


Direct distance measurements: parallax measurement:
$\Longrightarrow$ Measure stellar position several times over year with respect to background stars.
Parallax angle (small angle approximation):

$$
p=\frac{1 \mathrm{AU}}{d}
$$

( $p$ is measured in radians)
Typical values for $p$ are arcseconds
$\Longrightarrow$ define distance unit "Parsec" ("parallax second") such that $d=1 \mathrm{pc}$ for $p=1^{\prime \prime}$ :

The parsec is the distance at which 1 AU subtends an angle of $1^{\prime \prime}$.

Some people use $\pi$ instead of $p$ for the parallax.

Observational Properties: Distances

## Distances

How far is one parsec?

From $p=1 \mathrm{AU} / d$ follows with $p=1^{\prime \prime}$ :
$d=1 \mathrm{pc}=\frac{1 \mathrm{AU}}{1^{\prime \prime}}=\frac{1 \mathrm{AU}}{\pi /(180 \cdot 3600)}=206264 \mathrm{AU}=3.086 \times 10^{16} \mathrm{~m} \sim 3.26 \mathrm{ly}$

Note: If parallax $p$ is known and given in arcseconds, then distance can be immediately calculated:

$$
\frac{d}{1 \mathrm{pc}}=\frac{1}{p / 1^{\prime \prime}} \quad \text { or (sloppy notation) } \quad d=\frac{1}{p}
$$

## Distances

Best parallax measurements to date: ESA's Hipparcos satellite (with participation from Heidelberg and Tübingen).

- systematic error of position: $\sim 0.1$ mas
- effective distance limit: 1 kpc
- standard error of proper motion: $\sim 1 \mathrm{mas} / \mathrm{yr}$
- broad band photometry
- narrow band: $\mathrm{B}-\mathrm{V}, \mathrm{V}-\mathrm{J}$ (see later what this means)
- magnitude limit: 12 mag
- complete to 7.3-9.0 mag (see later)

Results available at http://astro.estec.esa.nl/Hipparcos/:
Hipparcos catalogue: 120000 objects with milliarcsecond precision.
Tycho catalogue: $10^{6}$ stars with 20-30 mas precision, two-band photometry

http://www.anzwers.org/free/universe/12lys.html $(12 \mathrm{ly}=3.7 \mathrm{pc})$

http://www. anzwers.org/free/universe/stardist.html


Known stars within $201 \mathrm{ly}=$
6.1 pc: 83 star systems with 109 stars and 3 brown dwarfs Note: We are probably missing many faint stars already in this small volume!
http://www. anzwers org/free/universe/ 201ys.html


## Definition: Luminosity of a star:

The total energy emitted by a star per second is called its luminosity
(=luminosity is a power)

In astronomy, luminosities are often measured in units of the solar luminosity,

$$
L_{\odot}=3.90 \times 10^{26} \mathrm{~J} \mathrm{~s}^{-1}=3.90 \times 10^{26} \mathrm{~W}=3.9 \times 10^{33} \mathrm{erg} \mathrm{~s}^{-1}
$$

Brightness and Luminosity


Flux: energy passing per second through area of $1 \mathrm{~m}^{2}$ at distance $r$ :

$$
F=\frac{L}{4 \pi r^{2}}
$$

(unit: $\mathrm{Wm}^{-2}$ or $\mathrm{erg} \mathrm{cm}^{-2} \mathrm{~s}^{-1}$ ).

Flux, IV
Fluxes from stars (apart from the Sun) are very small.
Example: $\alpha$ Centauri (closest star to the Sun).

- distance: $1.3 \mathrm{pc} \sim 4 \times 10^{16} \mathrm{~m}$
- luminosity: similar to the Sun $\left(4 \times 10^{26} \mathrm{~W}\right)$.
$\Longrightarrow$ Flux arriving on Earth:

$$
F=\frac{L}{4 \pi r^{2}}=\frac{3.9 \times 10^{26} \mathrm{~W}}{4 \pi \cdot 16 \cdot 10^{32} \mathrm{~m}^{2}}=2 \times 10^{-8} \mathrm{~W} \mathrm{~m}^{-2}
$$

(compare with solar constant, $F=1380 \mathrm{~W} \mathrm{~m}^{-2}$ !)
$\Longrightarrow$ your eye detects a power of

$$
P=A_{\text {eye }} F=5 \times 10^{-12} \mathrm{~W}
$$

from $\alpha$ Cen (assuming $A_{\text {eye }} \sim 25 \mathrm{~mm}^{2}$ )!
weakest visible stars: $\sim 100 \times$ weaker!


