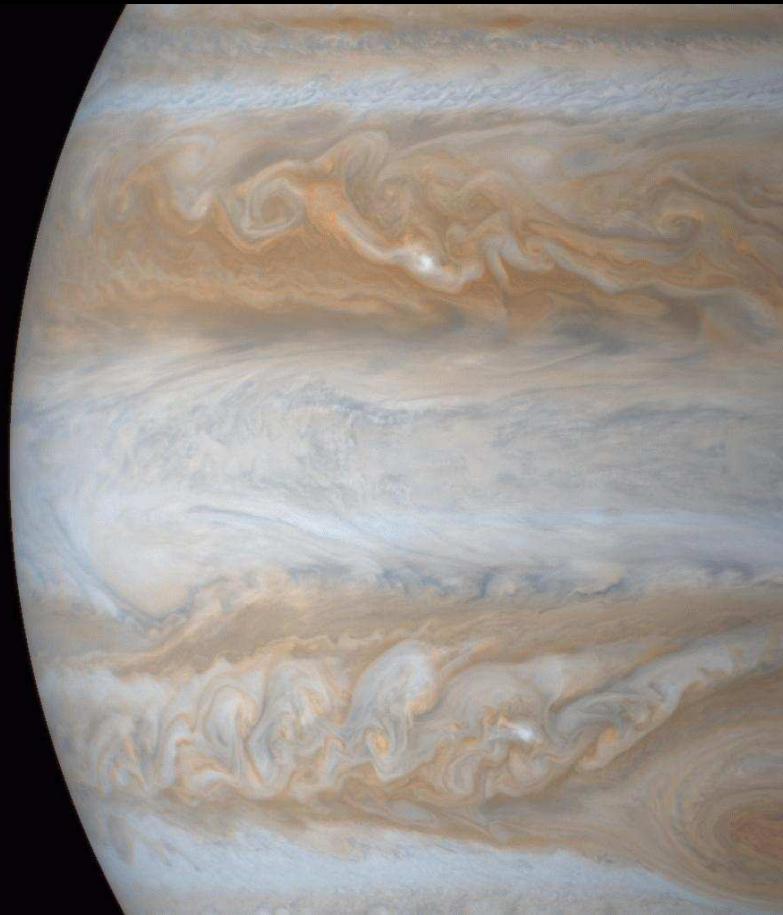


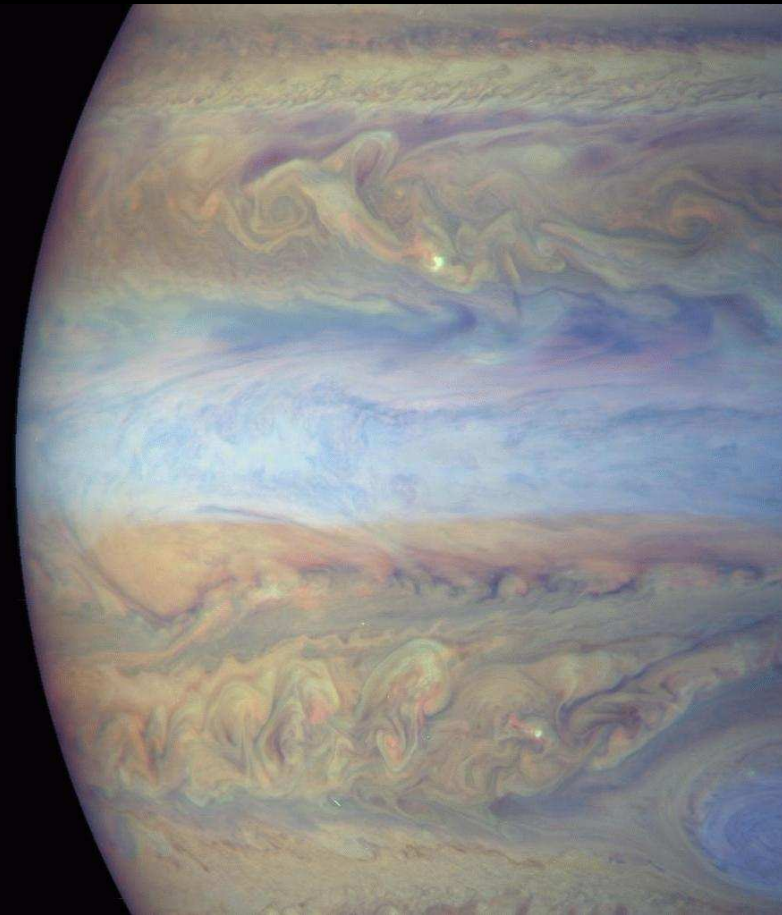


Mars: thin atmosphere (pressure on surface 1% Earth), but real seasonal variations.

2005 Feb 7,  
NASA/Malin  
Space Systems



**Jupiter:** true color image; colors likely from trace content of