World Models

Introduction

Cosmology: science of the universe as a whole

How did the universe evolve to what it is today?

Based on four basic facts:

The universe
- expands,
- is isotropic,
- and is homogeneous.

Isotropy and homogeneity of the universe: "*cosmological principle*".

Perhaps (for us) the most important fact is:

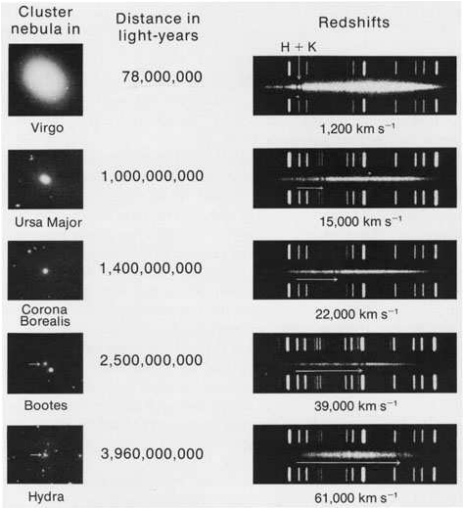
- **The universe is habitable for humans.**

("*anthropic principle*")

The one question cosmology does not attempt to answer is: How came the universe into being?

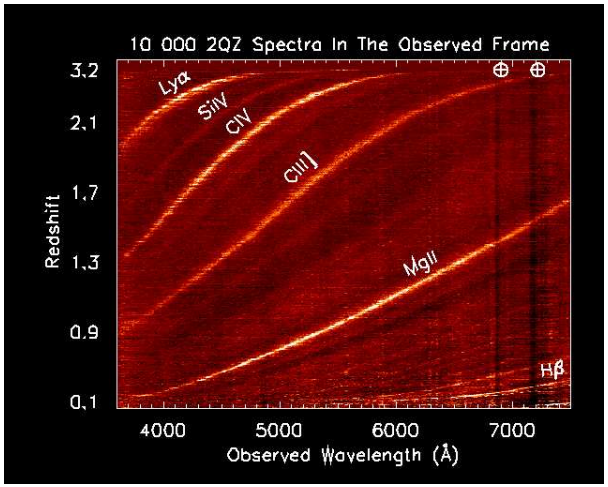
$\Longrightarrow$ Realm of theology!

Redshifts



Hubble: spectral lines in galaxies are more and more redshifted with increasing distance.

Redshifts



2dF QSO Redshift survey

Redshift:

$$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$$

interpreted as velocity:

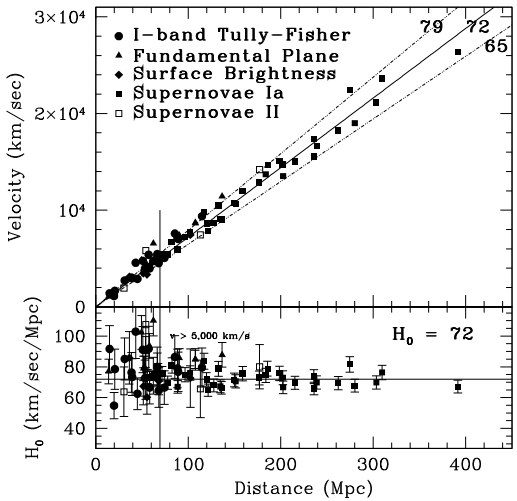
$$v = cz$$

where

$c = 300000\,\text{km s}^{-1}$

(speed of light)

Hubble Relation



(Freedman, 2001, Fig.4)

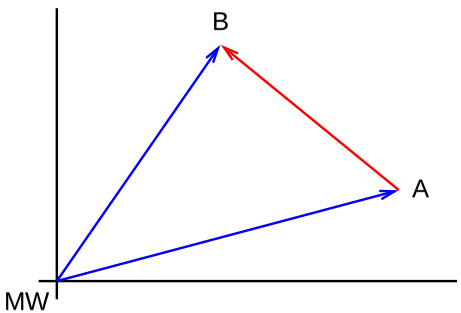
Hubble relation (1929):

The redshift of a galaxy is proportional to its distance: $v = cz = H_0 d$

where H_0: "Hubble constant".
Measurement: determine v from redshift (easy), d with standard candles (difficult)
$\Longrightarrow$ H_0 from linear regression.
Hubble Space Telescope finds

