Determination of Ω

Motivation

While most of the things happening during the Big Bang are rather insensitive to the detailed value of Ω, the evolution of the universe depends on

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} = \Omega_{\text{m}} + \Omega_{\Lambda} \tag{26.1}$$

where

$\Omega_{\mathbf{m}}$**:** Ω due to gravitating stuff (baryons and other things).
Ω_{Λ}**:** Ω due to vacuum energy or other exotic stuff.

Furthermore: inflation predicts $\Omega = 1$.

To decide whether that is true:

- need inventory of gravitating material in the universe,
- need to search for evidence of non-zero Λ

Will now work on Ω_{m}.

Motivation

Remember that

$$\Omega_{\text{m}} = \frac{\rho_{\text{m}}}{\rho_{\text{crit}}} = \frac{8\pi G\rho}{3H^2} \tag{24.14}$$

For a typical ensemble of stars,

$$\frac{M}{L} \approx \text{const.} \tag{26.2}$$

we therefore often express Ω in terms of a mass to luminosity ratio:

Using canonical luminosity density of universe, one can show that

$$\left.\frac{M}{L}\right|_{\text{crit}} = 1390\, h \frac{M_\odot}{L_\odot} \tag{26.3}$$

...which means that there *must* be lots of dark matter if $\Omega = 1$.

$h = H/100\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$

Introduction

Constituents of Ω_{m}:

- Radiation (3 K radiation)
- Neutrinos
- Baryons ("normal matter", Ω_{b})
- Other, non-radiating, gravitating material ("dark matter")

Radiation: From temperature of 3 K radiation, using $u = \rho c^2 = a_{\text{rad}} T^4$:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \tag{26.4}$$

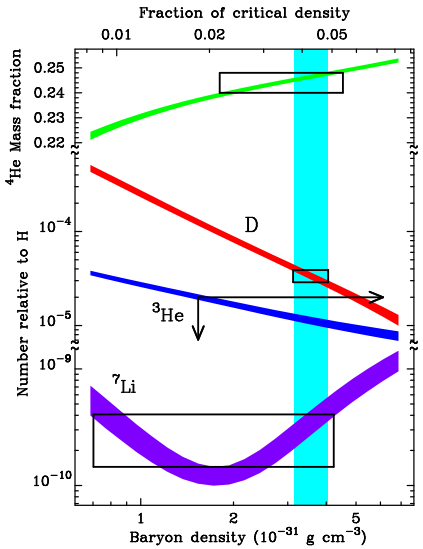
for $h = 0.72$: $\Omega_\gamma = 4.8 \times 10^{-5}$

Massless Neutrinos have

$$\Omega_\nu = 3 \cdot \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \Omega_\gamma = 0.68\, \Omega_\gamma \tag{26.5}$$

Photons and neutrinos are unimportant for todays Ω.

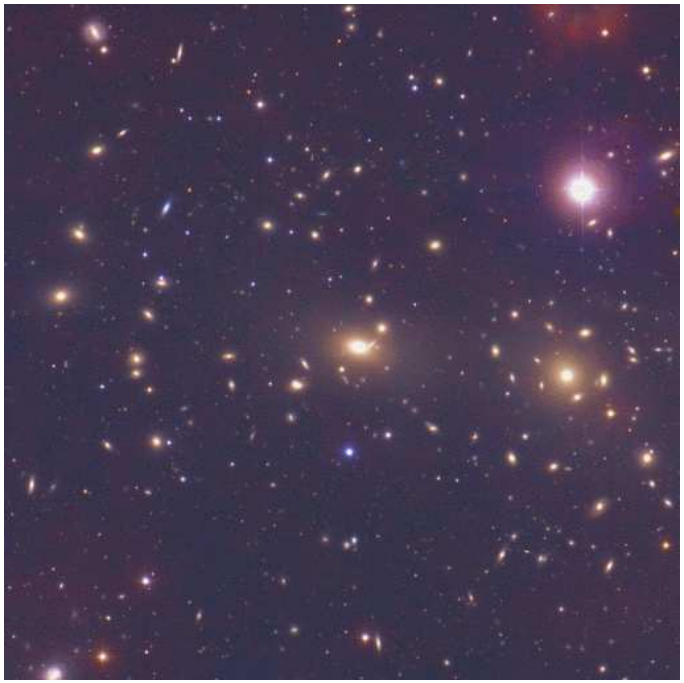
Baryons



Best evidence for mass in baryons, Ω_b: primordial nucleosynthesis.

$$\Omega_b h^2 = 0.02 \pm 0.002 \qquad (26.6)$$

(Burles, Nollett & Turner, 1999, Fig. 1)



Coma Cluster (O. Lopez-Cruz, I. K. Shelton, & KPNO)

Galaxy Clusters

For mass of galaxy clusters, make use of the virial theorem (see earlier homework):

$$E_{\text{kin}} = -E_{\text{pot}}/2 \qquad (26.7)$$

in statistical equilibrium.

Measurement: assume isotropy, such that

$$\left\langle v^2 \right\rangle = \left\langle v_x^2 \right\rangle + \left\langle v_y^2 \right\rangle + \left\langle v_z^2 \right\rangle = 3 \left\langle v_\parallel^2 \right\rangle \qquad (26.8)$$

Assuming that the velocity dispersion is independent of m_i gives:

$$E_{\text{kin}} = \frac{1}{2} \sum_i m_i \boldsymbol{v}_i^2 = \frac{3}{2} M \left\langle v_\parallel^2 \right\rangle \qquad (26.9)$$

where M is the total mass.