organic compounds in atmosphere



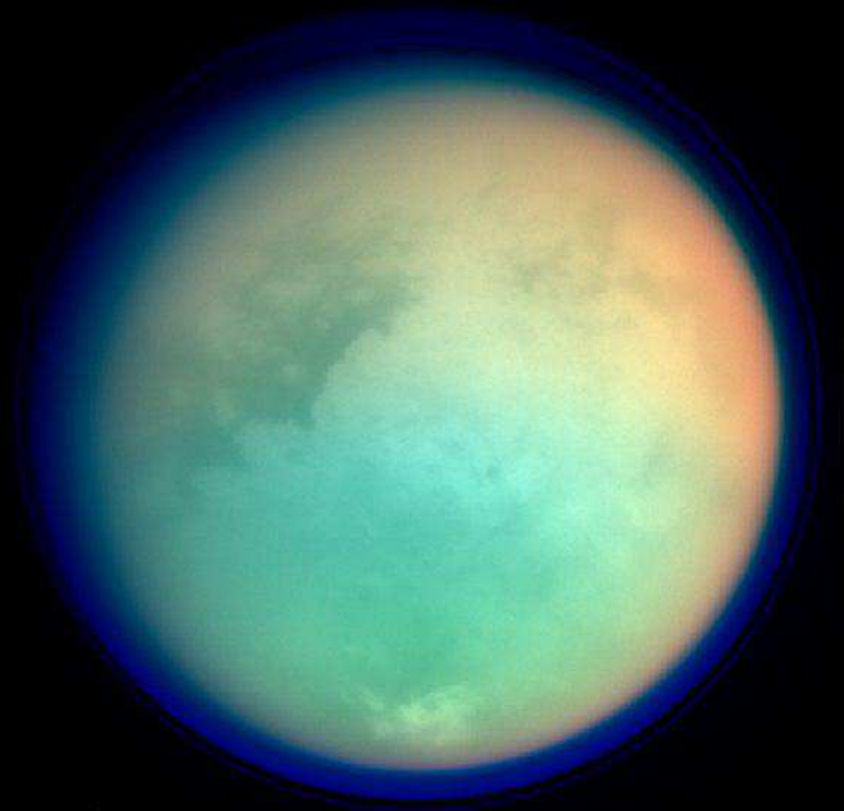
false color image, red: waterclouds, dark spots: deep hot spots

Overall atmospheric structure: **three layers:**

Ammonia – ammonia hydrosulfide ( $\text{NH}_4\text{HS}$ ) – water ice/water (deepest)



