

FAU Erlangen-Nürnberg: Introduction to Astronomy I

Anne Lohfink

January 7, 2008

Contents

I	The Solar System	4
1	The Planets - Properties	5
1.1	Introduction	5
1.2	The Terrestrial Planets	6
1.2.1	Interiors of the Terrestrial Planets	6
1.2.2	Impact Crater Formation	6
1.2.3	Mercury	7
1.2.4	Venus	8
1.2.5	Earth and Moon	8
1.2.5.1	Earth	8
1.2.5.2	Moon	8
1.2.5.3	Eclipses	8
1.2.6	Mars	8
1.3	The Jovian Planets	9
1.3.1	Introduction	9
1.3.2	Composition and Structure	9
1.3.3	Atmospheres	9
1.3.4	The Hydrostatic Equilibrium	10
1.3.4.1	Atmospheres	10
1.3.4.2	Gas Planets	10
1.3.5	Magnetic fields	11
1.3.6	Individual Properties	11
1.3.6.1	Jupiter	11
1.3.6.2	Saturn	11
1.3.6.3	Uranus	11
1.3.6.4	Neptune	11
1.3.7	The Moons of the Giants	12
1.3.7.1	The Galilean Moons	12
1.3.7.1.1	Io	12
1.3.7.1.2	Europa	12
1.3.7.1.3	Ganymede	12
1.3.7.1.4	Callisto	12
1.3.7.2	Titan	12
1.3.7.3	Triton	12
2	The Planets – Motion	13
2.1	Celestial Mechanics	13
2.1.1	Laws	13
2.1.2	Properties of Ellipses	13
2.1.3	Derivation of Kepler 2	14
2.1.4	Derivation of Kepler 3	14
2.2	Tidal Forces	15
2.2.1	Basic Concept	15

2.2.2	Moon-Earth	15
2.2.2.1	An Easy Tidal Model	15
2.2.2.2	A bit more complicated Tidal Model	16
2.2.2.3	Most Realistic Tidal Model	16
2.2.2.4	Tidal Locking	16
2.2.3	Tidal forces in the rest of the universe	16
2.3	Long-Term Evolution	17
3	Small Solar System Bodies (SSSBs)	19
3.1	Asteroids	19
3.2	Comets	19
3.3	TNOs	20
II	Astronomical Instruments	21
4	Telescopes	22
4.1	Types	22
4.1.1	Refractors or Reflectors	22
4.1.2	Newtonian Telescope	22
4.1.3	Cassegrain Telescope	22
4.1.4	Schmidt Telescope	22
4.2	Resolution	22
4.3	Active Optics	23
4.4	Adaptive Optics	23
5	Spectroscopy	24
III	Exoplanets	25
6	Exoplanets	26
6.1	General Stuff	26
6.2	Direct Imaging	26
6.2.1	Contrast	26
6.2.2	Angular Separation	26
6.3	Radial Velocity Measurements	27
6.4	Results	27
7	Formation of Planetary Systems	29
IV	Stars	30
8	Observations	31
8.1	Temperature and Spectrum	31
8.1.1	Planck's Radiation Law	31
8.1.2	Spectral Classification	31
8.2	Brightness and Luminosity	32
8.2.1	Luminosity	32
8.2.2	Magnitudes	32
8.2.2.1	easy	32
8.2.2.2	advanced	33
8.2.3	Colors	33
8.2.3.1	UBVRI Wavelength Filters	33

8.2.3.2	The Color Index	34
8.3	Distance	34
8.4	Masses	34
8.4.1	Visual Binaries	35
8.4.2	Photometric Binaries	35
8.4.3	Spectroscopic Binaries	35
8.4.4	Mass function	36
8.4.5	Mass-Luminosity Relation	36
9	Stellar Evolution	38
9.1	HRD	38
9.2	CMD	39
9.3	Star Birth	39
9.4	The Sun – Main Sequence Stars	40
9.4.1	Stellar Structure	40
9.4.1.1	4 coupled equations of stellar structure	40
9.4.1.2	Energy “Production”	42
9.4.1.2.1	Proton-Proton Chain	42
9.4.1.2.2	CNO-cycle	42
9.4.1.3	Structure of Sun	42
9.4.1.4	Example: Sun	43
9.4.1.5	Standard Solar Models	44
9.4.2	Solar Neutrinos	44

Part I

The Solar System

Chapter 1

The Planets - Properties

1.1 Introduction

terrestrial planets: Mercury, Venus, Earth, Mars

giant planets: Jupiter, Saturn, Uranus, Neptune

	Terrestrial	Giant
mean orbital distance	0.4–1.5 AU	5.0–30.0 AU
equatorial radius (R_{\oplus})	0.4–1.0	3.9–11.5
mass (M_{\oplus})	0.055–1.0	14.5–320
mean density in kg/m ³	3930–5520	690–1640
sidereal rotation period	24 h–243 d	9.9–17.2 h
number of moons	0–2	13–63
ring system?	no	yes
surface temperature (K)	215–730	70–165

eccentricities: almost zero, exceptions: Mercury 0.2 (+Pluto 0.2)

inclination angles to the ecliptic: very small (Pluto 17°)

Def.: Planet:

- orbits around Sun
- hydrostatic equilibrium (=round shape)
- cleared neighborhood around its orbit

Def.: Dwarf Planet:

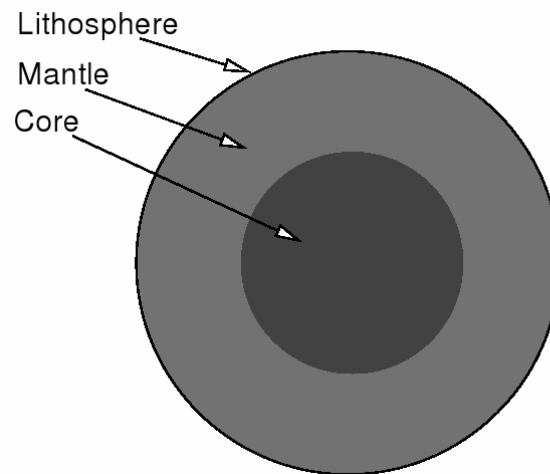
- orbits around Sun
- hydrostatic equilibrium (=round shape)
- has not cleared neighborhood around its orbit
- is not a satellite

Examples for dwarf planets:

- Eris (=largest TNO)
- Pluto
- Ceres (=largest asteroid)

1.2 The Terrestrial Planets

1.2.1 Interiors of the Terrestrial Planets



Core: high density (Fe)

Mantle: plastic materials, hot (e.g. Earth: molten rocks)

Lithosphere: rigid material, e.g., silicates

1.2.2 Impact Crater Formation

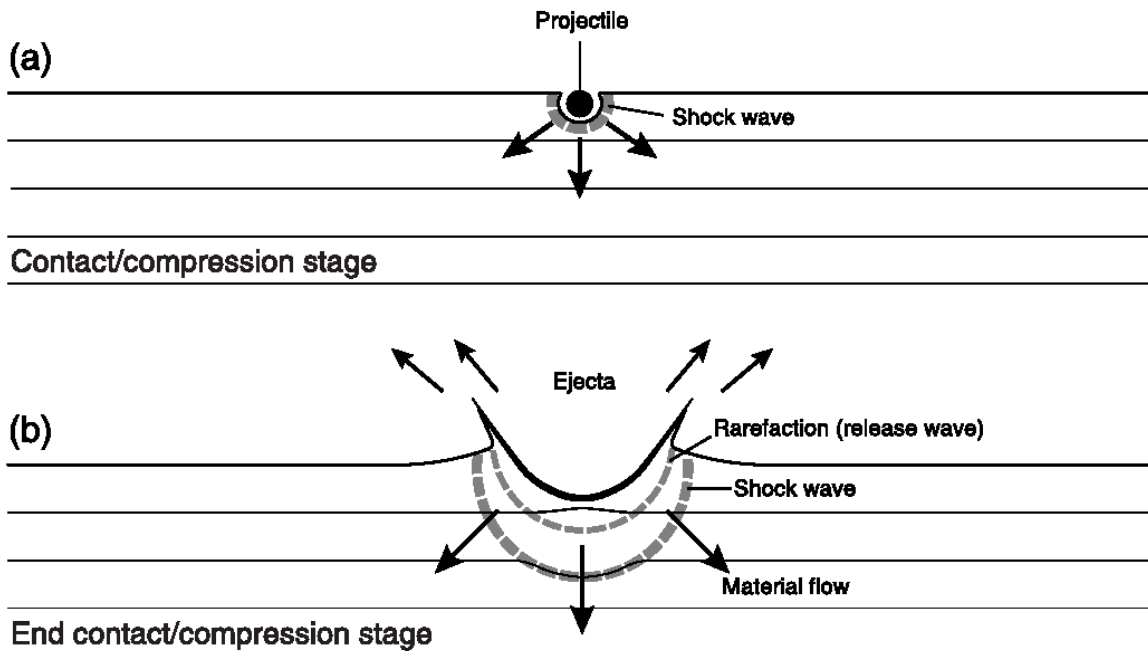
impact crater \neq volcanic crater

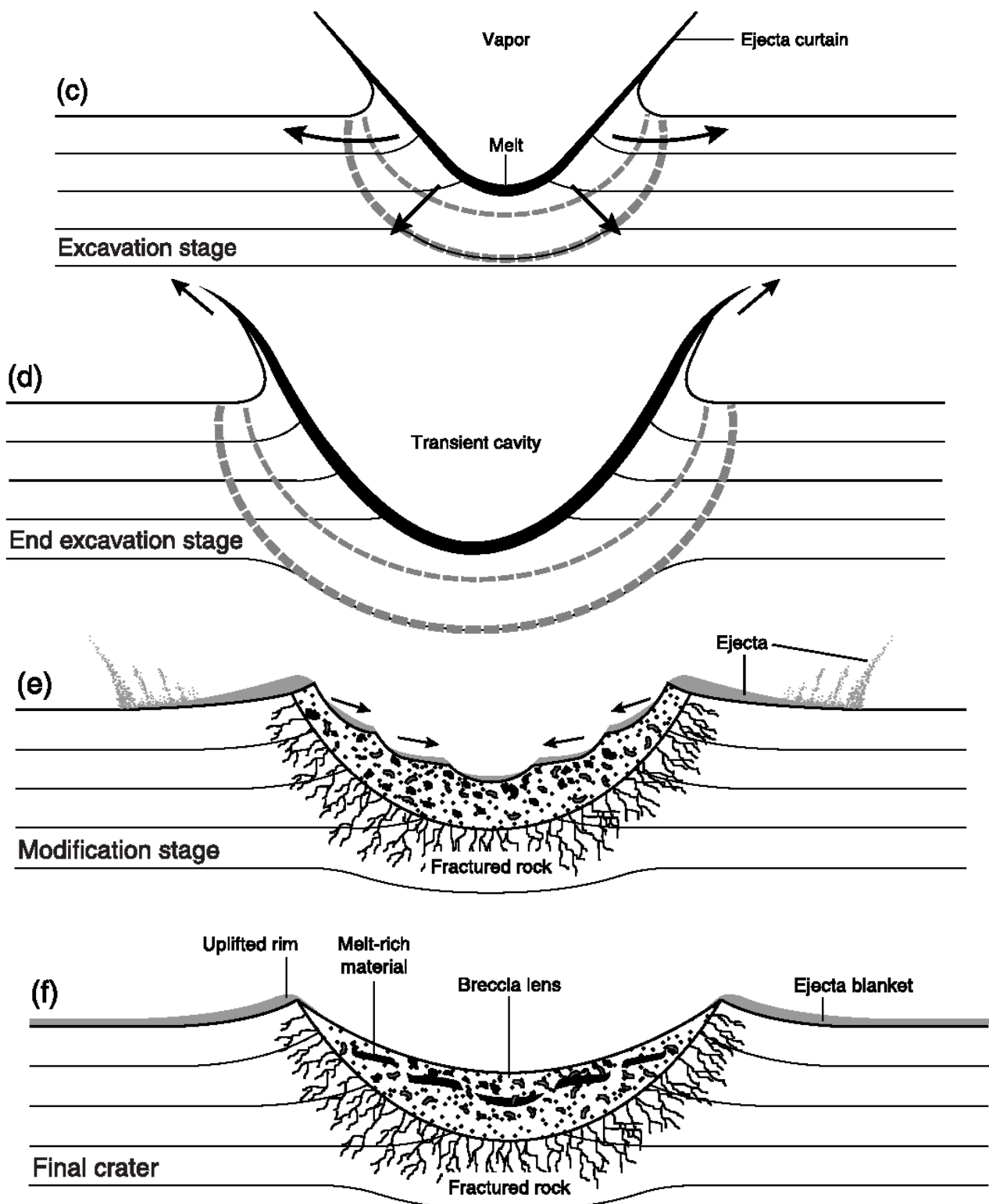
kinetic energy :

$$E = \frac{1}{2} m \cdot v^2 = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \rho \right) \cdot v^2 = \frac{\pi d^3 \rho v^2}{12} \quad (1.2.1)$$

important values: velocity of impact, structure, size of body

Process of Crater Formation:





1.2.3 Mercury

- rotation period:orbital period = 3:2 resonance ((partly relativistic) precession of the perihelion)
- surface filled with craters, but large smooth plains in between (→ lava flows) e.g. Caloris Basin 1300 km diameter → large impact event → other side: hilly, jumbled area because of seismic waves
- not much larger than the Moon (a bit similar but larger smooth areas)
- densest planet → 3/4 of diameter iron
- no atmosphere
- rotation + iron core + not a lot of liquid metal (otherwise stronger B -field) ⇒ weak to non-existent

magnetic field (no dynamo effect is possible)

1.2.4 Venus

- similar to Earth (size, mass, average density...)
- very slow rotation → no B -field
- retrograde rotation
- surface temperature very high because IR-light gets locked in + high reflection of atmosphere (96.5 % CO_2 , 3.5% N_2) strong Greenhouse effect → surface temperature $\approx 460^\circ \text{C}$
- 90× higher surface pressure than at Earth
- acid rain (H_2SO_4)
- few craters because of strong erosion (acid!), storms
- young surface → probably volcanism just “short” time ago (lava flows)
- observation: mainly radar

1.2.5 Earth and Moon

1.2.5.1 Earth

- can be considered as a double system
- surface: almost no craters (example: Nördlinger Ries) → dominated by plate tectonics, erosion, volcanos
- atmosphere: 80% N_2 , 20% O_2 → moderate Greenhouse effect → $T > 0^\circ \text{C}$
- water
- varying B -field

1.2.5.2 Moon

- very similar to Mercury
- rotation synchronous to orbit around Earth
- Mariae (dark-colored, smooth surfaced ← plains from massive impacts)
- lots of impact craters of all sorts ⇒ history of Moon/Solar System can be seen in craters, no changes since Moon became rigid
- some craters have rays
- far side of the Moon: few Mariae because of thicker crust, one large crater ⇒ otherwise more smaller craters

1.2.5.3 Eclipses

- sometimes Sun, Earth and Moon lie along a straight line ⇒ the shadow of the Earth can fall on the Moon and vice versa ⇒ this is called an eclipse
- lunar eclipse: occurs when the Moon is at full phase + Earth is located between Sun and Moon
- solar eclipse: occurs when Moon lies between Earth and Sun + Moon is at new moon
- to be in a straight line the line of nodes (intersection of Moon's orbit, which is inclined by 5° and Earth's orbit) must be pointed toward the Sun ⇒ for an eclipse: Sun and Moon must lie on or near the line of nodes, 2 seasons per year
- because angular diameters of Sun and Moon are equal as seen from Earth total eclipses are possible

1.2.6 Mars

- smaller than Earth
- Polar Caps (dry ice) → seasons similar to Earth because of 25° tilt of rotational axis → ice sublimates in summer and freezes again in winter (but: small water ice cap remains frozen)
- thin atmosphere: 95% CO_2 → weak Greenhouse effect (surface pressure 1% of Earth surface pressure)
- very low density → small or no core of Fe → no B -field
- water sublimates → no liquid water

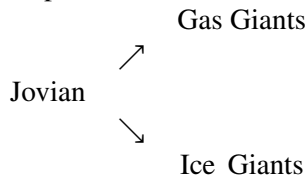
- two moons (captured asteroids)
- Olympus Mons (highest mountain in Solar System: 24 km) → shield volcano (all volcanos extinct)
- no plate tectonics
- sometimes dust storms cover whole planet → some are seasonal → erosion

1.3 The Jovian Planets

1.3.1 Introduction

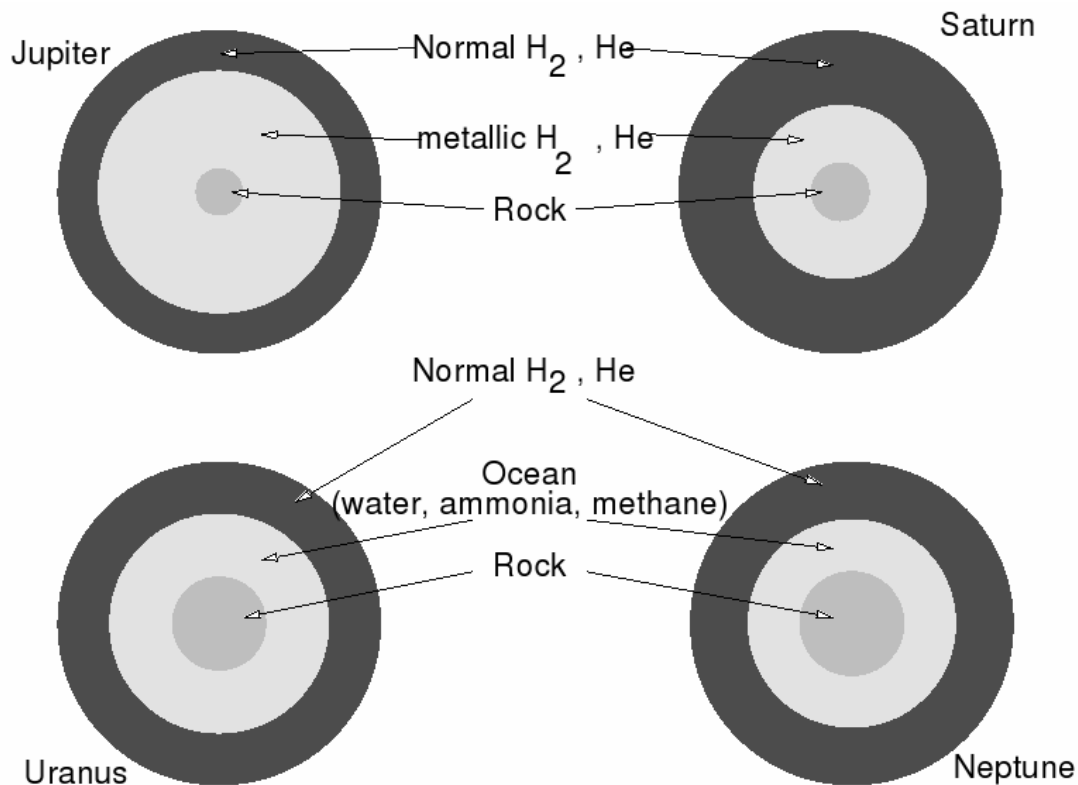
All have Rings!

composition \approx stars : 75% H, 24% He, 1% “metals”



1.3.2 Composition and Structure

Structure:



composition:

- Gas Giants \approx Sun
- Ice Giants: more metals than Gas Giants

1.3.3 Atmospheres

- Jupiter:
 - 3 layers: Ammonia, ammonia hydrosulfide, water ice/ water
 - colors from organic stuff → banded atmosphere
 - darker= deeper + hotter spots
 - storms: Great Red Spot (stable)
- Saturn: Atmospheres deeper than at Jupiter + not that dynamic

- Ice Giants:
 - H + He + a few % methane → absorbs red light → blueish color
 - banded atmosphere + clouds

1.3.4 The Hydrostatic Equilibrium

1.3.4.1 Atmospheres

Force of gas (above the area) with ρ on an area A is given by:

$$F = m \cdot g = A \cdot h \cdot \rho \cdot g \Rightarrow \text{Pressure: } p = \frac{F}{A} = \rho \cdot h \cdot g \quad (1.3.1)$$

For thin atmospheres $g = \text{const.}$! → p decreases when going upwards by Δh :

$$\Delta p = -\rho \cdot g \cdot \Delta h \rightarrow \text{infinitesimal } \lim_{\Delta h \rightarrow 0} : \frac{dp}{dh} = -\rho \cdot g \quad (1.3.2)$$

Because density and pressure aren't independent; need relationship between density and pressure ("equation of state")

$$\rightarrow \text{"Ideal Gas": } p = \frac{\rho}{\mu} \cdot k \cdot T \quad (\mu = \text{average mass of gas particle}) \quad (1.3.3)$$

$$\rightarrow \frac{dp}{dh} = -\left(\frac{\mu g}{k T}\right) \cdot p \quad (1.3.4)$$

At the beginning $h = 0$ and $p = p_0$. Assumption: $T = \text{const.}$!

Separation of Variables:

$$\int_{p_0}^{p(h)} \frac{1}{p} dp = - \int_0^h \left(\frac{\mu g}{k T}\right) dh \quad (1.3.5)$$

$$\rightarrow \ln\left(\frac{p(h)}{p(0)}\right) = -\frac{\mu g}{k T} \cdot h \quad (1.3.6)$$

such that

$$p(h) = p_0 \cdot e^{-\left(\frac{\mu g}{k T} \cdot h\right)} = p_0 \cdot e^{-\left(\frac{h}{H}\right)} \text{ with } H = \frac{k T}{\mu g} \quad (H \text{ is called scale height}) \quad (1.3.7)$$

⇒ only for isothermal atmospheres + ideal gases

$H_{\oplus} \approx 9 \text{ km}$

1.3.4.2 Gas Planets

estimation for supported material:

$$\frac{dp}{dr} = -\rho(r) \cdot g(r) \quad (1.3.8)$$

with: r = radial distance to center and $g(r) = \frac{G \cdot M(r)}{r^2}$

where $M(r)$ is the mass of the planet within r :

$$M(r) = \int_0^r 4\pi \rho(r) r^2 dr \quad (1.3.9)$$

(=summing up all shells)

⇒ now "equation of state" is needed → BUT: too complex

\Rightarrow Assumption: $\rho(r) = \text{const.} = \bar{\rho}$

$$\Rightarrow M(r) = \frac{4}{3} \pi r^3 \bar{\rho} \quad (1.3.10)$$

$$\Rightarrow \frac{dp}{dr} = -\bar{\rho}^2 G \frac{4}{3} \pi r \quad (1.3.11)$$

$$\Rightarrow dp = -\bar{\rho}^2 G \frac{4}{3} \pi r dr \quad (1.3.12)$$

boundary conditions: $r = 0, r = R, p(R) = 0, p_c = \text{center pressure}$

$$\int_0^R dp = p(R) - p(0) = 0 - p_c = -p_c \quad (1.3.13)$$

$$\Leftrightarrow - \int_0^R \bar{\rho}^2 G \frac{4}{3} \pi r dr = -\bar{\rho}^2 G \frac{2}{3} \pi (R^2 - 0) = -\bar{\rho}^2 G \frac{2}{3} \pi R^2 \quad (1.3.14)$$

\Rightarrow

$$\boxed{p_c = \bar{\rho}^2 G \frac{2}{3} \pi R^2} \quad (1.3.15)$$

(\approx factor 10 wrong)

1.3.5 Magnetic fields

differential rotation + metallic hydrogen \Rightarrow B -field \Rightarrow synchrotron radiation \Rightarrow strong radio emission

1.3.6 Individual Properties

1.3.6.1 Jupiter

- largest planet
- rapid motion \rightarrow flattend, banded atmosphere
- differential rotation (equator: slower rotation)
- strong B -field
- 4 “Galilean” moons

1.3.6.2 Saturn

- see Jupiter
- Rings! (+ gaps in between)
- six major moons

1.3.6.3 Uranus

- cold atmosphere \rightarrow frozen ammonia
- less He than Gas Giants
- inclination of rotation axis : 98° (“rolling on the ecliptic plane”)
- five major moons

1.3.6.4 Neptune

- atmosphere like Uranus but more active , bright methane clouds + cloud layers
- 2 major moons (Triton, Nereid)
- dark spot \rightarrow new spot

1.3.7 The Moons of the Giants

1.3.7.1 The Galilean Moons

all moons show the same face to Jupiter \Leftarrow synchronized rotation (tidally locked)

build-up similar to terrestrial planets + ice

Io and Europa: moonsize

Ganymede and Callisto: Mercury sized

1.3.7.1.1 Io

- colorful sulfur layer deposited by explosive eruptions from volcanic vents (\rightarrow somewhat like terrestrial geysers) \Leftarrow because: interior is heated by tidal forces with Jupiter (gets flexed) \rightarrow volcanism
- very high temperatures!
- because of movement in Jupiter's B -field \rightarrow radio emission

1.3.7.1.2 Europa

- composed of rock + covered with smooth ice layer
- no craters but cracks and ruffled crust \rightarrow volcanism in the past
- \rightarrow possibly: water ocean below ice (because of internal heat \leftarrow tidal forces)

1.3.7.1.3 Ganymede

- icy surface:
 - areas of dark ancient cratered surface
 - young, heavily grooved, lighter-colored terrain
- probably metallic core \leftarrow because: has strong magnetic field

1.3.7.1.4 Callisto

- pocked with craters, no geologic activity and tidal heating
- covered with dark, dusty substance
- no Fe-core!