If the cluster is spherically symmetric $\Longrightarrow$ Define weighted mean separation R_{cl}, such that

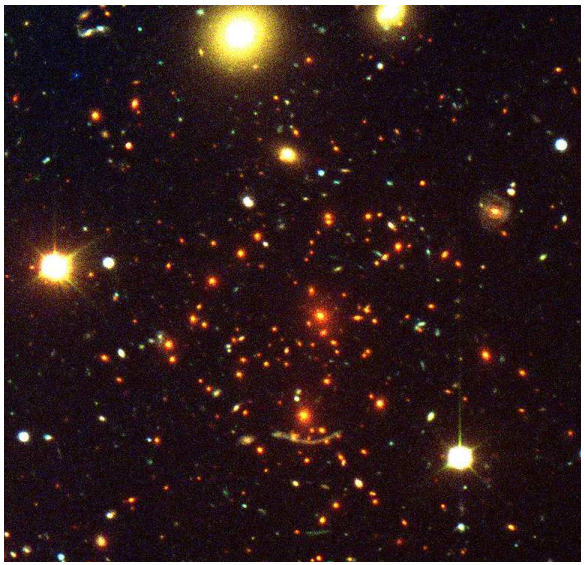
$$E_{\text{pot}} = \frac{GM^2}{R_{\text{cl}}} \qquad (26.10)$$

From Eqs. (26.9) and (26.10):

$$M = \frac{3}{G} \left\langle v_\parallel^2 \right\rangle R_{\text{cl}} \qquad (26.11)$$

E.g.: $v_\parallel \sim 1000\,\text{km s}^{-1}$, $R \sim 1\,\text{Mpc} \Longrightarrow M = 1.4 \times 10^{48}\,\text{g} = 7 \times 10^{14}\,M_\odot$ (MW: $6 \times 10^{11}\,M_\odot$).

Galaxy Clusters



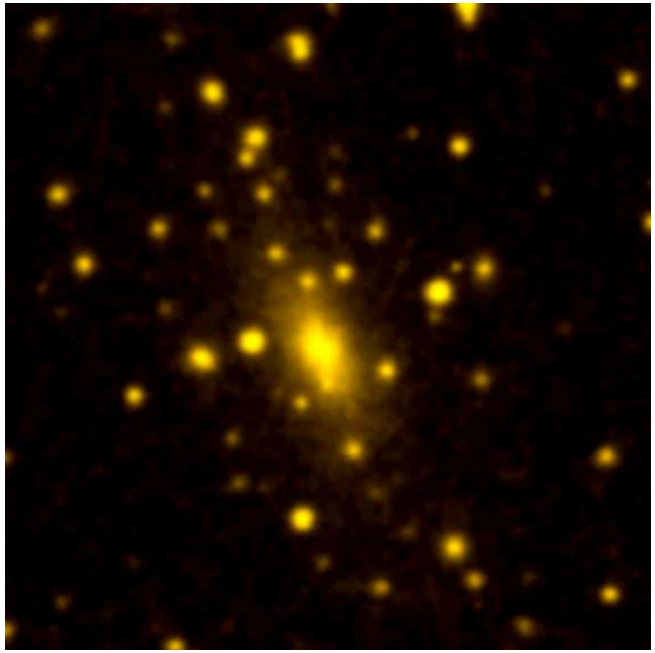
More detailed analysis using more complicated mass models gives (Merritt, 1987):

$$\frac{M}{L} \sim 350 h^{-1} \frac{M_\odot}{L_\odot} \qquad (26.12)$$

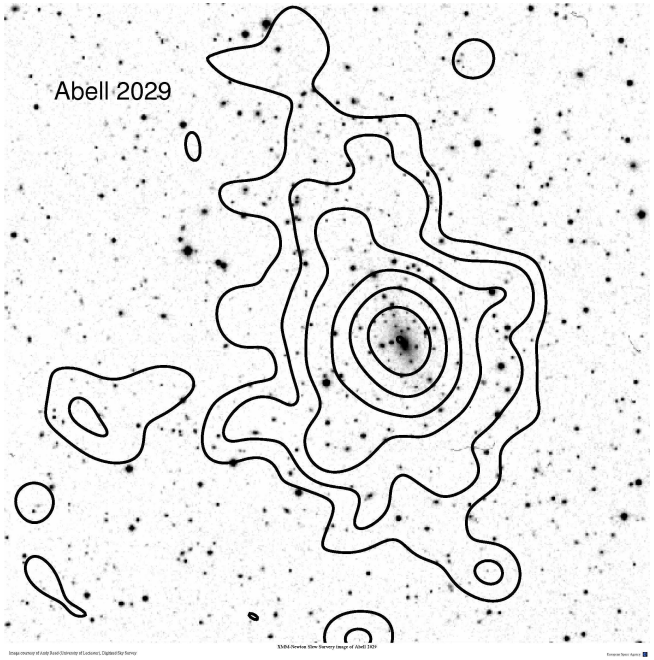
while we would have expected $M/L = 10 \ldots 20$ as for galaxies

Dark matter is an important constituent in galaxy clusters

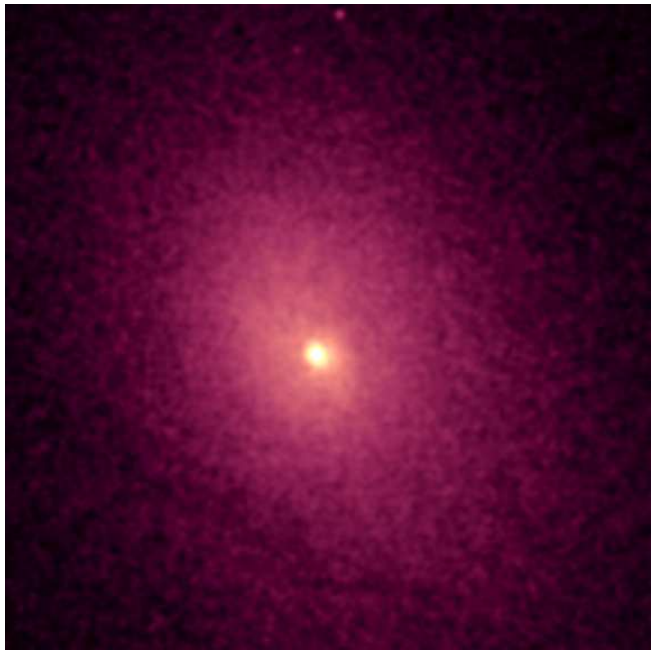
Abell 370 (VLT UT1+FORS)



Abell 2029, Palomar Schmidt [DSS]



Abell 2029, Optical and X-rays (*XMM-Newton*; Andy Read [Leicester]/DSS/ESA; larger FoV)



Abell 2029, Soft X-rays (*Chandra*; NASA/CXC/UCI/A.Lewis et al.)

26–12

X-ray emission

X-ray emission from galaxy clusters gives mass to higher precision:

Assume gas in potential of galaxy cluster. If gas is in hydrostatic equilibrium:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2} \tag{26.13}$$

where the pressure P:

$$P = nkT = \frac{\rho kT}{\mu m_{\mathrm{H}}} \tag{26.14}$$

where m_{H}: mass of H-atom, μ mean molecular weight of gas ($\mu = 0.6$ for fully ionized).