$$H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Hubble Relation

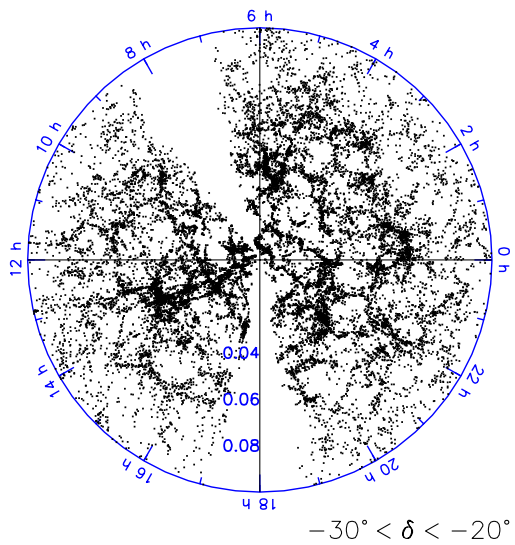


The expansion law $\boldsymbol{v} = H_0 \boldsymbol{r}$ is unchanged under rotation and translation: isomorphism.
Proof: consider two galaxies at positions $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$:
Hubble: $\boldsymbol{v}_A = H_0\,\boldsymbol{r}_A$ and $\boldsymbol{v}_B = H_0\,\boldsymbol{r}_B$
Galaxy B as seen from galaxy A:
$\boldsymbol{v}_B - \boldsymbol{v}_A = H_0\,\boldsymbol{r}_B - H_0\,\boldsymbol{r}_A = H_0\,(\boldsymbol{r}_B - \boldsymbol{r}_A)$
$\Longrightarrow$ observer at A derives the same Hubble law as we on Milky Way.
This is a direct consequence of the homogeneity of the universe.

Despite everything receding from us, we are not at the center of the universe $\Longrightarrow$ Copernicus principle still holds.

Copernicus principle: We are not at a special place in the universe in time or space.

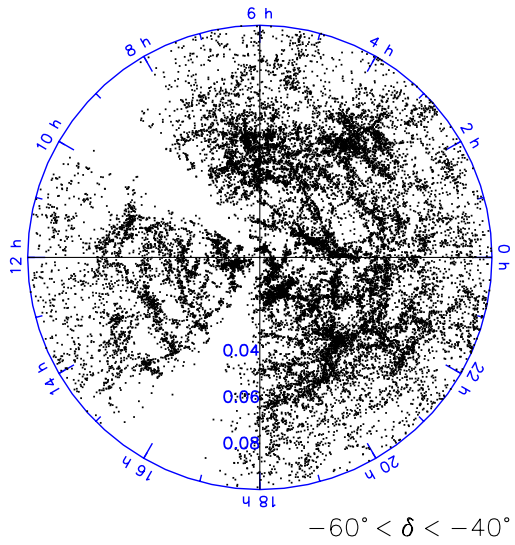
Homogeneity



Homogeneity: "The universe looks the same, regardless from where it is observed" (on scales $\gg$100 Mpc).

6dF QSO Redshift survey

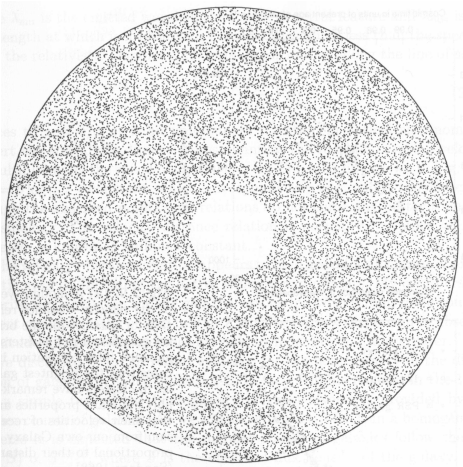
Homogeneity



Homogeneity: "The universe looks the same, regardless from where it is observed" (on scales $\gg$100 Mpc).

6dF QSO Redshift survey

Isotropy

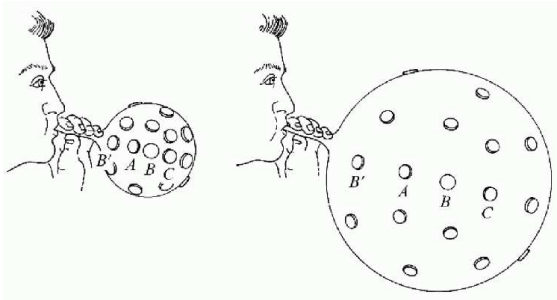


Peebles (1993): Distribution of 31000 radio sources on northern sky (wavelength $\lambda = 6$ cm)

Isotropy $\Longleftrightarrow$ The universe looks the same in all directions.

N.B. Homogeneity *does not* imply isotropy, and isotropy around one point does not imply homogeneity!

World Models

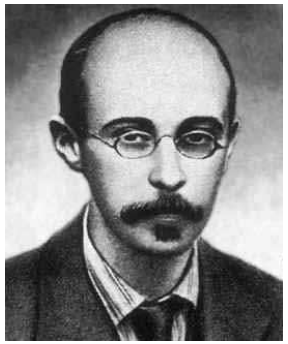


R small R large

Misner, Thorne, Wheeler

Friedmann: Mathematical description of the Universe using normal “fixed” coordinates (“comoving coordinates”), plus scale factor R which describes evolution of the Universe.

