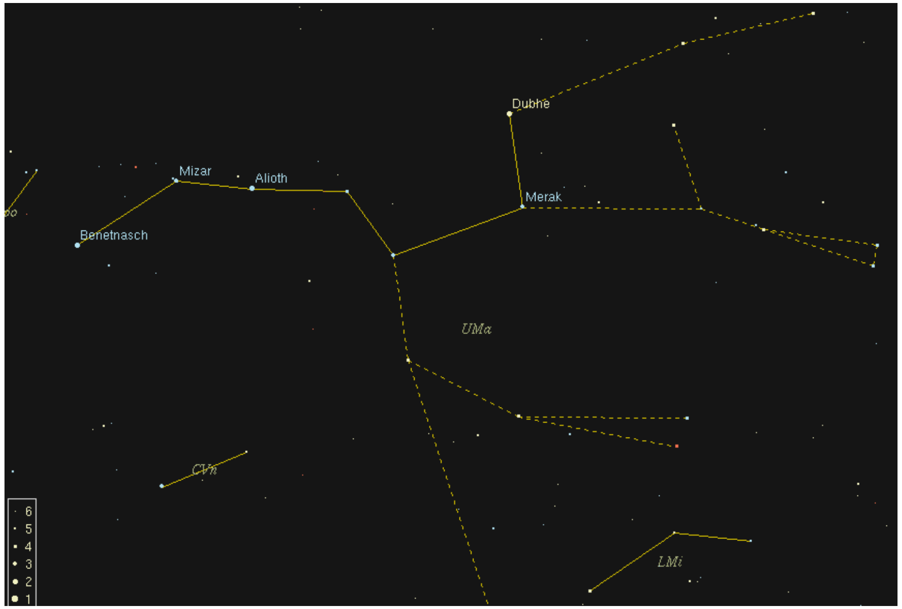
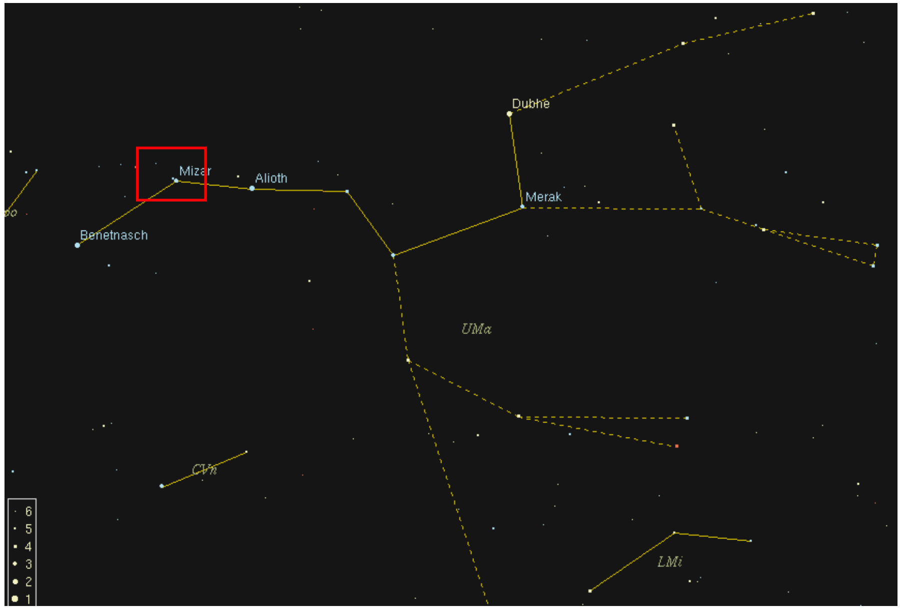
Stars: Masses

V. van Gogh: Starry Night over the Rhône (1888)

The WebMuseum (`http://www.ibiblio.org/wm/`; original: Paris, Musée d'Orsay)



