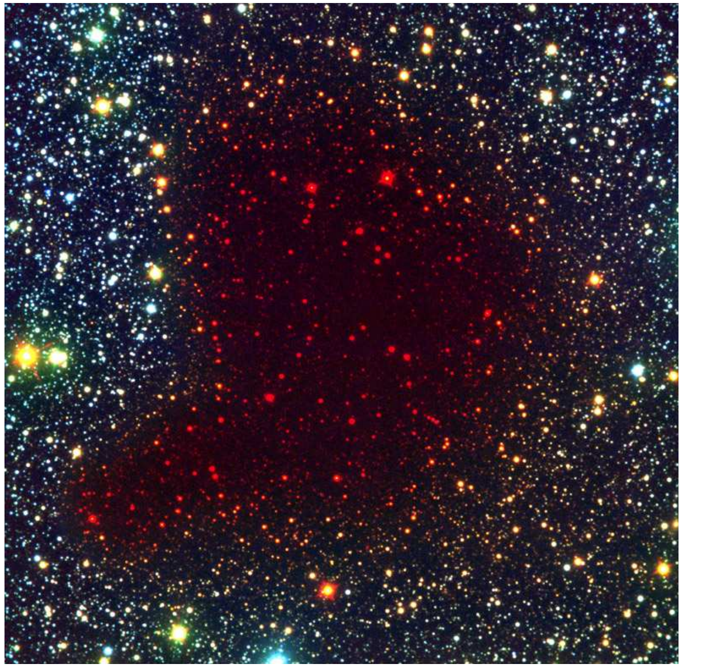
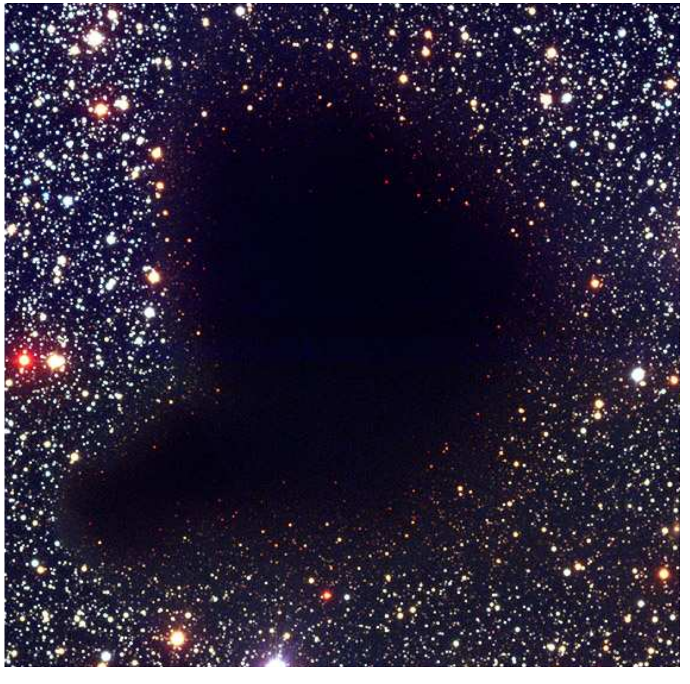
Stellar Structure and Evolution



IR View of B68 (ESO; VLT/FORS1 + NTT/SOFI)



Optical View of B68 (ESO; VLT/FORS1)

Stellar Birth

Stars are born in "Giant Molecular Clouds"

Typical GMC parameters (e.g., Orion):

- large clouds: typical diameters 50–100 pc
- contain lots of molecular gas (H_2, CO, alcohol,...).
- typical temperatures: 10–20 K (coolest regions in the interstellar medium)
- typical particle densities $n \sim 10^6$–$10^{10}\,\mathrm{cm}^{-3}$

Stars are born in groups out of collapsing Molecular Clouds.

Collapse triggered, e.g., by collisions of clouds or shocks caused by nearby supernovae.

Stellar Birth

Criterion for collapse: Cloud is instable, i.e., gravitation is stronger than thermal pressure.

In terms of thermal and gravitational energies, this means

$$\frac{3}{2}\frac{M}{m_\mathrm{p}}kT - \frac{3}{5}\frac{GM^2}{R} \leq 0 \tag{11.1}$$

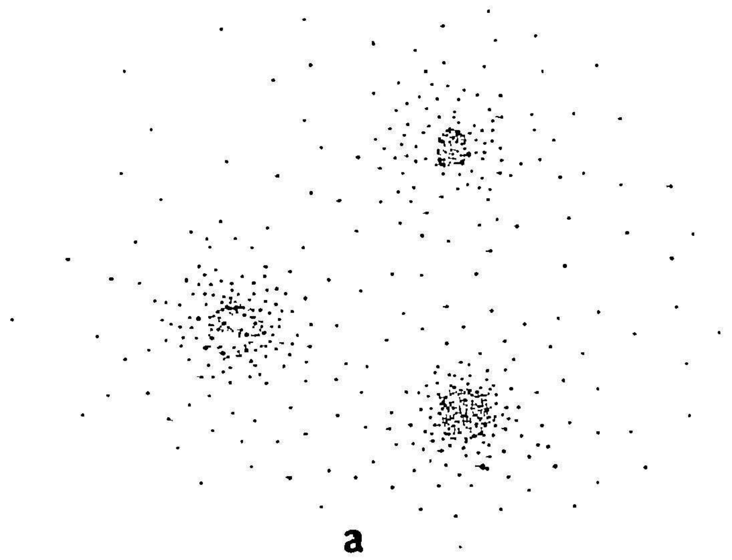
This can be expressed as

$$\frac{M}{R} \geq \frac{5}{2}\frac{kT}{Gm_\mathrm{p}} \quad \text{or} \quad \frac{4\pi}{3}\rho R^2 \geq \frac{5}{2}\frac{kT}{Gm_\mathrm{p}} \tag{11.2}$$

$\Longrightarrow$ Depends on R, collapse thus possible for

$$R > R_\mathrm{J} = \sqrt{\frac{15kT}{8\pi G m_\mathrm{p}\rho}} \sim \sqrt{\frac{kT}{Gm_\mathrm{p}\rho}} \tag{11.3}$$

where R_J is called the Jeans radius.



Shu et al. (1987, ARAA 25, 23, Fig. 7)

Stellar mass cores form from fragmentation of larger pieces.

Note: fragmentation only along B-field lines.

Stellar Birth

Plugging in typical typical numbers, i.e., $T \sim 50$ K, particle density $n = 10^5$ H-atoms cm^{-3} (that is a mass density of $\rho = nm_\mathrm{p} \sim 1.7 \times 10^{-9}$ g cm^{-3}) gives $R_\mathrm{J} \sim 0.2$ pc.

For a given Jeans radius, the mass within R_J is the Jeans mass

$$M_\mathrm{J} \sim \frac{4\pi}{3}R_\mathrm{J}^3\rho$$

...which has typical values of 50–100 $M_\odot$, i.e., larger than one star!

In reality things are more complicated: ISM contains magnetic fields

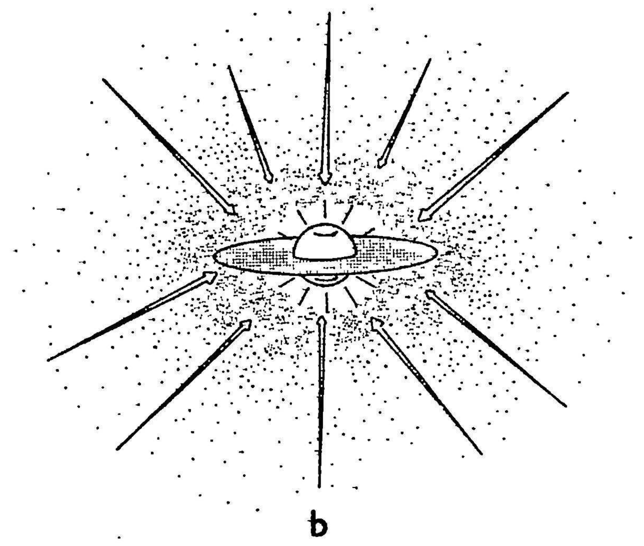
$\Longrightarrow$ Particle motion $\perp$ B-field lines difficult

$\Longrightarrow$ stops gas from collapsing.

This is good since Jeans formalism alone predicts too strong star formation.

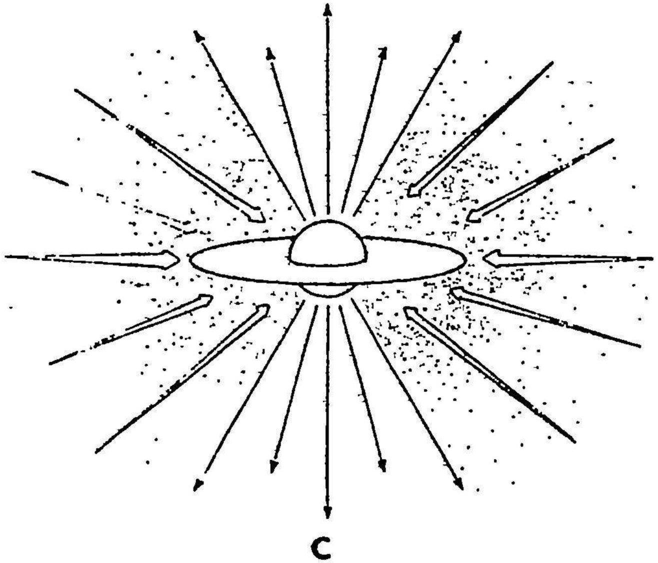
$\Longrightarrow$ Need star formation with magnetic fields

See Shu et al. (1987, Annual Reviews of Astronomy and Astrophysics 25, 23) for the gory details.



