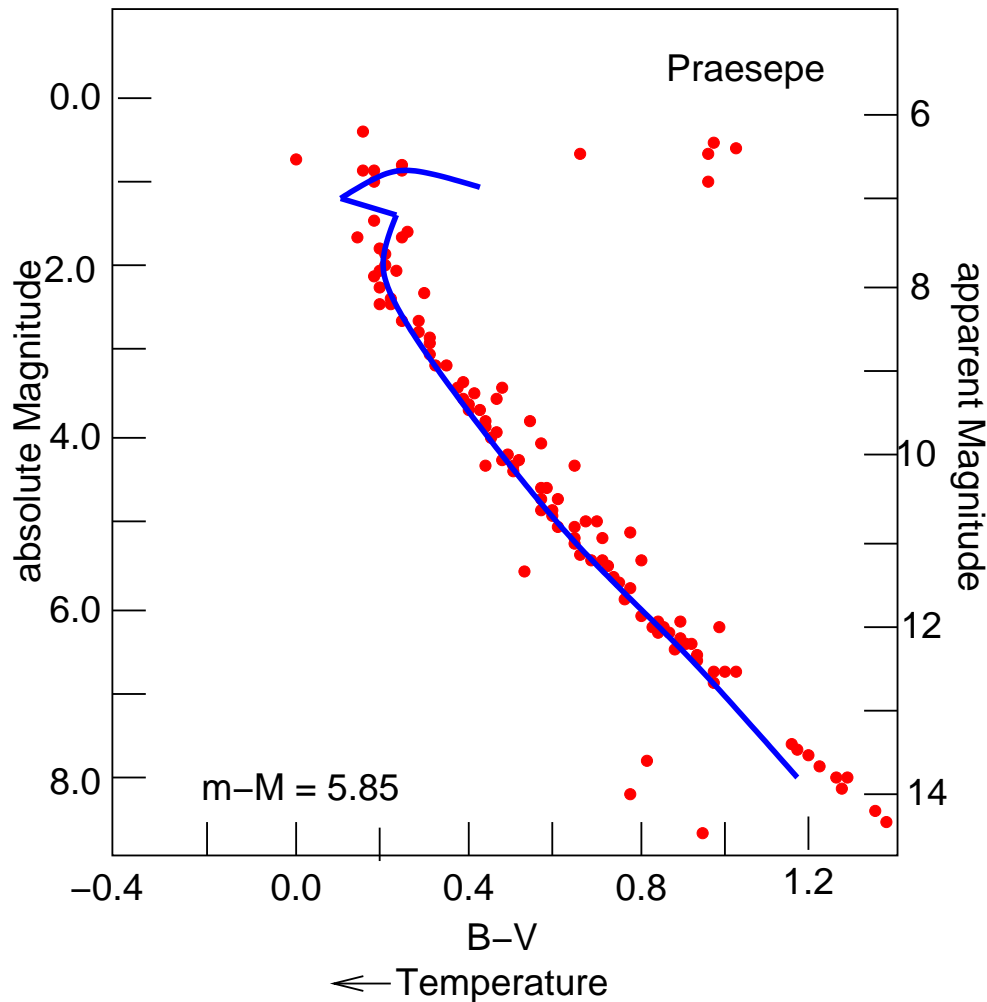
Shift observed HRD until main sequence agrees with location of MS measured for stars in solar vicinity  $\implies$  distance modulus.

Currently: distances to  $\sim 200$  open clusters known

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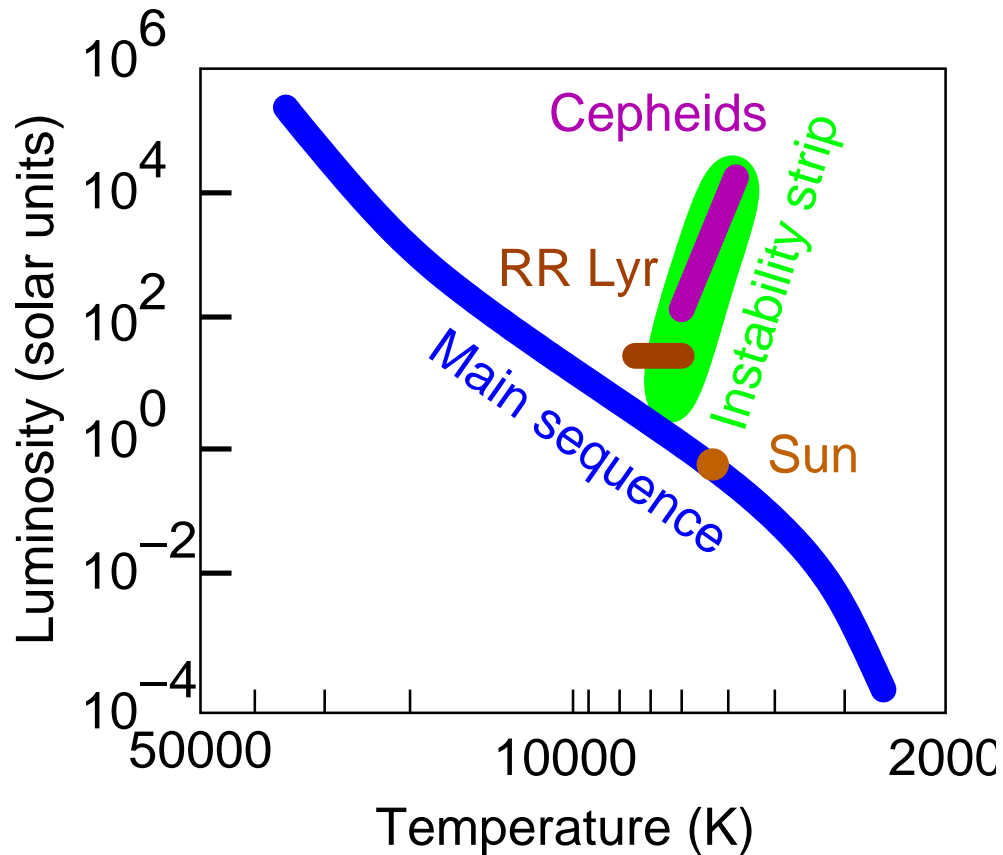
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## Introduction



Instability strip in the Hertzsprung-Russell Diagram

Certain regions of HRD: stars prone to **instability**:

Ionisation of Helium: transparency of outer parts of star changes

⇒ size of star changes

⇒ surface temperature and luminosity variations

Most important variables of this kind:

### 1. RR Lyr variables

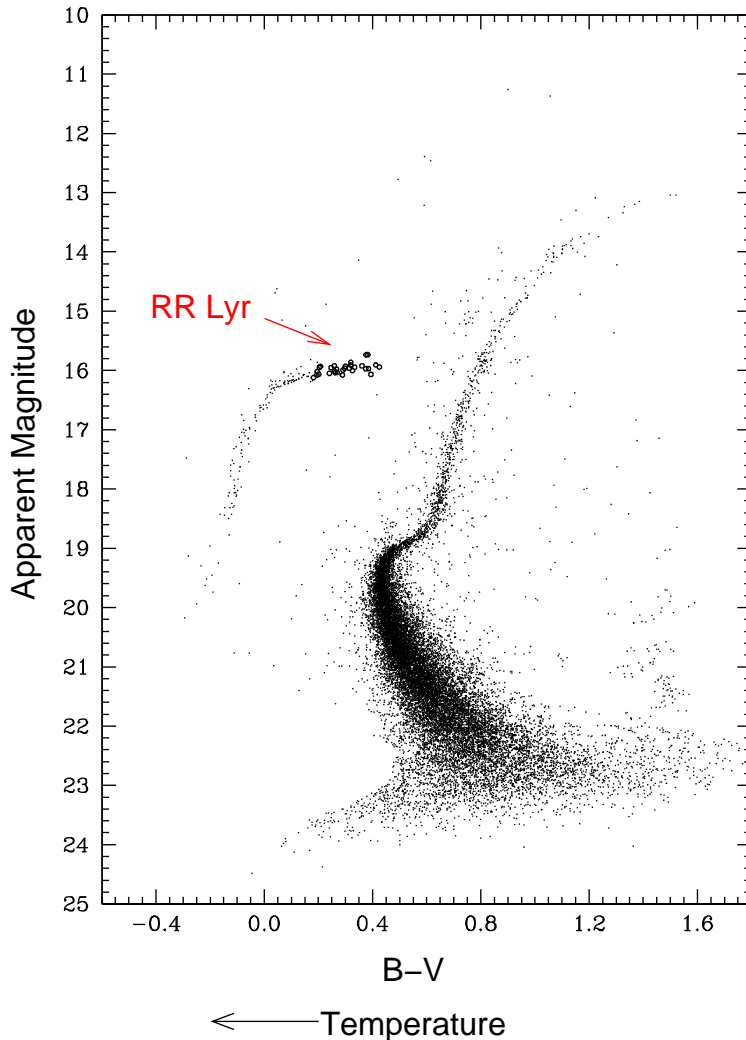
mainly in globular clusters: lower metallicity of clusters ("population II") allows stars to enter instability strip