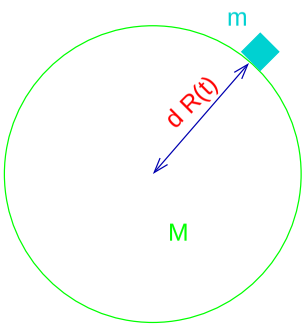
World Models



A.A. Friedmann (1888–1925)

Friedmann: Mathematical description of the Universe using normal “fixed” coordinates (“comoving coordinates”), plus scale factor R which describes evolution of the Universe.

Friedmann Equations



Evolution of universe described with Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (McCrea, 1937)

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \tag{24.1}$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d\,R(t)) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}m \tag{24.2}$$

Canceling $m \cdot d$ gives momentum equation:

$$\ddot{R}(t) = -\frac{4\pi G}{3}\frac{\rho_0}{R(t)^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{24.3}$$

Multiplying Eq. (24.3) with $\dot{R}$ and integrating yields the energy equation:

$$\frac{1}{2}\dot{R}(t)^2 = +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.} = +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.} \tag{24.4}$$

where the constant can only be obtained from GR.

Friedmann Equations

General relativistic approach: Insert metric into Einstein equation to obtain differential equation for $R(t)$:

Einstein equation:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathscr{R} g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{24.5}$$

where

$g_{\mu\nu}$: Metric tensor ($\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$)
$R_{\mu\nu}$: Ricci tensor (function of $g_{\mu\nu}$)
$\mathscr{R}$: Ricci scalar (function of $g_{\mu\nu}$)
$G_{\mu\nu}$: Einstein tensor (function of $g_{\mu\nu}$)
$T_{\mu\nu}$: Stress-energy tensor, describing curvature of space due to fields present (matter, radiation,...)
Λ: Cosmological constant

$\Longrightarrow$ greatly simplified by cosmolgical principle: Robertson-Walker metric

Friedmann Equations

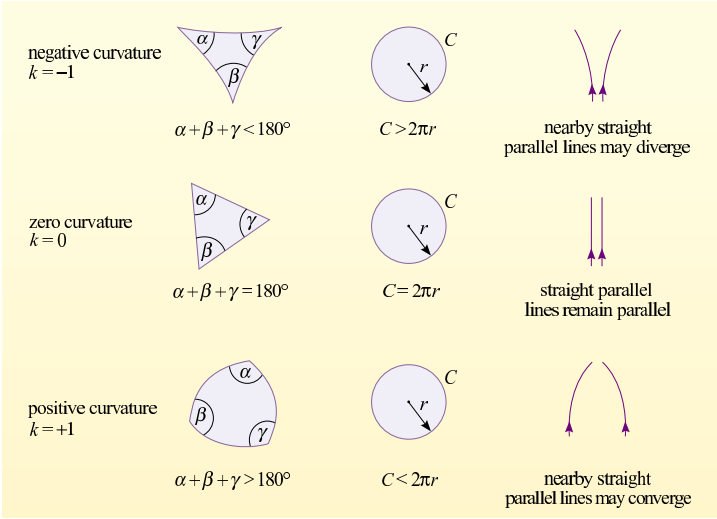
The exact GR derivation of Friedmanns equation gives:

$$\begin{aligned} \ddot{R} &= -\frac{4\pi G}{3} R \left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right] \\ \dot{R}^2 &= +\frac{8\pi G\rho}{3} R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right] \end{aligned} \tag{24.6}$$

Notes:

1. For $k = 0$: Eq. (24.6) $\longrightarrow$ Eq. (24.4).
2. k determines the curvature of space:
   - $k > 0$: closed universe (finite volume)
   - $k = 0$: flat universe
   - $k < 0$: open universe (infinite volume)
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. cosmological constant Λ introduced by Einstein to ensure stability of the universe. Physics unknown.

Friedmann Equations



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Jones & Lambourne: An Introduction to Galaxies and Cosmology

Hubble's Law

The variation of $R(t)$ implies Hubble's Law:

Small scales $\Longrightarrow$ Euclidean geometry

Proper distance between two observers with comoving distance d:

$$D(t) = d \cdot R(t) \tag{24.7}$$

Expansion $\Longrightarrow$ D changes:

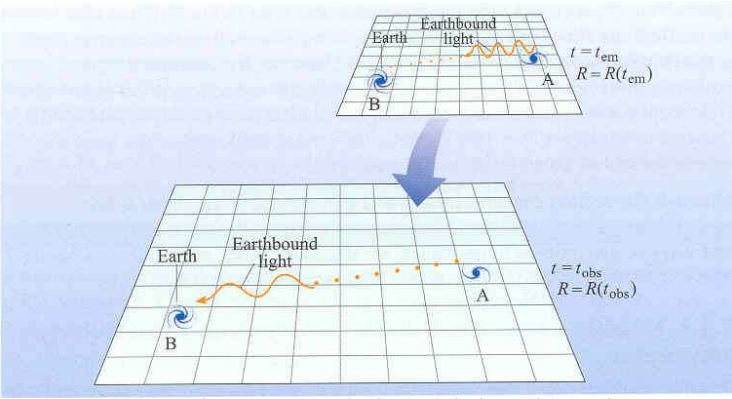
$$\frac{\Delta D}{\Delta t} = \frac{R(t+\Delta t)d - R(t)d}{\Delta t} \quad \text{and for} \quad \lim_{\Delta t \to 0} \quad v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}\,d = \frac{\dot{R}}{R} D =: H\,D \tag{24.8}$$

$\Longrightarrow$ Identify local Hubble "constant" as

$$H = H(t) = \frac{\dot{R}(t)}{R(t)} \tag{24.9}$$

$\Longrightarrow$ Hubble "constant" is time-dependent! $\Longrightarrow$ "Hubble parameter"

Hubble's Law



Jones & Lambourne: An Introduction to Galaxies and Cosmology

Expansion of space leads to the redshift:

$$z = \frac{R(t_0)}{R(t_{em})} - 1 \quad \Longrightarrow \quad \frac{R(t_{em})}{R(t_0)} = \frac{1}{z+1} \tag{24.10}$$

Critical Density

Looking at the energy equation for $\Lambda = 0$,

$$\dot{R}^2 = +\frac{8\pi G\rho}{3}R^2 - kc^2 \tag{24.11}$$

we find that the evolution of the Hubble parameter is:

$$\left(\frac{\dot{R}}{R}\right)^2 = H(t)^2 = \frac{8\pi G\rho(t)}{3} - \frac{kc^2}{R^2} \tag{24.12}$$

and therefore

$$k \cdot \frac{c^2}{R(t)^2 H(t)^2} = \frac{8\pi G}{3H(t)^2}\rho(t) - 1 = \frac{\rho(t)}{\rho_{crit}} - 1 = \Omega - 1 \tag{24.13}$$

where Ω is called the critical density:

$$\Omega = \frac{\rho}{\rho_{crit}} \quad \text{where} \quad \rho_{crit} = \frac{3H^2}{8\pi G} \tag{24.14}$$

currently: $\rho_{crit} \sim 1.67 \times 10^{-24}\,\mathrm{g\,cm^{-3}}$ (3...10 H-Atoms $\mathrm{m^{-3}}$).

Ω describes the curvature of the universe:

$\Omega > 1 \Longrightarrow k > 0$: closed ❙ $\Omega = 1 \Longrightarrow k = 0$: flat ❙ $\Omega < 1 \Longrightarrow k < 0$: open

Critical Density

World Model: Evolution of R as a function of time

Solution of Friedmann equations depends on boundary conditions:

1. Value of H as measured today (H is time dependent!)
2. Density Parameter of universe

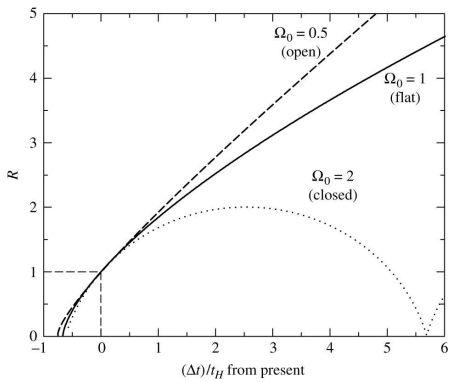
Note: total Ω is sum of:

1. Ω_m: Matter, i.e., everything that leads to gravitative effects
   Ω_m in baryonic matter is $\lesssim$3%, but note there might be "nonbaryonic dark matter" as well!
2. $\Omega_\Lambda = \Lambda c^2/3H^2$: contribution by cosmological constant Λ
   (Λ is often called "dark energy" for PR reasons)