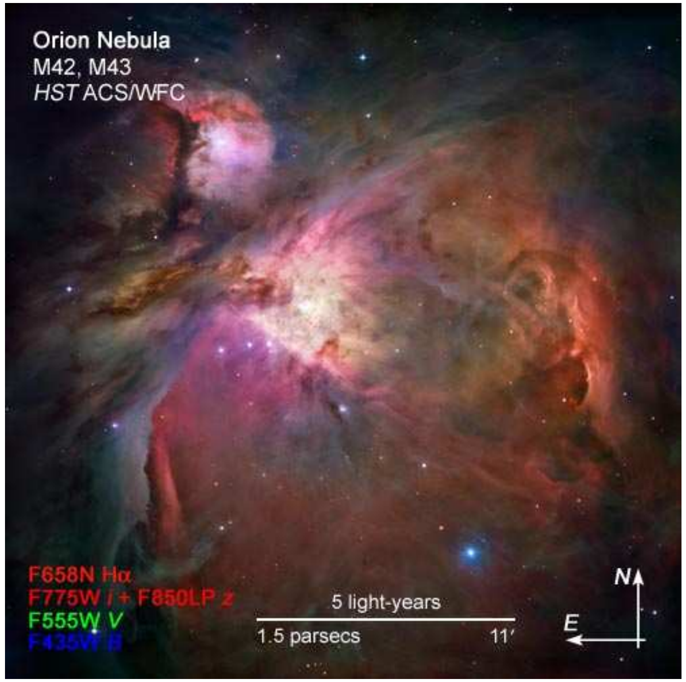
Shu et al. (1987, ARAA 25, 23, Fig. 7)

Protostar forms with surrounding disk ("inside out collapse") once core hot enough to allow fusion ($T > 10^6$ K)



Shu et al. (1987, ARAA 25, 23, Fig. 7)

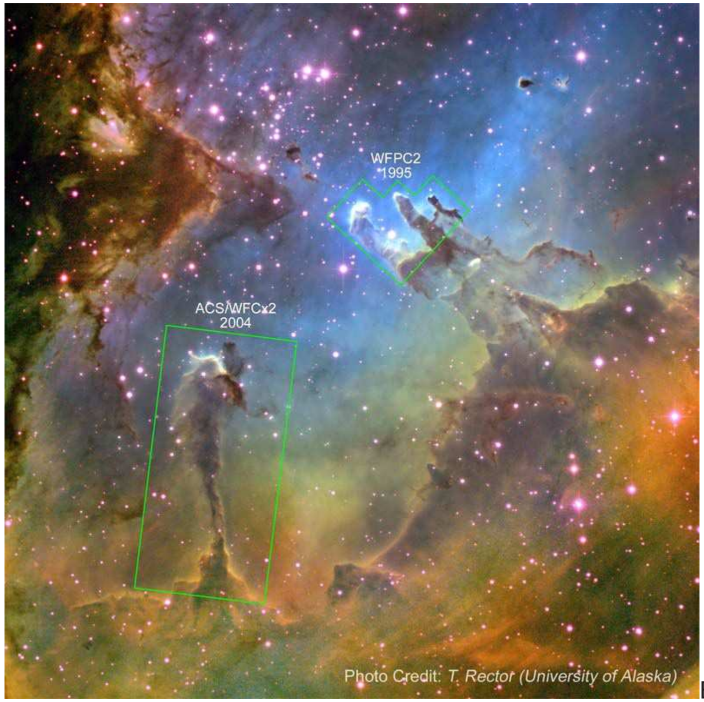
Stellar wind forms bipolar outflow



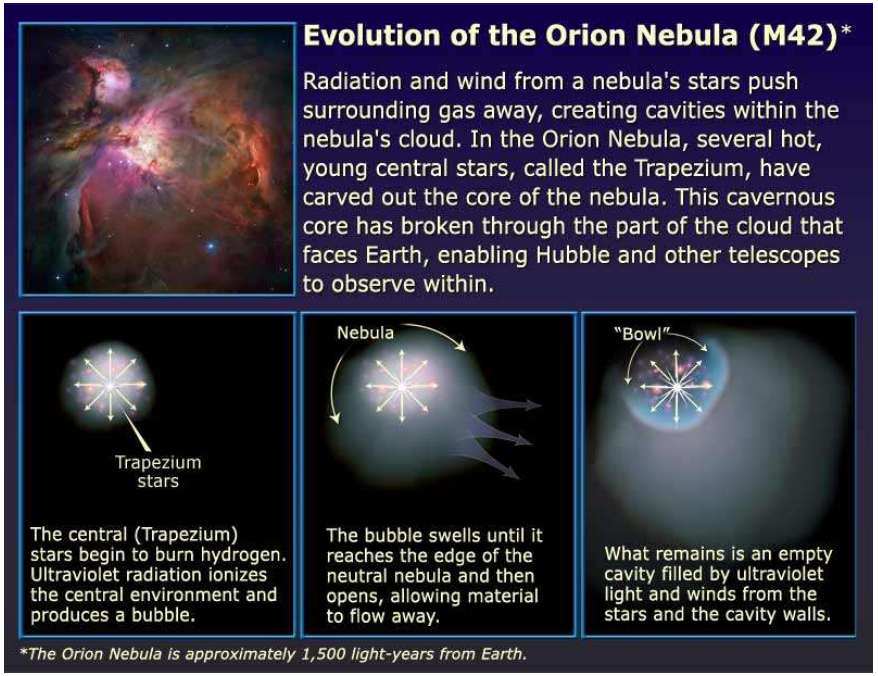
Orion (Bayer's Uranometria; image ©USNO)

Orion Nebula; R. Gendler

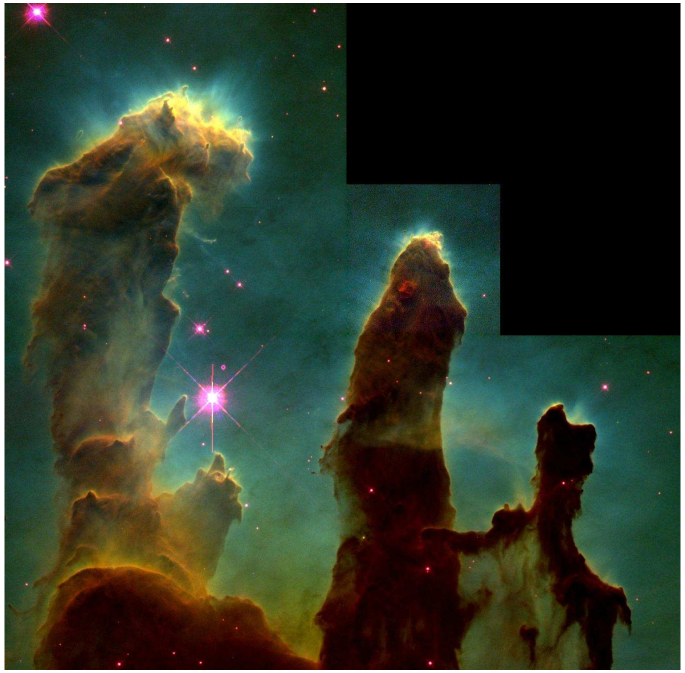
Orion Nebula; R. Croman



Eagle-nebula (M16)

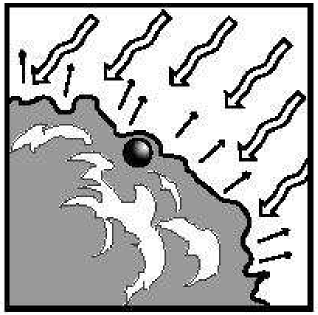


STScI

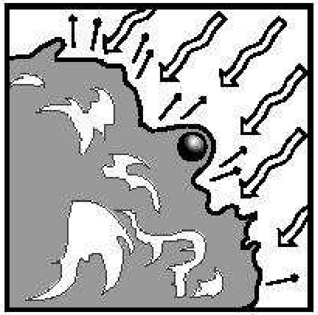


“pillars of creation” in Eagle Nebula (M16)

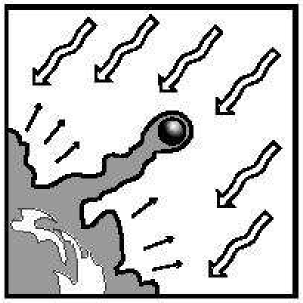
Stellar EGGs in M16



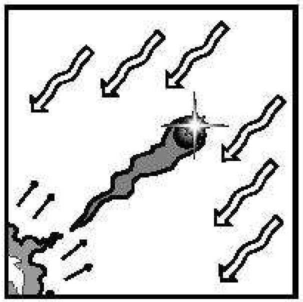
The surface of a molecular cloud is illuminated by intense ultraviolet radiation from nearby hot stars. The radiation evaporates material off of the surface of the cloud.



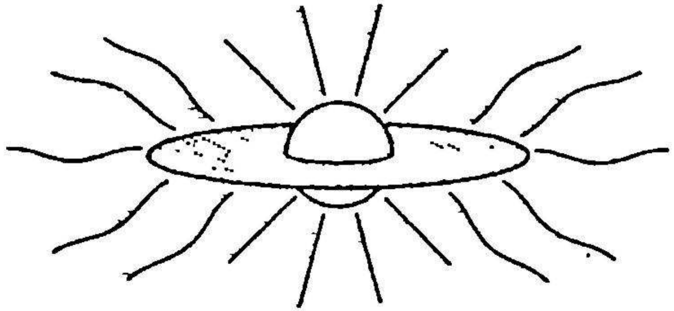
As the cloud is slowly eaten away by the ultraviolet radiation, a denser than average globule of gas begins to be uncovered



The EGG has now been largely uncovered. The shadow of the EGG protects a column of gas behind it, giving it a fing er-like appearance.



Eventually the EGG may become totally separated from teh molecular cloud in which it formed. As the EGG itself slowly evaporates, the star within is uncovered and may appear sitting on the front surface of the EGG.