NASA Voyager



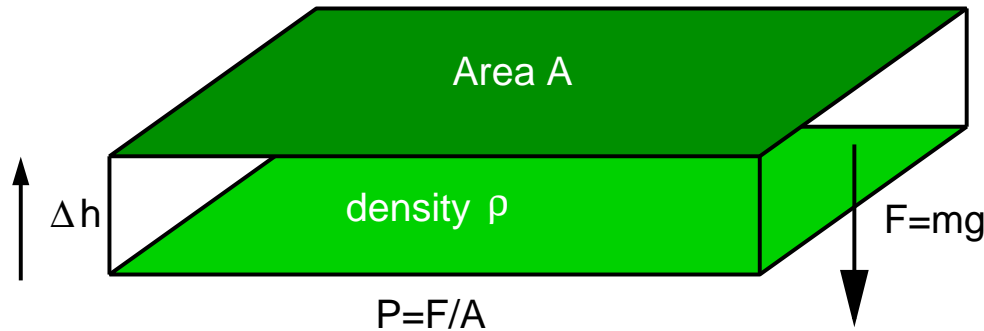
27.10.2004, false colour IR/UV; NASA/ESA

**Titan:** dense atmosphere, 99% nitrogen, 1% methane, some hydrocarbons, thought to be similar to primordial atmosphere of Earth.

Radius: 2575 km (~ Mercury!)

ESA probe *Huygens* landed on Titan on 2005 January 14

The structure of an atmosphere is defined by the concept of **hydrostatic equilibrium**:



The force exerted by gas with density  $\rho$  sitting on top of an area  $A$  is given by

$$F = mg = \rho Vg = Ah\rho g \quad (5.1)$$

Such that pressure becomes

$$P = \frac{F}{A} = \rho hg \quad (5.2)$$

where  $g$  is the gravitational acceleration.

For a thin atmosphere ( $g$  constant): Decrease of  $P$  when going upwards by  $\Delta h$ :

$$\Delta P = -\rho g \Delta h \quad \text{and for } \lim_{\Delta h \rightarrow 0} : \frac{dP}{dh} = -\rho g \quad (5.3)$$

To solve this differential equation, we need a relationship between density and pressure (“**equation of state**”). For an “ideal gas”, this relationship is given by

$$P = (\rho/\mu)kT \quad (5.4)$$

where  $T$  is the Temperature (K!),  $\mu$  the average mass of a gas particle, and  $k$  is Boltzmann’s constant ( $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$ ). Therefore:

$$\frac{dP}{dh} = -\left(\frac{\mu g}{kT}\right) P \quad (5.5)$$

In order to obtain  $P$  as a function of height,  $h$ , we need to solve this differential equation with the boundary condition that for  $h = 0$ ,  $P = P_0$ . This can be done easily using the technique of “separation of variables”, assuming that  $T$  does not change.

First, divide by  $P$  and integrate both sides of the equation with respect to height:

$$\int_0^h \frac{1}{P} \frac{dP}{dh} dh = - \int_0^h \left(\frac{\mu g}{kT}\right) dh$$

We can now substitute  $P(h)$  for  $h$  on the left hand side. Using the chain rule gives

$$\int_{P_0}^{P(h)} \frac{dP'}{P'} = - \int_0^h \left(\frac{\mu g}{kT}\right) dh$$

such that

$$\ln \left( \frac{P(0)}{P(h)} \right) = - \left( \frac{\mu g}{kT} \right) h$$

and exponentiating then gives

$$P(h) = P_0 \exp \left( - \frac{\mu g}{kT} \cdot h \right) = P_0 \exp \left( - \frac{h}{H} \right) \quad (5.6)$$

The pressure in the atmosphere thus decreases exponentially, the characteristic height scale of the decrease is given by the **scale height**,

$$H = \frac{kT}{\mu g} \quad (5.7)$$

Typical values for the planets are for Earth:  $H \sim 9$  km.