1.3.7.2 Titan

- Saturn's largest satellite
- terrestrial structure
- dense atmosphere: 99% N_2 , 1% CH_4 , some hydrocarbons \rightarrow perhaps: similar to early atmosphere of Earth

1.3.7.3 Triton

- moon of Neptune
- young icy surface \rightarrow volcanism (not a lot craters) \rightarrow nitrogen geysers
- frozen N_2 + ice cap of frozen methane
- strange orbit (retrograde)
- nitrogen atmosphere (very thin)

Chapter 2

The Planets – Motion

2.1 Celestial Mechanics

2.1.1 Laws

Kepler's Laws:

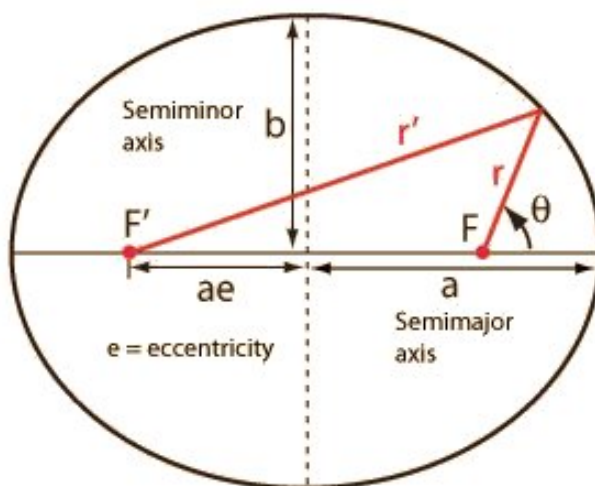
1. Each planet moves around the Sun in an ellipse, with the Sun at one focus.
2. The radius vector from the Sun to the planet sweeps out equal areas in equal intervals of time.
3. The squares of the periods of any two planets are proportional to the cubes of the semimajor axes of their respective orbits: $P \sim a^{\frac{3}{2}}$.

Newton's Law of Gravitation:

$$\mathbf{F}_1 = -\frac{G m_1 m_2}{r_{12}^2} \frac{\mathbf{r}_{21}}{r_{12}} \quad (2.1.1)$$

(F_1 is the force exerted on object 1)

2.1.2 Properties of Ellipses



Def.: Ellipse:

$$r + r' = 2a$$

Def.: Eccentricity e :

ratio between distance from center of ellipse to focal point and semimajor axis ($e = \frac{ae}{a}$)

In polar coordinate form:

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta} \quad (2.1.2)$$

(θ is called true anomaly)

for planets and comets/asteroids:

- nearest point from focus is called *perihelion*
- farthest point from focus is called *aphelion*

for stars:

- *periastron*
- *apastron*

calculation:

$$d_{\text{peri}} = a(1 - e) \quad \text{and} \quad d_{\text{ap}} = a(1 + e) \quad (2.1.3)$$

2.1.3 Derivation of Kepler 2

In polar coordinates the radius vector is $\mathbf{r} = r \hat{\mathbf{e}}_r$.

If the planet moves with angular velocity $\dot{\theta}$ the direction of $\hat{\mathbf{e}}_r$ changes also at the same rate:

$$\dot{\hat{\mathbf{e}}}_r = \dot{\theta} \hat{\mathbf{e}}_\theta \quad (2.1.4)$$

\Rightarrow velocity of the planet:

$$\dot{\mathbf{r}} = \dot{r} \hat{\mathbf{e}}_r + r \dot{\hat{\mathbf{e}}}_r = \dot{r} \hat{\mathbf{e}}_r + r \dot{\theta} \hat{\mathbf{e}}_\theta \quad (2.1.5)$$

\Rightarrow angular momentum:

$$\mathbf{L} = \mu(\mathbf{r} \times \dot{\mathbf{r}}) = \mu r^2 \dot{\theta} \hat{\mathbf{e}}_z \quad (2.1.6)$$

($\hat{\mathbf{e}}_z$ is unit vector perpendicular to orbital plane)

\Rightarrow magnitude of \mathbf{L} :

$$L = \mu r^2 \dot{\theta} \quad (2.1.7)$$

an infinitesimal area in polar coordinates is:

$$dA = dr(r d\theta) = r dr d\theta \quad (2.1.8)$$

\Rightarrow integration from focus to distance $r \Rightarrow$ area swept out when there is an infinitesimal change in θ is:

$$dA = \frac{1}{2} r^2 d\theta \quad (2.1.9)$$

\Rightarrow time rate of change:

$$\dot{A} = \frac{1}{2} r^2 \dot{\theta} \quad (2.1.10)$$

Comparison with L :

$$\dot{A} = \frac{1}{2\mu} L \quad (2.1.11)$$

Since L is constant in a conservative force field the swept out area is constant!

Because the radius vector to the planet varies, the velocity does so too. The planet is the fastest at perihelion and the slowest at aphelion.

2.1.4 Derivation of Kepler 3

$$\dot{A} = \frac{1}{2\mu} L \Leftrightarrow dA = \frac{1}{2\mu} L dt \quad (2.1.12)$$

to get the area swept over in one orbit, integration is necessary:

$$\int dA = \frac{1}{2\mu} L \int_0^P dt \quad (2.1.13)$$

since the area of an ellipse is:

$$\pi a b = \pi a^2 \sqrt{1 - e^2} \quad (2.1.14)$$

\Rightarrow

$$\pi a^2 \sqrt{1 - e^2} = \frac{1}{2\mu} L P \quad (2.1.15)$$

\Rightarrow squared:

$$\pi^2 a^4 (1 - e^2) = \frac{1}{4\mu^2} L^2 P^2 \quad (2.1.16)$$

for a closed planetary orbit L is:

$$L = \mu \sqrt{G M a (1 - e^2)} \quad (2.1.17)$$

(where M is the total mass)

\Rightarrow inserting L :

$$P^2 = \frac{\pi^2 4 a^3}{G M} \Leftrightarrow P^2 = \frac{4 \pi^2}{G (m_1 + m_2)} a^3 \quad (2.1.18)$$

if $m_1 \gg m_2$:

$$\boxed{\frac{P^2}{a^3} = \frac{4 \pi^2}{G m_1}} \quad (2.1.19)$$

2.2 Tidal Forces

2.2.1 Basic Concept

The force of gravity caused by an heavy object gets weaker as you move farther away from it this leads to a force on a body in this gravitational field - a tidal force.

The tidal force between two test masses m along an infinitesimal distance dr is

$$dF_g = \left(\frac{dF_g}{dr} \right) dr = \frac{2G M m}{r^3} dr \quad (2.2.1)$$

where r is the distance between M and m .

2.2.2 Moon-Earth

The tidal forces of Moon and Sun act on the Earth's body but it is rather rigid, so such distortion effects are small. However, the fluid in the Earth's oceans is much more easily deformed and this leads to significant tidal effects.

2.2.2.1 An Easy Tidal Model

basic ideas:

- planet is completely covered by an ocean of uniform depth
- negligible friction
- only effects by Moon taken into account

\Rightarrow The gravitational attraction of the Moon produces two tidal bulges on opposite sides of the Earth.

As Earth rotates under these bulges, a given point on the surface will experience two high and two low tides for each rotation of the planet.

2.2.2.2 A bit more complicated Tidal Model

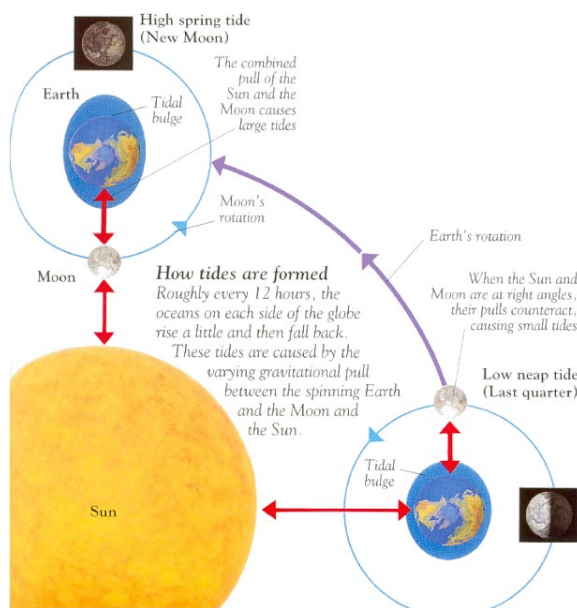
Why more complicated?

- Earth and Moon are not static but instead are in orbit around the center of mass for the system
- oceans have varying depth
- substantial friction between the oceans and the Earth exists
→ tidal bulges are ahead of Moon by 2.5 h

⇒ now it would be possible to explain the different tides at different places on Earth

2.2.2.3 Most Realistic Tidal Model

To be even more precise when making tidal models the effects of the other members of the Solar System have to be taken into account. For the purpose of a rough sketch of what is going on it is sufficient to allow for only the effects of the Sun (the effect of the Sun on Earth tides is less than half that of the Moon).



Large tides are experienced when the Sun and the Moon are lined up with the Earth at new/full phases of the Moon. These are called spring tides. The amount of heightening is about the same whether the Sun and Moon are lined up on opposite sides of the Earth or on the same side.

Conversely, when the Moon is at first/last quarter phase, the Sun and Moon interfere with each other in producing tidal bulges and tides are generally weaker; these are called neap tides.

2.2.2.4 Tidal Locking

Tidal forces are not only responsible for the tides but have various effects.

For example, as a consequence of tidal interactions with the Moon, the Earth is slowly decreasing its rotational period because of friction on the ocean floor. Eventually Earth and Moon, which is accelerated will have exactly the same rotational period, and these will also exactly equal the orbital period (the length of day increases by 1.6 ms per century). Thus, billions of years from now the Earth will always keep the same face turned toward the Moon. At the same time, the separation between the Earth and Moon will slowly increase (about 4 cm per year) in just such a way as to conserve angular momentum for the entire system.

Furthermore the interior of the Earth and Moon are heated by the tides in their bodies. This effect is very small for the Earth and Moon.

2.2.3 Tidal forces in the rest of the universe

Another application of tidal forces is the fact that there is a limiting radius for the orbit of one body around another, inside of which the tidal forces are so large that no large solid objects can exist that are held together only by gravitational forces.

