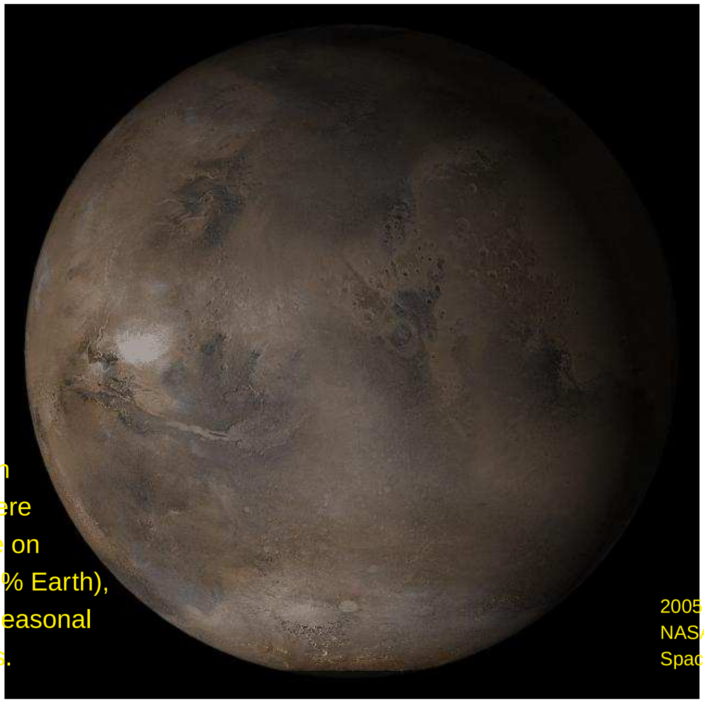
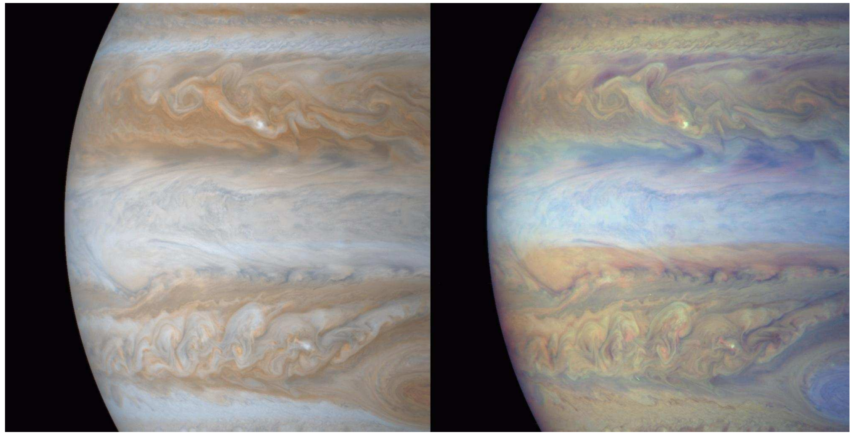
# *Planets: Atmospheres*



Mars: thin atmosphere (pressure on surface 1% Earth), but real seasonal variations.

2005 Feb 7, NASA/Malin Space Systems



NASA/JPL, Cassini, 2000 Dec 31

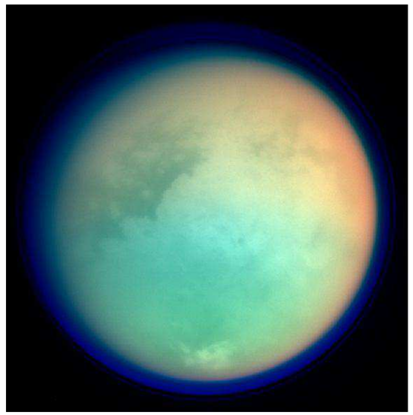
Jupiter: true color image; colors likely from trace content of organic compounds in atmosphere

false color image, red: waterclouds, dark spots: deep hot spots

Overall atmospheric structure: three layers:

Ammonia – ammonia hydrosulfide ($NH_4HS$) – water ice/water (deepest)

NASA Voyager



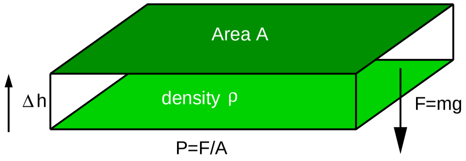
27.10.2004, false colour IR/UV; NASA/ESA

Titan: dense atmosphere, 99% nitrogen, 1% methane, some hydrocarbons, thought to be similar to primordial atmosphere of Earth.

Radius: 2575 km ($\sim$ Mercury!)