X


Dubhe
Mizar
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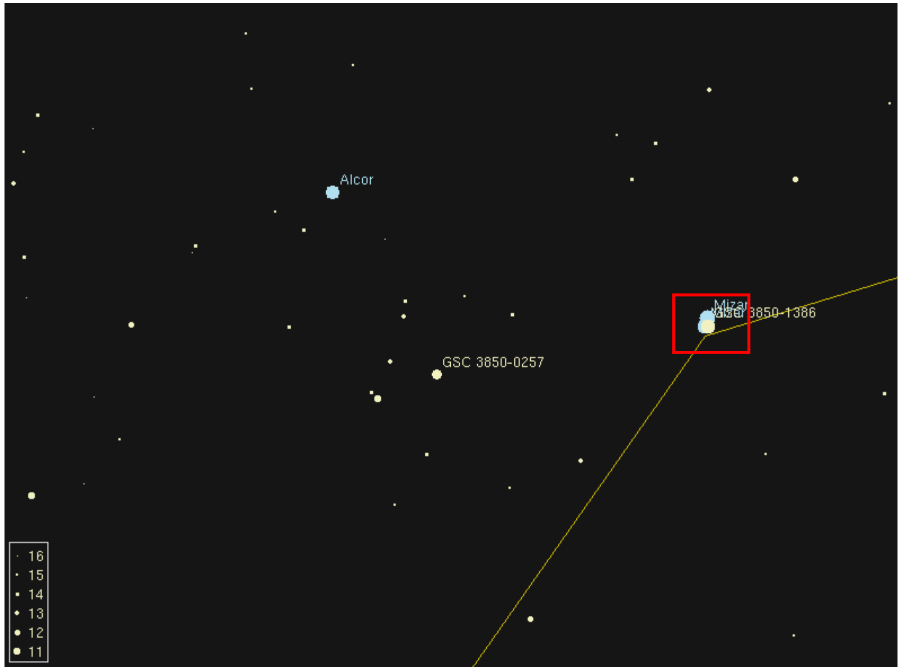
Alcor
Mizar
3850-1386
GSC 3850-0257
16
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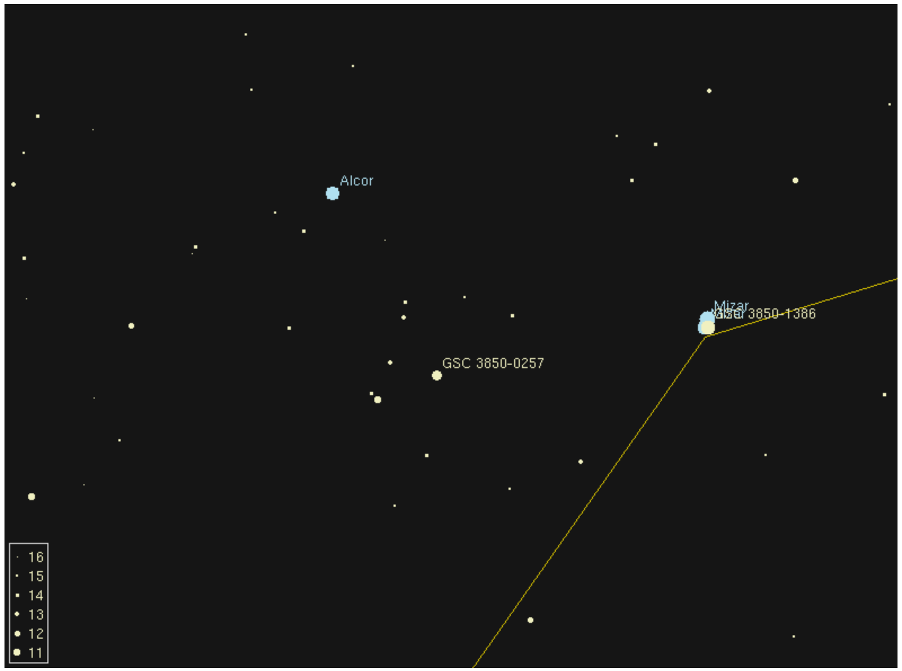
X