Differentiating Eq. (26.14) wrt r gives

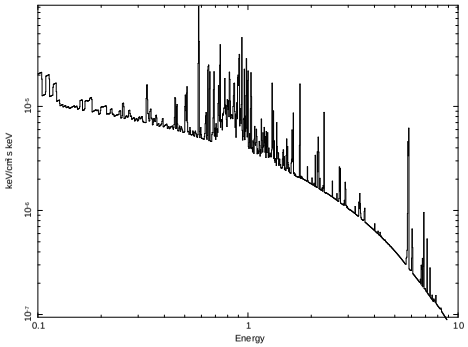
$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{k}{\mu m_{\mathrm{H}}}\left(T\frac{\mathrm{d}\rho}{\mathrm{d}r} + \rho\frac{\mathrm{d}T}{\mathrm{d}r}\right) = \frac{\rho kT}{\mu m_{\mathrm{H}}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{26.15}$$

Inserting $\mathrm{d}P/\mathrm{d}r$ into Eq. (26.13) and solving for M_r gives

$$M_r = -\frac{kTr^2}{G\mu m_{\mathrm{H}}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{26.16}$$

X-ray emission

To determine M_r, we need to measure $T(r)$ and $\rho(r)$. These quantities can be obtained from the observed X-ray spectrum:



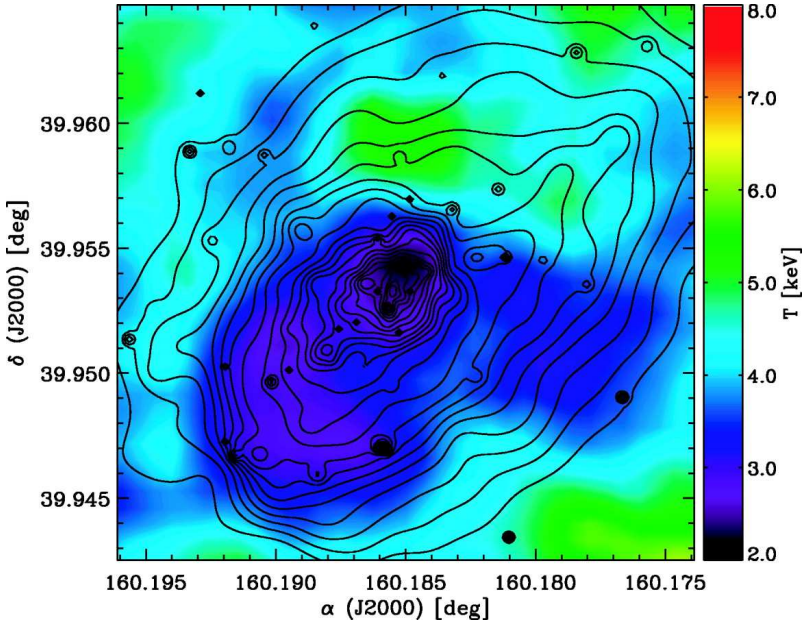
Theoretical X-ray spectrum of a cluster.

Cluster gas mainly radiates by bremsstrahlung emission, with a spectral continuum shape

$$\epsilon(E) \propto \left(\frac{m_e}{kT}\right)^{1/2} g(E,T)\, n\, n_e \exp\left(-\frac{E}{kT}\right) \quad (26.17)$$

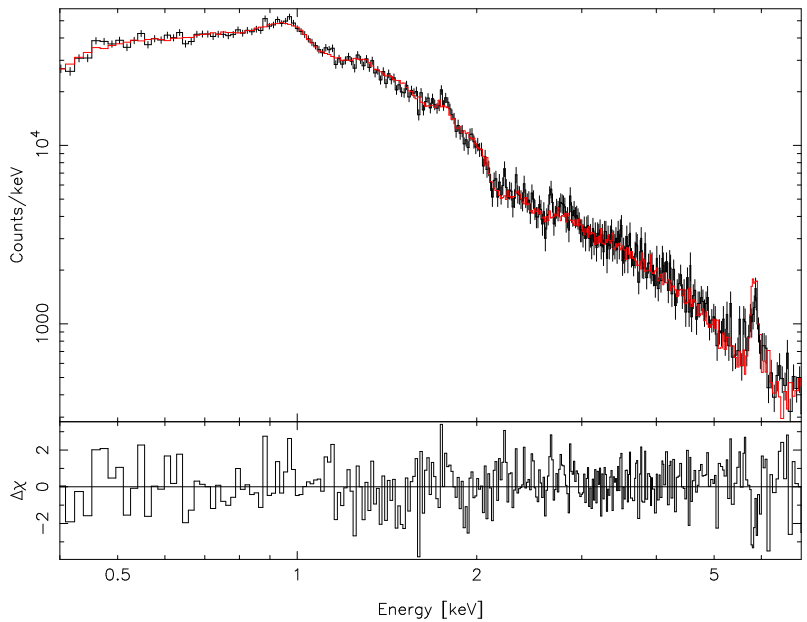
where
n: number density of nuclei,
n_e: number density of electrons,
$g(E,T)$: Gaunt factor (QM correction factor, roughly constant).
plus emission lines. . .

$\Longrightarrow$ $T(r)$ can be obtained from the X-ray spectral shape, n and n_e from the measured flux
$\Longrightarrow$ M_r.



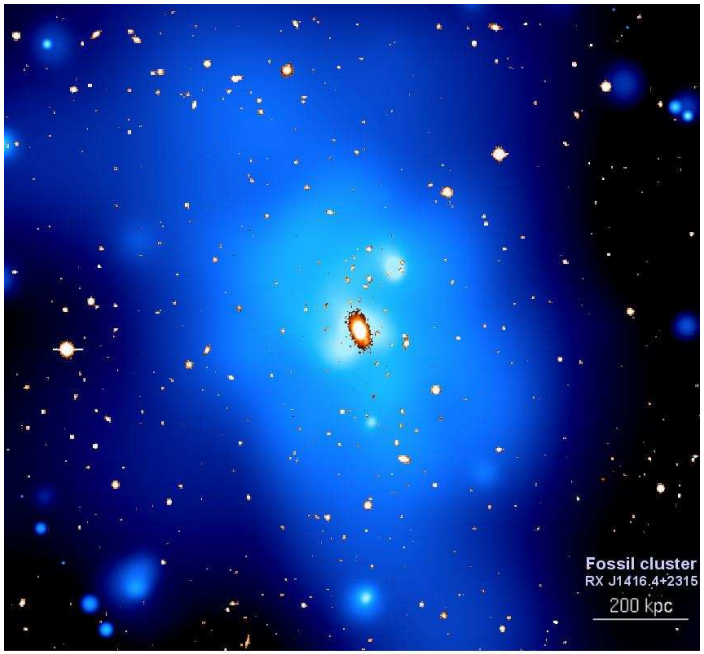
(Wise, McNamara & Murray, 2004, Fig. 8)