To calculate this limit we assume a spherical satellite (suppose that it is made up of two spheres of radius r and mass m) around a planet/central body with mass M and radius R . The force between the two spheres F_s is given

by:

$$F_s = \frac{G m^2}{(2r)^2} \quad (2.2.2)$$

assume that each part of the satellite is at a distance D of the planet. The attraction force F_1 , between the planet and the nearest sphere, will be greater than the force F_2 between the planet and the more distant sphere. The force F_1 is

$$F_1 = \frac{G M m}{D^2} \quad (2.2.3)$$

and force F_2 is

$$F_2 = \frac{G M m}{(D + 2r)^2} \quad (2.2.4)$$

The resulting force F_{res} is the force which tends to separate the two parts.

$$F_{\text{res}} = F_1 - F_2 \quad (2.2.5)$$

because $D \gg r$

$$F_{\text{res}} = -\frac{4G M m r}{D^3} \quad (2.2.6)$$

The two masses will be separated if

$$F_s < F_{\text{res}} \quad (2.2.7)$$

$$\frac{2^4 M}{D^3} > \frac{m}{r^3} \quad (2.2.8)$$

now replace the mass by the densities \Rightarrow so if D satisfies the following equation the satellite will be torn apart

$$D < 2^{\frac{4}{3}} R_{\text{planet}} \left(\frac{\rho_{\text{planet}}}{\rho_s} \right)^{\frac{1}{3}} \quad (2.2.9)$$

This is a quite good approximation since $2^{\frac{4}{3}}$ is 2.51, whereas the exact value is 2.456.

D is called the Roche Limit. Thus, solid objects put into orbit inside the Roche limit will be torn apart by tidal forces, and conversely, solid objects cannot grow by accreting into larger objects if they orbit inside the Roche limit.

A famous example are the rings of Saturn because they lie inside the Roche limit, they cannot be solid objects held together by gravitation and must be composed of many small particles.

Obviously solid objects can exist inside the Roche limit (e.g. a spacecraft) but they must be held together by forces other than gravity. In case of a spacecraft chemical forces between the atoms and molecules are much larger than the gravitational forces.

Tidal forces are present in lots of places in the universe, near black holes and neutron stars, in close binary systems, etc.

2.3 Long-Term Evolution

The Sun's gravitational attraction is the main force acting on each planet, but there are much weaker gravitational forces between the planets, which produce perturbations of their elliptical orbits; these make small changes in a planet's orbital elements with time.

Based on these principles mathematicians like Leibniz, Gauß, Euler, Lagrange, Poincaré and many more, in the 19th century, spent a lot of time calculating the precise orbits of the planets (celestial mechanics).

The planetary motion can be considered, to a first approximation, as taking place along Kepler's orbits.

This view can be advanced in a way that the constants which describe the motion of a planet around the Sun are "perturbed" by the motion of other planets and vary as a function of time; hence the name "perturbation theory".

The difficulty involved in the development of this particular theory is the fact that the terms of the expansions obtained contain the time outside the trigonometric functions. The contribution of such terms to the series

of perturbation theory becomes significant only for long periods of time. If relations of this kind have to be allowed for, both secular ($A t''$) and mixed ($B t \cos(\omega t + \psi)$) terms appear in the solutions.

So the use of mathematical series for the orbital elements as functions of time can accurately describe perturbations of the orbits of solar system bodies for limited time intervals. For longer intervals, the series must be recalculated.

Today computers are used to solve directly the equation of motion by numerical means. The computers can be programmed to make allowances for the important perturbations on all the orbits of the member bodies. Such calculations have now been made for the Sun and the major planets over time intervals of up to several tens of millions of years.

The result is that the motion of the inner planets is chaotic.

Chapter 3

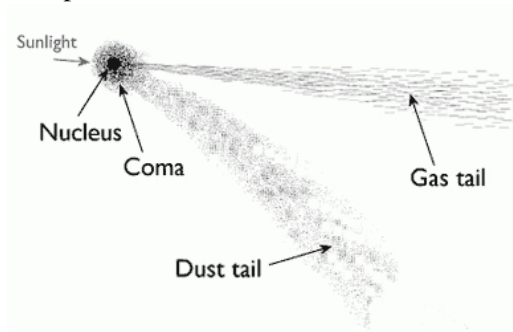
Small Solar System Bodies (SSSBs)

3.1 Asteroids

- minor planets: between Mercury and Neptune (esp. between Mars and Jupiter)
- Trojans, Greeks in Lagrangian Points of Jupiter and Sun
- Types of Asteroids:
 - S-type: 1/6 of known ones (silicaceous) (2–3.5 AU)
 - M-type: 8% iron and nickel dominated (metals) (2–3.5 AU)
 - C-type: 75% carbonaceous (2–4 AU)
 - also P, D (=Trojans)
- special gaps in asteroid belt called Kirkwood Gaps because of special resonances (orbits not very stable)
- diameter < 1000 km

3.2 Comets

Components:



- Nucleus: “Dusty Snowball” (1–50 km) water ice + 15–20% CO₂, CO
- Coma: 10⁴–10⁵ km evaporated gas surrounding nucleus → interacts with sunlight and solar winds → produces long, familiar tails (up to 1 AU length)
Coma is surrounded by hydrogen gas (halo) envelope ($\approx 10^{10}$ m diameter)
- Tail consists of 2 parts:
 - Dust Tail: evaporated dust away from nucleus; size $\approx 10^6$ –10⁷ km; behind comet slightly affected by centrifugal forces
 - Ion Tail: ionized gas, extends up to 10⁸ km, often blueish; moves perpendicular to direction of movement

Examples for famous comets:

- Halley: short-term comet
- Shoemaker-Levy: was in orbit around Jupiter; broke apart ⇒ fell onto Jupiter
- Sungrazers: loose material by sublimation processes when being too near to the Sun

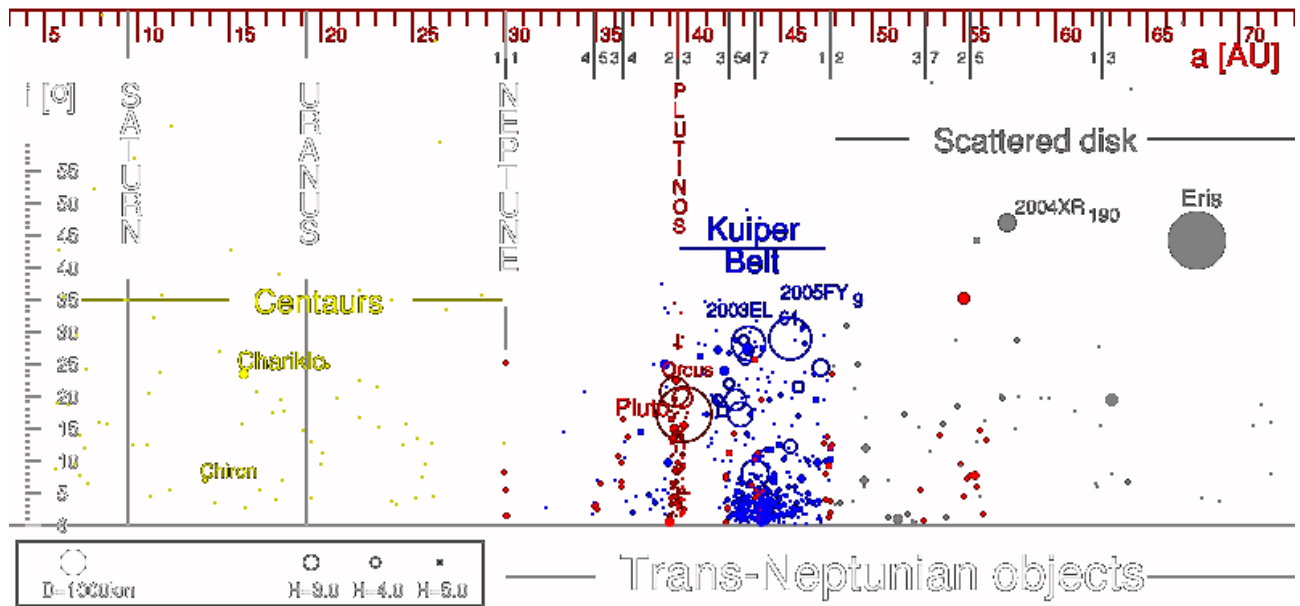
Types of comets:

Long-Period Comets: $P \geq 200$ yrs have very eccentric orbits \Leftarrow originate out of Oort cloud \rightarrow come inwards because of interaction with bypassing stars

Short-Period Comets: $P < 200$ yrs have angular momenta like planets; mostly in plane of Solar System → come from Kuiper Belt (30–50 AU)

3.3 TNOs

prototypes: Pluto/Charon: icy surface which is probably cratered, further out than Neptune



Part II

Astronomical Instruments

Chapter 4

Telescopes

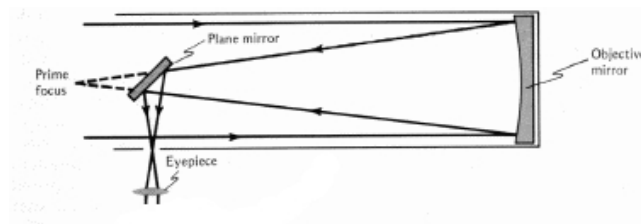
4.1 Types

4.1.1 Refractors or Reflectors

A) Lenses = Refractors: old fashioned way, because: max. diameter ≤ 1 m due to weight \rightarrow can not be supported

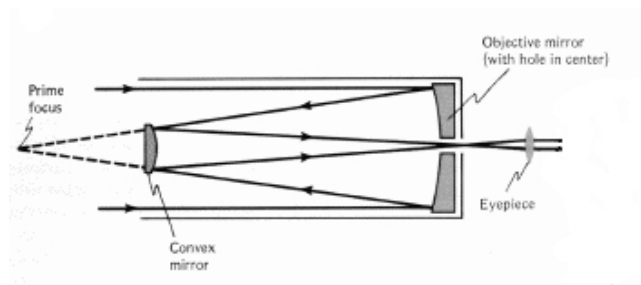
B) Mirrors = Reflectors: diameters up to 11 m, use of parabolic mirrors (spherical aberration \leftarrow would need correction)

4.1.2 Newtonian Telescope



parabolic mirror; common cheaper ones; \ominus large size (\approx focal length)

4.1.3 Cassegrain Telescope



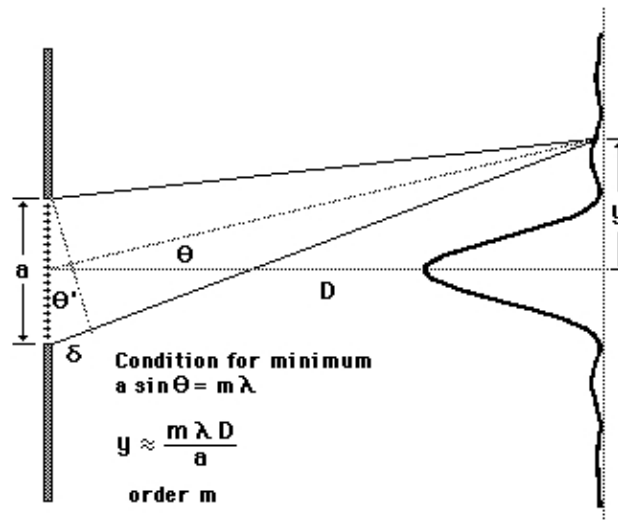
“folded optical path” \Rightarrow smaller=shorter \Rightarrow Telescope of choice today

4.1.4 Schmidt Telescope

spherical mirror; mostly for: wide angle ; needs corrector plate

4.2 Resolution

interference occurs due to light waves:



At a telescope rings → bright maximum in the middle called Airy disk!

⇒ Rayleigh criterion: maximum of diffraction pattern of one source must fall into minimum of diffraction pattern of other source.

Therefore the diffraction limited resolution is:

$$\alpha = \frac{1.220 \lambda}{d} \quad \alpha_{\text{opt}} = \frac{12''}{D/1\text{cm}} \quad (4.2.1)$$

where α is the minimum angle that can still be resolved. Therefore the resolution is better if the telescope has a larger diameter or if observation is performed at shorter wavelengths.

In reality: 3× better is achievable

4.3 Active Optics

Active Optics systems correct the shape and alignment of the telescope optics once every few minutes. This allows the optics to be continually adjusted in order to compensate for flexure (due to the weight of the mirror), temperature differences, etc. as the telescope moves around the sky.