Alcor
Mizar
3850-1386
GSC 3850-0257
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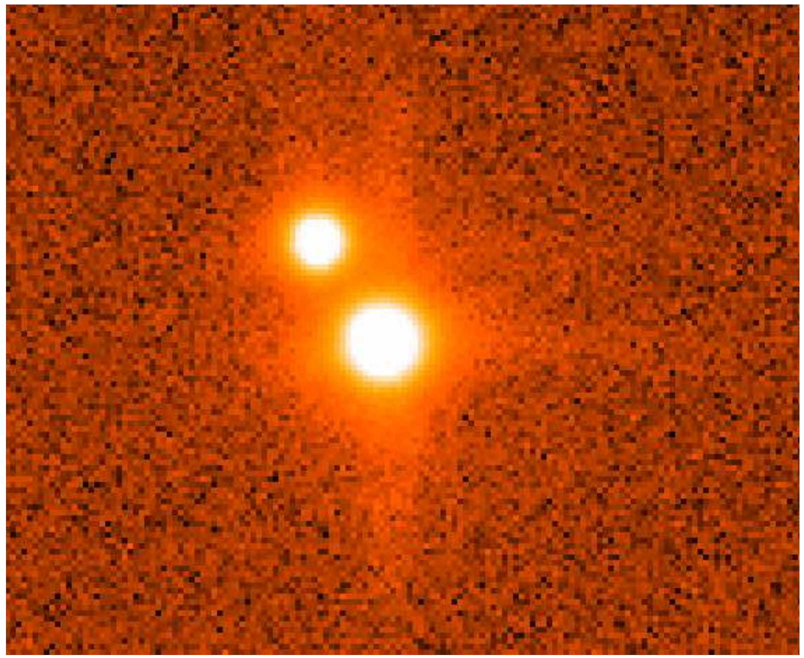

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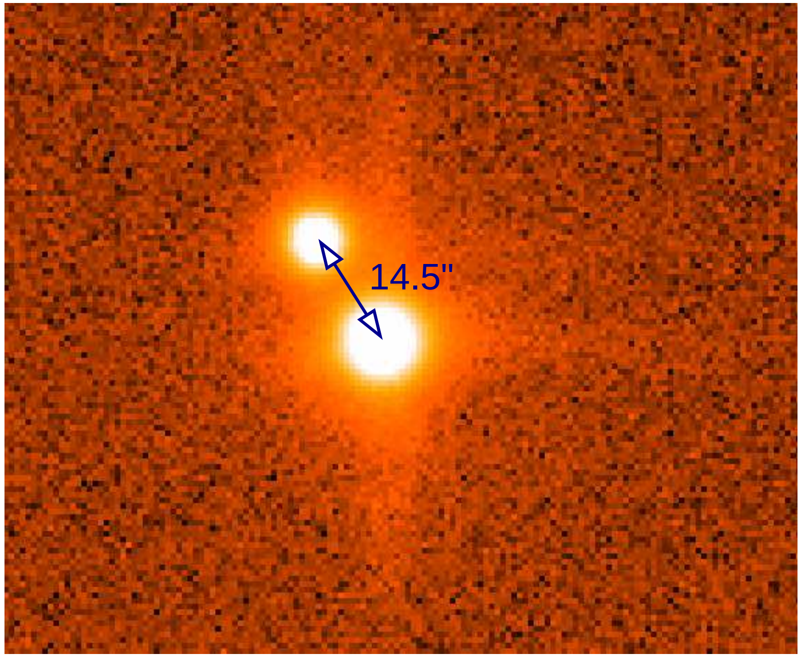

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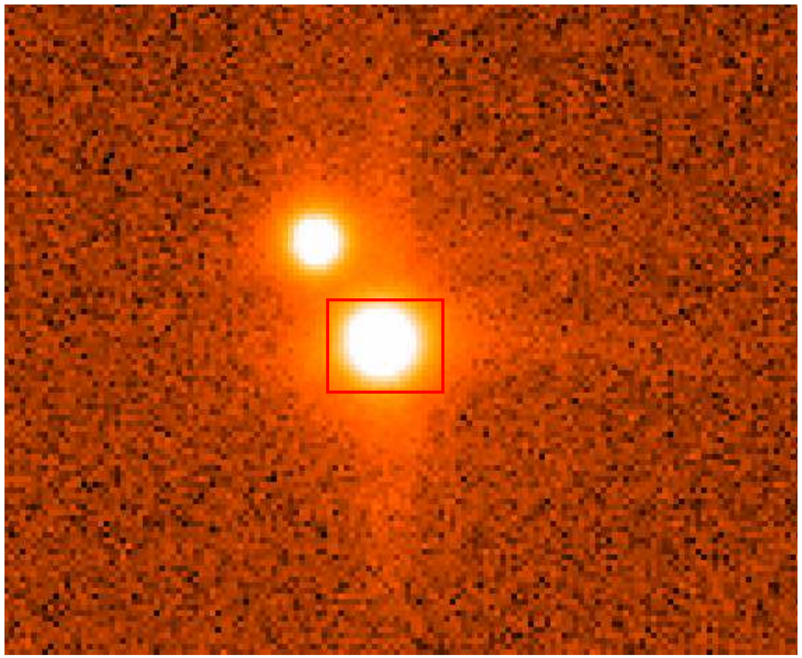