Temperature distribution in A1068 obtained with *Chandra*

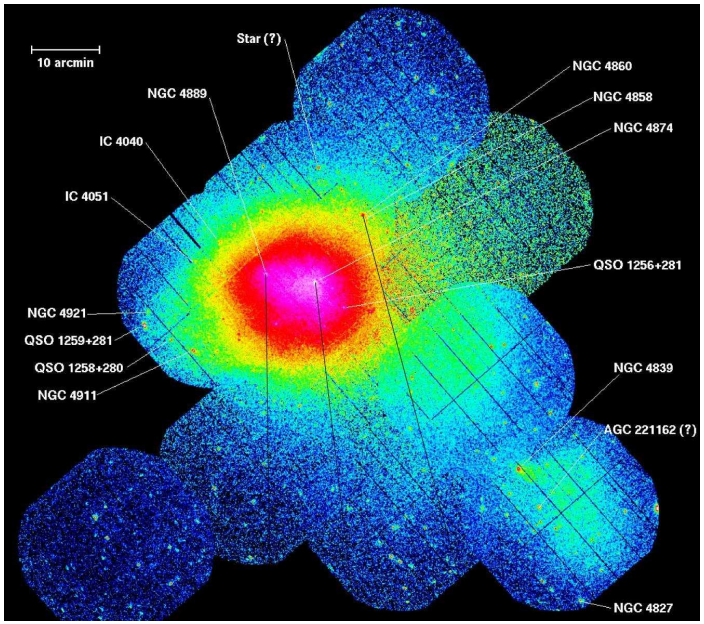


(Wise, McNamara & Murray, 2004, Fig. 2)

X-ray spectrum of A1068 obtained from *Chandra*



XMM-Newton explores the fossil galaxy cluster RX J1416.4+2315
Image courtesy of Habib Khosroshahi (University of Birmingham)
European Space Agency



Technical problems:

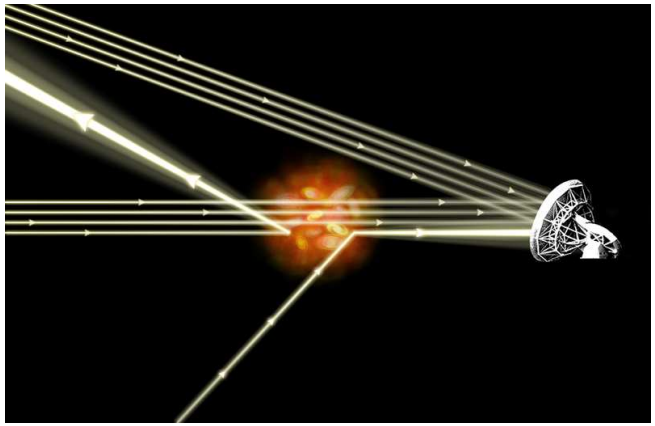
- see through cluster
  $\Longrightarrow$ integrate over line of sight, assuming spherical geometry.
- spherical geometry is assumed
- it is unclear whether gas is in hydrostatic equilibrium (cooling flows? – but note, there is sparse evidence for a "flow")

XMM-Newton, EPIC-pn

General Result:

$$\Omega_{\mathrm{m}} = \Omega_{\mathrm{b}}/f_{\mathrm{gas}} = (0.3 \pm 0.05)\, h^{-1/2} \qquad (26.18)$$

Sunyaev-Zeldovich



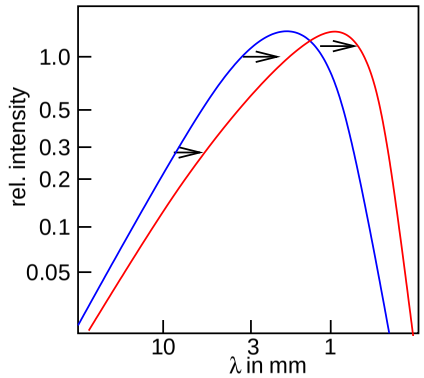
NASA/CXC/M.Weiss

Gas in cooling flow influences CMBR by Compton upscattering

$\Longrightarrow$ Sunyaev-Zeldovich effect (1970).

Sunyaev-Zeldovich

The quantitative derivation of the SZ-effect cannot be done in an introductory lecture.



after Schneider

The basic ingredients are the optical depth for Compton scattering (Compton y-parameter):

$$y = \int \left(\frac{kT_{\mathrm{e}}}{m_{\mathrm{e}}c^2}\right) \sigma_{\mathrm{T}} N_{\mathrm{e}}\, \mathrm{d}l \qquad (26.19)$$

From this follows in the Rayleigh-Jeans regime that the intensity due to Compton upscattering changes as follows:

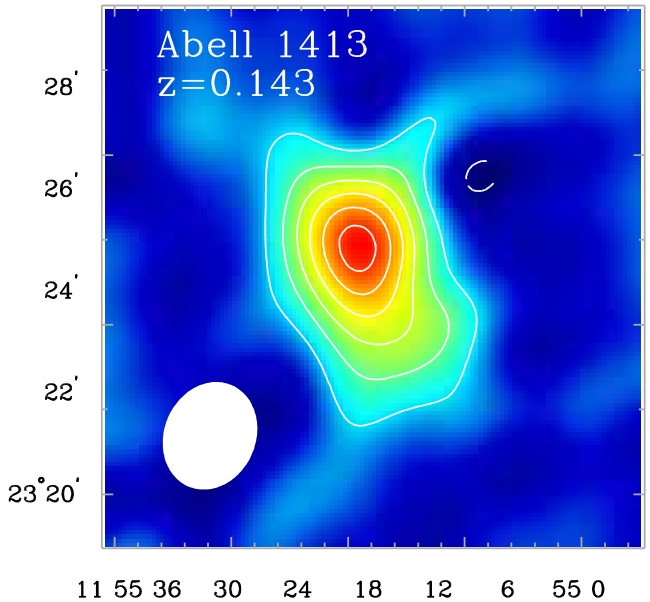
$$\frac{\Delta I}{I} = -2y \sim 10^{-4} \qquad (26.20)$$

(for typical parameters).