## Magnitudes,

First classification of stars:

- Stars of "magnitude 1": brightest (visible) stars
- Stars of "magnitude 6": faintest (visible) stars

Pogson (1865): Eye sensitivity is logarithmic, such that
A brightness difference of 5 magnitudes corresponds to a ratio of 100 in detected flux

So, if magnitudes of two stars are $m_{1}$ and $m_{2}$, then

$$
\frac{f_{1}}{f_{2}}=100^{\left(m_{2}-m_{1}\right) / 5}
$$

This means:

$$
\log _{10}\left(f_{1} / f_{2}\right)=\frac{m_{2}-m_{1}}{5} \log _{10} 100=\frac{2}{5}\left(m_{2}-m_{1}\right)
$$

or

$$
m_{2}-m_{1}=2.5 \log _{10}\left(f_{1} / f_{2}\right)=-2.5 \log _{10}\left(f_{2} / f_{1}\right)
$$

Note: Larger Magnitude = FAINTER Stars

Brightness and Luminosity

## Luminosity (revisited)

10-16

Inverse square law links flux $f$ at distance $d$ to flux $F$ measured at another distance $D$ :

$$
\frac{F}{f}=\frac{L / 4 \pi D^{2}}{L / 4 \pi d^{2}}=\left(\frac{d}{D}\right)^{2}
$$

Convention: to describe luminosity of a star, use the absolute magnitude $M$, defined as magnitude measured at distance $D=10 \mathrm{pc}$.

Therefore,

$$
m-M=2.5 \log (F / f)=2.5 \log (d / 10 p c)^{2}=5 \log d-5
$$

$m-M$ is called the distance modulus, $d$ is measured in pc.
Next observable: Temperature
Obtained using spectroscopy
In the following: rough outline, as stellar spectroscopy is rather complicated
Outline:

1. Planck's Radiation Laws
2. Stellar Continuum Spectra
3. Spectral Classification
... unfortunately, need to be a little bit formal first

## Spectroscopy, I

Quantum mechanics: atoms have discrete energy levels
Energy levels in Hydrogen:

$$
E_{n}=-\frac{2 \pi^{2} \mu e^{4}}{\hbar^{2}} \cdot \frac{1}{n^{2}} \propto-\frac{1}{n^{2}}
$$

( $n \in \mathbb{N}$; Balmer formula)


In hydrogen atom: electrons typically found in ground state.
if temperature is higher, can also be in 1st excited state, but the physical principles following remain the same... Spectroscopy, IV


Photon hitting atom has energy $E_{\text {phot }}=h \nu=h c / \lambda$. If $E_{\text {phot }}=E_{2}-E_{1}$, then photon can be absorbed. . . and electron has higher energy (is excited).


After absorption event, absorbing photon has disappeared and hydrogen atom remains in excited state.

Stellar Spectra

N.A.Sharp, NOAO/NSO/Kitt Peak FTS/AURA/NSF

Absorption line spectrum of the Sun: Fraunhofer Lines

| HD 12993 |  |
| :--- | :--- |
| HD 158659 |  |
| HD 30584 |  |
| HD1 16608 |  |
| HD 9547 |  |
| HD 10032 |  |
| BD 610367 |  |
| HD 28099 |  |
| HD 70178 |  |
| HD 23524 |  |
| SAO 76803 |  |
| HD 260655 |  |
| Yale 1755 |  |
| HD 94082 |  |
| SAO 81292 |  |
| HD 13256 |  |

Annie Cannon (around 1890): Stars have different spectra.
NOAO


Annie Jump Cannon (1863-1941) Biography: http:
//www.sdsc .edu/Sci encelomen/cannon. html

Annie Jump Cannon: There are spectral types.

Henry Draper catalogues (Cannon plus $\sim 10$ female "computers"): 225000 spectral classifications.



Annie Cannon: Strength of absorption lines varies with spectral type.

Summary spectral classes as a temperature sequence.


Silva \& Cornell, 1992, ApJ Suppl. 81, 865
Cecilia Payne-Gaposchkin: Spectral sequence is temperature sequence.


Cecilia Payne-Gaposchkin (1900-1979) Biography: http://www.harvardsquarelibrary org/unitarians/payne2.html

10-32
Spectroscopy, XVI

BSc, Cambridge, left UK because of situation of women in astronomy
$1^{\text {st }}$ person to obtain PhD in Astronomy at Harvard: "Stellar Atmospheres, A Contribution to the Observational Study of High Temperature in the Reversing Layers of Stars"
Otto Struve: "undoubtedly the most brilliant Ph.D. thesis ever written in astronomy."

```
Spectral types are a temperature sequence.
```

later: 1st female full professor at Harvard

plus subtypes: B0 . . B9,A0. . A9, etc.
Sun is G2.


Note: "early" and "late" has nothing to do with age!
Mnemonics:
(http://lheawww.gsfc.nasa.gov/users/allen/obafgkmrns.html)

O Be A Fine $\underset{\text { Guy }}{\text { Girl }}$ Kiss Me
Only Boys Accepting Feminism Get Kissed Meaningfully
Only Bold Astronomers Forge Great Knowledgeable Minds
Stellar Spectra


Brown Dwarfs $\Longrightarrow$ extension of spectral types to lower temperature objects
L dwarfs: objects with temperatures of $1200-2500 \mathrm{~K}$, low mass, some do not support fusion. Spectra peak in IR, optical spectra contain prominent lines from metal hydrides and alkali metals.
Designation: $L$ is character closest to $M$ that was still available.
T dwarfs: brown dwarfs with temperatures of $\sim 1000 \mathrm{~K}$, strong lines from molecules such as methane in the spectrum.