ESA probe *Huygens* landed on Titan on 2005 January 14

The structure of an atmosphere is defined by the concept of hydrostatic equilibrium:




The force excerted by gas with density $\rho$ sitting on top of an area $A$ is given by

$$F = mg = \rho V g = A h \rho g \tag{5.1}$$

Such that pressure becomes

$$P = \frac{F}{A} = \rho h g \tag{5.2}$$

where $g$ is the gravitational acceleration.
For a thin atmosphere ($g$ constant): Decrease of $P$ when going upwards by $\Delta h$:

$$\Delta P = -\rho g \Delta h \quad \text{and for} \lim_{\Delta h \to 0}: \quad \frac{\mathrm{d}P}{\mathrm{d}h} = -\rho g \tag{5.3}$$

To solve this differential equation, we need a relationship between density and pressure ("equation of state"). For an "ideal gas", this relationship is given by

$$P = (\rho/\mu) kT \tag{5.4}$$

where $T$ is the Temperature (K!), $\mu$ the average mass of a gas particle, and $k$ is Boltzmann's constant ($k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$). Therefore:

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\left(\frac{\mu g}{kT}\right) P \tag{5.5}$$

In order to obtain $P$ as a function of height, $h$, we need to solve this differential equation with the boundary condition that for $h = 0$, $P = P_0$. This can be done easily using the technique of "separation of variables", assuming that $T$ does not change.

First, divide by $P$ and integrate both sides of the equation with respect to height:

$$\int_0^h \frac{1}{P} \frac{\mathrm{d}P}{\mathrm{d}h} \mathrm{d}h = -\int_0^h \left(\frac{\mu g}{kT}\right) \mathrm{d}h$$

We can now substitute $P(h)$ for $h$ on the left hand side. Using the chain rule gives

$$\int_{P_0}^{P(h)} \frac{\mathrm{d}P'}{P'} = -\int_0^h \left(\frac{\mu g}{kT}\right) \mathrm{d}h$$



such that

$$\ln\left(\frac{P(h)}{P_0}\right) = -\left(\frac{\mu g}{kT}\right) h$$

and exponentiating then gives

$$P(h) = P_0 \exp\left(-\frac{\mu g}{kT} \cdot h\right) = P_0 \exp\left(-\frac{h}{H}\right) \tag{5.6}$$

The pressure in the atmosphere thus decreases exponentially, the characteristic height scale of the decrease is given by the scale height,

$$H = \frac{kT}{\mu g} \tag{5.7}$$

Typical values for the planets are for Earth: $H \sim 9\,\mathrm{km}$.

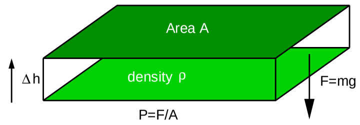
*Remark:* The method employed above is called "separation of variables" since people often jump from the first (linear) equation to the third one in one step, by "separating the dependent from the independent variable":

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\left(\frac{mg}{kT}\right) P \implies \frac{\mathrm{d}P}{P} = -\left(\frac{mg}{kT}\right) \implies \int_{P_0}^{P(h)} \frac{\mathrm{d}P'}{P'} = -\int_0^h \left(\frac{mg}{kT}\right) \mathrm{d}h$$



## Hydrostatic Equilibrium

Structure of atmosphere defined through hydrostatic equilibrium:




Force on area $A$ by slab of gas of area $A$ and density $\rho$ ($g$ = gravitational acceleration):

$$F_G = -mg = \rho V g = -A \Delta h \rho g \tag{5.8}$$

Buoyancy:

$$F_P = A(P(r + \Delta h) - P(r)) = A \Delta P \tag{5.9}$$

Gravitational force balanced by buoyancy:

$$\frac{\mathrm{d}P}{\mathrm{d}h} = -\rho g$$

Assuming ideal gas, $P = (\rho/\mu)kT$, and isothermal atmosphere:

$$P(h) = P_0 \exp\left(-\frac{\mu g}{kT} \cdot h\right) = P_0 \exp\left(-\frac{h}{H}\right)$$

scale height $H$, on Earth, $H \sim 9\,\mathrm{km}$.





## Atmospheric Composition, I

Atmospheric composition of selected terrestrial objects

| | Venus | Earth | Mars | Titan |
|---|---|---|---|---|
| $O_2$ | 0.0 | 21.0 | 0.0 | 0.0 |
| $H_2O$ | 50ppm | 1.0 | $<$ 100ppm | 0.4ppb |
| $CO_2$ | 96.5 | 0.0 | 95.3 | 10ppb |
| $N_2$ | 3.5 | 78.0 | 2.7 | 90 |
| Ar | 0.0 | 0.9 | 1.6 | |

Values are given as percentages by volume.