### 2. $\delta$ Cepheids





## RR Lyrae, I



HRD of Globular Cluster M2  
(after Lee et al., 1999, Fig. 2)

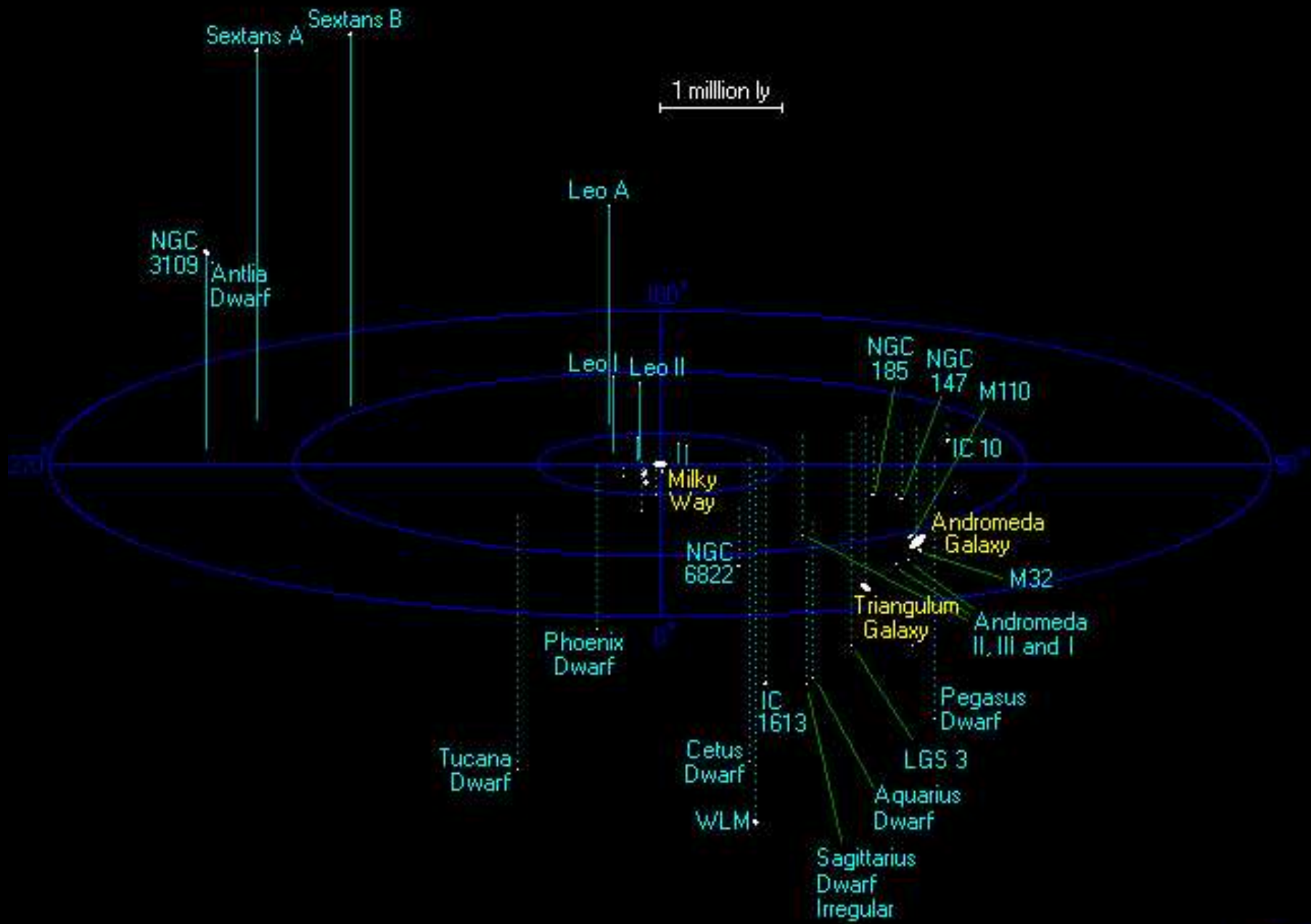
### RR Lyrae variables:

- Variability ( $P \sim 0.2 \dots 1$  d)
- Mainly temperature change
- RR Lyr gap clearly observable in globular cluster HRD

Absolute magnitude of RR Lyr gap:  
 $M_V = 0.6$  mag,  $M_B = 0.8$  mag, i.e.,  
 $L_{RR} \sim 50 L_{\odot}$ .

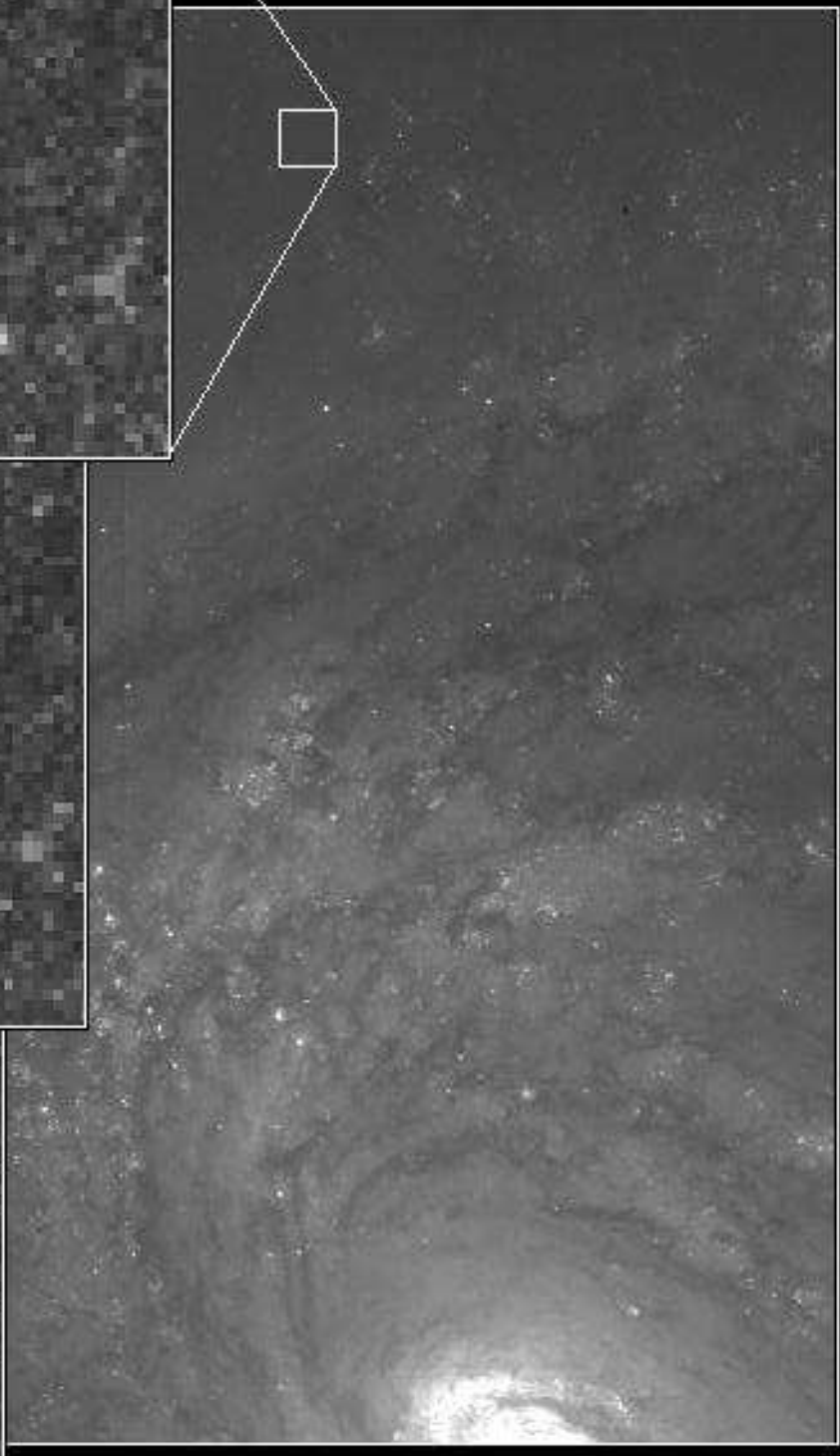
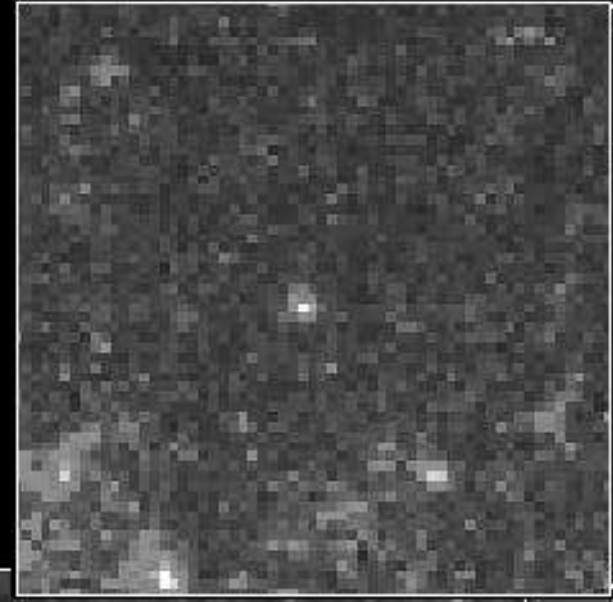
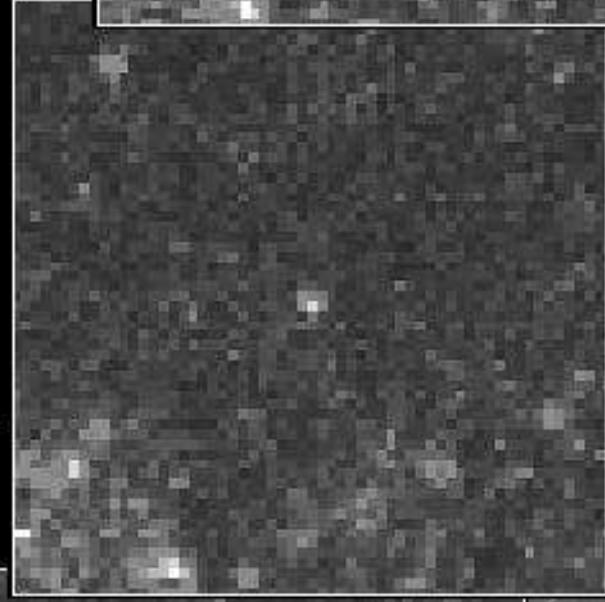
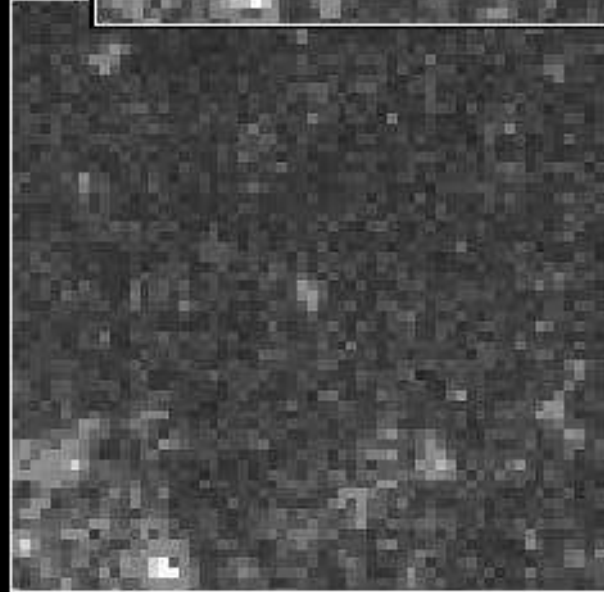
Works out to LMC ( $d \sim 50$  kpc) and other dwarf galaxies of local group, mainly used for globular clusters and local group.