Hubble time: Assume an empty universe ($\omega = 0$): linear expansion

$\Longrightarrow$ age of the Universe: $t_H = v/d = 1/H_0$ is called Hubble time

Critical Density

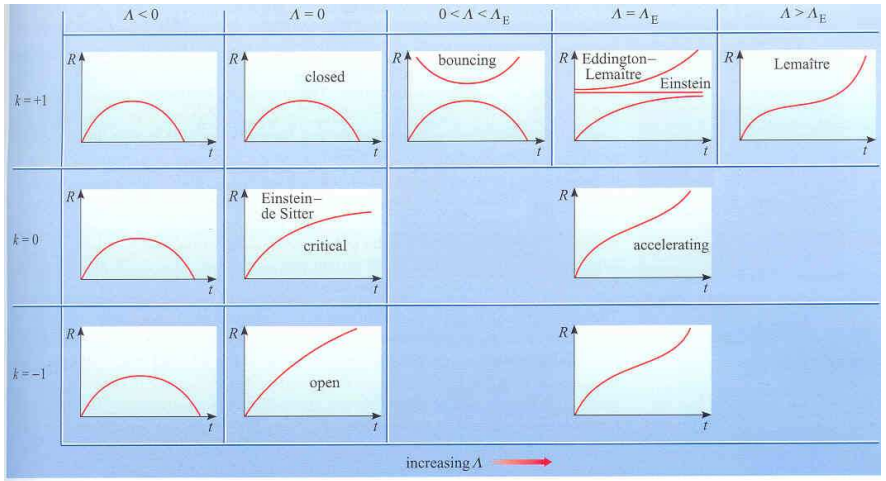


Carroll & Ostlie, Fig. 29.5

$\Omega > 1 \Longrightarrow$ bounces back ❙ $\Omega = 1 \Longrightarrow$ expands forever, stops at $t = \infty$

❙ $\Omega < 1 \Longrightarrow$ expands forever

Critical Density



Jones & Lambourne: An Introduction to Galaxies and Cosmology

Many different kinds of world models are possible, depending on Ω und Λ.

The Big Bang

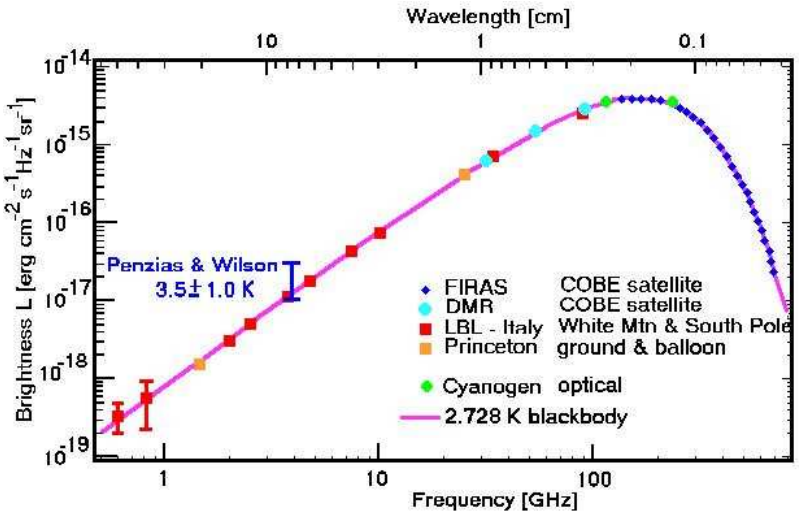
CMBR

Arno Penzias & Robert Wilson
(Nobel prize 1978)

Discovery of the Cosmic Microwave Background (CMB):

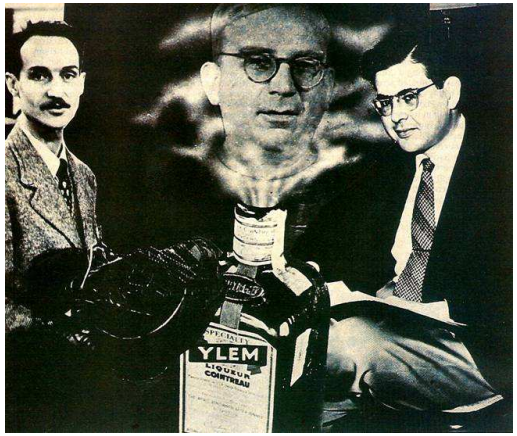
"A Measurement of Excess Antenna Temperature at 4080 Mc/s."
A.A. Penzias & R.W. Wilson (1965, ApJ 142, 419):
Abstract: *Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna . . . at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value of about 3.5 K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and free from seasonal variations (July, 1964 – April, 1965). A possible explanation for the observed excess noise temperature is the one given by Dicke, Peebles, Roll, and Wilkinson (1965) in a companion letter in this issue.*

CMBR



NASA COBE: CMB is a perfect black body with $T = 2.725 \pm 0.002$ K

CMBR



George Gamov: Thermodynamics implies that the young, dense Universe must have been extremely hot $\Longrightarrow$ The Big Bang

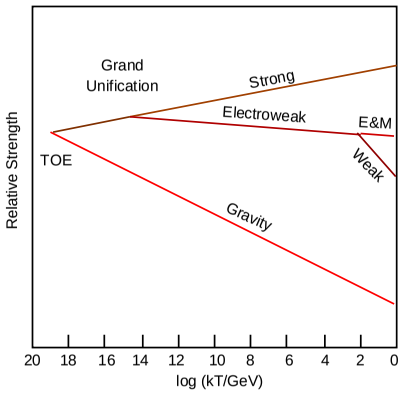
Temperature then decreased after BB as universe expanded,

$$T(z) = T_0(1+z) \qquad (25.1)$$

Gamov called original state the ylem. This term has (thankfully) not caught on.