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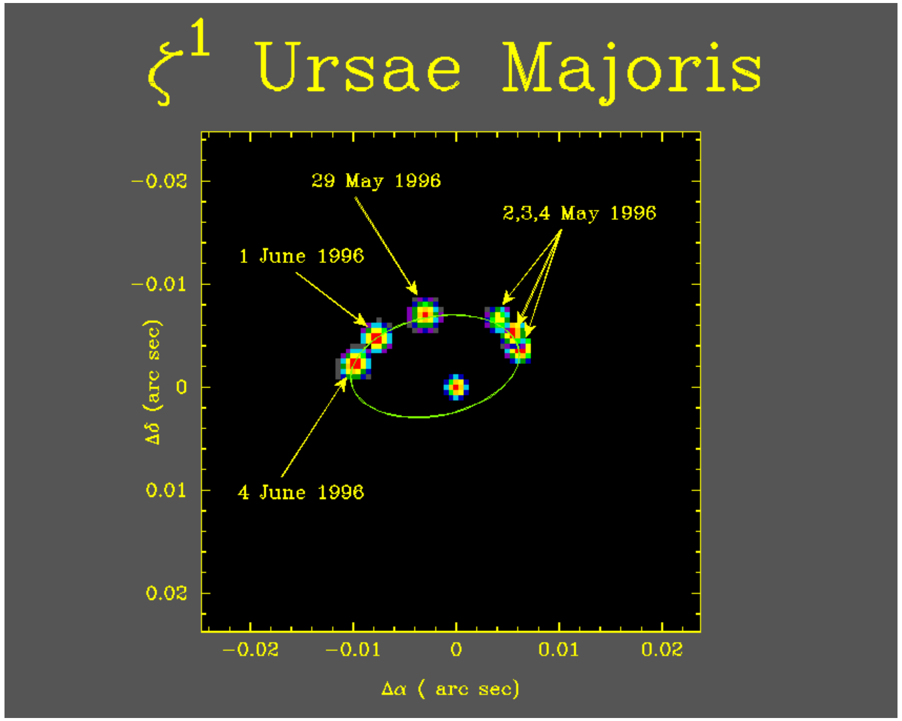


Masses

Mizar A and B are rather typical stars:

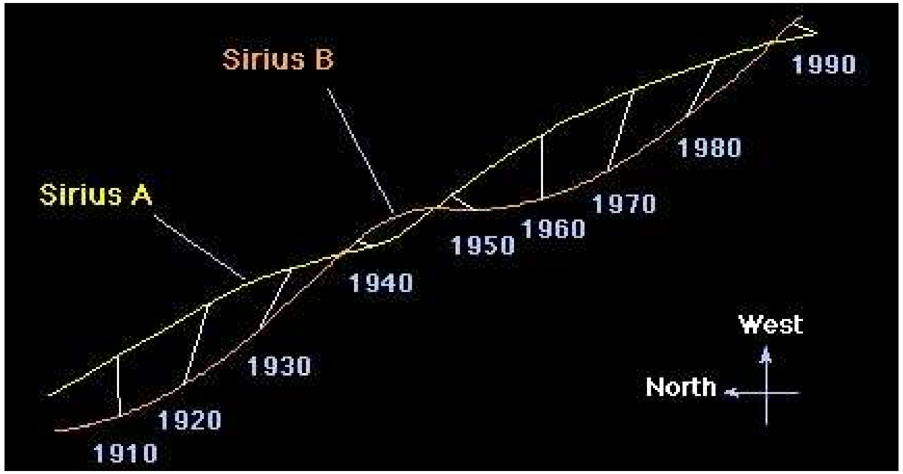
50% – 80% of all stars in the solar neighbourhood belong to multiple systems.

Rough classification:

apparent binaries: stars are *not* physically associated, just happen to lie along same line of sight (“optical doubles”).