Example: M5: gap at  $m = 16$  mag  $\implies m - M = 15.4$  mag  
 $\implies d = 12$  kpc.



source: <http://www.anzwers.org/free/universe/localgrp.html>  
 The neighbourhood of the Milky Way: the Local Group

# Cepheid Variable in M100 HST-WFPC2

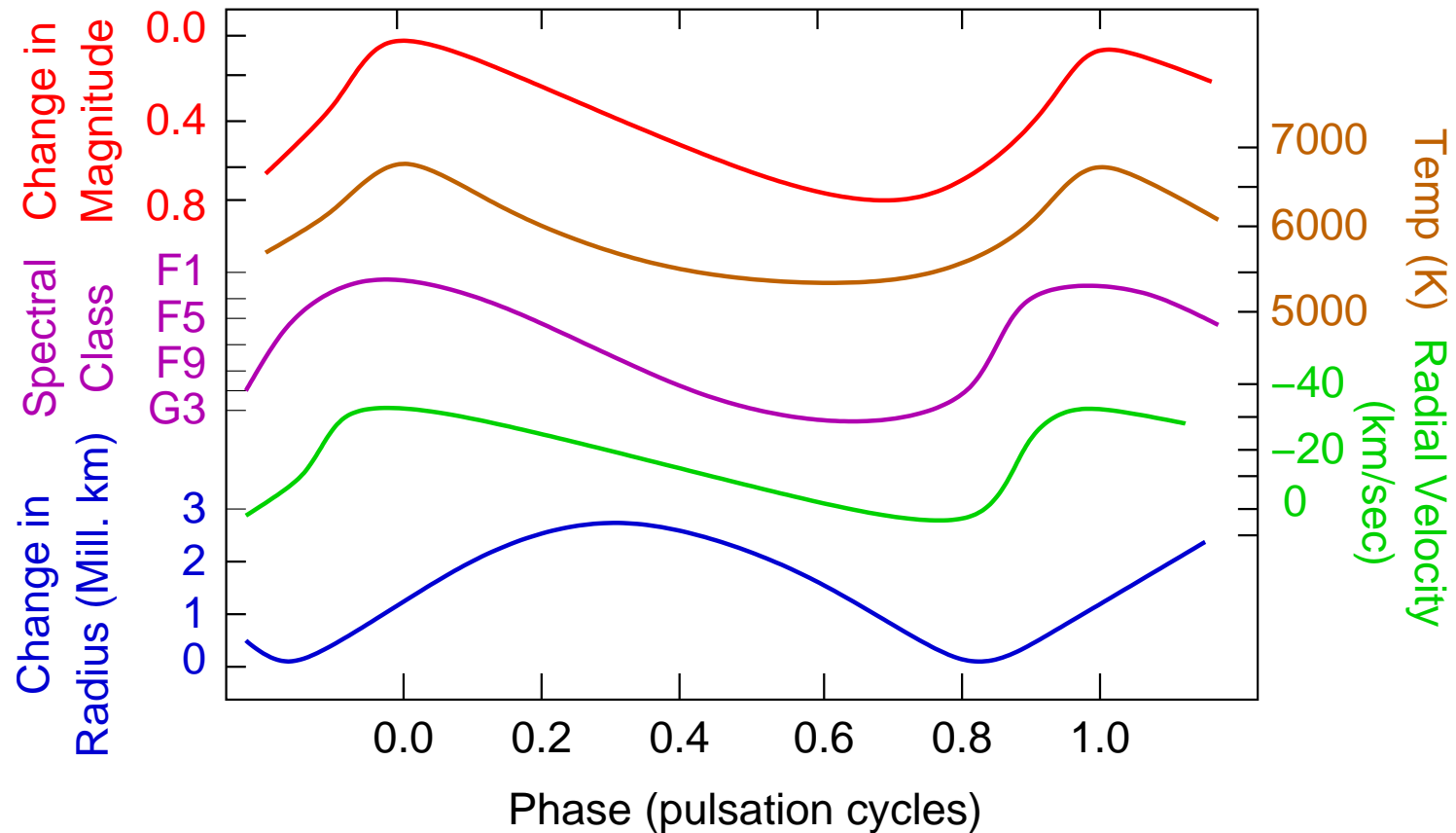




STScI



# Cepheids



after <http://csep10.phys.utk.edu/astr162/lect/index.html>

**Cepheids:** Luminous stars ( $L \sim 1000 L_{\odot}$ ) in **instability strip** with **large luminosity amplitude variation**,  $P \sim 2 \dots 150$  d (easily measurable).