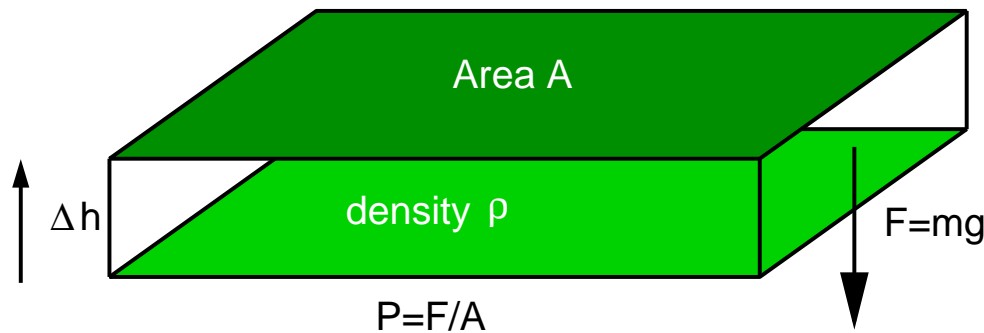
*Remark:* The method employed above is called “separation of variables” since people often jump from the first (linear) equation to the third one in one step, by “separating the dependent from the independent variable”:

$$\frac{dP}{dh} = - \left( \frac{mg}{kT} \right) P \quad \implies \quad \frac{dP}{P} = - \left( \frac{mg}{kT} \right) dh \quad \implies \quad \int_{P_0}^{P(h)} \frac{dP'}{P'} = - \int_0^h \left( \frac{mg}{kT} \right) dh$$



## Hydrostatic Equilibrium

Structure of atmosphere defined through **hydrostatic equilibrium**:



Force on area  $A$  by slab of gas of area  $A$  and density  $\rho$ :

$$F = mg = \rho V g = Ah\rho g \quad (5.8)$$

With  $P = F/A$  one finds:

$$\frac{dP}{dh} = -\rho g$$

where  $g$  gravitational acceleration.

Assuming ideal gas,  $P = (\rho/\mu)kT$ , and **isothermal atmosphere**:

$$P(h) = P_0 \exp\left(-\frac{\mu g}{kT} \cdot h\right) = P_0 \exp\left(-\frac{h}{H}\right)$$

The pressure in the atmosphere thus decreases exponentially, the characteristic height scale of the decrease is given by the **scale height**  $H$ .



## Atmospheric Composition

Atmospheric composition of selected terrestrial objects

	Venus	Earth	Mars	Titan
O <sub>2</sub>	0.0	21.0	0.0	0.0
H <sub>2</sub> O	50ppm	1.0	< 100ppm	0.4ppb
CO <sub>2</sub>	96.5	0.0	95.3	10ppb
N <sub>2</sub>	3.5	78.0	2.7	90
Ar	0.0	0.9	1.6	

Values are given as percentages by volume.

**Titan has atmospheric structure similar to Earth!**

after de Pater and Lissauer



## Atmospheric Composition

Typical  $H$  for the inner planets (Karttunen)

Gas	$\mu$	Venus km	Earth km	Mars km
H <sub>2</sub>	2	360	120	290
O <sub>2</sub>	32	23	7	81
H <sub>2</sub> O	18	40	13	32
CO <sub>2</sub>	44	16	5	13
N <sub>2</sub>	28	26	8	20
$T$ [K]		275	750	260
$g$ [m s <sup>-2</sup> ]		9.81	8.61	3.77