$\Longrightarrow$ ΔI allows to measure of $\int N_{\mathrm{e}} T_{\mathrm{e}}\, \mathrm{d}l$

$\Longrightarrow$ Mass!

T is known from X-ray spectrum.



(temperature decrement from 3 K background, Carlstrom et al., 2000, Fig. 3)

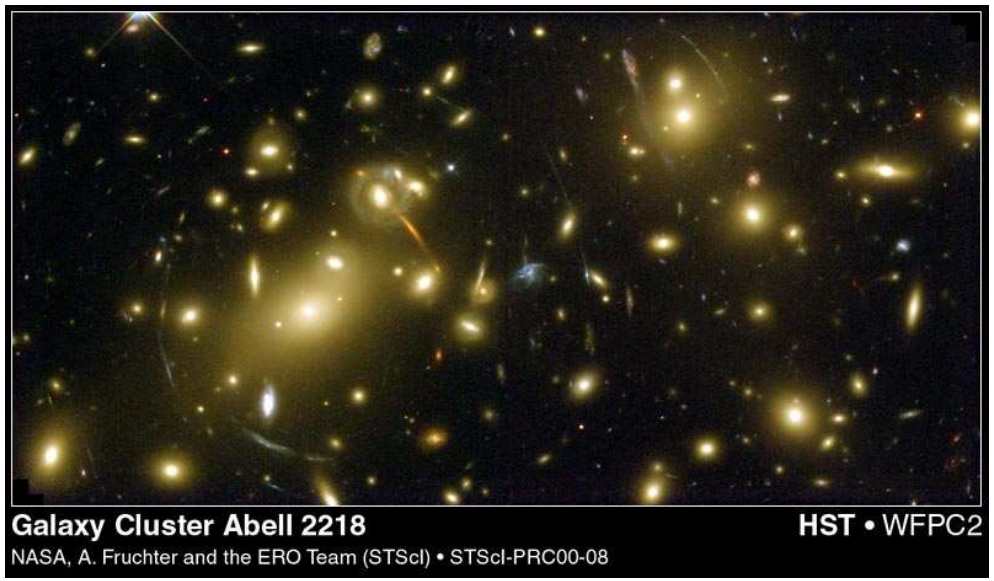
SZ analysis gives gas fraction for 27 clusters

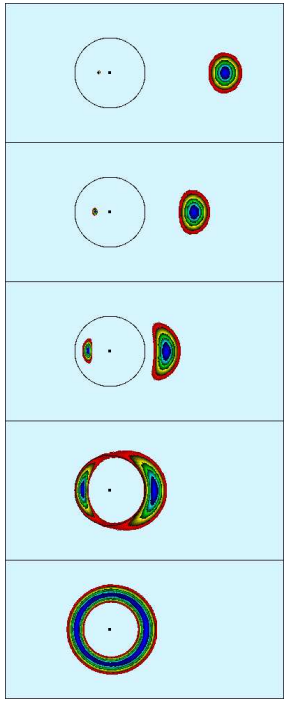
$$f_{\mathrm{gas}} = (0.06 \pm 0.006) h^{-3/2} \qquad (26.21)$$

remarkably similar to X-ray result $\Longrightarrow$ clumping of gas does not influence results! (SZ only traces real gas…)

f_{gas} translates to

$$\Omega_{\mathrm{m}} = (0.25 \pm 0.04)\, h^{-1} \qquad (26.22)$$





Einstein ring: source directly behind lens,
Lens Radius is found to be:

$$\theta_{\mathrm{E}}^2 = \frac{4GM}{c^2}\frac{1}{D} \qquad (26.24)$$

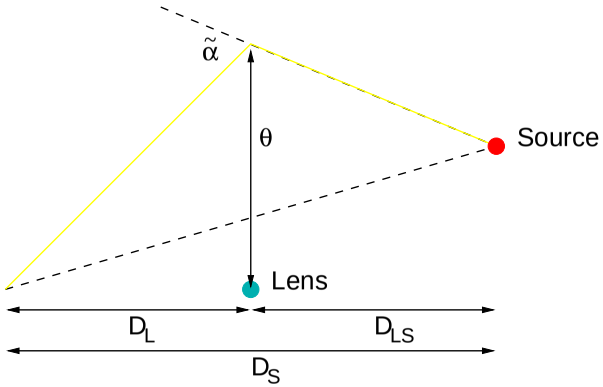
i.e.,

$$\theta_{\mathrm{E}} = 98.9'' \left(\frac{M}{10^{15}\,M_\odot}\right)^{1/2} \frac{1}{(D/1\,\mathrm{Gpc})^{1/2}} \qquad (26.25)$$

Mass measurements possible by observing "giant luminous arcs" and Einstein rings.

26–22

Gravitational Lenses

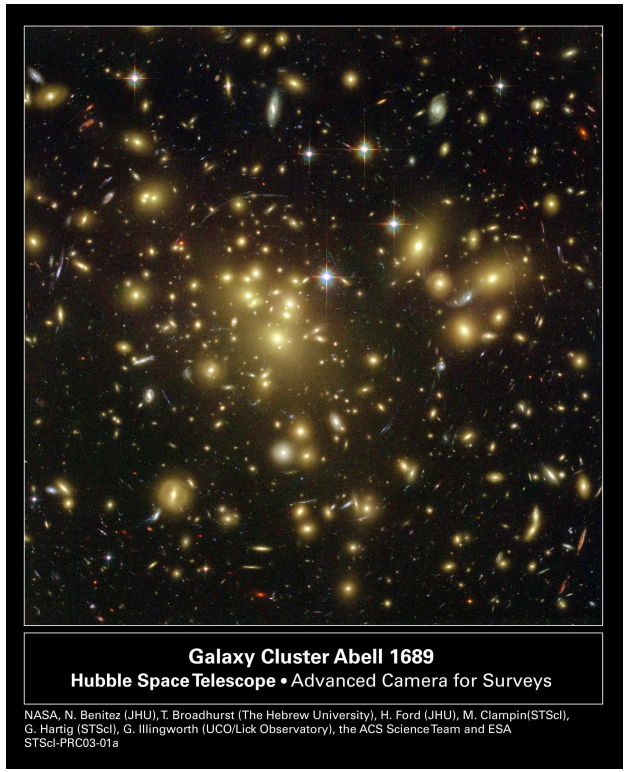


(after Longair, 1998, Fig. 4.8a)

GR: Angular deflection of light due to presence of mass M:

$$\tilde{\alpha} = \frac{4GM}{\theta c^2} = \frac{2}{c^2}\cdot\frac{2GM}{\theta} \qquad (26.23)$$

where θ: distance of closest approach (*twice* the classical result).