## Cepheids



© ASP

### Henrietta Leavitt (1868–1921):

- Graduated from Radcliffe College
  - from 1895: volunteer at Harvard Observatory
  - was ill, and partially deaf as a result
  - 1902: back at Harvard Obs
- discovered 1777 variable stars in LMC
  - 1912: discovered **Period-Luminosity relation** of Cepheids in SMC, but was not allowed to follow this up
  - later: defined Harvard photographic magnitude system

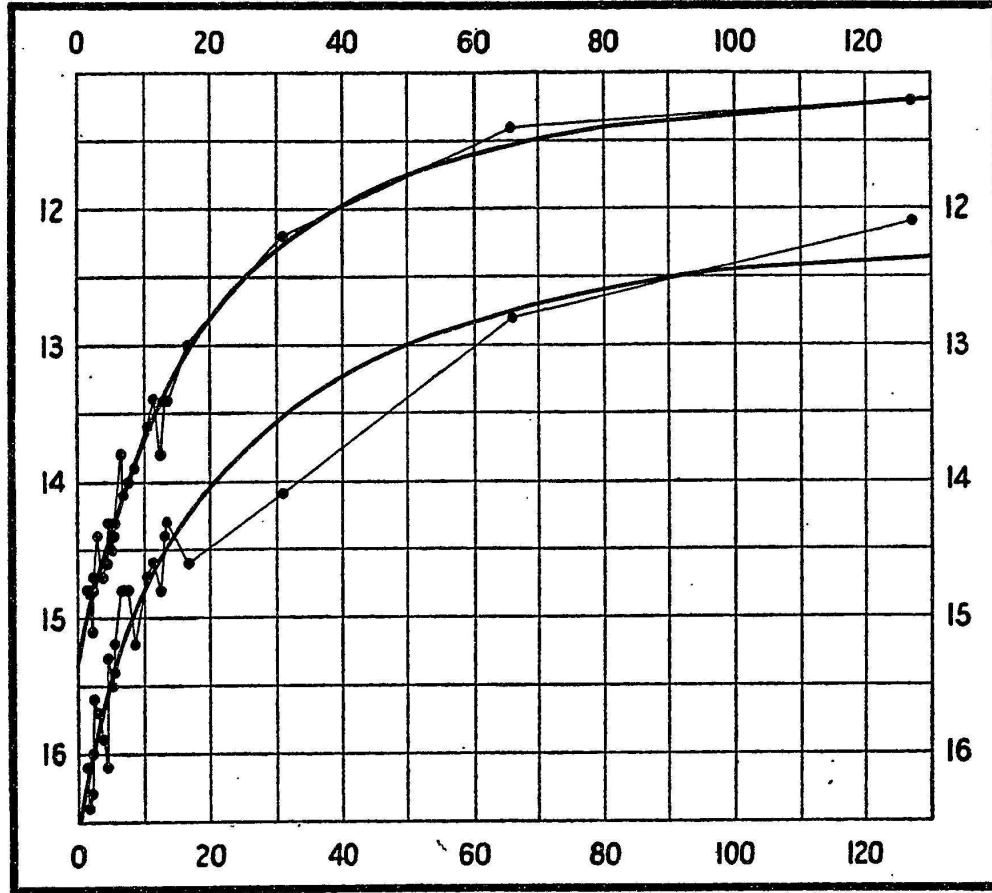


FIG. 1.

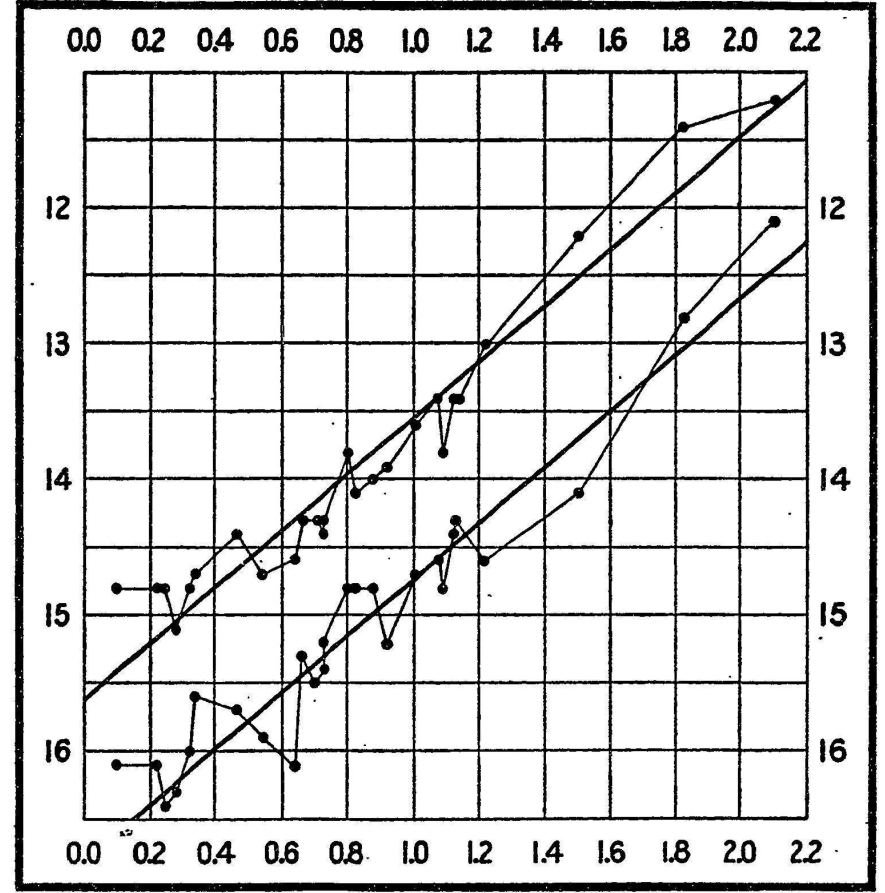


FIG. 2.

*X*-axis: period in days, *Y*-axis: magnitude

Leavitt & Pickering, 1912, Periods of 25 Variable Stars in the Small Magellanic Cloud,  
 Harvard College Observatory Circular, vol. 173, pp. 1–3





# Cepheids

Henrietta Leavitt(1912):

Cepheids have a **period luminosity relationship**:  $M \propto -\log P$

Low luminosity Cepheids have lower period

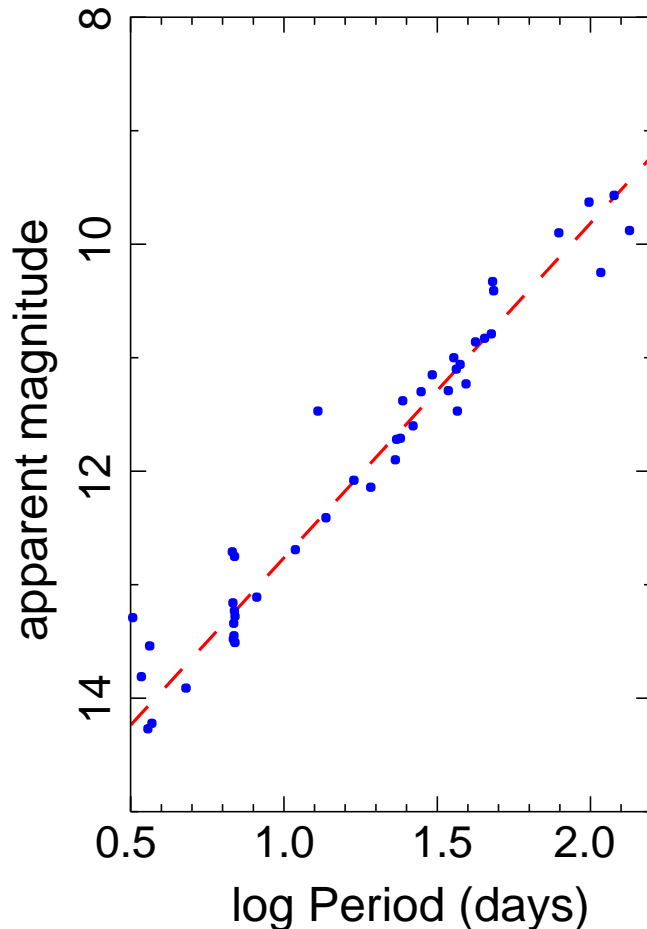
Observations find:

$$M = -2.76 \log P - 1.40$$

( $P$  in days)

Calibrated from observing Large Magellanic Cloud Cepheids (see figure), and determining LMC distance from other means (MS fitting, RR Lyr, ...) to find absolute magnitudes...

With HST: **works out to Virgo cluster**  
( $d = 18.5$  Mpc).



Period-Luminosity relation for the LMC Cepheids  
after Mould et al. (2000, Fig. 2)



The origin of the Period-Luminosity relationship is in the Helium ionisation instability discussed before. The details of this are rather messy, however, it is easy to see that a Period-Luminosity relationship as that observed for the Cepheids is a simple consequence of the fact that the pulsating star is not disrupted by its oscillation.