The very early universe

Big Bang Theory: Initially, the universe was very hot and as it expanded, it cooled down. Theory gives $T(t) = 1.52 \times 10^{10}\,\mathrm{K} \cdot (t/1\,\mathrm{s})^{-1/2}$



after Carroll & Ostlie, Fig. 30.2

Fundamental physics: unification of forces

- $t = 10^{-44}$ s: GUT & gravitation
  $\Longrightarrow$ Theory of Everything (TOE)
- $t = 10^{-34}$ s: electroweak & strong nuclear force
  $\Longrightarrow$ Grand Unifying Theory (GUT)
- $t = 10^{-11}$ s: electromagnetic & weak nuclear forces
  $\Longrightarrow$ electroweak force

Physics is understood from $t = 10^{-11}$ s after the Big Bang

The hot Big Bang

- 10^{-4} s after Big Bang: Temperature has decreased to $T = 10^{12}$ K, corresponding energy $E = kT = 86$ MeV
- Universe consists of only a few particle types:
  - photons γ, electrons e^-, positrons e^+, neutrinos: ν_e, ν_μ, ν_τ and anti-neutrinos $\bar\nu_e, \bar\nu_\mu, \bar\nu_\tau$
  - few hadrons: protons p & neutrons n: $N(p) \sim 5 \times 10^{-10} N(\gamma)$
- reactions at work:
  - pair formation and annihilation:

$$\gamma \rightleftharpoons e^- + e^+ \quad \text{and} \quad \gamma \rightleftharpoons \nu_e + \bar\nu_e$$

  - transformation of particles:

$$n \rightleftharpoons p + e^- + \nu_e$$
$$n + e^+ \rightleftharpoons p + \bar\nu_e$$
$$n + \nu_e \rightleftharpoons p + e^-$$

- resulting thermodynamic equilibrium: neutron-proton-ratio:

$$\frac{N(n)}{N(p)} = \exp\left(-\frac{(m_p - m_n)c^2}{kT}\right) \qquad (25.2)$$

where $(m_p - m_n)c^2 = 1.293$ MeV $\Longrightarrow N_n/N_p = 0.985$

The hot Big Bang

- $t \sim 2$ s: Temperature has decreased to $t \sim 2 \times 10^{10}$ K.
  At this time: Timescale for nuclear reactions $<$ Expansion time scale

Result: End of thermodynamical equilibrium, "freeze out"

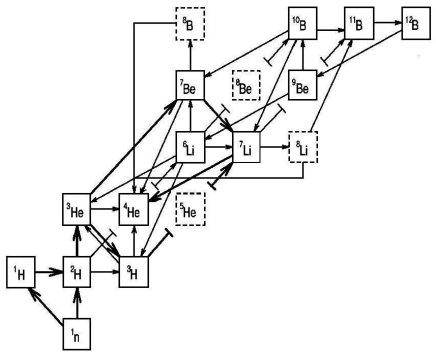
  - neutron-proton ratio now $N_n/N_p = 0.223$
  - Neutrino freeze-out: neutrinos decouple, no interaction with other particles
  - electron-positron pair formation stops at threshold 1.22 MeV (1.4×10^{10} K)
  - electrons and positrons annihilate $\Longrightarrow$ very few electrons remain
  - $\Longrightarrow$ no neutrons can be formed neutrons decay to protons (half-life 10.3 min.)
- $T = 10^9$ K, $t = 230$ s:
  - Deuterium formation: $p + n \rightleftharpoons d + \gamma$
  - Helium formation:

$$d + d \rightleftharpoons t + p \quad \text{and then} \quad t + d \rightleftharpoons {}^4He + n$$

and also

$$d + d \rightleftharpoons {}^3He + n \quad \text{and} \quad {}^3He + d \rightleftharpoons {}^4He + p$$

The hot Big Bang



Ohio State University

- Helium formation: neutrons start to decay

$$N_n(t) = 0.223 \exp(-\lambda t) \qquad (25.3)$$

where $\lambda = \ln 2/\tau$. Nucleosynthesis starts once $N_n/N_p = 0.164$.
Most neutrons still around manage to get bound in He before they decay:

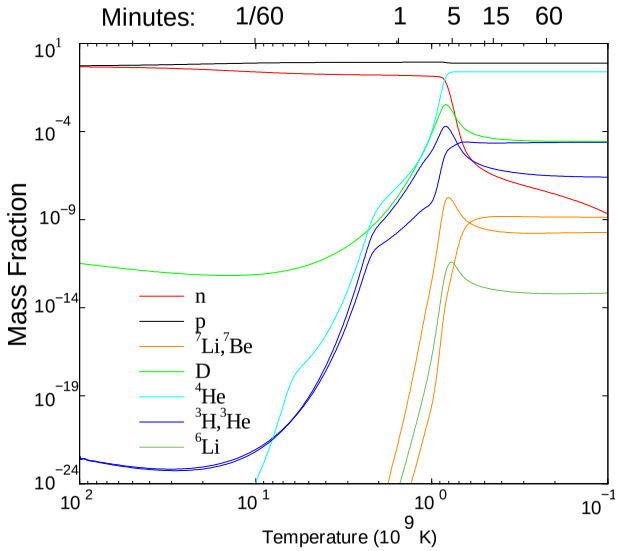
$$N_{He}/N_H \sim 0.1 \qquad (25.4)$$

(turns out to be almost independent of Ω)

- Some higher elements are also formed, especially ^{7}Li Abundances of deuterium, tritium, ^{3}He and ^{7}Li arestrongly dependent on Ω

Since there are no stable nuclei with A = 5 & 8, heavier elements can not form in Big Bang Nucleosynthesis.

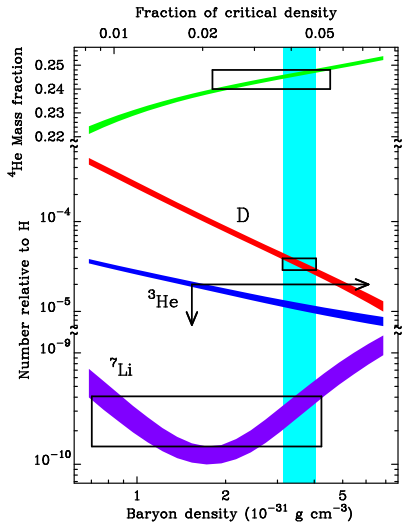
The hot Big Bang



Big Bang Nucleosynthesis finishes after a quarter of an hour!

(Burles, 1999)

The hot Big Bang



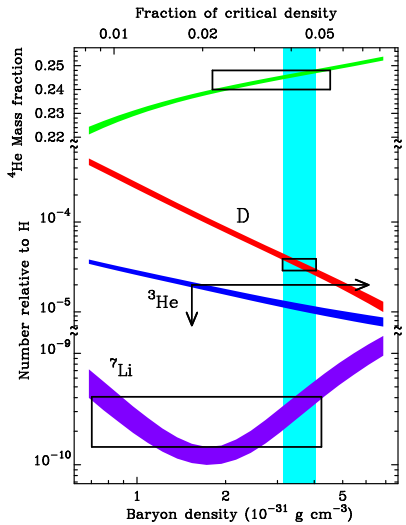
Synthesis of the elements in stars: Burbidge, Burbidge, Fowler & Hoyle (1957, B^{2}FH):

Stars produce much less He than observed

$\Longrightarrow$ The helium (^{4}He) abundance is a smoking gun for the hot big bang

(Burles, 1999)

The hot Big Bang



Deuterium, ^{3}He & Lithium abundances: primordial abundances are very difficult to determine as these elements are easily destroyed in stars by thermonuclear reactions.
Can be approximately measured from old stars and the intergalactic medium
Result:

Density parameter $\Omega_m = 0.04$: Normal matter ("baryons") contributes 4% of the critical density.

(Burles, 1999)

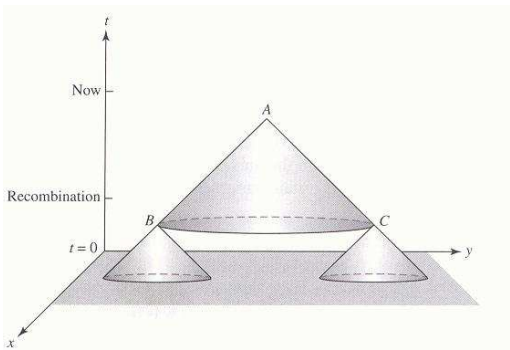
The hot Big Bang

BB works remarkably well in explaining the observed universe.

However, there are many problems with classical BB theory:

- Horizon problem:
  Why ist the CMB so isotropic?
- Flatness problem:
  Why is and was the density so close to critical, i.e., $\Omega = 1$?
- Baryogenesis:
  Why is there virtually no antimatter in the universe?
- What is the nature of Dark Matter & Dark Energy?
- Structure formation:
  The fluctuations of the cosmic microwave background are too small to allow stars & galaxies to form as early as observed. Why?

The Horizon Problem

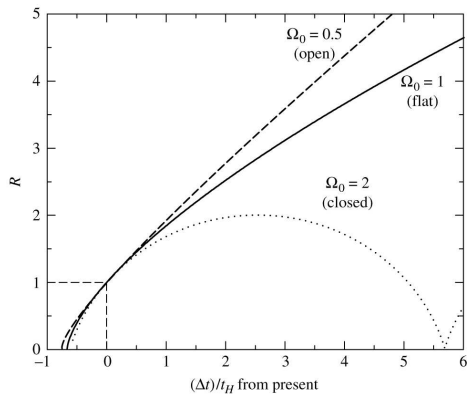


Carroll & Ostlie, Fig. 30.3

At the time of decoupling ($T_{\rm dec} = 300000$ yrs) spatial regions of the CMB that are now separated by more than 2 degrees on the sky could never have been in causal contact.
Only a very small spatial variation of the CMB radiation is observed, i.e., the same CMB spectrum observed at point A (today) originating at points B and C.