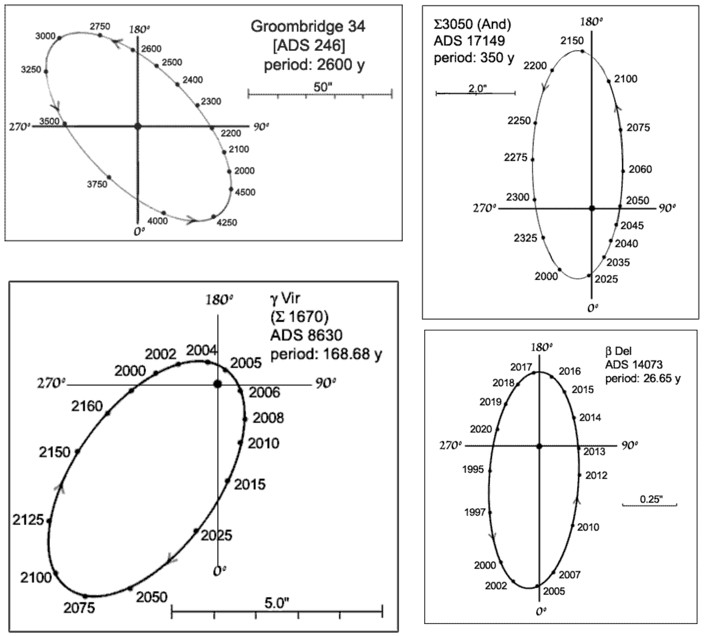
visual binaries: bound system that can be resolved into multiple stars (e.g., Mizar); can image orbital motion, periods typically 1 year to several 1000 years.

spectroscopic binaries: bound systems, cannot resolve image into multiple stars, but see Doppler effect in stellar spectrum; often short periods (hours. . . months).



http://csep10.phys.utk.edu/astr162/lect/binaries/astrometric.html

Astrometric binaries: Motion of stars around common center of mass results in a "wobble" around the CM (since CM is moving along a straight line).



Taking out proper motion leaves us with binary star orbits.

Masses

To determine stellar masses, use Kepler's 3rd law:

$$\frac{a^3}{P^2} = \frac{G}{4\pi^2}(m_1 + m_2)$$

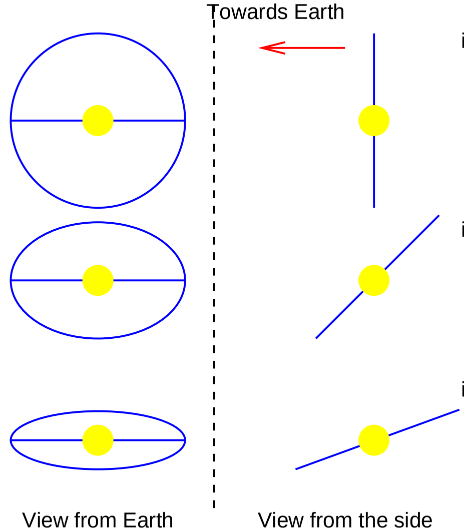
where

- $M_{1,2}$: masses
- P: period
- a semimajor axis

Observational quantities:

- P – directly measurable
- a – measurable from image *if and only if* distance to binary and the inclination are known

Masses



Problem when analysing orbits: orientation of orbit in space: "inclination"

In simplest case: real semi-major axis:

$$a_{\text{observed}} = a_{\text{real}} \cos i$$

Inclination typically found using Kepler's 2nd law plus geometry. . .

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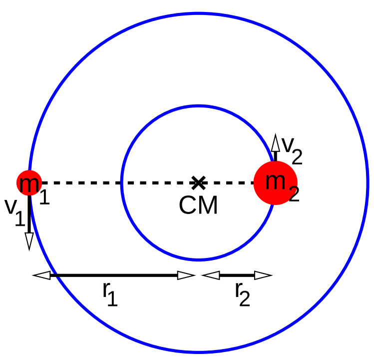
Masses



Kepler's 3rd law gives $M_1 + M_2$.