**Titan has atmospheric structure similar to Earth!**

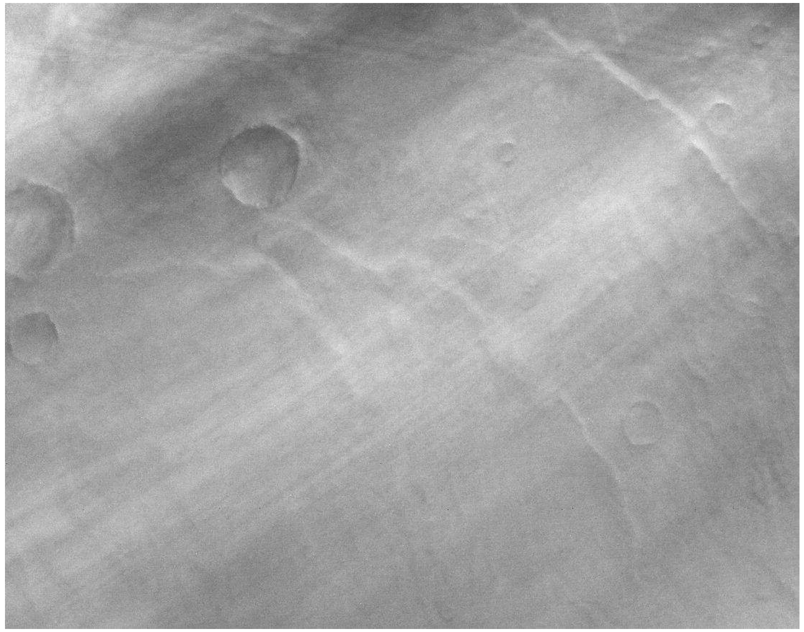
after de Pater and Lissauer

## Atmospheric Composition, II

Typical $H$ for the inner planets (Karttunen)

| Gas | $\mu$ | Venus km | Earth km | Mars km |
|---|---|---|---|---|
| $H_2$ | 2 | 360 | 120 | 290 |
| $O_2$ | 32 | 23 | 7 | 81 |
| $H_2O$ | 18 | 40 | 13 | 32 |
| $CO_2$ | 44 | 16 | 5 | 13 |
| $N_2$ | 28 | 26 | 8 | 20 |
| $T$ [K] | | 750 | 275 | 260 |
| $g$ [m s$^{-2}$] | | 8.61 | 9.81 | 3.77 |





NASA/C.J.Hamilton

Mars: Streaky clouds



## Atmospheric Composition, III

Atmospheric composition of gas giants and the Sun

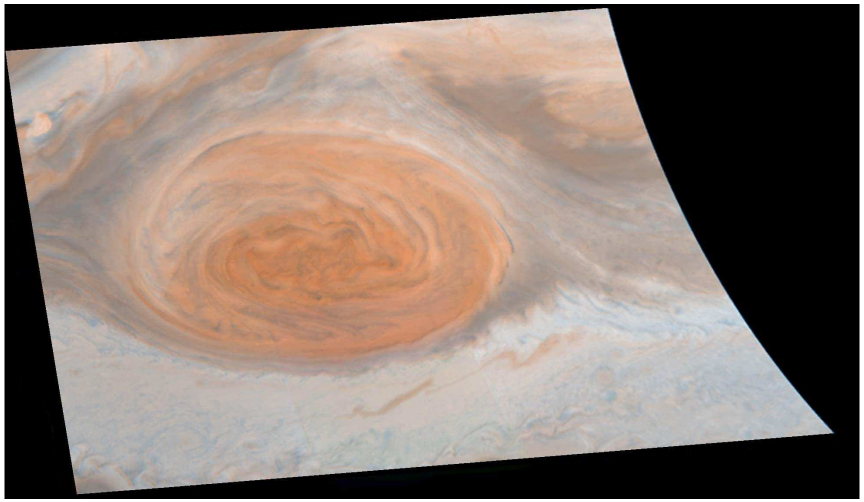
| | Sun | Jupiter | Saturn | Uranus | Neptune |
|---|---|---|---|---|---|
| $H_2$ | 83.5 | 86.4 | 96.3 | 85±5 | 85±5 |
| He | 19.5 | 15.7 | 3.4 | 18±5 | 18±5 |

H: volume percent relative to total atmosphere
He: volume percent relative to H

**Gas giants have approximately same composition as the Sun.**

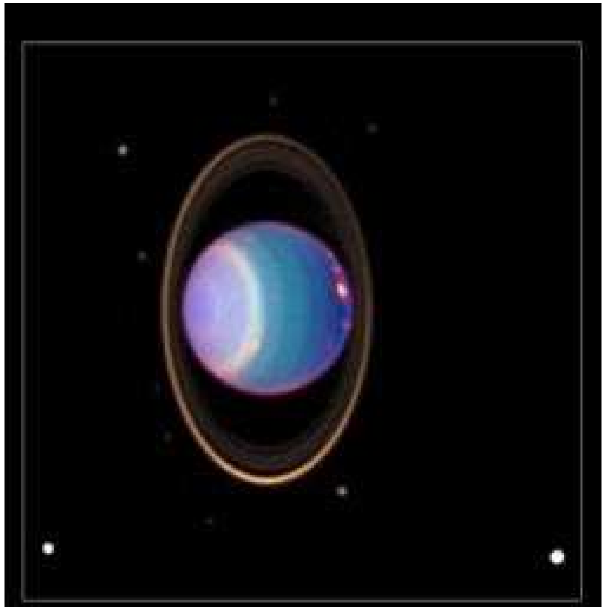
after de Pater and Lissauer





Great Red Spot
NASA Galileo, 1996 June 26

Jupiter's Great Red Spot: Storm System, $\sim 2\times$ Earth diameter, exists since more than 300 years, 8 km above and $10^\circ$ cooler than surrounding region (rising high pressure region), rotates counterclockwise (Coriolis force on Southern hemisphere).



HST Image (image enhanced) of Uranus ring system,
plus evidence for banded atmosphere and clouds