4.4 Adaptive Optics

Adaptive Optics means correcting the atmospheric turbulences by adapting the shape of the mirror to its optimal parabolic shape. The mirror is placed on so called “actuators”, which can change their size. A computer then controls the pressure on the mirror to keep its shape optimal. Because of turbulences it’s not possible to see stars with $\theta < 0.3''$ (stars → disks).

So if telescope diameter is larger than 40 cm, the resolution does not increase automatically.

Adaptive Optics improves astronomical seeing but works on Earth only in IR. (→ highest resolution optical and UV observations are done in space)

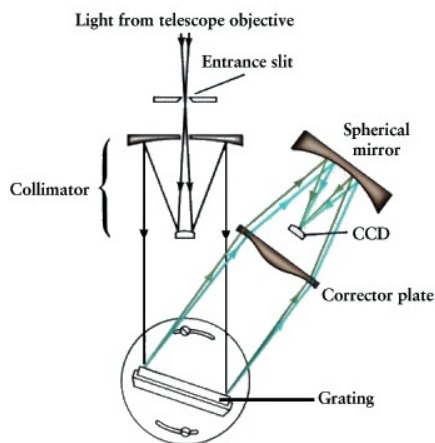
For knowing how to change you need a guide star or an artificial star made with a laser.

Chapter 5

Spectroscopy

Spectroscopy is an important tool in astronomy. We can get information about the temperature, density, composition, and important physical processes of an astronomical object.

For obtaining the spectrum of an astronomical object a spectrograph is needed.



For precise measurements the spectrograph must be able to distinguish between slightly different wavelengths. The smallest difference is

$$\Delta\lambda = \frac{\lambda}{nN} \quad (5.0.1)$$

where N is the number of lines illuminated and n is the order of the spectrum.

The number $\frac{\lambda}{\Delta\lambda}$ is called resolving power of the spectrograph.

Part III

Exoplanets

Chapter 6

Exoplanets

6.1 General Stuff

2 ways:

A) direct method = direct imaging

B) indirect method:

- gravitational interaction with star in radial velocity (spectroscopic)
- gravitational interaction with star in motion of star (astrometric)
- influence of star on light from behind planet (gravitational lensing)
- transits can be detected in photometry

6.2 Direct Imaging

Need good telescope! (Resolution power + contrast)

6.2.1 Contrast

Energy passing through 1 m^2 per second at distance r (“flux”) when assuming isotropic spread:

$$F = \frac{L}{4\pi r^2} \quad (6.2.1)$$

at Earth the luminosity is 10^{10} times weaker than at the Sun and in IR 10^7 (for Jupiter: 10^9 times)
 \Rightarrow It's necessary to get contrasts of $1:10^9 \Rightarrow$ for Solar System planets \Rightarrow not possible today

6.2.2 Angular Separation

$$\tan \theta = \frac{r}{d} \quad (6.2.2)$$

(where r is distance between planet and star and d is the distance to the star)
 \Rightarrow because of small-angle approximation:

$$\theta \approx \frac{r}{d} \quad (6.2.3)$$

Typical distance to stars: $d \approx 100 \text{ lyrs}$

Typical distances in planetary systems: $r \approx 1 \text{ AU}$

$\Rightarrow \theta = 0.03''$

\Rightarrow resolving power of telescope:

$$\alpha = \frac{12''}{D/1\text{cm}} \quad (6.2.4)$$

$\Rightarrow 0.03'' \hat{=} 4\text{m}=D \rightarrow$ works

BUT NO: Atmosphere \rightarrow resolution $\approx 0.5''$ (“seeing”)
 \rightarrow only from space possible

⇒ NO DIRECT IMAGING FROM EARTH

(only in IR: with adaptive optics, but only dim stars because of contrast = it's possible to display regions close to stars but no good resolution)

6.3 Radial Velocity Measurements

The radial velocity or Doppler shift method has been the most successful extrasolar planet detection method to date, detecting the vast majority of planets.

2-body problem: Assumption circular orbit (CMS system):

$$\frac{m_1}{r_2} = \frac{m_2}{r_1} \quad (6.3.1)$$

velocity of star due to action of planet:

$$v_1 = \frac{2\pi r_1}{P} = \frac{2\pi m_2}{P m_1} r_2 \quad (6.3.2)$$

→ with Jupiter $v_1 = 13.1 \text{ m s}^{-1} \approx 50 \text{ km h}^{-1}$

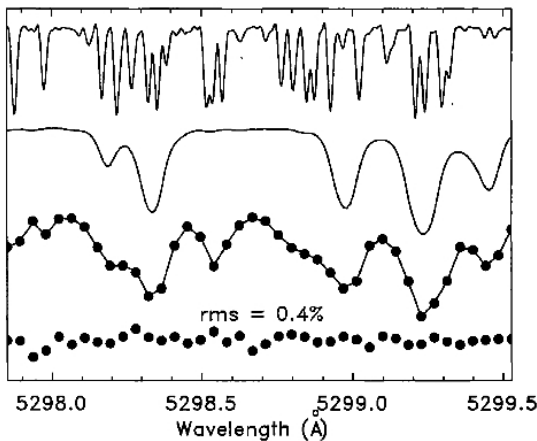
→ need to be able to measure star velocities with better than 13 m s^{-1}

→ works with spectroscopic methods via Doppler effect:

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{v}{c} \quad (6.3.3)$$

→ good spectrographs needed → $\frac{\Delta\lambda}{\lambda} = 4 \cdot 10^{-8}$ → even such a good spectrograph may not be useful because of the intrinsic line width (thermal motion)!

However, very high precision spectral line measurements (one part in a ten thousandth of a spectral line width) can be performed by superimposing a comparison spectrum with many lines (like an iodine cell in the light path at the observatory) on to the stellar spectrum.



When starlight passes through the cell on its way into the spectrograph, the gaseous iodine absorbs light at very specific wavelengths, adding many, deep, sharp absorption features to the spectrum. The measured lines get shifted against the lines of the iodine cell and the actual position of the lines can be fixed very precisely.

The precision possible for this detection method is about one meter per second, this limit imposed by intrinsic stellar surface fluctuations i.e., variations present in even the most stable solar-type stars. Thus this method is limited to detection of planets around fairly stable, single star systems.

6.4 Results

about 200 exoplanets found

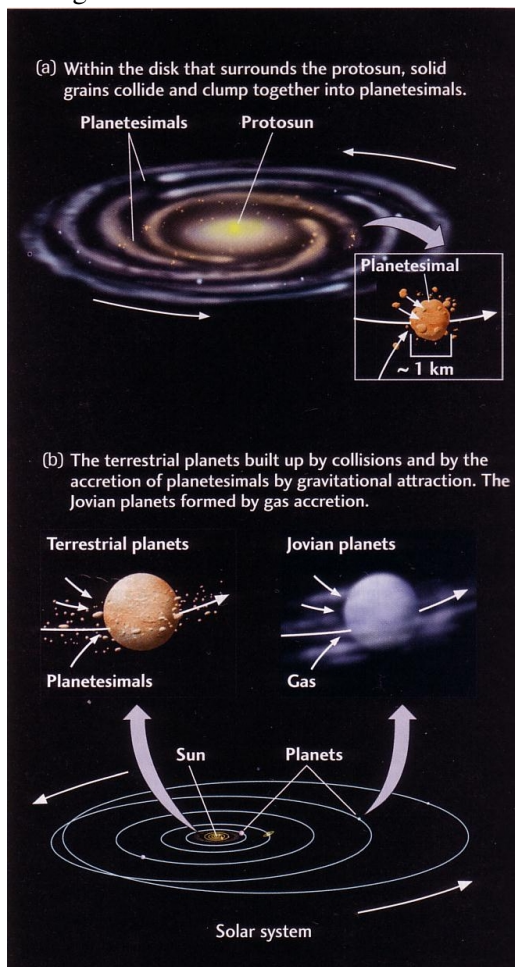
1. mass: $M > M_{\text{Jupiter}}$ for most ones → selection effect (more change in v); can be determined via the mass function which will be introduced in section 8.4.4; lowest mass found today is $5 M_{\oplus}$
2. semimajor axis: small ($P = \text{years}$) → selection effect (e.g. “hot Jupiters” like sungrazers) because of short observation times

3. eccentricities: most in eccentric orbits → different from Solar System → ???
4. multiple planets

Chapter 7

Formation of Planetary Systems

A good theory of planetary formation is based on the nebular hypothesis, which will be explained in the following.



1. within interstellar gas and dust cloud collapse begins \Rightarrow stars form out of biggest fragments \Rightarrow first stars evolve \Rightarrow explode as SNe \Rightarrow heavy elements + gas is pushed outwards \rightarrow cools down

2. smaller fragments condense \Rightarrow formation of a prestellar cloud which is spheroidal and slowly rotating \Rightarrow while condensing: its rotation rate increases (conservation of angular momentum) \rightarrow becomes increasingly flattened \Rightarrow in the middle of the disk: protostar forms \Rightarrow By the time fusion starts, the protoplanetary disk has started to form agglomerations at various distances from the center.

3. dust grains grow by collisions into planetesimals (see figure) \Rightarrow planetesimals begin to interact with each other through gravity \rightarrow grow rapidly

4. What kind of planet will be the result?

What kind of planet a protoplanet will become is determined by its mass and its distance from the central star.

Planets of low mass cannot hold hydrogen and helium. The other crucial factor, the distance of the planet from the star, also influences the escape of hydrogen and helium from the planet's gravity, because inner planets become hotter and so have more difficulty in retaining the lightest gases.

These considerations explain well the overall structure of the Solar System. However, the majority of extrasolar planets do not conform at all to this system. Planets with masses even greater than that of Jupiter have been found in near-circular orbits of their host stars. Since it is hard to understand how they could have formed in situ, the current understanding is that they were formed further out, beyond several AU, but then underwent inward orbital migration.

Part IV

Stars

Chapter 8

Observations

Def.: Star: gas ball → consist of H, He; produce energy by fusion

8.1 Temperature and Spectrum

8.1.1 Planck's Radiation Law

approximately: thermodynamic equilibrium

→ Max Planck's Blackbody Radiation:

$$F_{\lambda} = \frac{2 \frac{hc^2}{\lambda^5}}{e^{\frac{hc}{\lambda kT}} - 1} \quad (8.1.1)$$

(F_{λ} is the energy emitted per second and wavelength interval)

Stefan-Boltzmann Law: Power emitted per 1 m² surface of blackbody

$$P = \sigma T^4 \quad (8.1.2)$$

(hotter body → higher luminosity)

Wien's displacement law: maximum blackbody radiation

$$\lambda_{\max} T = 2.898 \cdot 10^{-6} \text{ K} \quad (8.1.3)$$

(hotter → peak at shorter wavelength)

8.1.2 Spectral Classification

blackbody → stellar atmosphere: absorbing → line spectrum

(Sun: Fraunhofer lines)

Spectral types are a temperature sequence!

Spectral classes:

O B A F G K M

30 000 K “early type”

3 000 K “late type”

subtypes 0...9

sun: G2

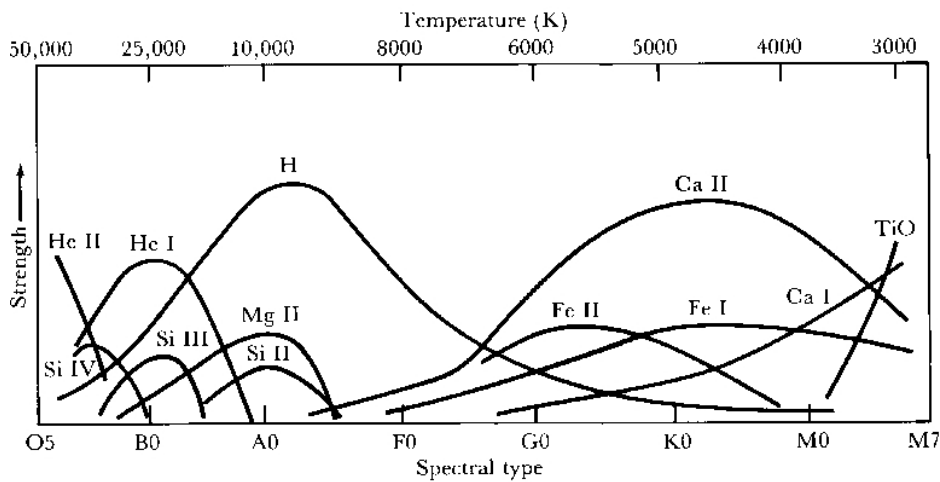


Figure 12-9
Kaufmann
DISCOVERING THE UNIVERSE
Second Edition
© 1990, W. H. Freeman and Company

7-36

L, T - Stars: Brown dwarfs

L-dwarfs:

- $T = 1200 \text{ K} - 2500 \text{ K}$
- low mass
- some do not have fusion
- peak in IR
- optical: prominent lines from metal hydrides and alkali metals

T-dwarfs:

- $T \approx 100 \text{ K}$
- strong molecule lines such as methane

8.2 Brightness and Luminosity

8.2.1 Luminosity

Def.: Luminosity: The total energy emitted by a star per second is called its luminosity.