General results of mass determinations from lensing agree with other methods.

Summary

So far, we have seen:

Photons:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \tag{26.26}$$

Neutrinos:

$$\Omega_\nu h^2 = 1.69 \times 10^{-5} \tag{26.27}$$

Baryons (from nucleosynthesis):

$$\Omega_\mathrm{b} h^2 = 0.02 \quad \text{where} \quad \Omega_\mathrm{stars} \sim 0.005 \ldots 0.01 \tag{26.28}$$

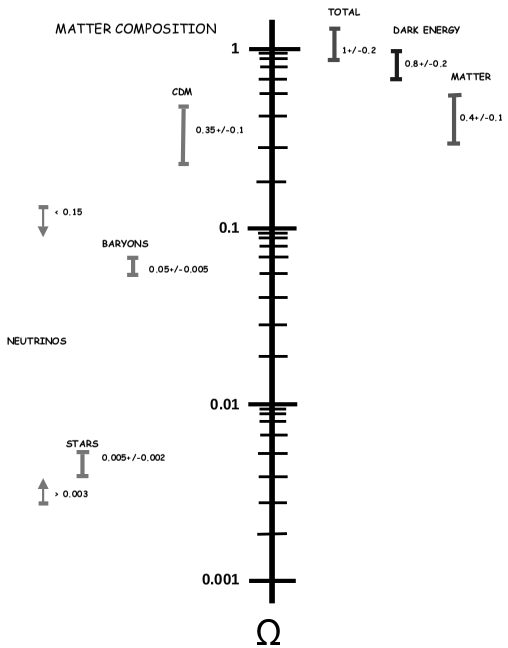
Baryons+dark matter (from clusters):

$$\Omega_\mathrm{m} \sim 0.25 \tag{26.29}$$

(of which $\sim 10\%$ in baryons)

If we believe in $\Omega_\mathrm{total} \equiv 1 \Longrightarrow \Omega_\Lambda \sim 0.7$.

MATTER / ENERGY in the UNIVERSE



(Turner, 1999, Fig. 1, numbers slightly different to ours. . .)

The Nature of Dark Matter

Big Bang Nucleosynthesis:

Normal (baryonic) matter density: only $\sim 4\%$ of the critical density ($\Omega_m \sim 0.04$).

Observed (concordance) model: $\Omega = 0.28$

What is the missing Dark Matter?

Nonbaryonic dark matter

Requirements:

- gravitating
- no or very weak other interaction with baryons (="us")

The Nature of Dark Matter

$\Longrightarrow$ Grab-box of elementary particle physics:

1. Neutrinos with non-zero mass

   Pro: It exists, mass limits are a few eV, need only $\langle m_\nu c^2 \rangle \sim 10\,\mathrm{eV}$

   Contra: ν are relativistic ($v \sim c$), this has implications for galaxy formation that make it unlikely that they form a major part of dark matter.

2. WIMPs (weakly interacting massive particles; masses $mc^2 \sim$ GeV)

   Pro: may be identified with Super-symmetric particles predicted by elementary particle theory.

   Contra: We do not know they exist. . . (but they might soon be detectable by the Large Hadron collider (LHC) at CERN.)

WIMPS are heavy and therefore non-relativistic ("move slowly"). $\Longrightarrow$ helps to explain formation of first stars and galaxies.

WIPMS = Cold Dark Matter

Burles, S., Nollett, K. M., & Turner, M. S., 1999, Big-Bang Nucleosynthesis: Linking Inner Space and Outer Space, APS Centennial Exhibit, astro-ph/9903300

Carlstrom, J. E., Joy, M. K., Grego, L., Holder, G. P., Holzapfel, W. L., Mohr, J. J., Patel, S., & Reese, E. D., 2000, Phys. Scr., T85, 148

Merritt, D., 1987, ApJ, 313, 121

Peacock, J. A., 1999, Cosmological Physics, (Cambridge: Cambridge Univ. Press)

Turner, M. S., 1999, in The Third Stromlo Symposium: The Galactic Halo, ed. B. K. Gibson, T. S. Axelrod, M. E. Putmann, ASP), in press (astro-ph/9811454)

Wise, M. W., McNamara, B. R., & Murray, S. S., 2004, ApJ, 601, 184

Determination of Λ

Friedmann with $\Lambda \neq 0$

$\Longrightarrow$ So far we have ignored Λ

Friedmann equation with $\Lambda \neq 0$:

$$H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3} \tag{25.6}$$

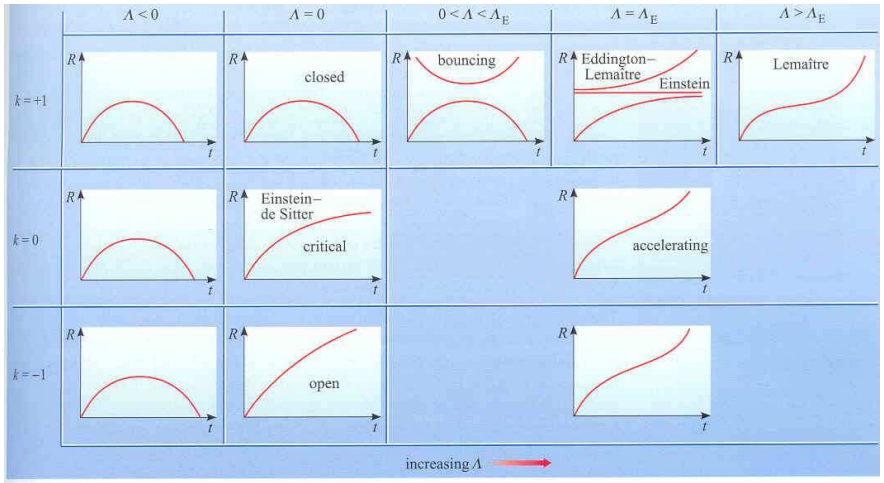
And define the Ωs:

$$\Omega_{\mathrm{m}} = \frac{8\pi G\rho_{\mathrm{m}}}{3H_0^2}, \qquad \Omega_\Lambda = \frac{\Lambda c^4}{3H_0^2}, \qquad \Omega_k = -\frac{kc^2}{R_0^2 H_0^2} \tag{27.1}$$