Why is the CMB so isotropic although there is no causal relation?

The Flatness Problem



Carroll & Ostlie, Fig. 29.5

Because of expansion, Ω is changing with time:

$$\frac{1}{\Omega_0} - 1 = \left(\frac{1}{\Omega} - 1\right) \cdot (1 + z) \qquad (25.5)$$

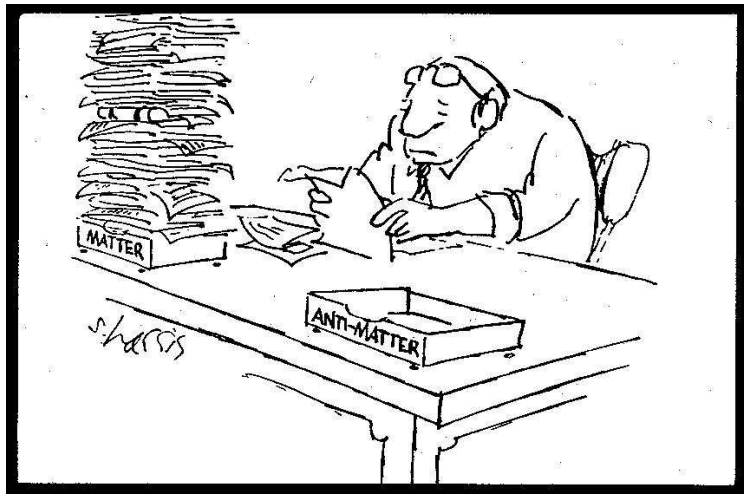
What was Ω at time when radiation and matter decoupled ($T_{\rm dec} = 380000$ yrs; $z = 1100$)?

$\Longrightarrow$ assume $\Omega(T_{\rm dec}) = 2$:
Results in big crunch after $\sim$ 3 milllion years

$\Longrightarrow$ assume $\Omega(T_{\rm dec}) = 0.5$:
Results in a very fast evolution of the universe, no stars & galaxies can form.

Why is the universe (nearly) flat?

The matter/antimatter problem



The matter/antimatter problem

Early universe was in thermodynamic equilibrium

$\Longrightarrow$ there should have been as much matter as antimatter

But: If there were as much matter as antimatter formed in the Big Bang, they would have annihilated very soon.

$\Longrightarrow$ Only photons would have remained, we should not exist.

Observations of Big Bang nucleosynthesis: ratio of hadrons to photons: $\sim 5 \cdot 10^{-10}$

$\Longrightarrow$ slightly more matter than antimatter must have been formed.

There is no observational evidence for antimatter

(except for production by high energy cosmic rays)

$\Longrightarrow$ There must have been some kind of symmetry breaking in the production of elementary particles in the very early universe!

Inflation

Possible solution of these problems: inflation

Basic assumption of inflationary cosmology:

During the big bang there was a phase where Λ dominated the Friedmann equation.

Use the Friedmann equation with a cosmological constant:

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{25.6}$$

where $a = R(t)/R_0$

For $\Lambda =$ const. this gives

$$H(t) = \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}} = \text{const.} \tag{25.7}$$

Solution of Eq. (25.7):

$$a \propto \mathrm{e}^{Ht} \tag{25.8}$$

Inflation

When and why did inflation happen?

Typical assumption: Inflation = phase transition of a scalar field ("inflaton") associated with Grand Unified Theories = GUT.

(GUT predicts that the electromagnetic, weak nuclear, and strong nuclear forces are fused into a single unified field at energies above 10^{14} GeV)

$\Longrightarrow$ temperature $kT_{\text{GUT}} = 10^{15}$ GeV, when $1/H \sim 10^{-34}$ sec ($t_{\text{start}} \sim 10^{-34}$ s).

$\Longrightarrow$ inflation lasted for 100 Hubble times, i.e., for $\Delta T = 10^{-32}$ s.

With Eq. (25.8): Inflation: Expansion by factor $\mathrm{e}^{100} \sim 10^{43}$... corresponding to a volume expansion by factor $\sim 10^{130}$

$\Omega \neq 1$ (curved) before inflation $\Longrightarrow \Omega = 1$ (flat) thereafter.

Universe was so small before inflation that all parts of it were in causal contact.

Inflation solves horizon, antimatter, and flatness problem.

... but it was *constructed* to do so, and the real physical cause for inflation is still unclear.