See Kirkpatrick (2005, Ann. Rev. Astron. Astrophys., 43, 195) for an overview and the formal definition of these spectral types.

## Ursae Majoris




Masses, XIII
Mizar A and B are rather typical stars:
$50 \%-80 \%$ of all stars in the solar neighbourhood belong to multiple systems.

## Rough classification:

apparent binaries: stars are not physically associated, just happen to lie along same line of sight ("optical doubles").
visual binaries: bound system that can be resolved into multiple stars (e.g., Mizar); can image orbital motion, periods typically 1 year to several 1000 years.
spectroscopic binaries: bound systems, cannot resolve image into multiple stars, but see Doppler effect in stellar spectrum; often short periods (hours. . . months).


Astrometric binaries: Motion of stars around common center of mass results in a
'wobble" around the CM (since CM is moving along a straight line).


Taking out proper motion leaves us with binary star orbits.

Masses, XVII
To determine stellar masses, use Kepler's 3rd law:

$$
\frac{a^{3}}{P^{2}}=\frac{G}{4 \pi^{2}}\left(m_{1}+m_{2}\right)
$$

where

- $M_{1,2}$ : masses
- $P$ : period
- $a$ semimajor axis

Observational quantities:

- $P$ - directly measurable
- $a$ - measurable from image if and only if distance to binary and the inclination are known

Masses


Masses



HDE 226868/Cyg X-1; Pottschmidt (2001)

Motion of star visible through Doppler shift in stellar spectrum:
$\frac{\Delta \lambda}{\lambda}=\frac{v_{r}}{c}=\frac{v}{c} \sin i \cos \omega t$
For virtually all stars, classical Doppler effect is enough; once
$v \gtrsim 0.1 c$, however, use relativistic Doppler effect,

$$
\nu_{\mathrm{obs}}=\nu_{\mathrm{em}} \sqrt{\frac{1+v / c}{1-v / c}}
$$



To derive the mass function, we start as usual with Kepler's 3rd law.

$$
\frac{G}{4 \pi^{2}}\left(M_{1}+M_{2}\right)=\frac{R^{3}}{P^{2}}
$$

In the following, we will assume that we observe the spectral lines from star number 1 only.
Because of the center of mass definition,
$M_{1} r_{1}=M_{2} r_{2}$
such that

$$
R=r_{1}+r_{2}=r_{1}\left(1+\frac{r_{2}}{r_{1}}\right)=r_{1}\left(1+\frac{M_{1}}{M_{2}}\right)
$$

In the case that the orbits are circular, the velocity of the star whose spectrum we see is

However, due to the unknown inclination, we only observe the radial velocity component, that is
rms of the observables, $r_{1}$ is
such that finally

$$
\begin{gathered}
r_{1}=\frac{P}{2 \pi} v_{1}=\frac{P}{2 \pi} \frac{v_{\text {os }}}{\sin i} \\
R=r_{1}\left(1+\frac{M_{1}}{M_{2}}\right)=\frac{P}{2 \pi} \frac{v_{\text {oss }}}{2 \pi \sin i}\left(1+\frac{M_{1}}{M_{2}}\right)
\end{gathered}
$$

and obtain after some straightorward algebra

$$
\frac{G}{4 \pi^{2}}\left(M_{1}+M_{2}\right)=\frac{1}{P^{2}} \frac{P^{3}}{(2 \pi)^{3}} \frac{v_{\text {obs }}^{3}}{\sin ^{3} i}\left(1+\frac{M_{1}}{M_{2}}\right)^{3}
$$

$$
\frac{M_{2}^{3}}{\left(M_{1}+M_{2}\right)^{2}} \sin ^{3} i=\frac{P_{0}{ }_{0}^{3} b_{s}}{2 \pi G}
$$

the mass function. On the right side are the observables $P$ and $v_{\text {obs }}$, on the left hand side the unknowns $i, M_{1}$, and $M_{2}$.

## Mass Function

If only one star visible: can only determine limits for mass: mass function

$$
\frac{P v_{\mathrm{obs}}^{3}}{2 \pi G}=\frac{M_{2}^{3} \sin ^{3} i}{\left(M_{1}+M_{2}\right)^{2}}=: f_{\mathrm{M}}
$$

with observables:

- $v_{\text {obs }}$ : (velocity amplitude of $M_{1}$ )
- $P$ : period


## and unknowns

- $M_{1}$ : mass of "primary star"
- $M_{2}$ : mass of (unseen) "secondary star"
- $i$ : inclination
$\Longrightarrow f_{\mathrm{M}}$ is lower limit for $M_{2}$, since for $M_{1}=0, M_{2}=f_{\mathrm{M}} / \sin ^{3} i \geq f_{\mathrm{M}}$
Often used for neutron star and black hole binaries. .



Finally, stars also classified in luminosity classes la, lb, II,
III, IV, V, VI
"Morgan-Keenan classes"
M-K class is appended to spectral class: Sun: G2 V, Beteigeuze: M2 lab


for stellar evolution