To determine individual masses, M_1 and M_2, we make use of the fact that the stars move around their common center of mass (CM):

$$M_1 a_1 = M_2 a_2 \quad \text{such that} \quad \frac{M_1}{M_2} = \frac{a_2}{a_1}$$

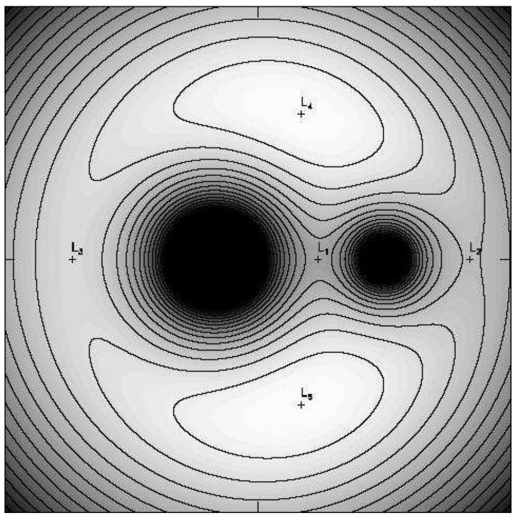
where a_1, a_2: semi-major axes of orbits around CM (observable from imaging).

MOVIE TIME: vbin0.mpg, vbin4.mpg

Also recommended: Interactive program:
http://instruct1.cit.cornell.edu/courses/astro101/java/binary/b

12–18

Photometric Binaries



R. Hynes

In a close binary system: Gravitational potential described by the Roche potential:

$$\Phi_R(\boldsymbol{r}) = -\frac{GM_1}{|\boldsymbol{r}-\boldsymbol{r}_1|} - \frac{GM_2}{|\boldsymbol{r}-\boldsymbol{r}_2|} - \frac{1}{2}(\boldsymbol{\omega}\times\boldsymbol{r})^2$$

and where

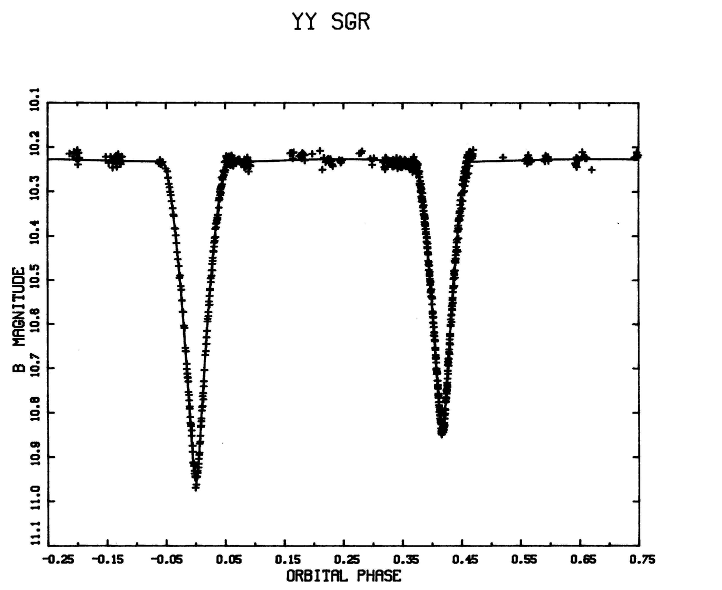
$$\boldsymbol{\omega} = \left(\frac{GM}{a^3}\right)^{1/2} \hat{e}$$

Stellar surfaces are isosurfaces of this potential

$\Longrightarrow$ stars are non-spherical

$\Longrightarrow$ Stellar magnitude changes with orbit.

MOVIE TIME: output.mpg

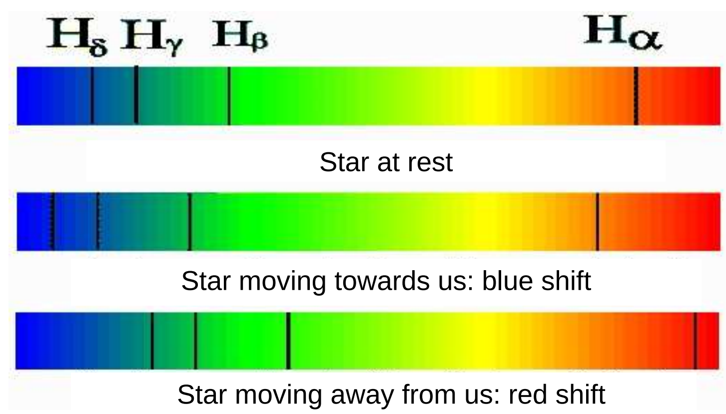


(YY Sgr, $M_1/M_2 = 0.95$, $P = 2.6285372(8)$ d; Lacy, 1993, AJ 106, 738; B5/B6 stars)

Eclipsing binaries: photometric binaries where the orbital plane is perpendicular to the celestial plane.