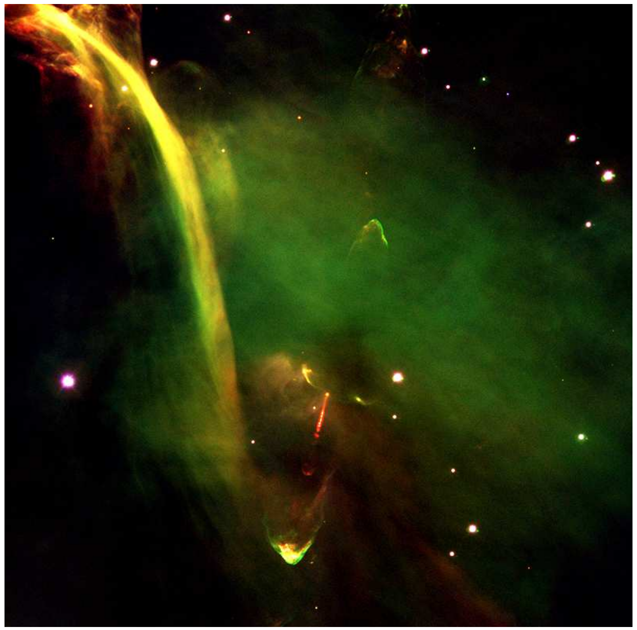


d

Shu et al. (1987, ARAA 25, 23, Fig. 7)

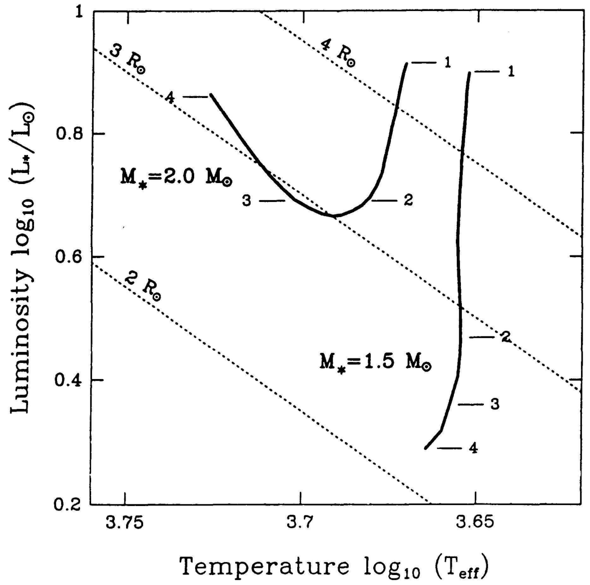
Star has reached zero age main sequence (ZAMS) plus circumstellar disk.

Some disks produce fast collimated outflows (jets): Herbig Haro Objects



HH34 in Orion (ESO VLT KUEYEN/FORS2)

Herbig Haro Objects: shocks and jets/outflows produced during formation of stars.



Palla & Stahler (1993, ApJ 418, 414; numbers are time in 10^6 years)

Stellar Evolution from protostar to ZAMS takes a few million years.

Zero Age Main Sequence

Once star has collapsed and nuclear fusion has started: zero age main sequence (ZAMS) is reached

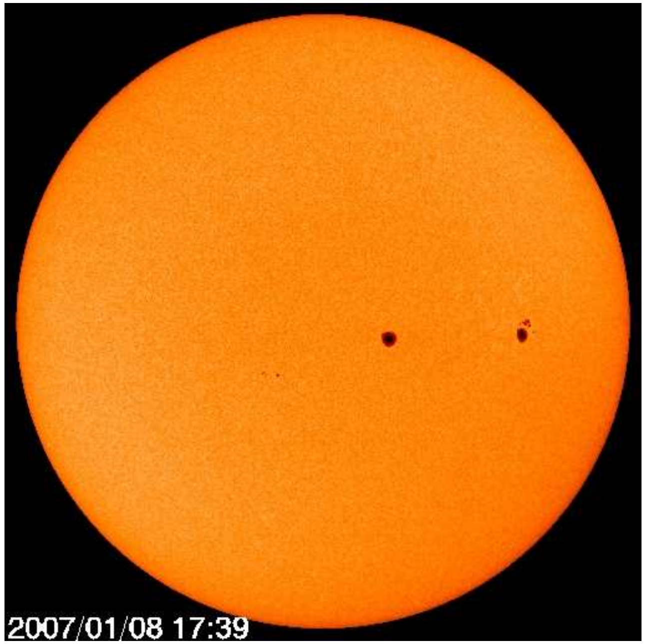
The Main Sequence is the result of steady state fusion (“burning”) of hydrogen into helium in stellar centers.

. . . longest phase of stellar evolution (10 billion years for Sun)

Stellar structure defined by balance between pressure inwards due to gravitation and pressure outwards due to energy release (“hydrostatic equilibrium”).

Pleiades (R. Gendler; $d = 150$ pc, diameter: 5 pc, 3000 stars

Once stars have formed, strong UV radiation removes residual dust (still seen as a reflection nebula) and an open cluster is formed.



The Sun: A typical star (ESA/NASA SOHO)

In the following we derive the equations of stellar structure.

In *hydrostatic equilibrium*, gravitation pulls material in directon of the center of the star. This gravitational pressure is counteracted by gas pressure.

Force balance on a particle dm:

Gravitational force downwards:

$$dF_g = -\frac{GM_r dm}{r^2} = -\frac{GM_r \rho}{r^2}\, dA\, dr \tag{11.4}$$

where M_r is the mass within the radius r.

Let the pressure at the base of the volume element be P, and that at the top $P + dP$. Then the pressure force acting on the volume element is

$$dF_P = PdA - (P + dP)dA = -dP\, dA \tag{11.5}$$

Since pressure decreases towards the outside, dP is negative and F_P positive.

In equilibrium:

$$dF_g = -dF_P \tag{11.6}$$

and therefore

$$-\frac{GM_r\rho}{r^2}\, dA\, dr = dP\, dA \tag{11.7}$$

such that

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} \tag{11.8}$$

The *mass distribution* is obtained by mass conservation. The mass within a spherical shell is

$$dM_r = 4\pi r^2 \rho\, dr \tag{11.9}$$

and therefore

$$\frac{dM_r}{dr} = 4\pi r^2 \rho \tag{11.10}$$

Conservation of energy is taken into account by asking that all mass generated within the star has to be transported to its surface (where it is radiated away).

We call ϵ the energy production coefficient, i.e., the energy released within the star per time and mass. Then the change in luminosity due to energy generation within a small shell is

$$dL_r = \epsilon dM_r = 4\pi r^2 \rho \epsilon\, dr \tag{11.11}$$

Stellar Structure, II

Stellar structure governed by four coupled differential equations:

Mass structure
(mass conservation)

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

Pressure structure
(hydrostatic equilibrium)

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\frac{GM(r)}{r^2}$$

Temperature structure
(energy transport)

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\kappa\rho(r)}{T^3}\frac{L(r)}{4\pi r^2}$$

Energy conservation
(energy transport)

$$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho(r)\epsilon(r)$$

plus “equation of state” ($P = P(T, \rho)$), energy generation ($\epsilon = \epsilon(T, \rho, Z)$),...

Stellar model: numerical solution of stellar structure equations.

and therefore

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon \tag{11.12}$$

Lastly, we need to take a look at the *temperature gradient*, i.e., the transport of energy.

Energy is transported in stars by

- conduction
- convection
- or radiation

In most stars, either convection or radiation is important, conduction can usually be ignored.

The derivation of the temperature gradient equation is rather complicated and will not be done here. In the end one obtains for the case of energy transport by radiation

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\kappa\rho(r)}{T^3}\frac{L(r)}{4\pi r^2} \tag{11.13}$$

where a, κ, and c are constants.

For the case of convection

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr} \tag{11.14}$$

To solve these four equations, one needs the *boundary conditions*:

- at $r = 0$: $M_r = 0$ and $L(0) = 0$
- at $r = R$: $P(R) = 0$, $T(R) = 0$, $M_r(R) = M$

Furthermore one needs to know:

- *equation of state*: in the simplest case: $P = nkT = \rho kT/(\mu m_p)$ where μ is the mean molecular weight
- *energy generation*: $\epsilon = \epsilon(T, \rho, \text{chemical composition})$
- *opacity*: $\kappa = \kappa(T, \rho, \text{chemical composition})$

Energy generation: Overview

Main sequence: Nuclear fusion of Hydrogen into Helium:

$$4\mathrm{p} \longrightarrow {}^4_2\mathrm{He} + E$$

How much energy is gained?

Particle physics: express mass as “rest energy equivalent” via $E = mc^2$

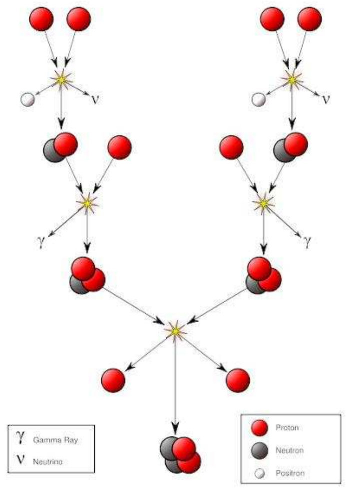
(and call it “mass”...); usually use energy units of MeV, 1 MeV $= 1.602 \times 10^{-13}$ J

	mass of 4 protons (4 × 938 MeV):	3752 MeV
−	mass of ${}^4_2$He:	3727 MeV
	mass defect Δmc^2:	25 MeV

In the fusion of hydrogen to helium, 0.7% of the available rest mass energy is converted to energy.

Two main burning cycles: proton-proton chain and the CNO cycle.

Energy generation: Proton-Proton chain



For moderate central temperatures, He is produced using the proton proton chain.

First, two protons create a deuteron:

$$^1\mathrm{H} + {}^1\mathrm{H} \longrightarrow {}^2\mathrm{H} + \mathrm{e}^+ + \nu_\mathrm{e} \qquad (11.15)$$

This process is slow (happens once for a nucleon per 10^{10} years)

Then an additional proton is attached:

$$^2\mathrm{H} + {}^1\mathrm{H} \longrightarrow {}^3\mathrm{He} + \gamma \qquad (11.16)$$

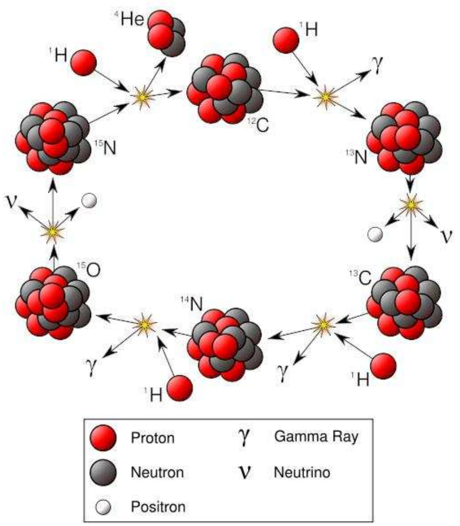
and two helium nuclei can form an alpha particle:

$$^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + 2{}^1\mathrm{H} \qquad (11.17)$$

This is the so called pp I-cycle, minor variations of the theme exist (pp II, pp III cycles), but pp I dominates.

pp chain dominates for $T \lesssim 2 \times 10^7$ K, $\epsilon_{\mathrm{pp}} \propto T^5$; Sun: 98.4%.