For the outer parts of the star to remain bound, the kinetic energy of the pulsating outer parts of the stars has to remain smaller than their binding energy:

$$\frac{1}{2}mv^2 \lesssim \frac{GMm}{R}$$

But we know that for the velocity

$$v < \frac{2R}{P}$$

where  $P$  is the period of the star and  $R$  its radius at maximum extension (we observe the star to expand to a radius  $R$  once every  $P$  seconds, so the maximum distance the expanding material can go during that time is  $2R$ ). Inserting  $v$  into the above equation gives

$$\frac{1}{2} \frac{4R^2}{P^2} \lesssim \frac{GM}{R} \iff P^2 \gtrsim \frac{2}{G} \frac{R^3}{M} = \frac{2}{G} \frac{1}{M/R^3}$$

If we assume that the pulsation is close to the break-up speed, and noting that  $M/R^3$  is proportional to the average density of the star, then it is easy to see that

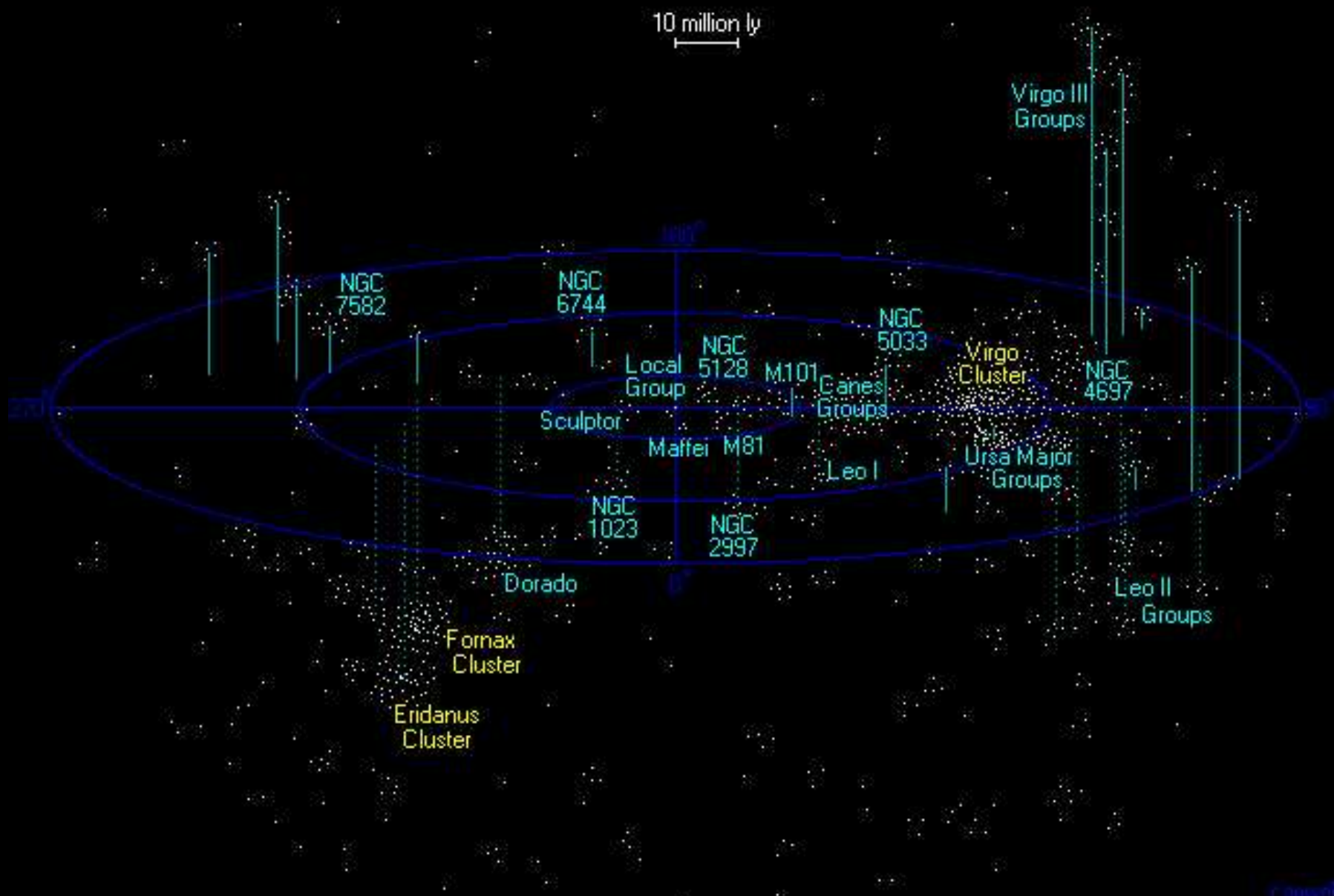
$$P \propto (G\rho)^{-1/2}$$

In the homework for this week you are asked to convince yourself that  $(G\rho)^{-1/2}$  has the dimension of a period, i.e., for all gas balls oscillating close to the break up speed, we expect that  $P \propto \rho^{-1/2}$ . To obtain the period luminosity relationship, you need to remember that the emissivity per square-metre of the surface of a star with temperature  $T$  is  $\sigma T^4$  (per the Stefan-Boltzmann law), while the surface of the star is proportional to  $R^2$ . Therefore, the luminosity of the star is  $L \propto R^2 T^4$ .

This week's homework asks you to use  $L \propto R^2 T^4$  and  $P \propto \rho^{-1/2}$  to show that from these the absolute magnitude of a pulsating star is related to the period through

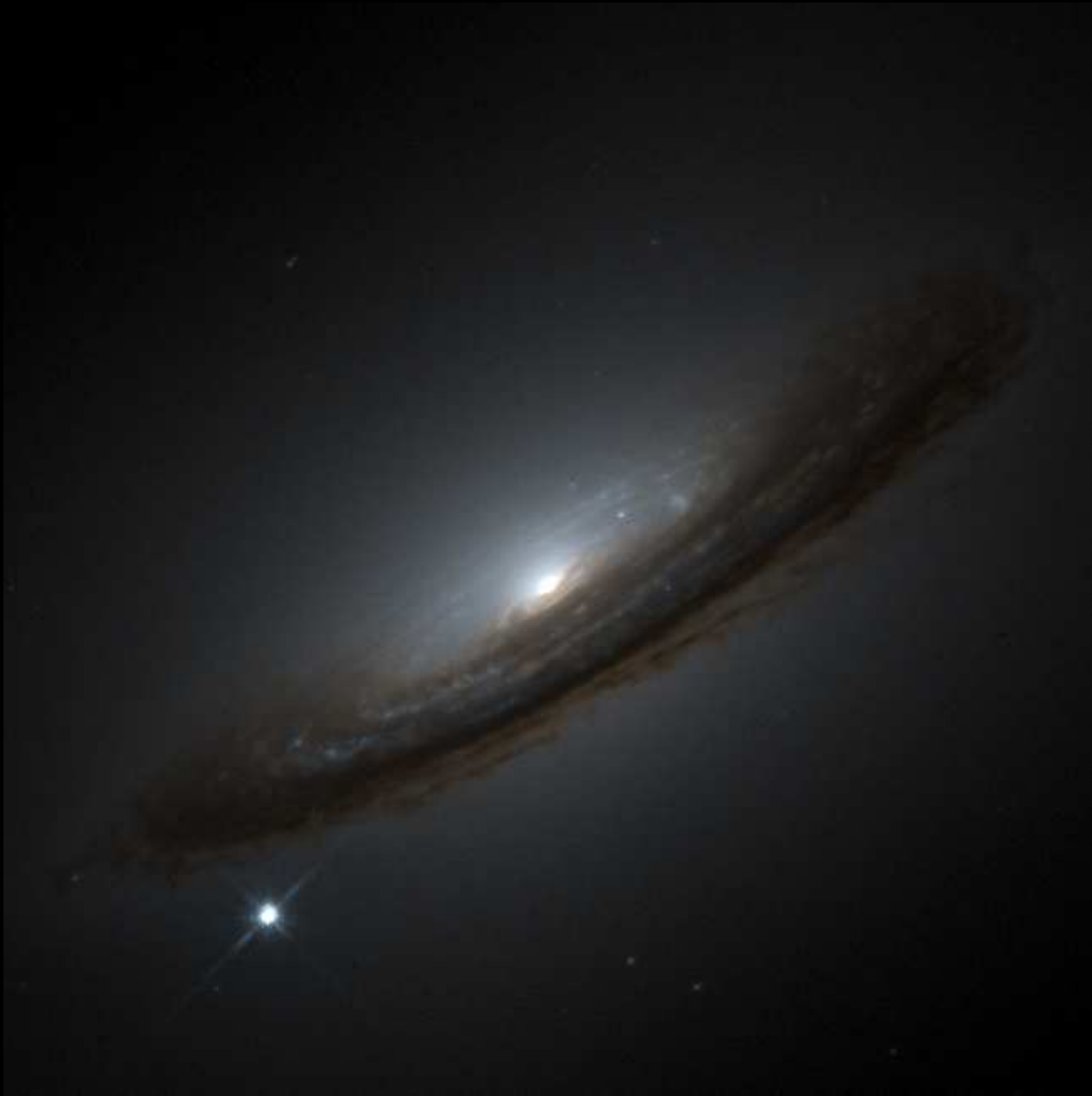
$$\log P \propto -m$$

as observed for Cepheids.