## Atmospheric Composition

Atmospheric composition of gas giants and the Sun

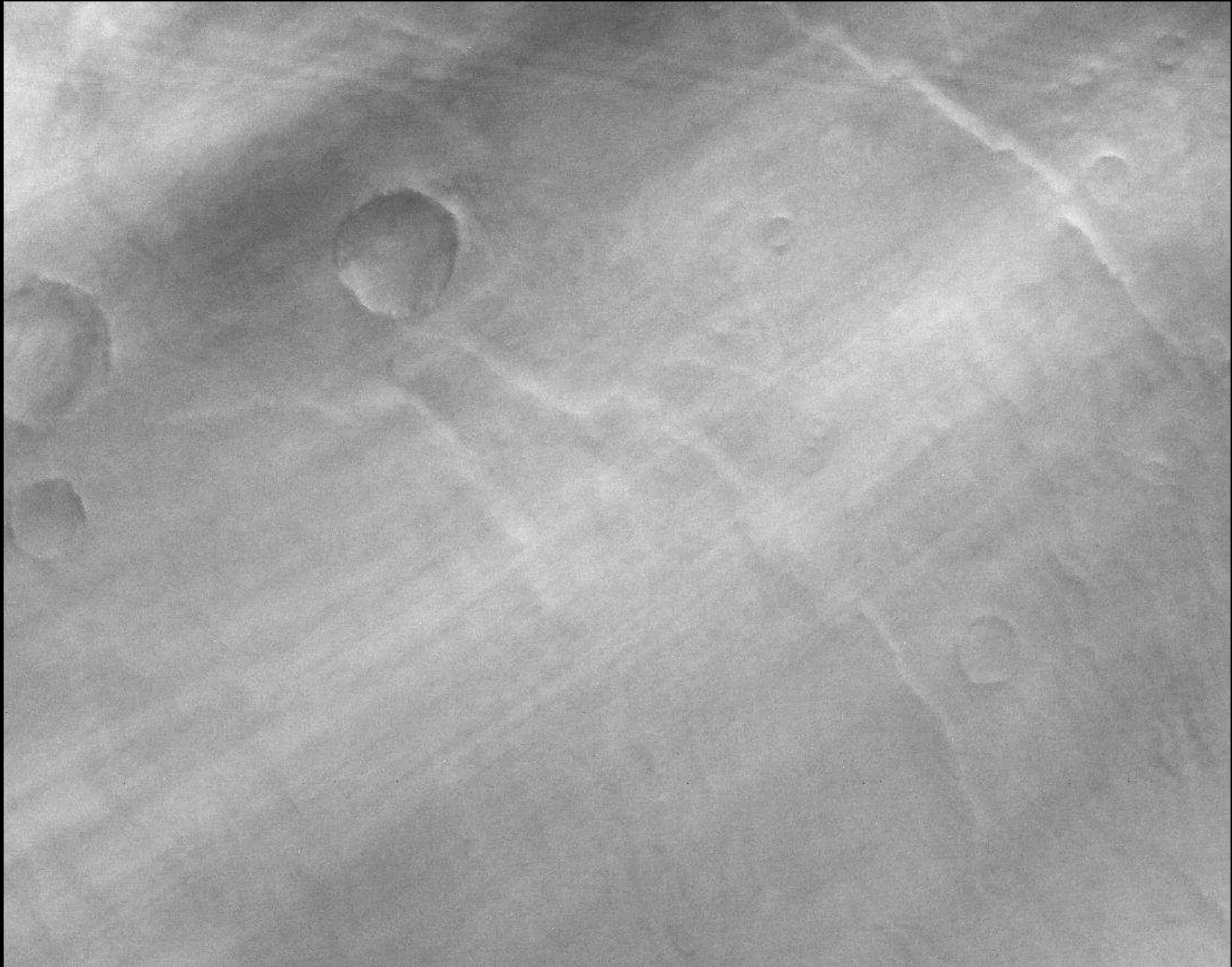
	Sun	Jupiter	Saturn	Uranus	Neptune
H <sub>2</sub>	83.5	86.4	96.3	85±5	85±5
He	19.5	15.7	3.4	18±5	18±5