Energy generation: CNO cycle



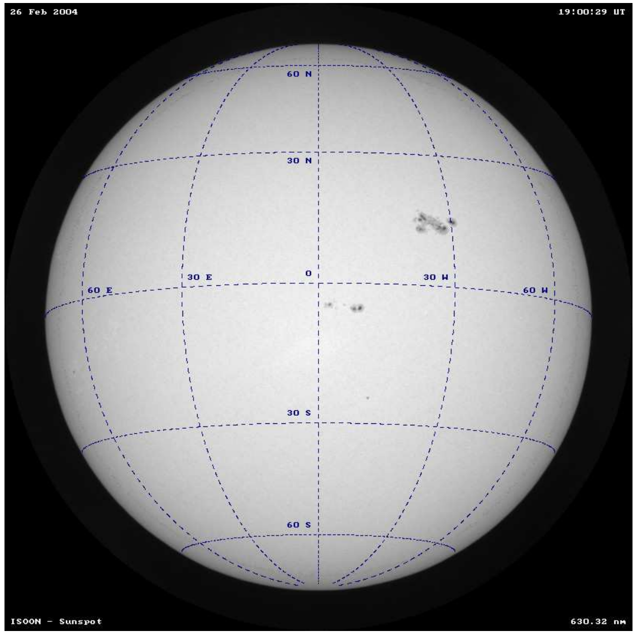
The CNO cycle (Bethe-Weizsäcker-cycle) requires the presence of C, N, and O isotopes as catalysts.

CNO cycle has slightly smaller energy release than pp-cycle because of higher neutrino losses.

Reaction $^{14}\mathrm{N} + p \longrightarrow {}^{15}\mathrm{O} + \gamma$ is th slowest reaction (one million years).

CNO cycle dominates above 2×10^7 K, $\epsilon_{\mathrm{CNO}} \propto T^{17}$; Sun: 1.6%.

Wikipedia



The Sun

Solar Structure, II

Based on observations of

- Solar Mass: $1\,M_\odot = 1.997 \times 10^{30}\,\mathrm{kg} = 1.997 \times 10^{33}\,\mathrm{g}$
- Solar Luminosity: $1\,L_\odot = 3.127 \times 10^{26}\,\mathrm{W} = 3.127 \times 10^{33}\,\mathrm{erg\,s^{-1}}$
- Solar chemical composition (=elemental abundances):

it is possible to use the equations of stellar structure to determine a model for the structure of the Sun, i.e., M_r, L_r, $\rho(r)$, $T(r)$, abundances(r).

The best models available are called "standard models".

To get a feel for what is going on, let's do some order of magnitude astrophysics. See, Karttunen , chapter 11, for more on this.

Sun: $M = 2 \times 10^{30}$ kg and $R = 700000$ km.

Therefore, the surface acceleration on the Sun is

$$g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \cdot 2 \times 10^{30}\,\mathrm{kg}}{(7 \times 10^8\,\mathrm{m})^2} = 274\,\mathrm{m\,s^{-1}} = 28 g_\oplus \tag{11.18}$$

and the mean density of the Sun is

$$\langle\rho\rangle = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{2 \times 10^{30}\,\mathrm{kg}}{\frac{4}{3}\pi \cdot 3.4 \times 10^{26}\,\mathrm{m^3}} = 1410\,\mathrm{kg\,m^{-3}} = 1.4\,\mathrm{g\,cm^{-3}} \tag{11.19}$$

so not much denser than water.

To obtain an estimate for the pressure use the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} \tag{11.20}$$

Now assume that $\rho = \langle\rho\rangle$. Then

$$M_r = \frac{4}{3}\pi\langle\rho\rangle r^3 \tag{11.21}$$

such that

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = \frac{4\pi G\langle\rho\rangle^2 r}{3} \tag{11.22}$$

This can be used to estimate the pressure at $r = R/2$:

Separation of Variables gives

$$\int_P^0 dP = -\frac{4}{3}\pi G\langle\rho\rangle^2 \int_{R/2}^R r\,dr \tag{11.23}$$

$$= -\frac{4}{3}\pi G\langle\rho\rangle^2 \cdot \frac{1}{2}\left(R^2 - \frac{R^2}{4}\right) \tag{11.24}$$

$$= -\frac{1}{2}\pi G\langle\rho\rangle R^2 \tag{11.25}$$

such that

$$P = \frac{1}{2}\pi G\langle\rho\rangle R^2 = 10^{14}\,\mathrm{Pa} \tag{11.26}$$

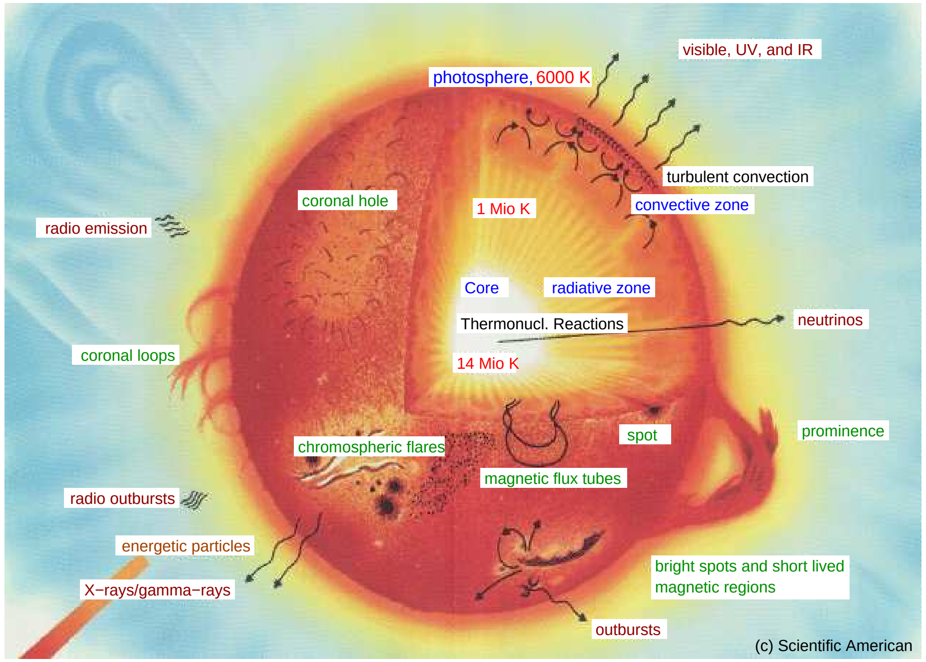
From this, the mean temperature can be obtained from the equation of state

$$P = \frac{\langle\rho\rangle kT}{\mu m_\mathrm{p}} \implies T = \frac{\mu m_\mathrm{p} P}{k\langle\rho\rangle} \tag{11.27}$$

For this we need to know the mean molecular weight, μ, i.e., the mean mass per proton. For a pure ionized hydrogen gas, since we can ignore the electron mass, $\mu = 0.5$. In reality, the Sun also contains helium and some heavier elements, such that $\mu = 0.61$.

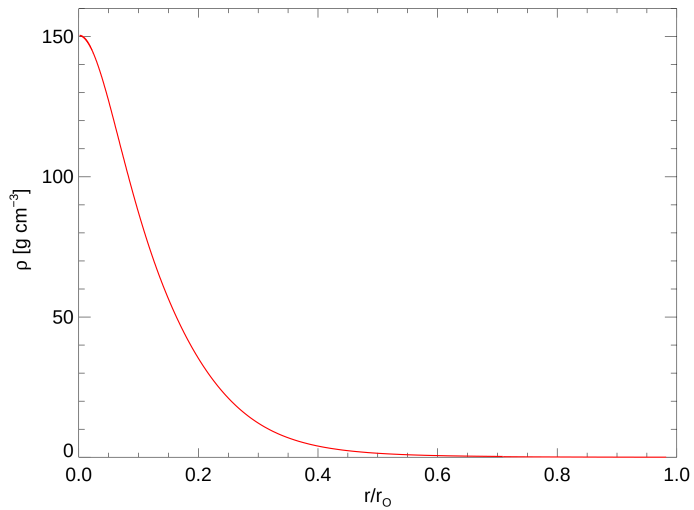
Inserting $\langle\rho\rangle$ and μ into Eq. (11.27) then gives

$$T = \frac{\mu m_\mathrm{p} P}{k\langle\rho\rangle} = \frac{0.61 \cdot 1.67 \times 10^{-27}\,\mathrm{kg} \times 10^{14}\,\mathrm{Pa}}{1.38 \times 10^{-23}\,\mathrm{m^3\,Pa\,K^{-1}} \times 1400\,\mathrm{kg\,m^{-3}}} = 5 \times 10^6\,\mathrm{K} \tag{11.28}$$