## The universe out to the Virgo Cluster

source: <http://www.anzwers.org/free/universe/virgo.html>

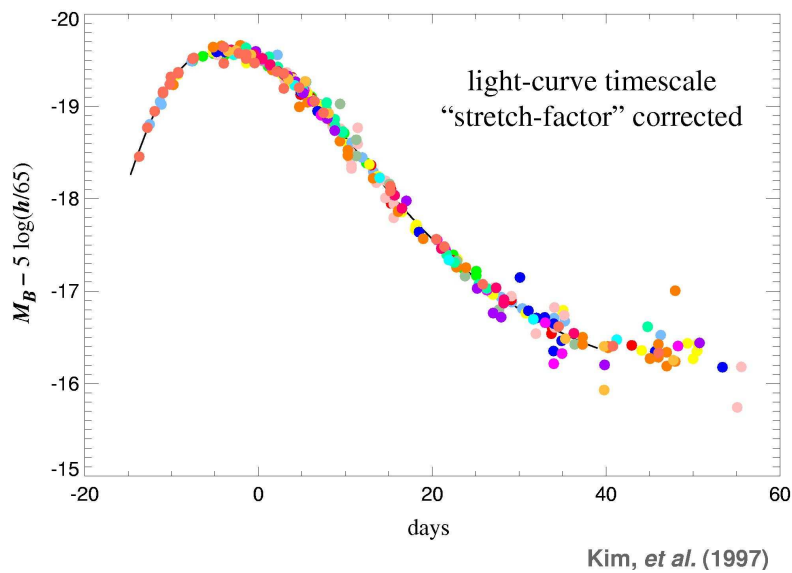
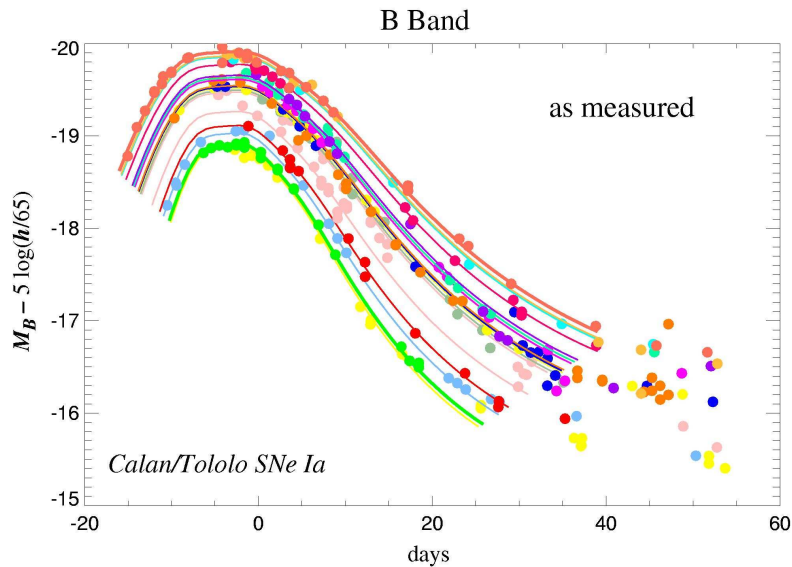


SN1994d (HST WFPC)

Supernovae have luminosities comparable to whole galaxies:  $\sim 10^{51}$  erg/s in light, 100 $\times$  more in neutrinos.



# Supernovae



After correction of systematic effects and time dilatation (expansion of the universe, see later):

SN Ia lightcurves all look the same

⇒ standard candle



## Supernovae

SN Ia = Explosion of CO white dwarf when pushed over Chandrasekhar limit ( $1.4 M_{\odot}$ ) (via accretion?).

⇒ Always similar process

⇒ Very **characteristic light curve**: **fast rise**, **rapid fall**, **exponential decay** (“FRED”) with half-time of 60 d.

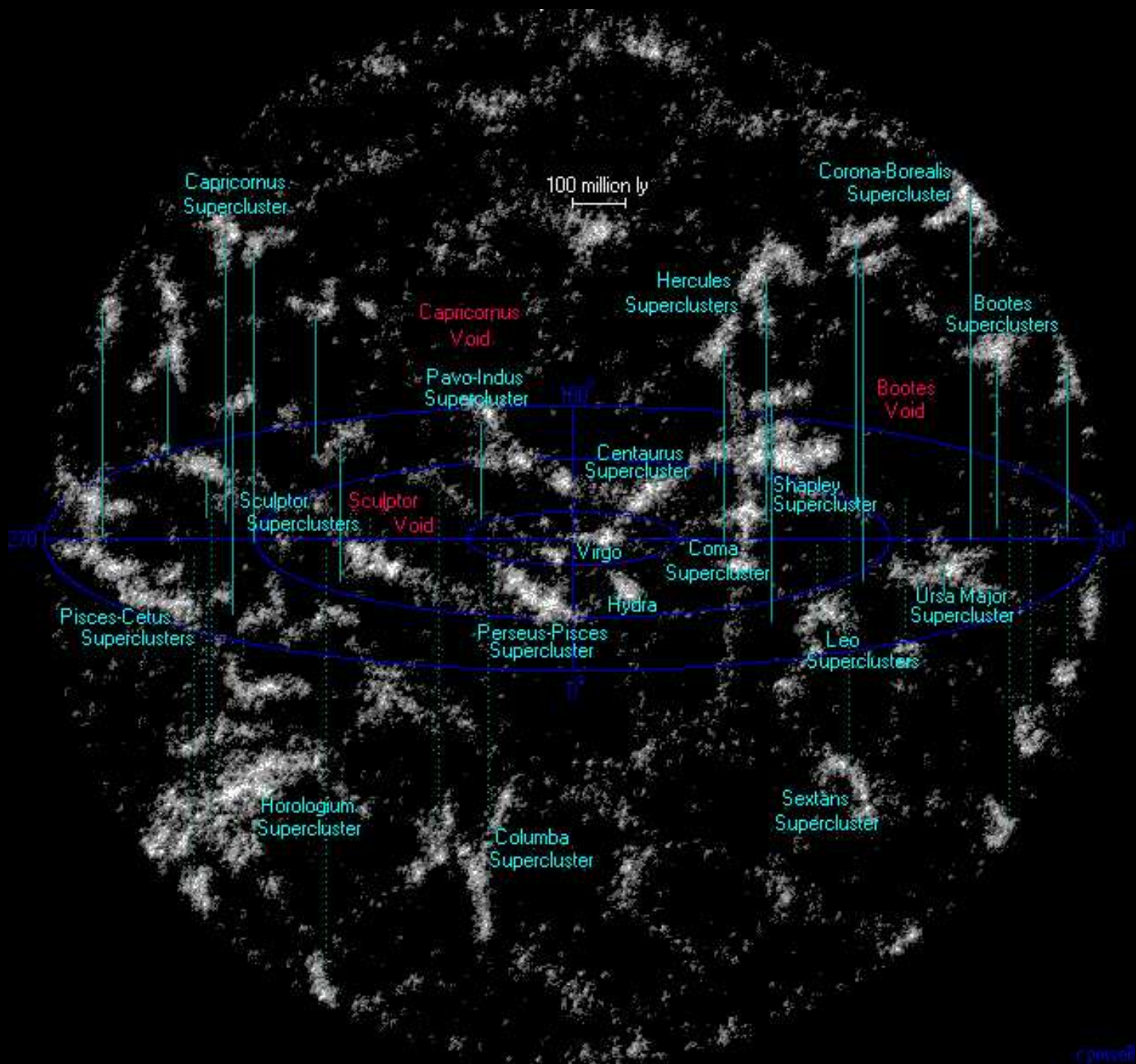
60 d time scale from radioactive decay  $\text{Ni}^{56} \rightarrow \text{Co}^{56} \rightarrow \text{Fe}^{56}$   
 (“self calibration” of lightcurve if same amount of  $\text{Ni}^{56}$  produced everywhere)

**Calibration**: SNe Ia in nearby galaxies where Cepheid distances known.

At **maximum light**:

$$M_B = -18.33 \pm 0.11 \quad \Longleftrightarrow \quad L \sim 10^{9 \dots 10} L_{\odot}$$

Observable **out to  $\gtrsim 1$  Gpc** ⇒ covers almost **the whole universe**...



