PX 144: Introduction to Astronomy: Useful formulae

The following is a collection of formulae that are useful to remember. Please see the handouts for an explanation of the symbols used. While you will be given all the constants in exams and in the homework, for sanity checking answers and for getting a feel for the order of magnitude of things, the numbers given are useful as well[1].

1 Gravitation

1.1 Ellipses

Eccentricity:

$$e = \frac{\text{distance between foci}}{\text{major axis}} = \frac{d}{a}$$

Perihelion and Aphelion distances:

$$d_{\text{perihelion}} = a(1-e) \quad \text{and} \quad d_{\text{aphelion}} = a(1+e)$$

1.2 Gravitation

Newton's law of gravitation:

$$F_{\text{grav}} = G\frac{m_1 m_2}{R^2}$$

Gravitational constant:

$$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$$

Center of mass:

$$m_1 r_1 = m_2 r_2$$

Centripetal force on a circular orbit:

$$F = \frac{mv^2}{r} = \frac{4\pi^2 mr}{P^2}$$

Newton's form of Kepler's 3rd law (generally it is useful to know how this one is derived, at least for $m_2 \longrightarrow 0$):

$$\frac{a^3}{P^2} = \frac{G(m_1+m_2)}{4\pi^2}$$

Kepler's 3rd law in units of $M_\odot$, AU, and years (set $M_2 = 0$ for planets)

$$\frac{a^3}{P^2} = M_1 + M_2$$

Mass function for binary stars:

$$f(M,m) = \frac{m^3 \sin^3 i}{(M+m)^2} = \frac{r_1^3}{P^2}$$

2 Properties of Planets

Typical densities for terrestrial planets and gas giants:

$$\langle\rho\rangle_{\text{terr.}} \sim 5.5\,\mathrm{g\,cm^{-3}} \quad \langle\rho\rangle_{\text{gas giants}} \sim 1.2\,\mathrm{g\,cm^{-3}}$$

Typical masses:

- $M_{\text{Earth}} = 6 \times 10^{24}\,\mathrm{kg}$
- $M_{\text{Jupiter}} = 318\,M_{\text{Earth}}$

Astronomical Unit:

$$1\,\mathrm{AU} = 150 \times 10^6\,\mathrm{km}$$

Typical size scales for Solar system:

- Mercury: 0.4 AU
- Earth: 1 AU
- Mars: 1.5 AU
- Jupiter: 5 AU
- Saturn: 10 AU
- Neptune: 30 AU

3 Properties of Stars

3.1 Distances

Angular distance between extrasolar Planet and star or between two double stars:

$$\theta = \frac{r}{d}$$

Parallax angle (2nd equation for p in arcseconds and d in parsecs):

$$p = \frac{1\,\mathrm{AU}}{d} = \frac{1}{d}$$

Parsec:

$$1\,\mathrm{pc} = 206264\,\mathrm{AU} = 3 \times 10^{16}\,\mathrm{m} = 3.26\,\mathrm{ly}$$

3.2 Magnitudes

Luminosity and flux:

$$F = \frac{L}{4\pi d^2}$$

Apparent magnitude:

$$m_2 - m_1 = -2.5\log_{10}\left(\frac{F_2}{F_1}\right)$$

Brightest star: Sirius: $m = -1.4\,\mathrm{mag}$
Faintest stars: $m \sim 6\,\mathrm{mag}$
Absolute magnitude M:

magnitude object would have if at $d = 10\,\mathrm{pc}$

Distance modulus:

$$m - M = 2.5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right)^2 = 5\log_{10} d - 5$$

3.3 Stellar Structure

Typical "solar" composition (by mass):

- 75% Hydrogen,
- 24% Helium,
- 1% "metals"

Equation of hydrostatic equilibrium:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\frac{GM(r)}{r^2}$$

Ideal gas law:

$$P = nkT$$

Mass-Luminosity Relationship (solar units):

$$L \propto M^4$$

[1] As a general rule of thumb for doing physics, knowing detailed numbers by heart is not necessary since for day to day work these can be looked up. What *is* neccessary, though, and what makes a good physicist, is that you should develop a "feel" for order of magnitude estimates enabling you to check the outcome of more detailed computations.

3.4 Compact Objects

Schwarzschild radius:

$$r_S = \frac{2GM}{c^2} \sim 3\,\mathrm{km}\left(\frac{M}{1\,M_\odot}\right)$$

3.5 Typical Numbers

Properties of the Sun:

$$\begin{array}{rlrl} M_\odot & = 2\times10^{30}\,\mathrm{kg} & L_\odot & = 3.90\times10^{26}\,\mathrm{J\,s^{-1}} \\ R_\odot & = 700000\,\mathrm{km} & T_{\mathrm{eff}} & = 5800\,\mathrm{K} \\ m & = -26\,\mathrm{mag} & M & = 5\,\mathrm{mag} \end{array}$$

Properties of the stars:

$$\begin{array}{rcl} 0.01 \lesssim & L/L_\odot & \lesssim 10^4 \\ 0.01 \lesssim & M/M_\odot & \lesssim 100 \\ -7\,\mathrm{mag} \lesssim & M_\mathrm{v} & \lesssim +13\,\mathrm{mag} \\ 3000\,\mathrm{K} \lesssim & T_{\mathrm{eff}} & \lesssim 50000\,\mathrm{K} \end{array}$$

4 Radiation

Energy of photons:

$$E = h\nu = hc/\lambda$$

Doppler effect (nonrelativistic speeds):

$$\frac{\lambda_{\mathrm{observed}} - \lambda_{\mathrm{emitted}}}{\lambda_{\mathrm{emitted}}} = \frac{v}{c}$$

Doppler effect (relativistic speeds):

$$\lambda_{\mathrm{observed}} = \lambda_{\mathrm{emitted}}\sqrt{\frac{1+v/c}{1-v/c}}$$

Planck's law:

$$B_\lambda = \frac{2hc^2/\lambda^5}{\exp(hc/\lambda kT) - 1}$$

Stefan-Boltzmann law (power emitted per square metre by surface of temperature T):

$$P = \sigma T^4$$

Wien's law:

$$\lambda_{\max} T = 2.898\times10^{-3}\mathrm{m\,K}$$

5 Cosmology

Hubble relation: The redshift of a galaxy is proportional to its distance:

$$v = cz = H_0 d$$

where $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 2.3\times10^{-18}\,\mathrm{s^{-1}}$.
$1/H_0$ is roughly the age of the universe.
Critical density:

$$\Omega = \frac{\rho}{\rho_{\mathrm{crit}}} \qquad \text{where} \qquad \rho_{\mathrm{crit}} = \frac{3H_0^2}{8\pi G}$$