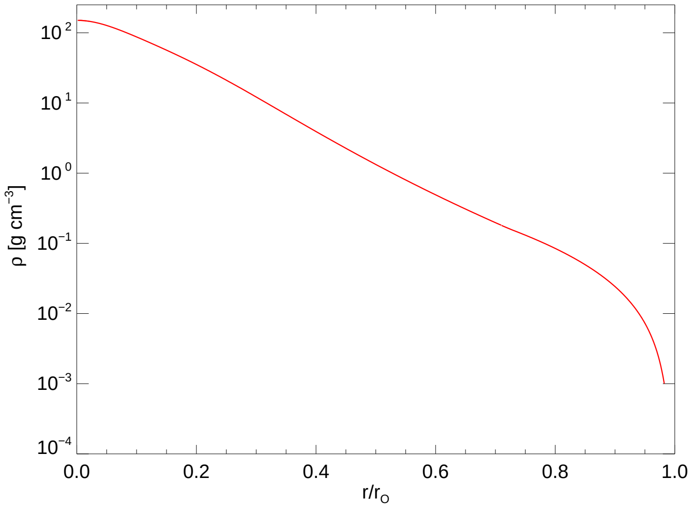
11–35

Standard Solar Model, I



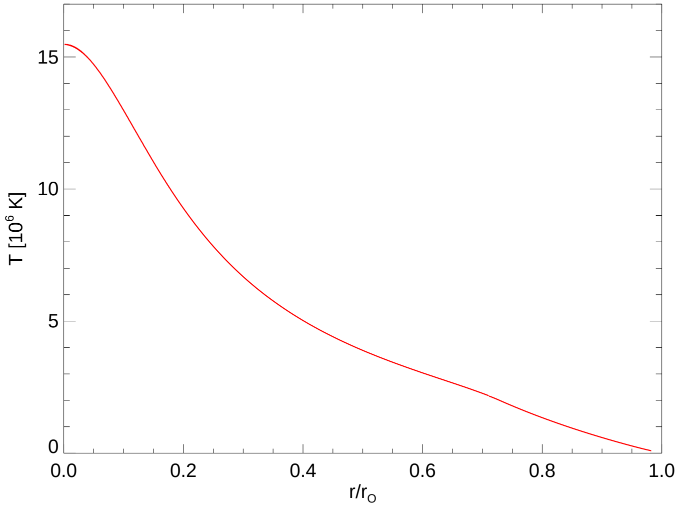
Standard solar model of Bahcall & Serenelli (2005, ApJ 626, 530)

Standard Solar Model, II



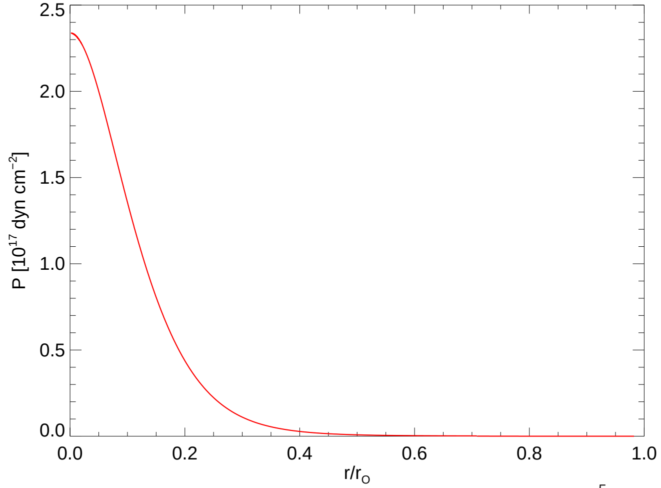
Standard solar model of Bahcall & Serenelli (2005, ApJ 626, 530)

Standard Solar Model, III



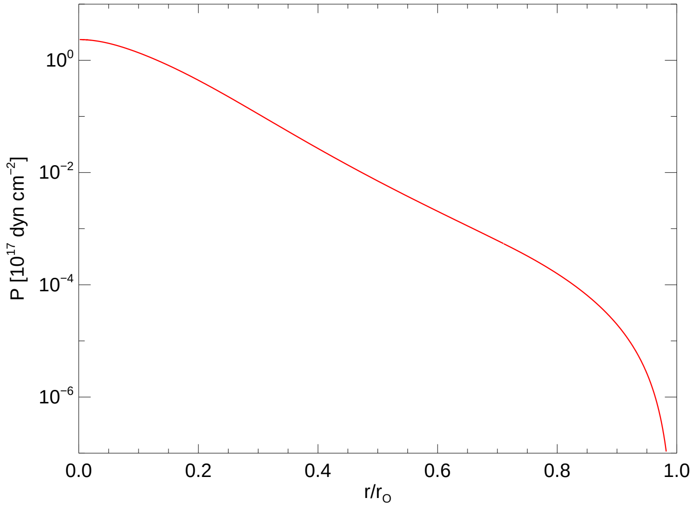
Standard solar model of Bahcall & Serenelli (2005, ApJ 626, 530)

Standard Solar Model, IV



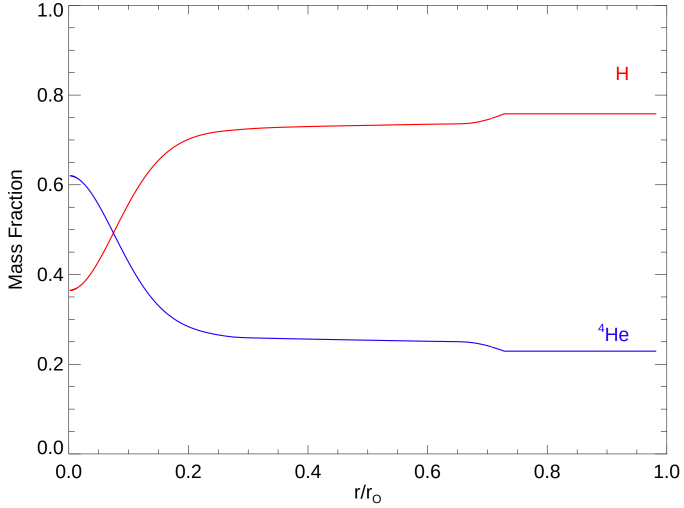
Standard solar model of Bahcall & Serenelli (2005, ApJ 626, 530; 1 dyn $= 10^{-5}$ N, 1 dyn cm^{-2} $= 0.1$ Pa)

Standard Solar Model, V



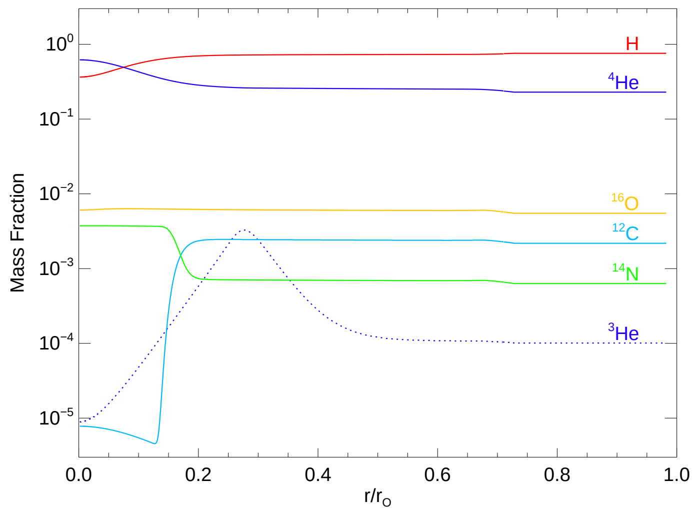
Standard solar model of Bahcall & Serenelli (2005, ApJ 626, 530)

Standard Solar Model, VI



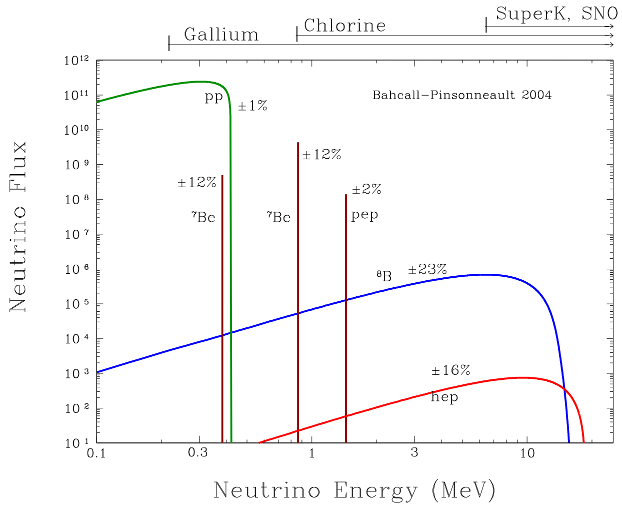
Standard solar model of Bahcall & Serenelli (2005, ApJ 626, 530)

Standard Solar Model, VII



Standard solar model of Bahcall & Serenelli (2005, ApJ 626, 530)

Solar Neutrinos, I



after Bahcall

The solar model predicts a solar neutrino spectrum that can be compared with Earth based measurements. This is the most direct test of theory of stellar structure known.

Problem: Neutrinos are difficult to detect since their interaction cross section is *very small*
$\Longrightarrow$ large detectors are needed.

The first neutrino experiment in the Homestake mine (J. Davis et al., 1968ff.).
Based on reaction

$$\nu_e + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar} + e^-$$

Use Chlorine in large tetrachloroethylene tank (615 T), detect Ar with radiochemical methods.

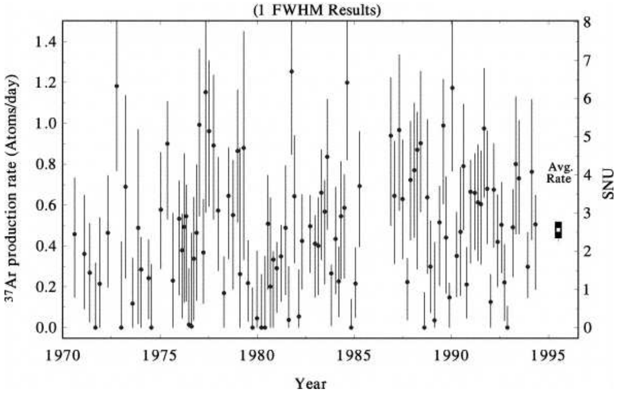
Sensitive for electron neutrinos at energies above ~0.8 MeV, which are rare.