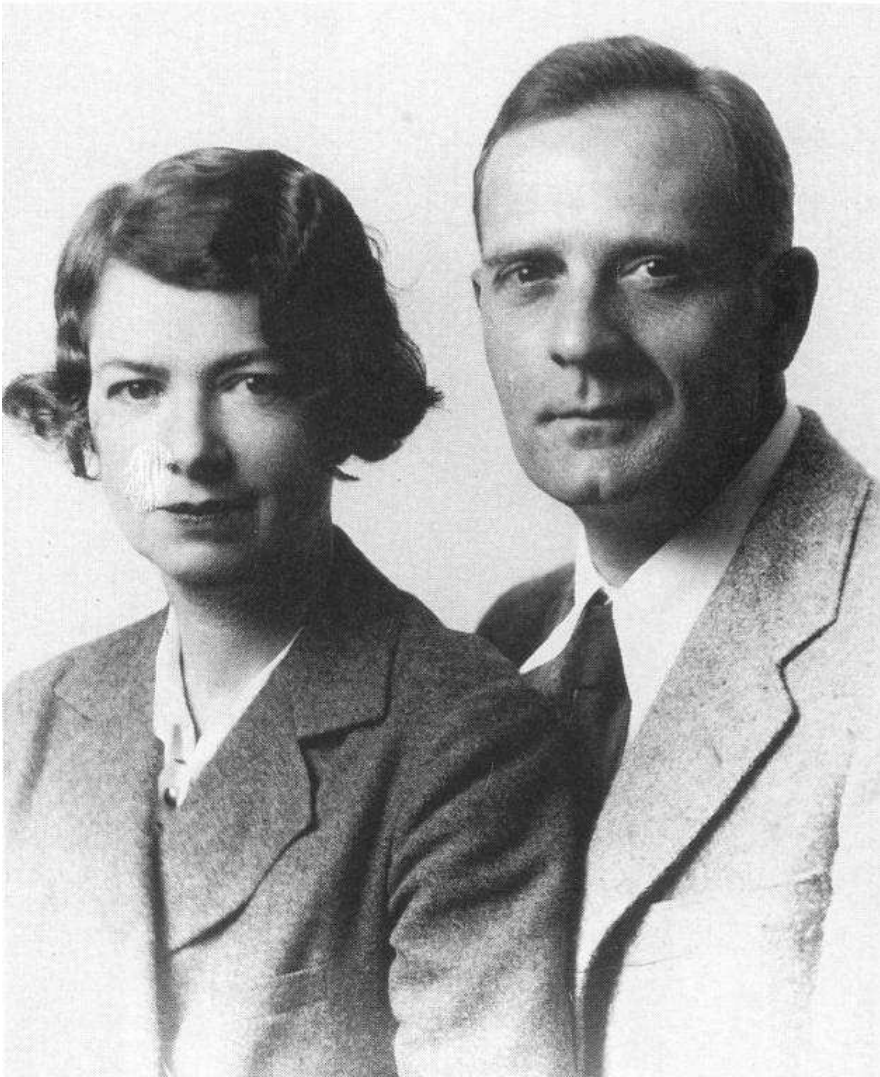
## Superclusters in our vicinity

source: <http://www.answers.org/free/universe/superc.html>





## Edwin Hubble



Christianson, 1995, p. 165

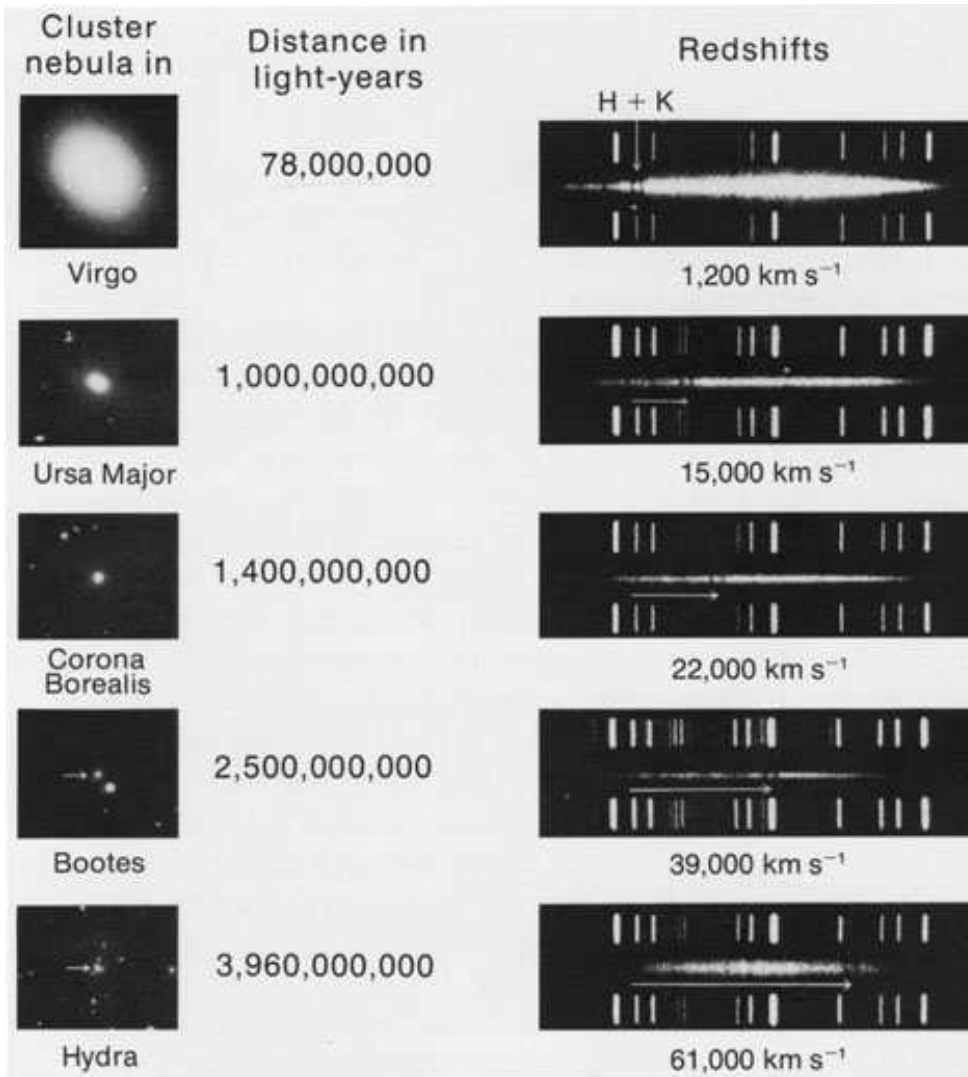
Edwin Hubble (1889–1953):

- Realisation of galaxies as being outside of the Milky Way
- Discovery that universe is expanding

Founder of modern extragalactic astronomy



# Redshifts, I

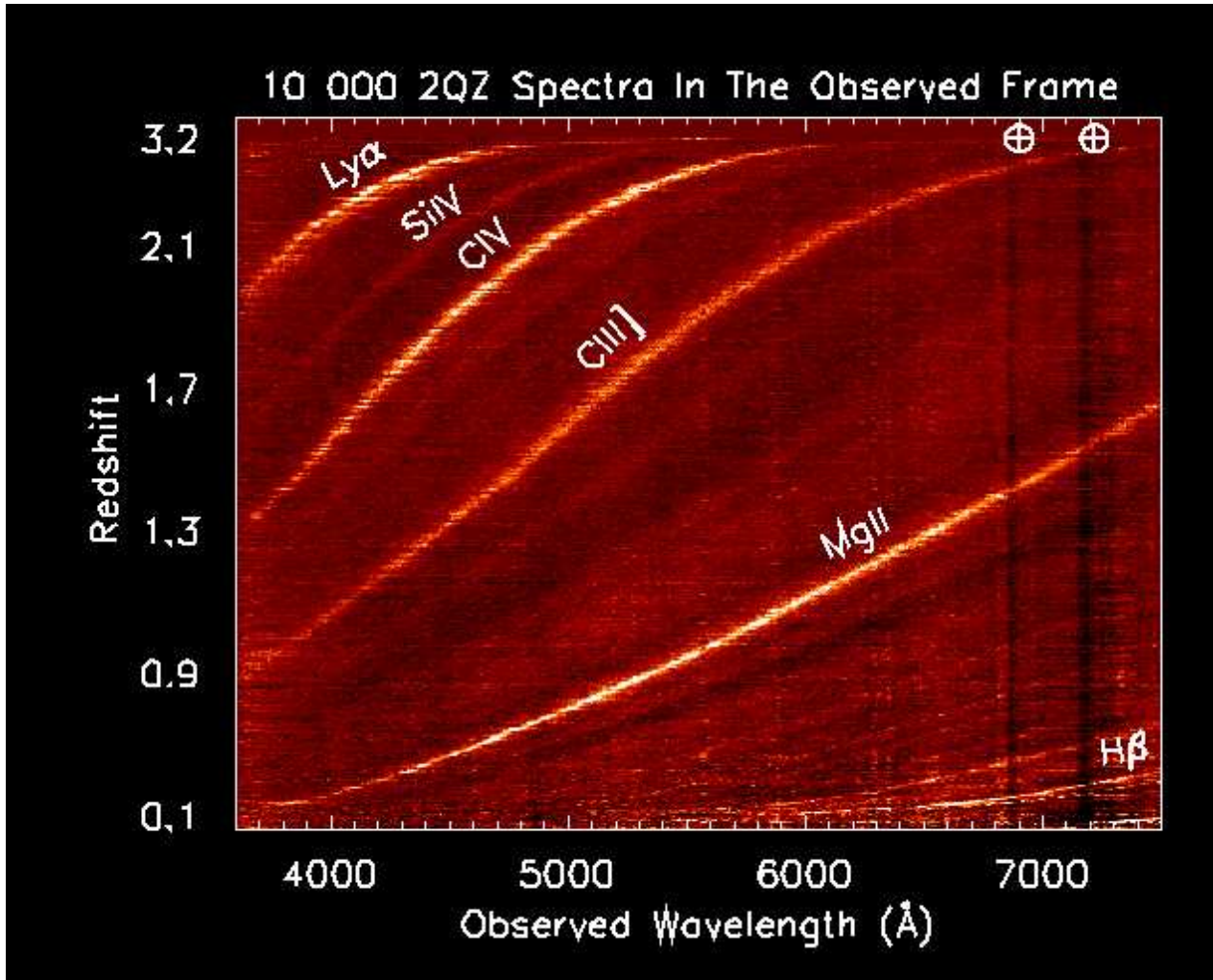


Hubble: spectral lines in galaxies are **more and more redshifted** with increasing distance.