Luminosity of the Sun: $L_{\odot} = 3.9 \cdot 10^{26} \text{ W}$

Assumption: isotropic radiation

\Rightarrow Flux is given by the *inverse square law*:

$$F = \frac{L}{4\pi r^2} \quad (8.2.1)$$

(star fluxes are very small $\approx 10^{-8} \text{ W m}^{-2}$)

8.2.2 Magnitudes

8.2.2.1 easy

Stellar fluxes are traditionally measured in *magnitudes*.

A brightness difference of 5 magnitudes corresponds to a ratio of 100 in detected flux.

Two stars have magnitudes m_1 and m_2 :

$$\frac{f_1}{f_2} = 100^{(m_2 - m_1)/5} \quad (8.2.2)$$

$$\Rightarrow \log\left(\frac{f_1}{f_2}\right) = \frac{m_2 - m_1}{5} \log_{10} 100 = \frac{2}{5}(m_2 - m_1) \quad (8.2.3)$$

or

$$m_2 - m_1 = 2.5 \log_{10} \left(\frac{f_1}{f_2} \right) = -2.5 \log_{10} \left(\frac{f_2}{f_1} \right) \quad (8.2.4)$$

LARGER MAG = FAINTER STAR!!!

⇒ Inverse square law links different distances to magnitudes:

$$\frac{F}{f} = \frac{L/(4\pi D^2)}{L/(4\pi d^2)} = \left(\frac{d}{D} \right)^2 \quad (8.2.5)$$

⇒ Absolute magnitude M : (distance= 10 pc)

$$\Rightarrow m - M = 2.5 \log_{10} \left(\frac{F}{f} \right) = 2.5 \log_{10} \left(\frac{d}{10pc} \right)^2 = 5 \log_{10} d - 5 \quad (8.2.6)$$

$m - M \triangleq$ distance modulus

(Sun: -26.7 mag, full moon: -12.6 mag, naked eye limit: $+6.0$ mag best achievable: $+30$ mag)

8.2.2.2 advanced

The brightness of an object (whether apparent or absolute) depends on the wavelength at which we observe it, as we saw in the discussion of radiation laws.

Generally, astronomical observations are made with an instrument that is sensitive to a particular range of wavelengths. For example, if we observe with the naked eye, we are sensitive only to the visible part of the spectrum, with the most sensitivity coming in the yellow-green portion of that.

Thus, to be precise in discussing brightness or magnitude, we must specify which region of the electromagnetic spectrum we are just talking about.

The magnitude integrated over all wavelengths is called bolometric magnitude.

This magnitude is defined relative to the Sun:

$$M_{\text{bol}} = M_{\text{bol(Sun)}} - 2.5 \log \left(\frac{L}{L_{\odot}} \right) \quad (8.2.7)$$

where $M_{\text{bol(Sun)}} = +4.75$

The bolometric magnitude can be related to the visible magnitude using a bolometric correction (BC)

$$M_{\text{bol}} = M_V + BC \quad (8.2.8)$$

or in apparent magnitudes

$$m_{\text{bol}} = m_V + BC \quad (8.2.9)$$

We now have an equation relating a stars luminosity to its absolute bolometric magnitude and so if we know the stars M_{bol} we can work out how much power the star is radiating from its surface (Actually this is an underestimate, as we have not taken into account the absorption of starlight by interstellar matter).

8.2.3 Colors

8.2.3.1 UBVRI Wavelength Filters

Filter transparent for:

U → ultraviolet = centered at 365 nm; bandwidth: 68 nm

B → blue = centered at 440 nm; bandwidth: 98 nm

V → visual = centered at 550 nm; bandwidth: 89 nm

R → red = centered at 638 nm; bandwidth: 138 nm

I → infrared = centered at 797 nm; bandwidth: 149 nm

8.2.3.2 The Color Index

The color index is the difference in brightness between two different spectral ranges(filters). By definition, the difference is brightness at shorter wavelength minus brightness at longer wavelength.

The star Vega is defined as an A0 star, and is the standard for:

$$B - V \equiv 0 \text{ and } B - U \equiv 0$$

The color index is negative for hotter and positive for cooler stars. There is a close connection between the surface temperature/spectral class and the color index.

Color Index	Spectral Class	Color
-0.33	O5	Blue
-0.17	B5	Blue-white
0.15	A5	White with bluish tinge
0.44	F5	Yellow-White
0.68	G5	Yellow
1.15	K5	Orange
1.64	M5	Red

(This table is only valid for the $B - V$ color index.)

The UBVRI system is not the only broad band system but the others aren't used that frequently.

8.3 Distance

parallax measurement (several times over year)

parallax angle (small-angle approximation):

$$p = \frac{1 \text{ AU}}{d} \quad (8.3.1)$$

The parsec (pc) is the distance at which 1 AU subtends 1".

$$1 \text{ pc} \approx 3.26 \text{ lyrs}$$

if p is known in arcsecs then distance:

$$p = \frac{1}{d} \quad (8.3.2)$$

best parallax measurements:

Hipparcos satellite: 120 000 objects in mas errors ($B - V, V - J$)

Tycho catalogue: 10^6 stars with 20–30mas precision (2 band photometry)

direct distant measures $\approx 1 \text{ kpc}$

accuracy $\approx 0.01''$ Earth, 1 mas space

but: the planned European Gaia satellite will bring better results

($\sim 4 \mu\text{arcsec}$ precision, 4 color to $V = 20 \text{ mag}$, 10^9 objects)

8.4 Masses

50–80% of all stars in solar neighborhood are part of multiple systems

apparent binaries: (“optical double”) just seem to belong together

visual binaries: bound system that can be imaged (e.g., Mizar) \rightarrow motion can be imaged (periods $\approx 1\text{--}100 \text{ yrs}$)

spectroscopic binaries: bound system, cannot resolve image into stars, but: Doppler effect in stellar spectrum
(short periods: hrs, months) \rightarrow wobble around center of mass (because CMS moves as a straight line)

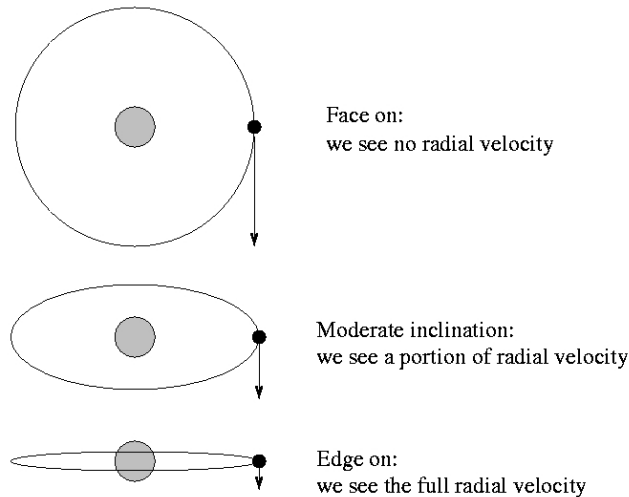
8.4.1 Visual Binaries

→ Stellar masses: Kepler 3 (see 2.1.4)

$$\frac{a^3}{P^2} = \frac{G}{4\pi^2} (m_1 + m_2) \quad (8.4.1)$$

(Observational Parameters: P – directly measurable, a – measurable from image if and only if distance to binary+inclination are known)

inclination:



simplest case: real major axis; if not: $a_{\text{obs}} = a_{\text{real}} \cos i$

To figure out individual masses:

$$\frac{M_1}{M_2} = \frac{a_2}{a_1} \quad (8.4.2)$$

(a are the semimajor axis around CMS)

8.4.2 Photometric Binaries

special case of spectroscopic binaries

describable by Roche potential (potentials of two stars + rotating system → Coriolis force)

Isosurfaces: only very near to star spherical, elsewhere not → stellar magnitude changes in orbit

Eclipsing binaries: photometric binaries where the orbital plane is perpendicular to the celestial plane

8.4.3 Spectroscopic Binaries

only possible to measure radial velocity in line of sight

for circular orbit, angle θ on orbit:

$$\theta = \omega t \text{ with } \omega = \frac{2\pi}{P} \quad (8.4.3)$$

observed radial velocity:

$$v_r = v \cdot \cos(\omega t) \quad (8.4.4)$$

from observed $v_r(t) \rightarrow v \cdot \sin i$ ("velocity amplitude")

Motion of star visible through Doppler shift in stellar spectrum:

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c} = \frac{v}{c} \sin i \cos(\omega t) \quad (8.4.5)$$

(for stars no relativistic Doppler effect needed)

8.4.4 Mass function

Because spectroscopic binaries cannot be resolved into separate stars, it is not possible to measure the period and the semimajor axis directly. But a numerical relation between the masses of the two components of the binary system can be found. It provides information on an lower limit for the unseen star when the spectral lines of only one component can be seen.

Kepler 3:

$$\frac{G}{4\pi^2}(M_1 + M_2) = \frac{R^3}{P^2} \quad (8.4.6)$$

(Assumption: observation of lines of star 1 only)

$$\rightarrow \text{CMS: } M_1 r_1 = M_2 r_2 \quad (8.4.7)$$

$$R = r_1 + r_2 = r_1 \left(1 + \frac{r_2}{r_1}\right) = r_1 \left(1 + \frac{M_1}{M_2}\right) \quad (8.4.8)$$

assumption: circular

$$v_1 = \frac{2\pi r_1}{P} \quad (8.4.9)$$

But: inclination unknown, we only observe radial component, i.e.:

$$v_{\text{obs}} = v_1 \sin i \quad (8.4.10)$$

$$\Leftrightarrow r_1 = \frac{P}{2\pi} v_1 = \frac{P v_{\text{obs}}}{2\pi \sin i} \quad (8.4.11)$$

$$\Rightarrow R = r_1 \left(1 + \frac{M_1}{M_2}\right) = \frac{P v_{\text{obs}}}{2\pi \sin i} \left(1 + \frac{M_1}{M_2}\right) \quad (8.4.12)$$

\Rightarrow Insert into Kepler 3:

$$\frac{G}{4\pi^2}(M_1 + M_2) = \frac{1}{P^2} \frac{P^3}{\sin^3 i} \left(1 + \frac{M_1}{M_2}\right) \quad (8.4.13)$$

$$\Rightarrow \boxed{\frac{M_2^3}{(M_1 + M_2)^2} \sin^3 i = \underbrace{\frac{P v_{\text{obs}}^3}{2\pi G}}_{\text{Observables}}} =: f_M \quad (8.4.14)$$

(mass function)