Expected rate: 8.5 ± 1.9 SNU
Detected rate: 2.6 ± 0.2 SNU

1 SNU: 10^{-37} captures target atom^{-1} s^{-1}.

Brookhaven National Laboratory

Solar Neutrinos, III

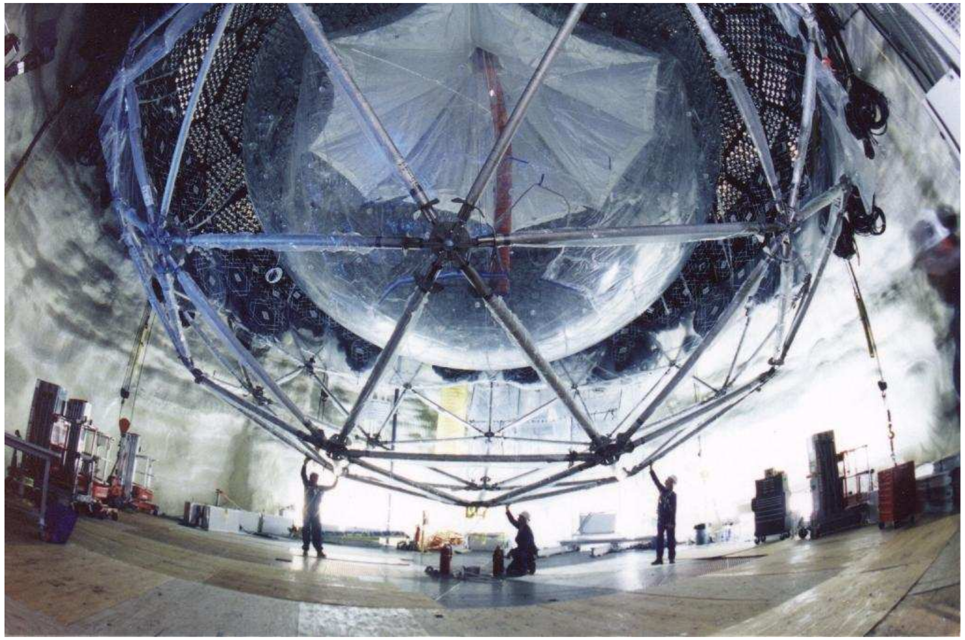


Cleveland et al. (1998, ApJ 596, 505; a total of 875 ^{37}Ar atoms were detected in the experiment, 766 of which were solar)

Low flux from early Homestake runs was confirmed in the early 1990s by Kamiokande.

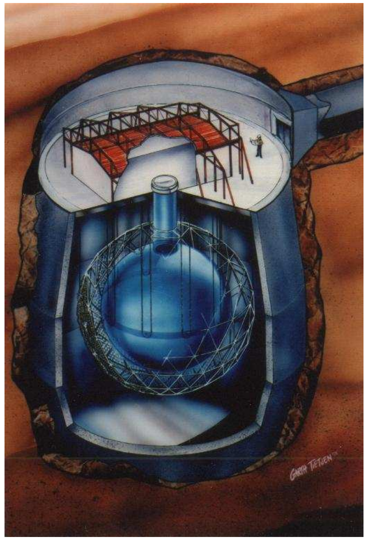
Solar Neutrino Problem: Solar neutrino flux is $\sim$ 1/3 of predicted neutrino flux.

Most particle physicists believed that reason for the solar neutrino problem is that the standard solar model is wrong. They were wrong.



courtesy SNO

Solar Neutrinos, IV



Sudbury Neutrino Observatory: uses 1000 T of heavy water, i.e., D_2O, 2000 m below ground. Possible neutrino reactions:

charged current: $\nu_e + D \rightarrow p + p + e^- - 1.442\,\mathrm{MeV}$

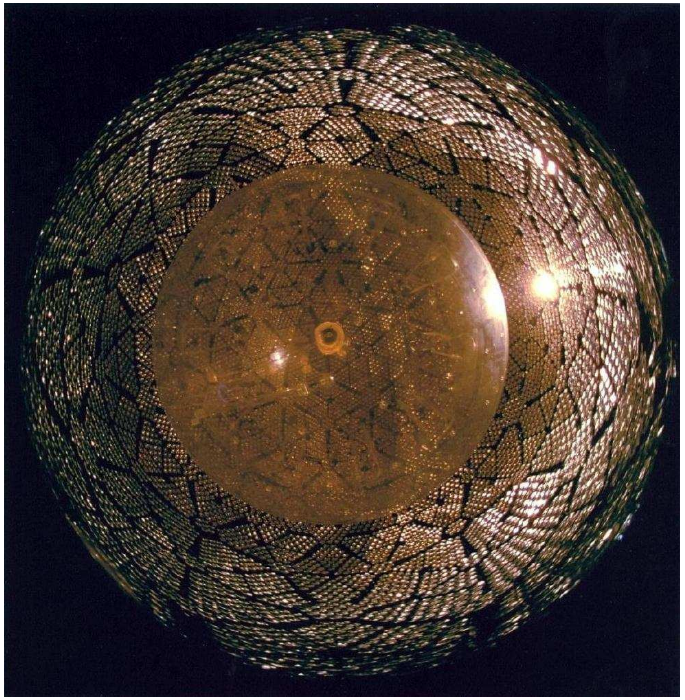
neutral current: $\nu + D \rightarrow p + n + \nu - 2.224\,\mathrm{MeV}$

elastic scattering: $\nu + e^- \rightarrow \nu + e^- - 2.224\,\mathrm{MeV}$

The neutral current reaction is sensitive to *any* flavor of neutrino.

SNO detects $\sim$5000 neutrino events per year.

courtesy SNO



Acrylic vessel surrounded by photomultiplier tubes.

View through fisheye lens.

courtesy SNO

VOLUME 87, NUMBER 7 PHYSICAL REVIEW LETTERS 13 AUGUST 2001

Measurement of the Rate of $\nu_e + d \rightarrow p + p + e^-$ Interactions Produced by ^{8}B Solar Neutrinos at the Sudbury Neutrino Observatory

Q. R. Ahmad,[15] R. C. Allen,[11] T. C. Andersen,[12] J. D. Anglin,[7] G. Bühler,[11] J. C. Barton,[13,*] E. W. Beier,[14] M. Bercovitch,[7] J. Bigu,[4] S. Biller,[13] R. A. Black,[13] I. Blevis,[3] R. J. Boardman,[13] J. Boger,[2] E. Bonvin,[9] M. G. Boulay,[9] M. G. Bowler,[13] T. J. Bowles,[6] S. J. Brice,[6,13] M. C. Browne,[15] T. V. Bullard,[15] T. H. Burritt,[15,6] K. Cameron,[12] J. Cameron,[13] Y. D. Chan,[5] M. Chen,[9] H. H. Chen,[11,†] X. Chen,[5,13] M. C. Chon,[12] B. T. Cleveland,[13] E. T. H. Clifford,[9,1] J. H. M. Cowan,[4] D. F. Cowen,[14] G. A. Cox,[15] Y. Dai,[9] X. Dai,[13] F. Dalnoki-Veress,[3] W. F. Davidson,[7] P. J. Doe,[15,11,6] G. Doucas,[13] M. R. Dragowsky,[6,5] C. A. Duba,[15] F. A. Duncan,[9] J. Dunmore,[13] E. D. Earle,[9,1] S. R. Elliott,[15,6] H. C. Evans,[9] G. T. Ewan,[9] J. Farine,[3] H. Fergani,[13] A. P. Ferraris,[13] R. J. Ford,[9] M. M. Fowler,[6] K. Frame,[13] E. D. Frank,[14] W. Frati,[14] J. V. Germani,[15,6] S. Gil,[10] A. Goldschmidt,[6] D. R. Grant,[3] R. L. Hahn,[2] A. L. Hallin,[9] E. D. Hallman,[4] A. Hamer,[6,9] A. A. Hamian,[15] R. U. Haq,[4] C. K. Hargrove,[3] P. J. Harvey,[9] R. Hazama,[15] R. Heaton,[9] K. M. Heeger,[15] W. J. Heintzelman,[14] J. Heise,[10] R. L. Helmer,[10,‡] J. D. Hepburn,[9,1] H. Heron,[13] J. Hewett,[4] A. Hime,[6] M. Howe,[15] J. G. Hykawy,[4] M. C. P. Isaac,[5] P. Jagam,[12] N. A. Jelley,[13] C. Jillings,[9] G. Jonkmans,[4,1] J. Karn,[12] P. T. Keener,[14] K. Kirch,[6] J. R. Klein,[14] A. B. Knox,[13] R. J. Komar,[10,9] R. Kouzes,[8] T. Kutter,[10] C. C. M. Kyba,[14] J. Law,[12] I. T. Lawson,[12] M. Lay,[13] H. W. Lee,[9] K. T. Lesko,[5] J. R. Leslie,[9] I. Levine,[3] W. Locke,[13] M. M. Lowry,[8] S. Luoma,[4] J. Lyon,[13] S. Majerus,[13] H. B. Mak,[9] A. D. Marino,[5] N. McCauley,[13] A. B. McDonald,[9,8] D. S. McDonald,[14] K. McFarlane,[3] G. McGregor,[13] W. McLatchie,[9] R. Meijer Drees,[15] H. Mes,[3] C. Mifflin,[3] G. G. Miller,[6] G. Milton,[1] B. A. Moffat,[9] M. Moorhead,[13,5] C. W. Nally,[10] M. S. Neubauer,[14] F. M. Newcomer,[14] H. S. Ng,[10] A. J. Noble,[3,‡] E. B. Norman,[5] V. M. Novikov,[3] M. O'Neill,[3] C. E. Okada,[5] R. W. Ollerhead,[12] M. Omori,[13] J. L. Orrell,[15] S. M. Oser,[14] A. W. P. Poon,[5,6,10,15] T. J. Radcliffe,[9] A. Roberge,[4] B. C. Robertson,[9] R. G. H. Robertson,[15,6] J. K. Rowley,[2] V. L. Rusu,[14] E. Saettler,[4] K. K. Schaffer,[15] A. Schuelke,[5] M. H. Schwendener,[4] H. Seifert,[4,6,15] M. Shatkay,[3] J. J. Simpson,[12] D. Sinclair,[3] P. Skensved,[9] A. R. Smith,[5] M. W. E. Smith,[15] N. Starinsky,[3] T. D. Steiger,[15] R. G. Stokstad,[5] R. S. Storey,[7,†] B. Sur,[1,9] R. Tafirout,[4] N. Tagg,[12] N. W. Tanner,[13] R. K. Taplin,[13] M. Thorman,[13] P. Thornewell,[6,13,15] P. T. Trent,[13,*] Y. I. Tserkovnyak,[10] R. Van Berg,[14] R. G. Van de Water,[14,6] C. J. Virtue,[4] C. E. Waltham,[10] J.-X. Wang,[12] D. L. Wark,[13,6,§] N. West,[13] J. B. Wilhelmy,[6] J. F. Wilkerson,[15,6] J. Wilson,[13] P. Wittich,[14] J. M. Wouters,[6] and M. Yeh[2]