## Redshifts, II



2dF QSO Redshift survey

Redshift:

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

interpreted as velocity:

$$v = cz$$

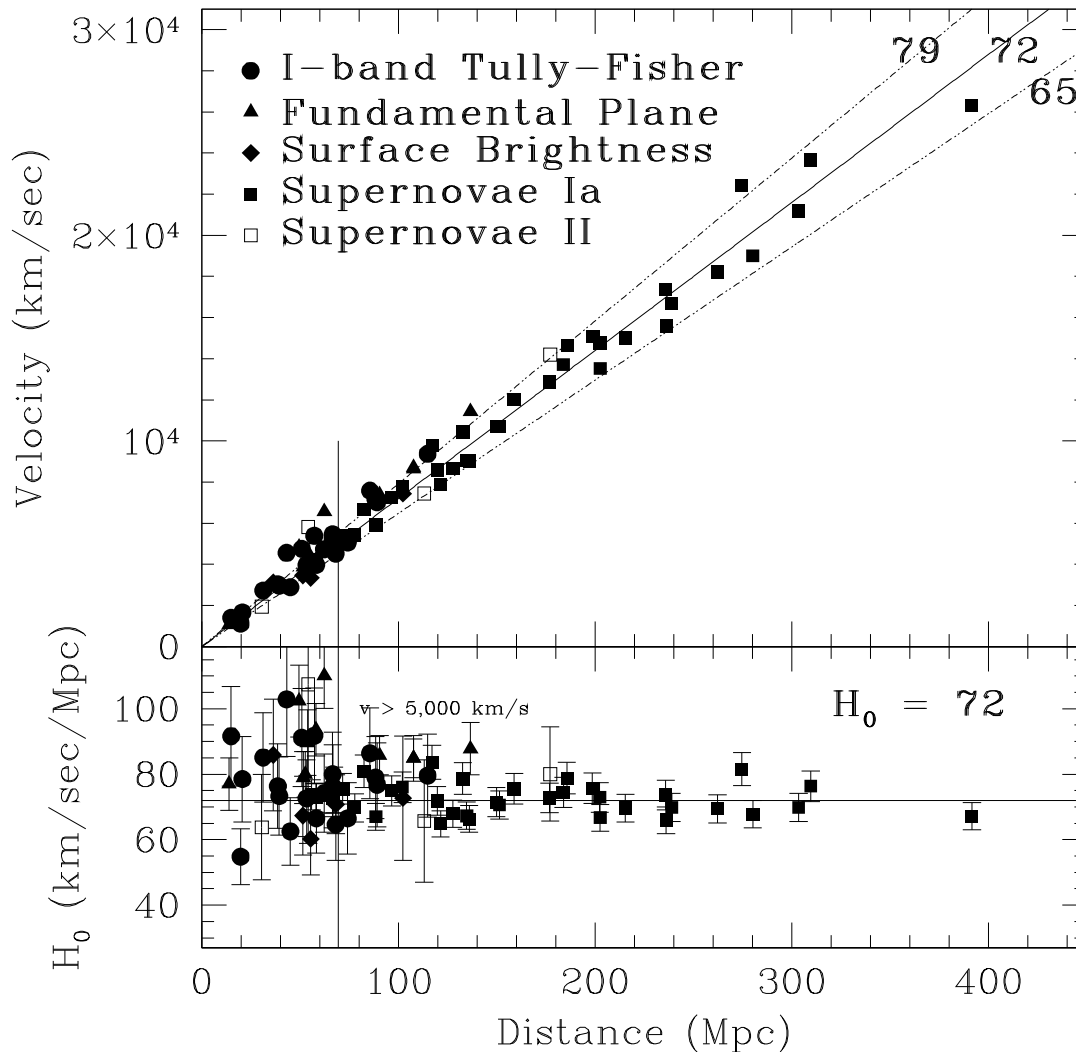
where

$$c = 300000 \text{ km s}^{-1}$$

(speed of light)



# Hubble Relation



(Freedman, 2001, Fig.4)

Hubble relation (1929):

The redshift of a galaxy is proportional to its distance:

$$v = cz = H_0 d$$

where  $H_0$ : “Hubble constant”.

*Measurement:* determine  $v$  from redshift (easy),  $d$  with standard candles (difficult)

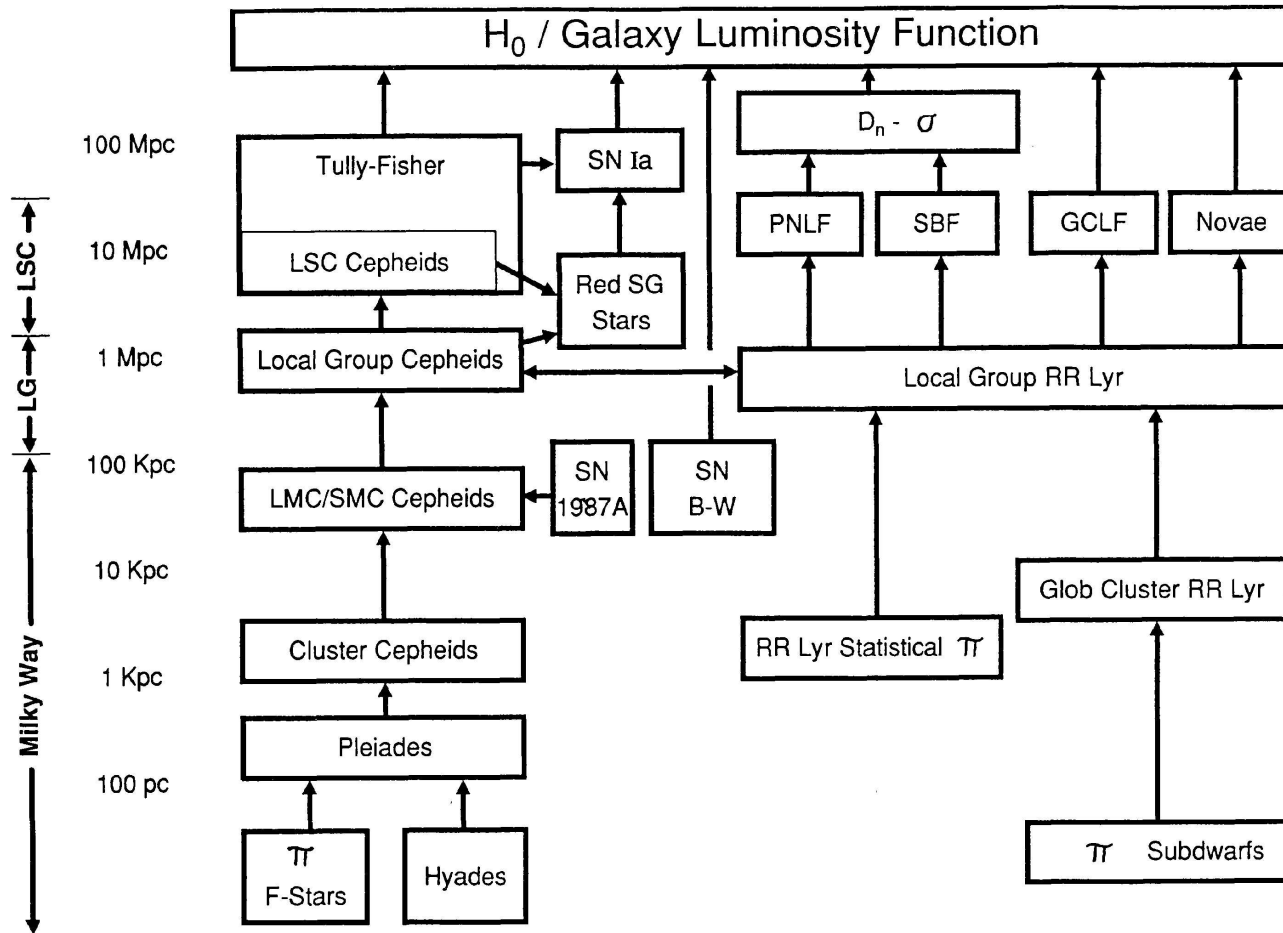
$\implies H_0$  from linear regression.

Hubble Space Telescope key project finds

$$H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$$



# Summary: Distance Ladder



*Pathways to Extragalactic Distances*

Jacoby (1992, Fig. 1)



## *Active Galactic Nuclei*