Because of Eq. (25.6),

$$\Omega_{\mathrm{m}} + \Omega_\Lambda + \Omega_k = \Omega + \Omega_k = 1 \tag{27.2}$$

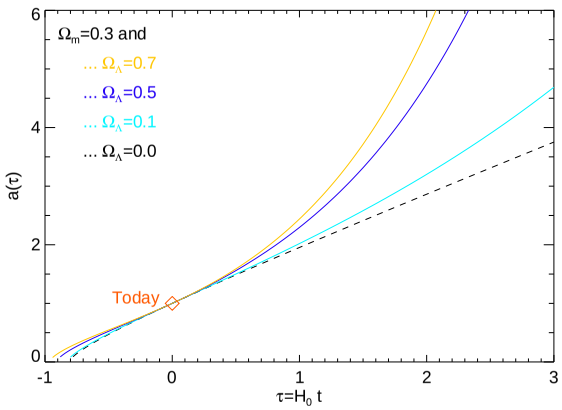
Friedmann with $\Lambda \neq 0$



Jones & Lambourne: An Introduction to Galaxies and Cosmology

Many different kinds of world models are possible, depending on Ω und Λ.

$\Omega_\Lambda < 1$



For $\Omega_\Lambda < 1$: first matter domination, similar to earlier results, then Λ domination, exponential rise.

Universes with $\Omega_\Lambda > 0$ are *older* than those with $\Omega_\Lambda = 0$.

This solves the age problem, that some globular clusters have age comparable to age of universe if $\Omega_\Lambda = 0$.

Motivation

We have already seen that evolution of the universe depends on

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} = \Omega_{\text{m}} + \Omega_\Lambda \tag{27.3}$$

where

$\Omega_{\mathbf{m}} = 0.3$**:** Ω due to gravitating stuff,
Ω_Λ**:** Ω due to vacuum energy or other exotic stuff.

If inflation is true and $\Omega = 1$, then

$$\Omega_\Lambda = \frac{\rho_{\text{vac}}}{\rho_{\text{crit}}} = \frac{\rho_{\text{vac}}}{3H^2/8\pi G} = \frac{\Lambda c^4}{3H^2} = 0.7 \tag{27.4}$$

Influence of Λ is most prominent at large distances!

$\Longrightarrow$ Expect influence on Hubble Diagram.

Luminosity Distance

For Hubble diagram: Need to find relation between measured flux, emitted luminosity, and redshift.

Assume source with luminosity L at comoving coordinate r, emitting isotropically into 4π sr.

At time of detection today, photons are

- on sphere with proper radius $R_0 r$,
- redshifted by factor $1 + z$,
- spread in time by factor $1 + z$.

$\Longrightarrow$ observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} =: \frac{L}{4\pi d_{\text{L}}^2} \quad \text{where} \quad d_{\text{L}} = R_0 \cdot r \cdot (1+z) \tag{27.5}$$

where d_{L} is called the luminosity distance

The calculation of d_{L} is somewhat technical, one can show that (Carroll, Press & Turner, 1992):

$$d_{\text{L}} = \frac{c}{H_0} |\Omega_k|^{-1/2} \cdot S_{-\text{sgn}(\Omega_k)} \left\{ |\Omega_k|^{1/2} \int_0^z \left[(1+z)^2 (1+\Omega_{\text{m}} z) - z(2+z)\Omega_\Lambda \right]^{1/2} \mathrm{d}z \right\} \tag{27.6}$$

Supernovae

Best way to determine Ω_Λ:

Type Ia supernovae

The distance modulus is

$$m - M = 5 \log \left(\frac{d_{\text{L}}}{1\,\text{Mpc}} \right) + 25 \tag{27.7}$$

Use SNe as standard candles $\Longrightarrow$ Deviations from $d_{\text{L}} \propto z$ are indicative of Λ.

Two projects:

- High-z Supernova Team (STSCI, Riess et al.)
- Supernova Cosmology Project (LBNL, Perlmutter et al.)

Both find SNe out to $z \sim 1$.

Supernovae

Basic observations: easy:

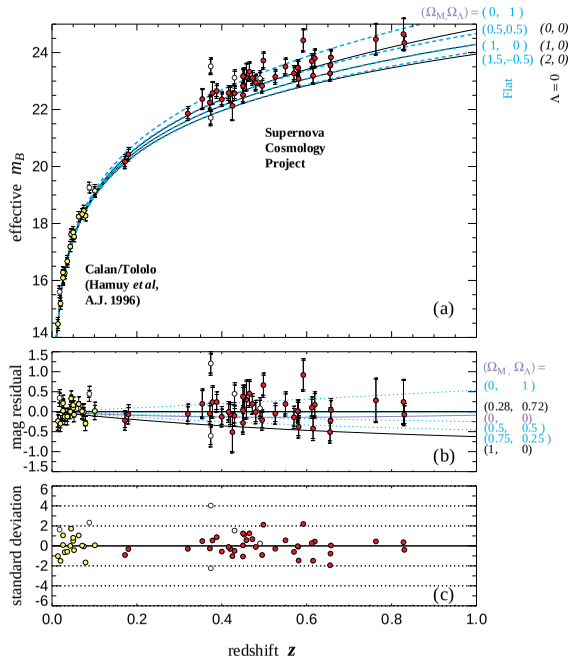
- Detect SN in rise $\Longrightarrow$ CTIO 4 m
- Follow SN for $\sim$ 2–3 months with 2–4 m class telescopes, HST, Keck. . .