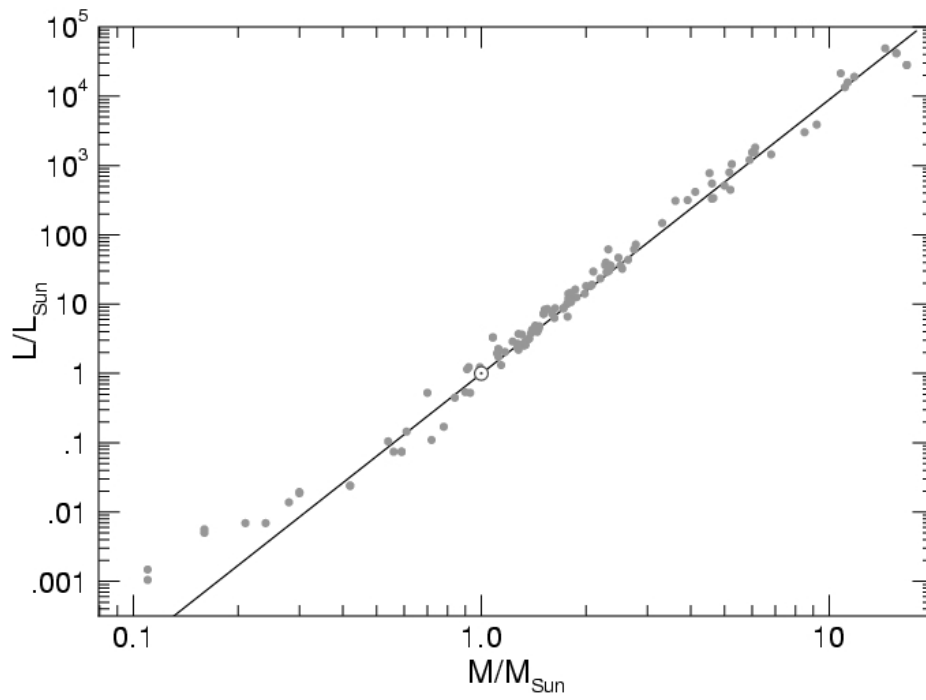
$\Rightarrow f_M$ is lower limit for M_2

\Rightarrow application:

8.4.5 Mass-Luminosity Relation

- apparent magnitude m and distance \Rightarrow luminosity
- mass from binary stars

\Rightarrow mass-luminosity relationship



Empirical results:

$$\frac{L}{L_{\odot}} = \begin{cases} 0.23 \left(\frac{M}{M_{\odot}} \right)^{2.4} & (M < 0.43 M_{\odot}) \\ \left(\frac{M}{M_{\odot}} \right)^{4.0} & (M \geq 0.43 M_{\odot}) \end{cases}$$

sometimes one also uses $L \propto M^{3.3}$ for the whole mass range

⇒ more massive stars ⇒ significantly higher L ! (factor 2 in M ⇒ factor 8 in L)

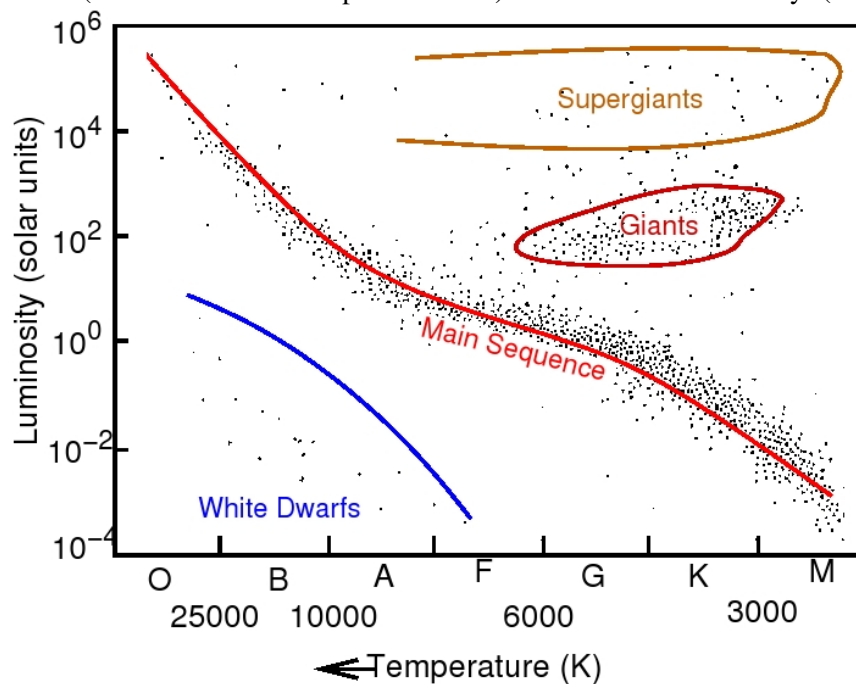
⇒ More massive stars live much shorter!

Chapter 9

Stellar Evolution

9.1 HRD

Stellar temperature (or color index or spectral class) vs. stellar luminosity (or absolute magnitude)



- most on main sequence (called “dwarfs”)
- luminosity: $L = 4\pi R^2 \sigma T^4 \propto R^2 T^4 \rightarrow$ cold but luminous stars \rightarrow “Giants”
- Hot, underluminous stars \rightarrow are small \rightarrow “white dwarfs”

Mass-luminosity relationship + HRD:

Main Sequence is a Mass Sequence!

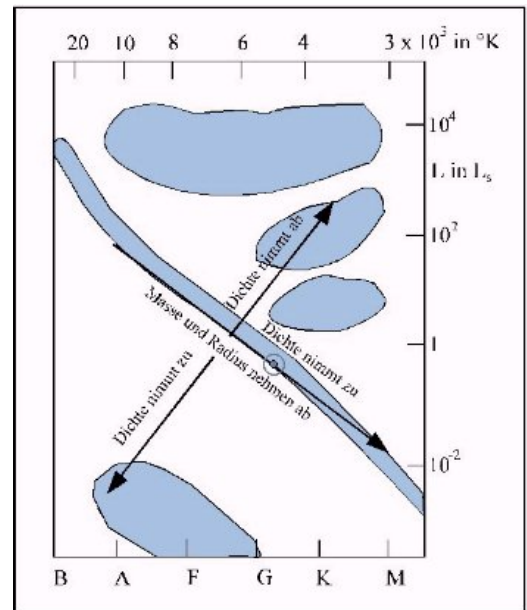
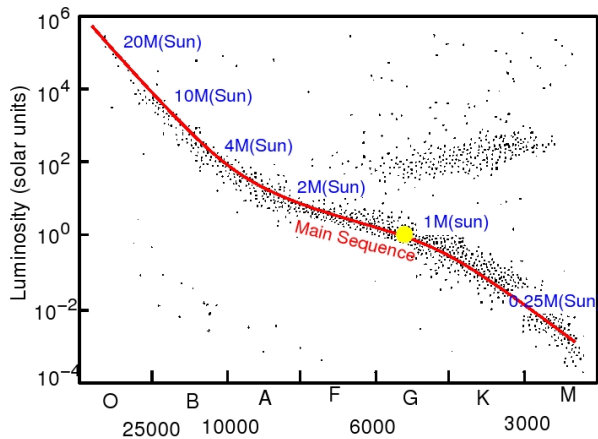
(M-dwarf $\approx 0.25 M_{\odot}$, G \approx Sun, O;B $\approx M \geq 20 M_{\odot}$)

Morgan-Keenan classes:

luminosity classes (Sun: G2 V)

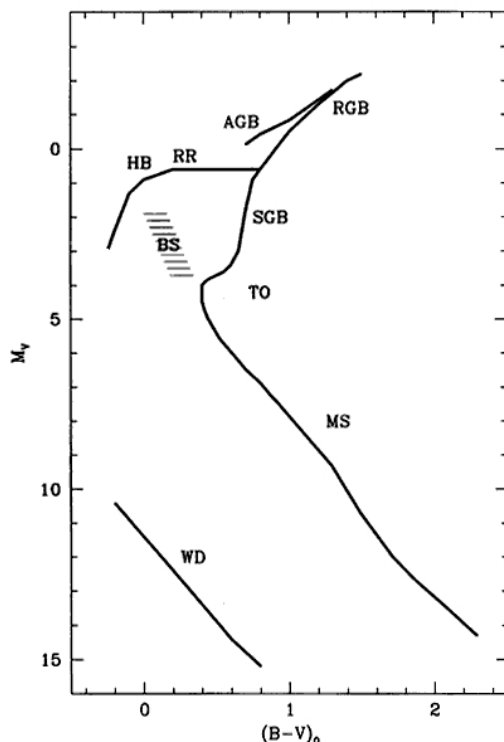
\rightarrow if you know: luminosity (apparent brightness + spectrum \rightarrow spectral class \rightarrow HRD \rightarrow luminosity) and apparent brightness/magnitude (measurement) \rightarrow distance: $d = 10^{(m-M+5)/5} \rightarrow$ not exact

\Rightarrow HRD points out stellar evolution.



9.2 CMD

It's possible to plot the absolute magnitude vs. the color index, such a diagram is called a color-magnitude diagram, which is similar to a HRD. (Instead of the absolute magnitude quite often the apparent V -magnitude is used which is equal for clusters.) This method is particularly useful with star clusters where taking the spectrum of thousands of closely-spaced stars to gain the spectral classes would be impossible.



AGB: asymptotic giant branch

BS: Blue Stragglers

HB: horizontal branch

MS: Main Sequence

RGB: red giant branch

RR: RR-Lyrae stars

SGB: subgiant branch

TO: Turn-off point

WD: White Dwarfs

(The intrinsic color index $(B - V)_0$ is the real color of the stars without interstellar extinction.)

The location of stars in the color-magnitude diagram is primarily due to the mass of the individual stars. The structure of the CMD can be explained from stellar evolution, see Introduction to Astronomy II.

9.3 Star Birth

born in "Giant Molecular Clouds"! (diameter: 50–1000 pc, molecular gas: H_2, CO, \dots , very cool (10–20 K), densities: $n \approx 10^6 - 10^{10} \text{ cm}^{-3}$)

→ collapse triggered by collision of clouds or supernovae

idea of: instable cloud grav > thermal pressure:

$$\Rightarrow R > R_J = \sqrt{\frac{15 k T}{8 \pi G m_p \rho}} \approx \sqrt{\frac{k T}{G m_p \rho}} \quad \text{Jeans Radius} \quad (9.3.1)$$

⇒ Problem: Masses derived from Jeans equation are too large!

Reality: ISM has B -fields

⇒ particle motion \perp to B -field difficult

⇒ stops gas collapsing

Good thing cause after Jeans stuff too much star formation!

⇒ more difficult theory including fields needed

Process of protostar development:

1. Stellar mass cores form because of fragmentation along B -fields
2. Material collapses inwards until material in center has enough pressure and heat ($T > 10^6$ K) to start fusion (process is called “inside out collapse” because at start density in middle was higher than around and therefore rose much faster in the middle) Around Protostar an accretion disk forms!
3. because of stellar wind “bipolar outflow” ⇒ radio lobes ⇒ O,B stars start hydrogen burning (Orion: Trapezium) ⇒ UV light ionises gas ⇒ winds push gas outwards ⇒ around newly formed stars empty space ⇒ compression of other gas ⇒ triggers more star formation ⇒ reflection nebula + open cluster is formed
 ⇒ Stars usually don't come to life alone!
4. Star now on the zero main sequence (ZAMS) (still has circumstellar disk) ⇒ sometimes disks make collimated outflows (jets) ⇐ Herbig Haro Objects

Protostar → ZAMS a few 10^6 years

The main sequence is state of fusion of hydrogen into helium.

($\approx 10^9$ years for Sun)

Star is in “hydrostatic equilibrium”.

9.4 The Sun – Main Sequence Stars

9.4.1 Stellar Structure

9.4.1.1 4 coupled equations of stellar structure

In stars the gravitational force is resisted by an internal pressure due to the thermal motion of the particles. These two forces, gravitational attraction and thermal pressure, play the most important role in determining the structure of stars. In addition to these forces, it is also necessary to consider the thermal properties of stars. Stars are continually radiating away energy into space. The origin of this energy and the way it is transported to the surface must be taken into account by any theory of stellar structure.

In this section equations for the stellar structure will be formulated. Two fundamental assumptions about the structure of stars are made. Although stars do change with time this change is so slowly that it is a good approximation to neglect it. Furthermore it will be assumed that all stars are spherical and symmetric. If these two assumptions are made, the structure of a star is governed by a set of 4 equations.