(SNO Collaboration)

[1]*Atomic Energy of Canada Limited, Chalk River Laboratories, Chalk River, Ontario K0J 1J0 Canada*
[2]*Chemistry Department, Brookhaven National Laboratory, Upton, New York 11973-5000*
[3]*Carleton University, Ottawa, Ontario K1S 5B6 Canada*
[4]*Department of Physics and Astronomy, Laurentian University, Sudbury, Ontario P3E 2C6 Canada*
[5]*Institute for Nuclear and Particle Astrophysics and Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720*
[6]*Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[7]*National Research Council of Canada, Ottawa, Ontario K1A 0R6 Canada*
[8]*Department of Physics, Princeton University, Princeton, New Jersey 08544*
[9]*Department of Physics, Queen's University, Kingston, Ontario K7L 3N6 Canada*
[10]*Department of Physics and Astronomy, University of British Columbia, Vancouver, BC V6T 1Z1 Canada*
[11]*Department of Physics, University of California, Irvine, California 92717*
[12]*Physics Department, University of Guelph, Guelph, Ontario N1G 2W1 Canada*
[13]*Nuclear and Astrophysics Laboratory, University of Oxford, Keble Road, Oxford, OX1 3RH, United Kingdom*
[14]*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6396*
[15]*Center for Experimental Nuclear Physics and Astrophysics, and Department of Physics, University of Washington, Seattle, Washington 98195*

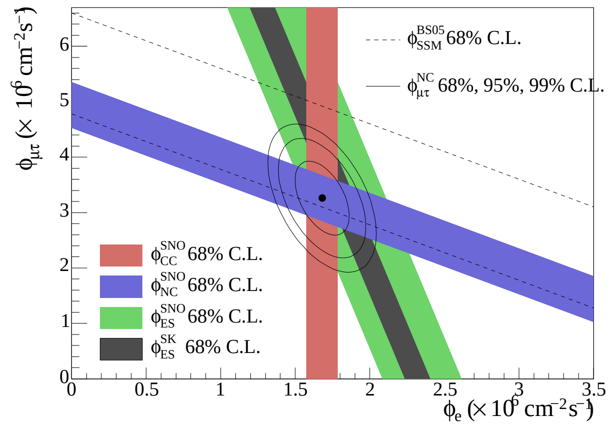
(Received 18 June 2001; published 25 July 2001)

Solar neutrinos from ^{8}B decay have been detected at the Sudbury Neutrino Observatory via the charged current (CC) reaction on deuterium and the elastic scattering (ES) of electrons. The flux of ν_e's is measured by the CC reaction rate to be $\phi^{CC}(\nu_e) = 1.75 \pm 0.07(\text{stat})^{+0.12}_{-0.11}(\text{syst}) \pm 0.05(\text{theor}) \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1}$. Comparison of $\phi^{CC}(\nu_e)$ to the Super-Kamiokande Collaboration's precision value of the flux inferred from the ES reaction yields a 3.3σ difference, assuming the systematic uncertainties are normally distributed, providing evidence of an active non-ν_e component in the solar flux. The total flux of active ^{8}B neutrinos is determined to be $5.44 \pm 0.99 \times 10^6\ \text{cm}^{-2}\,\text{s}^{-1}$.

DOI: 10.1103/PhysRevLett.87.071301 PACS numbers: 26.65.+t, 14.60.Pq, 95.85.Ry

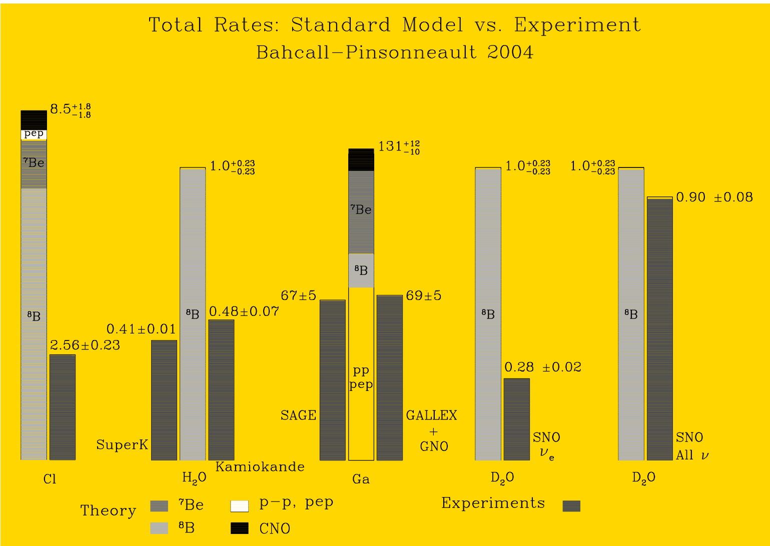
071301-1 0031-9007/01/87(7)/071301(6)$15.00 071301-1

Solar Neutrinos, IX



Aharmin et al., 2005 (dashed line: prediction of standard solar model)

SNO (2001): 2/3 of all ν_e produced in Sun change into ν_τ or ν_μ on their way from Sun to Earth: neutrino oscillations $\Longrightarrow$ physics beyond the standard model of particle physics!



Bahcall

SNO (2001): When taking *all* neutrino flavors into account, the measured and predicted neutrino fluxes agree $\Longrightarrow$ Neutrinos change their flavor.

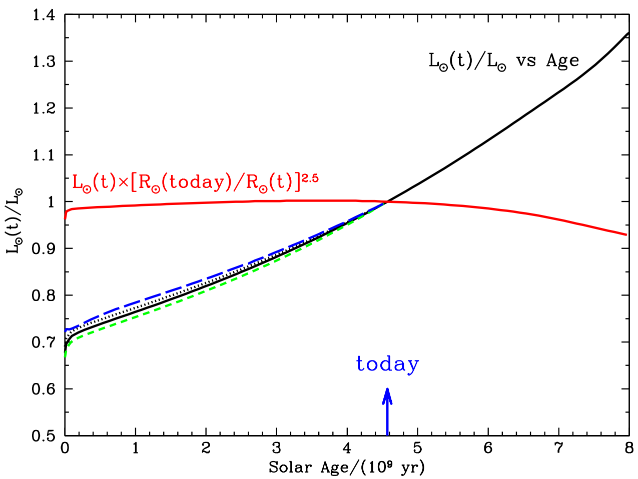
Evolution of the Sun

Now that we believe that the solar model is correct: evolution of the Sun

Principle:

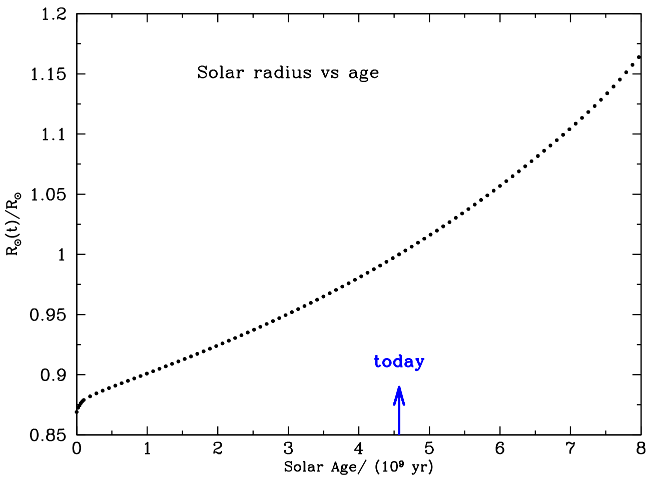
1. Construct stellar model by solving equations of stellar structure for given radial abundances.
2. Evaluate change in elemental abundances as a function of radius from energy generation equations.
3. Change abundances appropriately for a time step Δt.
4. goto step 1

Solar Evolution: Luminosity



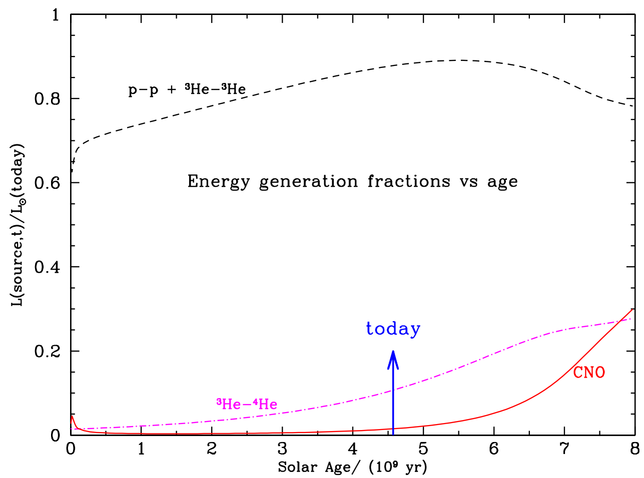
Bahcall, Pinsonneault & Basu (2001, ApJ 555, 990)

Solar Evolution: Radius



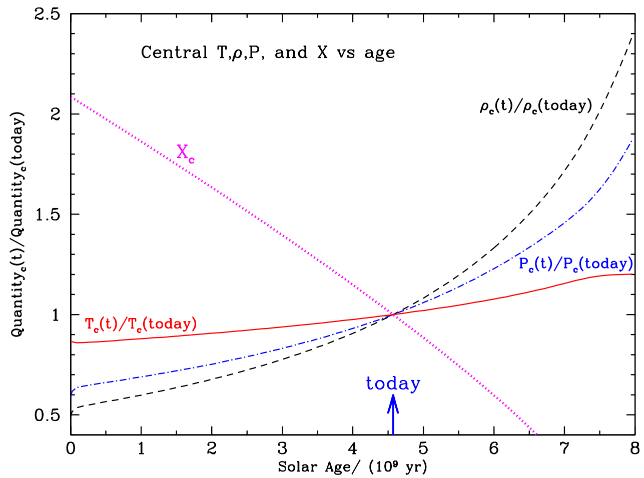
Bahcall, Pinsonneault & Basu (2001, ApJ 555, 990)

Solar Evolution: Energy Generation



Bahcall, Pinsonneault & Basu (2001, ApJ 555, 990)

Solar Evolution: Center



Bahcall, Pinsonneault & Basu (2001, ApJ 555, 990; X_c is the central H fraction)

Stars: Main Sequence, I

Main sequence: Hydrogen burning at the center.