H: volume percent relative to total atmosphere

He: volume percent relative to H

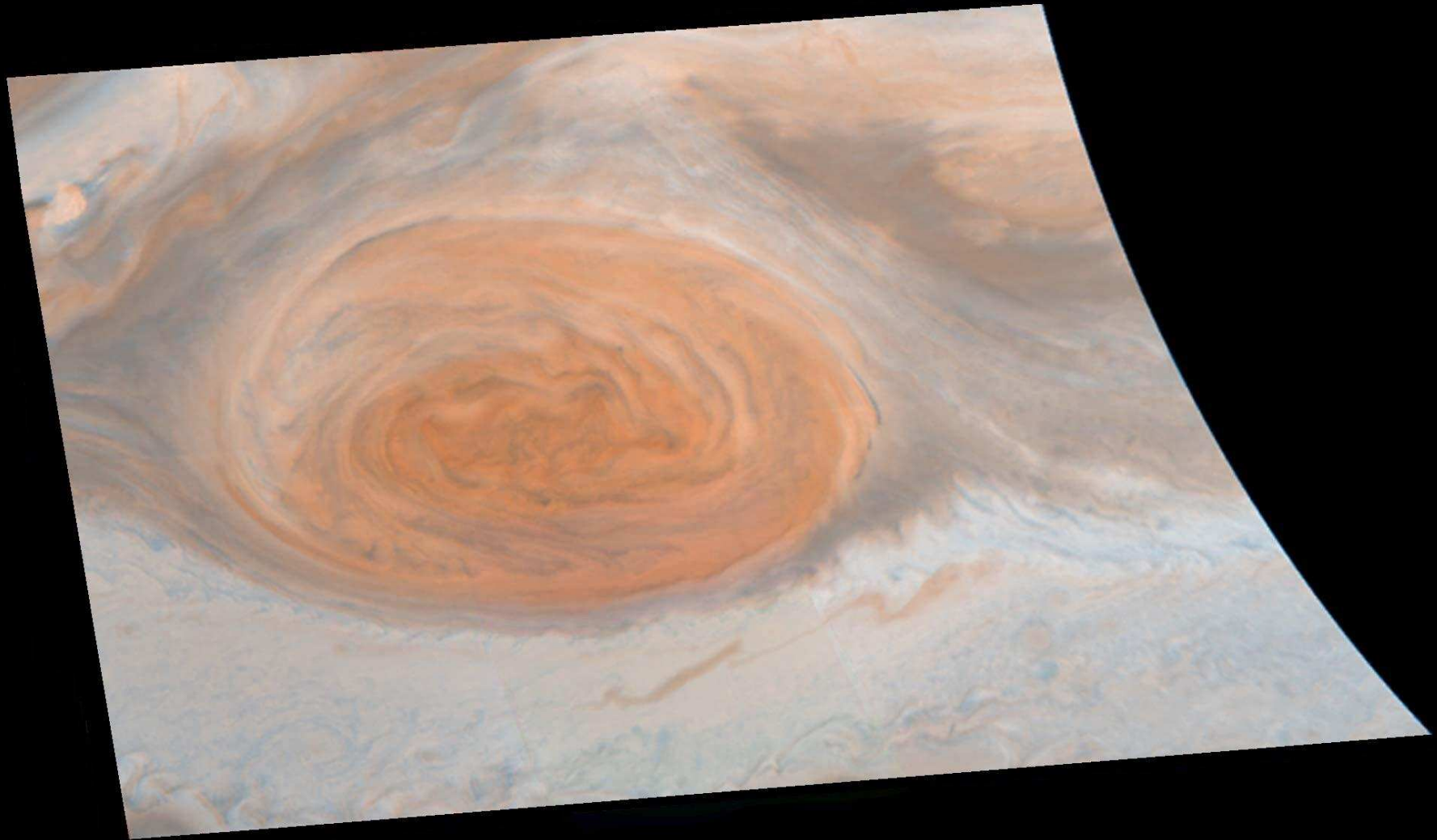
Gas giants have approximately same composition as the Sun.