1. “hydrostatic equilibrium”

$$F_G = \nabla p \quad (9.4.1)$$

$$dF_g = -\frac{G M_r dm}{r^2} = -\frac{G M_r \rho}{r^2} dA dr \quad M_r \text{ mass in shell} \quad (9.4.2)$$

Pressure: p bottom of shell, $p + dp$ top of shell (with $p = \frac{F}{A}$)

$$\Rightarrow dF_p = p dA - \underbrace{(p + dp) dA}_{\approx 0} = -dp dA \quad (9.4.3)$$

(= difference between top and bottom)

pressure decreases outwards \Rightarrow

$$dF_g = -dF_p \quad \text{Equilibrium} \quad (9.4.4)$$

$$\Leftrightarrow -\frac{G M_r \rho}{r^2} dA dr = dp dA \quad (9.4.5)$$

$$\Leftrightarrow \boxed{\frac{dp}{dr} = -\frac{G M(r) \rho(r)}{r^2}} = -g(r) \rho \quad (9.4.6)$$

(pressure structure)

2. Mass distribution = The Equation of Mass Conservation

$$dM_r = \underbrace{4\pi r^2}_{\text{shell}} \rho dr \quad (9.4.7)$$

$$\longrightarrow \boxed{\frac{dM_r}{dr} = 4\pi r^2 \rho(r)} \quad (9.4.8)$$

(mass structure)

3. Conservation of Energy

all energy “produced” has to leave the star; ε is amount of energy released per gram of stellar material in one unit time

$$\underbrace{dL_r}_{\text{change due to energy production in one “small” shell}} = \varepsilon dM_r = 4\pi r^2 \rho \varepsilon dr \quad (9.4.9)$$

$$\boxed{\frac{dL_r}{dr} = 4\pi r^2 \rho(r) \varepsilon(r)} \quad (9.4.10)$$

(energy transport)

4. Temperature Gradient = Transportation of Energy

3 ways:

- conduction
- convection
- or radiation

\longrightarrow especially: convection or radiation (conduction negligible, only important for white dwarfs etc.)

derivation quite complex:

$$\boxed{\frac{dT}{dr} = -\frac{3}{4} \frac{\kappa \rho(r) L(r)}{a c T^3 \pi r^2}} \quad (9.4.11)$$

(κ is the mass absorption coefficient; a is radiation constant)

for convection:

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{p} \frac{dp}{dr} \quad (9.4.12)$$

(adiabatic component $\gamma = \frac{c_p}{c_v} \hat{=}$ ratio of specific heats in constant pressure)

\Rightarrow for a stellar model all 4 equations 1,2,3,4 have to be solved (but they’re coupled):

- need boundary conditions:
center of star: $r = 0, M_r = 0, L(0) = 0$
surface of star: $r = R, p(R) = 0, T(R) = 0, M_r(R) = M$ (because p and T are small compared to center)
- equation of state of gas needed:
easiest option: $p = n k T = \rho k T \frac{1}{\mu m_p}$ (μ is molecular weight)
- energy generation: ε so need: T, ρ , chemical composition
- κ = opacity: need T, ρ , chemical composition

\Rightarrow only numerical solutions possible \rightarrow stellar model

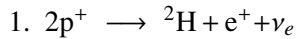
9.4.1.2 Energy “Production”

during main sequence: $4p \rightarrow {}^4\text{He} + E$

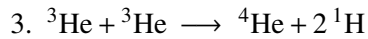
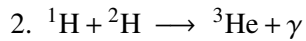
$$E = m \cdot c^2 = 4m_p \cdot c^2 - m_{{}_2^4\text{He}} = 25 \text{ MeV}$$

→ in this fusion ($\text{H} \rightarrow \text{He}$) 0.7% of the rest mass energy E is converted into energy

9.4.1.2.1 Proton-Proton Chain consists of 3 steps



very slow process (1 nucleon per 10^{10} yrs)



(there are other variations but that one dominates)

pp-chain especially for stars $T \leq 2 \cdot 10^7 \text{ K}$; $\varepsilon_p \propto T^5$

Sun: 98.4 % of processes

9.4.1.2.2 CNO-cycle requires catalysts: O,N,C (-isotopes)

a bit smaller energy release than pp-chain → neutrinos

slowest reaction $\approx 10^6$ yrs

Temperature dependant: $> 2 \cdot 10^7 \text{ K}$

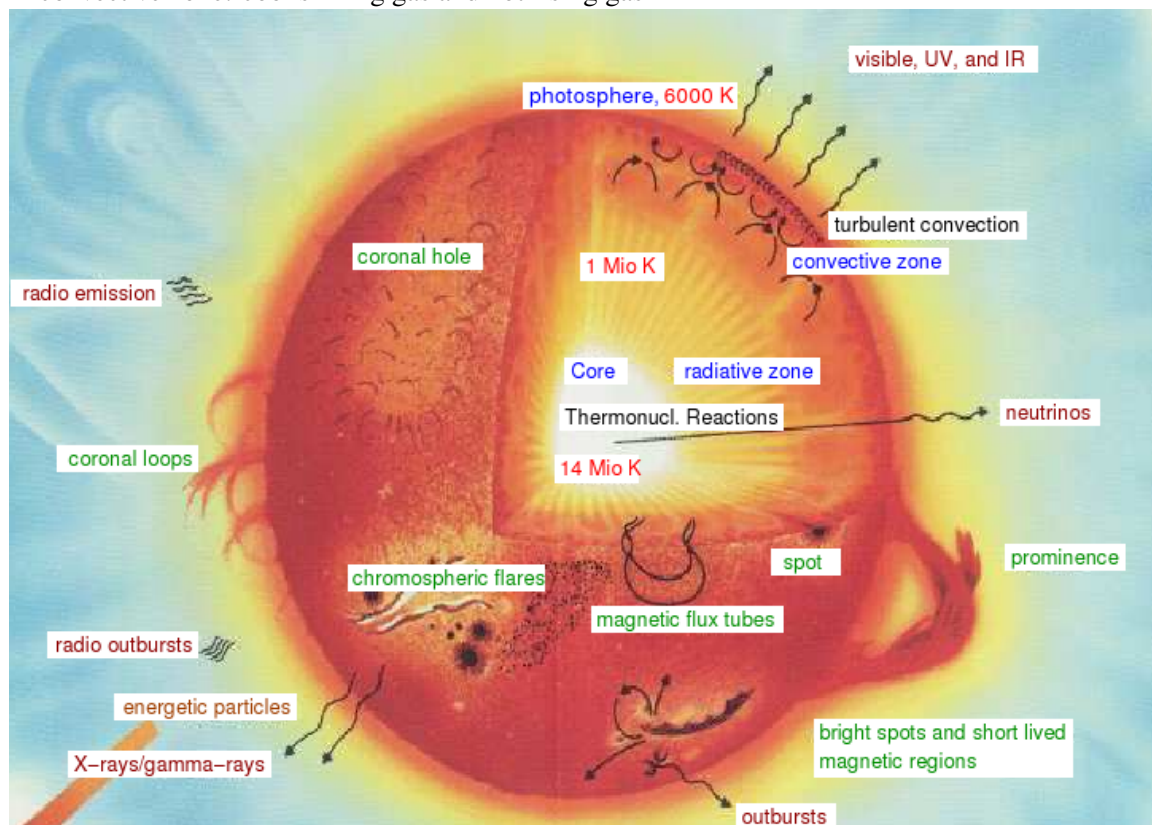
$\varepsilon_{\text{CNO}} \propto T^{17}$ (10% in Temp. → Energy output 50×)

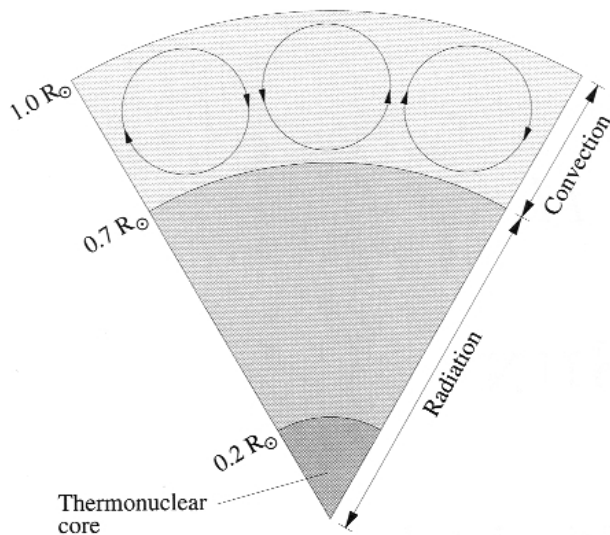
Sun: 1.6%

⇒ especially in older stars

9.4.1.3 Structure of Sun

in convective zone: cool sinking gas and hot rising gas





9.4.1.4 Example: Sun

- Mass: $1M_{\odot} = 1.997 \cdot 10^{30} \text{ kg}$
- Luminosity: $1L_{\odot} = 3.827 \times 10^{26} \text{ W}$
- chemical composition

→ models can be made!

$$g_{\odot} = 28 g_{\oplus} \quad \bar{\rho} = 1410 \text{ kg m}^{-3} = 1.41 \cdot 10^3 \text{ kg m}^{-3}$$

≈ pressure → need to use “hydrostatic equilibrium”:

$$\frac{dp}{dr} = -\frac{\rho M_r G}{r^2} \quad (9.4.13)$$

with $\rho = \bar{\rho}$
and $M_r = \frac{4}{3}\pi \bar{\rho} r^3$

$$\Rightarrow \frac{dp}{dr} = -\frac{G M_r \rho}{r^2} = -\frac{4\pi G \bar{\rho}^2 r}{3} \quad (9.4.14)$$

(decreases linearly)

Estimate at $r = \frac{R}{2}$

$$dp = -\frac{4}{3}\pi G \bar{\rho}^2 r dr \quad (9.4.15)$$

$$\int_p^0 dp = -\frac{4}{3}\pi G \bar{\rho}^2 \int_{R/2}^R r dr = \quad (9.4.16)$$

$$= \frac{1}{2}\pi G \bar{\rho} R^2 \quad (9.4.17)$$

$$p = \frac{1}{2}\pi G \bar{\rho} R^2 = 10^{14} \text{ Pa} \quad (9.4.18)$$

now: mean temperature

$$p = \underbrace{\frac{\bar{\rho}}{\mu m_p}}_n k T \quad \Rightarrow \quad T = \frac{\mu m_p p}{k \bar{\rho}} \quad (9.4.19)$$

(μ mean mass per $p^+ \Rightarrow$ ionised gas $\Rightarrow \mu=0.5$; reality = 0.61; cause of He and metals)

$$\Rightarrow T = 5 \cdot 10^6 \text{ K} \quad (9.4.20)$$

(=scales are alright)

9.4.1.5 Standard Solar Models

- density: rises extremely at $0.4 R_{\odot}$
→ logarithmic shape (linear)
- temperature: $0.2 R_{\odot}$ → $T > 10^6$ K hotter in center
- pressure: outer parts very small; strong rise to center $R = 0.4 R_{\odot}$
(logarithmic shape)
- composition:
 - center more He due to fusion → outer parts normal composition
 - sharp bend at $0.7 R_{\odot}$ cause of change from radiative zone to convection zone → in convection zone: straight line → cause: all gets mixed up
⇒ if convection changes: material from inner parts can come to the surface
 - other elements: inner zone: CNO is possible ⇒ very few C^{12} → but more N; ^3He abundance very low cause of nuclear reaction (^4He is formed or gets destroyed immediately); at $0.7 R_{\odot}$: more likely to stay than to be destroyed again ⇒ peak!

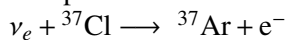
9.4.2 Solar Neutrinos

test for standard solar model by measuring neutrino flux

problem: almost no interaction → need of big detectors

3 different types of neutrinos: μ, e^-, τ

1.Experiment: Homestake mine



worked only for $E_{\nu} > 0.8$ MeV

⇒ so even less to detect because not pp1-chain neutrinos!

⇒ but: results: only 1/3 of expected found

(measured in SNU= 10^{-36} reactions per target atom per second)

2.Experiment: 1990 Kamiokande

→ confirmed results

⇒ assumption: Solar Physics wrong!

3.Experiment: Sudbury Neutrino Observatory

works with heavy water = D_2O

was sensitive to any flavor of neutrino and to energies from pp1-chain

⇒ change of understanding of neutrinos

→ if all flavors are taken into account number matches

⇒ neutrinos change their flavors on the way to Earth (neutrino oscillations)

⇒ Solar Model is correct!!