More technical problems in data analysis: Conversion into source frame:

- Correction of photometric flux for redshift: “K-correction”
- Correct for time dilatation in SN light curve

Further things to check

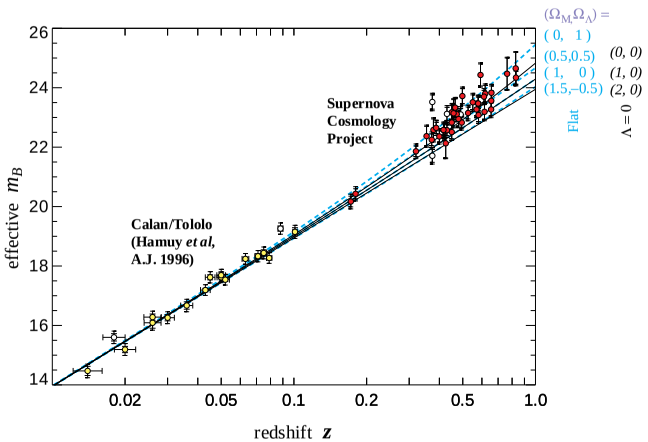
- SN internal extinction
- Galactic extinction
- Galactic reddening
- Photometric cross calibration
- Peculiar motion of SN



Best fit: $\Omega_{\mathrm{m,\ flat}} = 0.28^{+0.09}_{-0.08}$,
$\chi^2/\mathrm{DOF} = 56/50$
corresponding best free fit:
$(\Omega_\mathrm{m}, \Omega_\Lambda) = (0.73, 1.32)$.

(Perlmutter et al., 1999, Fig. 2)

Supernovae

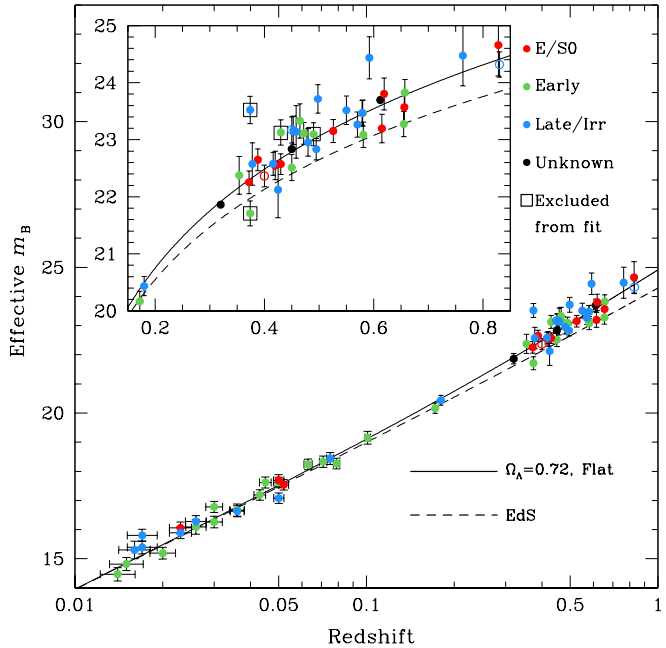


Vertical error bars: measurement uncertainty *plus* 0.17 mag intrinsic mag. dispersion

Horizontal error bars: 300 km s^{-1} peculiar velocity uncertainty

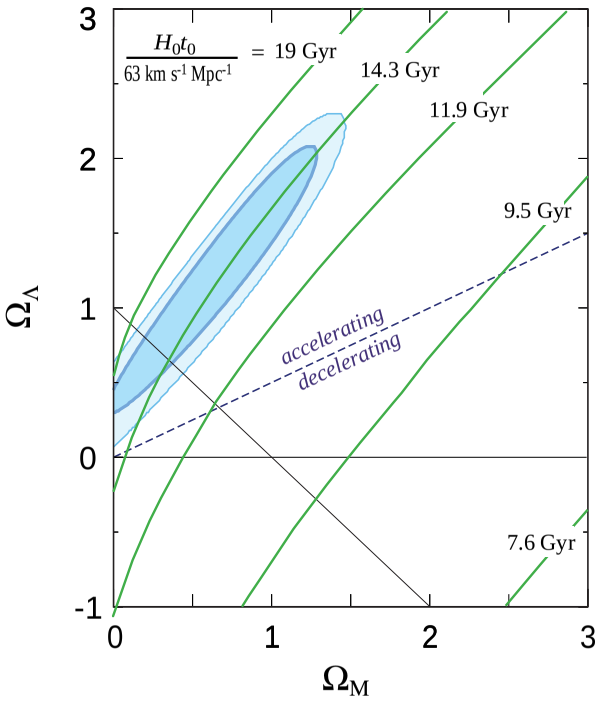
(Perlmutter et al., 1999, Fig. 1)

42 SNe from SCP, 18 low redshift from Calán/Tololo SN Survey



Updated 2002 Hubble diagram for SN Iae confirms Perlmutter (1999).

Sullivan et al., 2002



Isochrones for age of universe for $H_0 = 63\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (for $h = 0.7$: age 10% smaller).
$\Longrightarrow$ Consistent with globular cluster ages!

(Perlmutter et al., 1999, Fig. 9)

Summary

For all practical purposes, currently the best values of Ω_m and Ω_Λ are

$$\Omega_\mathrm{m} \sim 0.3 \qquad \text{and} \qquad \Omega_\Lambda = 0.7$$

Even if $\Omega \neq 1$:

$$\Omega_\Lambda \neq 0$$

And therefore

Baryons are an energetically unimportant constituent of the universe.

The Nature of Dark Energy

- What is the physical meaning of Einstein's cosmological constant Λ?
- candidate: vacuum energy (Ground state energy of the universe)
- Equation of state:

$$P_\mathrm{vac} = -\rho_\mathrm{vac} \cdot c^2 = -u_\mathrm{vac} \tag{27.8}$$

  where u_vac = energy density $\Longrightarrow$ negative pressure!
- quantum field theory predicts a huge energy density: $u_\mathrm{vac} = 10^{111}\,\mathrm{J\,m^{-3}}$
- Observation: $u_\mathrm{dark} = \rho_\Lambda \cdot c^2 = 6 \cdot 10^{-10}\,\mathrm{J\,m^{-3}}$
- mismatch by 120 orders of magnitude

The Nature of Dark Energy

Currently discussed to solve vacuum energy problem: quintessence: "rolling scalar field", corresponding to very lightweight particle ($\lambda_\text{de Broglie} \sim 1$ Mpc), looks like time varying cosmological "constant".

Why? $\Longrightarrow$ More naturally explains why Ω_Λ so close to 0 (i.e., why matter and vacuum have so similar energy densities)

Motivated by string theory and M theory. . .

Still VERY SPECULATIVE, decision Λ vs. quintessence should be possible in next 5. . . 10 years when new instruments become available.

Carroll, S. M., Press, W. H., & Turner, E. L., 1992, ARA&A, 50, 499

Perlmutter, S., et al., 1999, ApJ, 517, 565