after de Pater and Lissauer



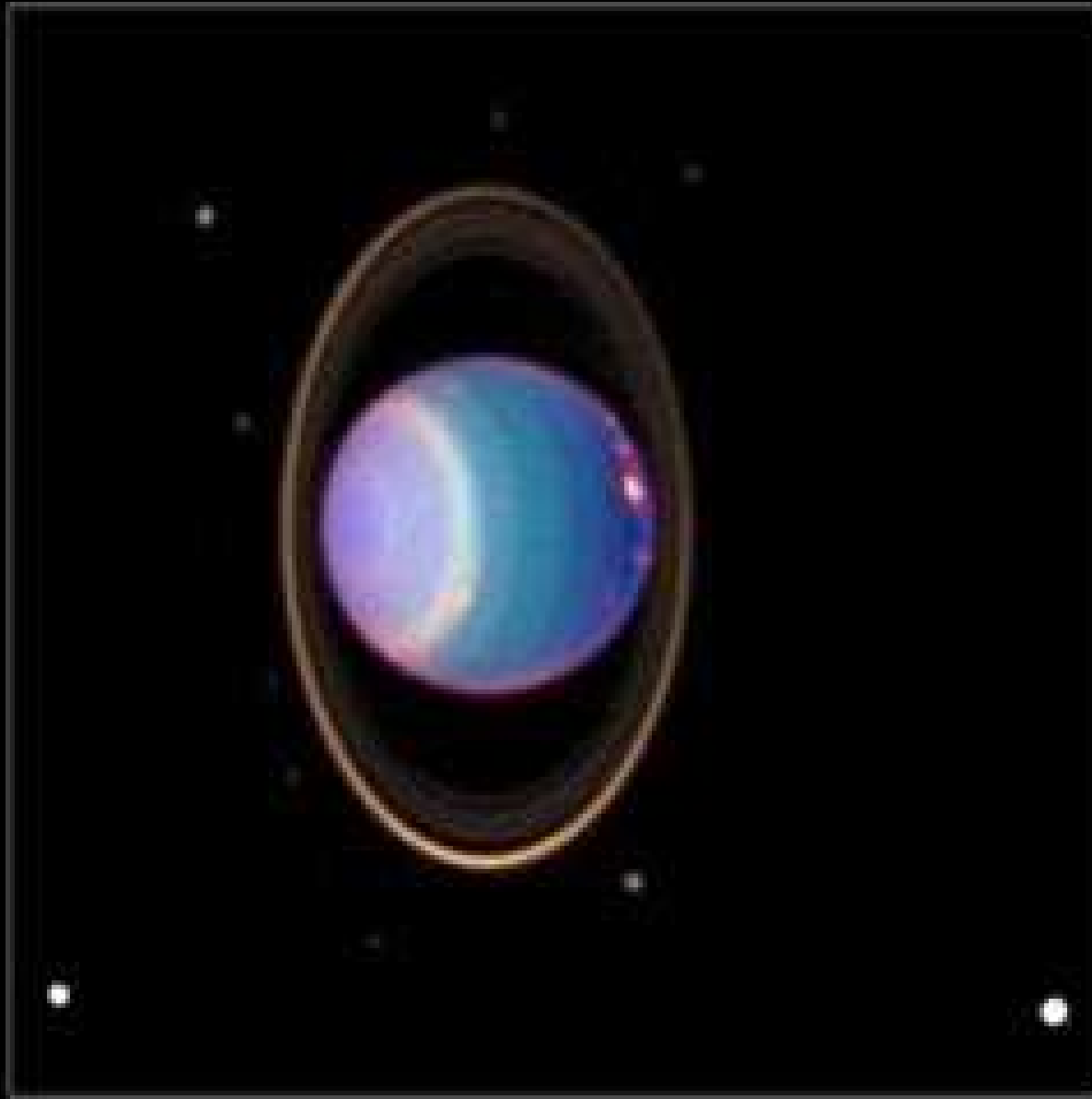
NASA/C.J.Hamilton

Mars: Streaky clouds



Great Red Spot  
NASA Galileo, 1996 June 26

**Jupiter's Great Red Spot:** Storm System,  $\sim 2 \times$  Earth diameter, exists since more than 300 years, 8 km above and  $10^\circ$  cooler than surrounding region (rising high pressure region), rotates counterclockwise (Coriolis force on Southern hemisphere).



HST Image (image enhanced) of Uranus ring system,  
plus evidence for banded atmosphere and clouds



# *Planets: Surfaces and Interiors*