Evolution timescale dominated by the nuclear timescale = timescale needed to use up the fuel in the center of the star.

According to simulations, this is ∼10% of the available Hydrogen. Since 0.7% of $M_{\text{core}}c^2$ converted into He, the nuclear timescale is

$$t_{\text{n}} = \frac{0.007 \cdot 0.1 M c^2}{L} = \frac{M/M_{\odot}}{L/L_{\odot}} \cdot 10^{10}\,\text{years} \tag{11.29}$$

A second important timescale is the timescale the star would need to radiate its stored thermal energy: thermal timescale

Roughly given as

$$t_{\text{t}} = \frac{0.5 G M^2/R}{L} = \frac{(M/M_{\odot})^2}{(R/R_{\odot})(L/L_{\odot})} \cdot 2 \times 10^{7}\,\text{years} \tag{11.30}$$

Stars: Main Sequence, II

Structure on the Main Sequence: Simulations show existence of two regimes:

lower main sequence : stars have structure similar to Sun:

- energy generation: pp-chain ($\epsilon \propto T^5$)
- inner radiative core
- convective hull

upper main sequence : for central temperatures of 18×10^6 K (1.5 $M_{\odot}$ stars): pp-chain and CNO-cycle produce equal amounts of energy. Above that: CNO dominates.

- energy generation: CNO-cycle ($\epsilon \propto T^{17}$)
- inner convective core since energy generation from CNO cycle strongly peaked towards center.
- outer radiative hull

Solar Evolution: Post Main Sequence

H-burning stars on main sequence: hydrostatic equilibrium, inwards gravitational pressure balanced by outwards gas pressure

Since the gas pressure is $P = nkT$: energy source needed to heat gas (=fusion)

End of H-burning: energy source ceases to work $\Longrightarrow$ core has to collapse!

BUT:

collapse cannot continue indefinitely:

increased density $\Longrightarrow$ quantum mechanical effects become important.

Different ways to write the equation of state of an ideal gas

Among the more confusing subjects of thermodynamics are the many different ways in which the ideal gas equation can be written.

The one I prefer for astronomy is

$$P = nkT$$

where

- P: Pressure (measured in $N\,\text{m}^{-2}$)
- n: particle density (i.e., number of particles per cubic meter, unit: m^{-3})
- $k = 1.38066 \times 10^{-23}\,\text{J}\,\text{K}^{-1}$: Boltzmann constant
- T: Temperature (measured in Kelvins)

This equation has the advantage that it counts all particles individually (thus using n). If you know the mass of the gas particles, m_{gas} then another way of writing the ideal gas equation is

$$P = \frac{n m_{\text{gas}}}{m_{\text{gas}}} kT = \rho k T \frac{1}{m_{\text{gas}}}$$

illustrating that for an ideal gas, $P \propto \rho$, where ρ is the mass density.

Another way to write the ideal gas equation is in terms of the total number of gas molecules, $N = nV$, where V is the volume. The ideal gas equation then is

$$P = \frac{N}{V} kT \quad \Longleftrightarrow PV = NkT \quad \Longleftrightarrow \frac{PV}{T} = Nk$$

This version has the problem, however, that the number of gar molecules is typically rather large (there are 6×10^{23} molecules in a volume of 22.4 liters of gas, this number of particles is called one *mole*). Because working with smaller numbers is generally thought a good idea, chemists prefer to work with moles. Per definition, the unit of particle number here is the Avogadro number $N_{\text{A}} = 6.0221 \times 10^{23}$. So, if you want to work with moles, then the above equation becomes

$$PV = \frac{N}{N_{\text{A}}} AkT = N_{\text{mol}} RT$$

where

- N_{mol}: the number of moles of the gas in the volume V,
- $R = N_{\text{A}} k 8.3145\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$: the universal gas constant

To summarise, each of these equations has its own uses, and which one you want to use, really depends on the circumstances of the problem you are solving. For your future life as physicists, try to remember one of them, and then understand how you get from this one to the others, instead of memorising all four ones. This approach will need less memory and lead to a better understanding of what is really going on behind the scenes.

QM interlude, I

Quantum mechanics: One of the stranger phenomena in QM is the **Pauli exclusion principle**:

> For particles such as electrons ("Fermions"), at least one of their quantum numbers must be different.

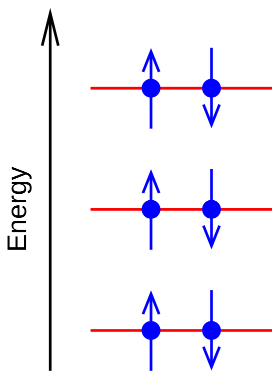
Quantum numbers are, e.g.,

- position (x, y, z),
- momentum (mv_x, mv_y, mv_z),
- angular momentum,
- spin (s)

All of these numbers are "**quantized**", i.e., can only have discrete values (e.g., spin: $+1/2$, $-1/2$).

In a typical gas, this is not a problem ("**phase space is (almost) empty**"), but once it becomes dense $\Longrightarrow$ exclusion principle kicks in.

QM interlude, VIII



Energy of electrons at the same position in space

Effect of high density on electron energy:

> In degenerate electron gases, electrons have much higher energies than in thermal gas.

Interaction of electrons results in **degeneracy pressure**:

$$P = \frac{\hbar^2}{m_e} n_e^{5/3} \propto \rho^{5/3}$$

Note: The degeneracy pressure is **independent of the temperature!**

Solar Mass Stars: Post Main Sequence

Once H is used up in center, H continues to burn in a **shell** around the He core.

Low mass stars ($\lesssim$ solar mass): Star reacts by expanding convective hull until it is almost fully convective: **First motion in HRD horizontally towards the right**, and then upwards to higher L: **red giant stage**.

Core continues to grow, gets compressed $\Longrightarrow$ ρ and T increase until core is degenerate.

Once central temperature $\sim$100 $\times$ 10^6 K: **Triple alpha process**:

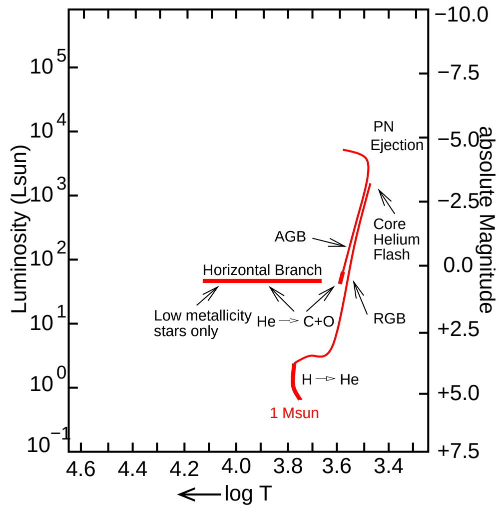
$$^4\mathrm{He} + {}^4\mathrm{He} \leftrightarrow {}^8\mathrm{Be}$$
$$^8\mathrm{Be} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C}$$

Since ^{8}Be has a half life of only 2.6 $\times$ 10^{-16} s: effectively this can only work if 3 α-particles collide.

$\Longrightarrow$ High thermal conductivity of electrons $\Longrightarrow$ core has uniform temperature
$\Longrightarrow$ 3α onset is rapid **He flash**

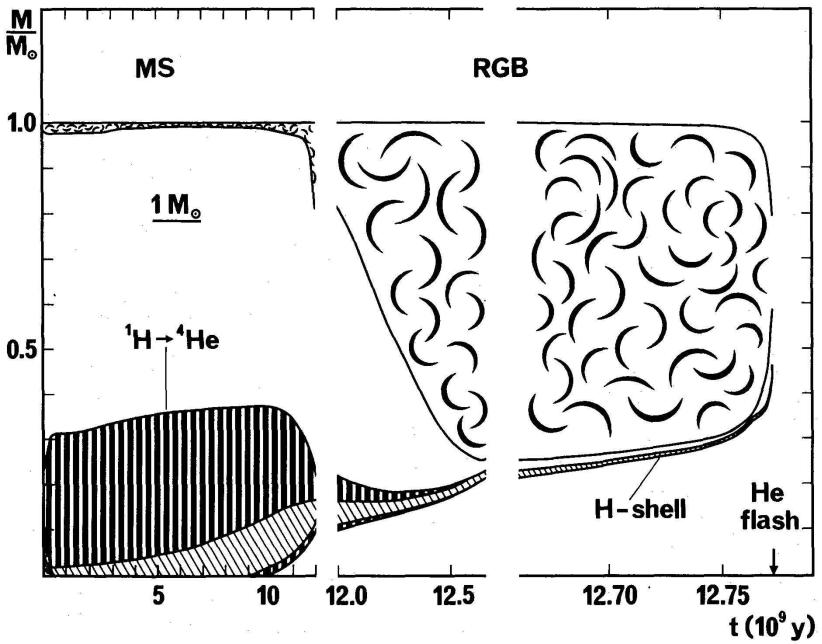
Not seen on surface ("buffered" by convective envelope).

Solar Mass Stars: Post Main Sequence



Evolution of a 1 $M_\odot$ star in the HRD from the main sequence to the AGB.

after Iben, 1991



Evolution of the structure of a 1 $M_{\odot}$ star to the Helium flash (Maeder & Meynet, 1989).