12–20

Spectroscopic Binaries

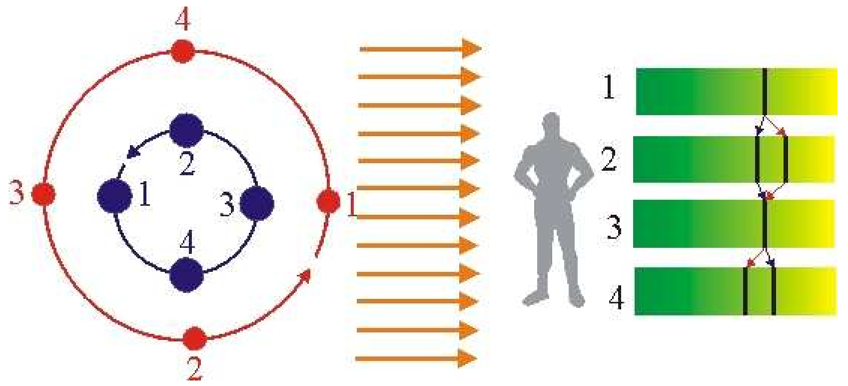


Doppler formula:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \qquad (12.1)$$

c: speed of light

Spectroscopic Binaries



Spectroscopic binaries: Motion of stars leads to periodic shift of spectral lines

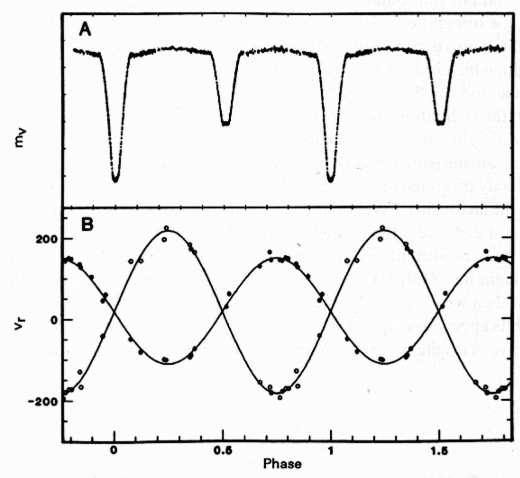
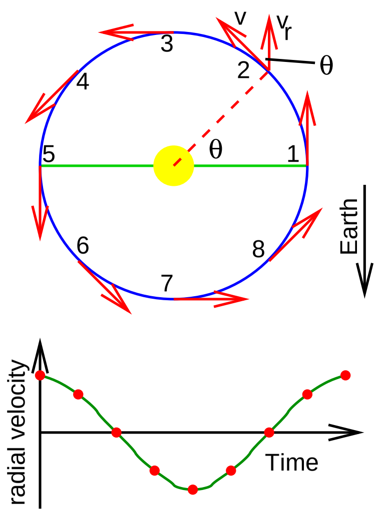


Fig. 9.6. Light and velocity curves for the eclipsing binary ζ Phoenicis are shown as a function of orbital phase. During the times when one star is behind the other we see a reduced light intensity. This happens twice during one period, once if star 1 is in front of star 2, and once if star 2 is in front of star 1. If star 2 has the larger surface brightness, i.e., if star 2 emits more light per cm², then less light will be received when star 2 is eclipsed during the first eclipse than will be received when star 1 is eclipsed, i.e., during the second eclipse. The first light minimum is then deeper than the second one. (From Paczinski 1985.)

Spectroscopic Binaries



For spectroscopic binaries: can only measure radial velocity along line of sight

For circular orbit, angle θ on orbit:

$$\theta = \omega t$$

where $\omega = 2\pi/P$.

Observed radial velocity:

$$v_r = v\cos(\omega t)$$

If orbit has inclination i, then

$$v_r(t) = v \sin i \cos(\omega t) =: K \cos(\omega t)$$

From observation of $v_r(t) \Longrightarrow v\sin i$.
("velocity amplitude", K)

Mass function

If only one star is visible: obtain mass limits from the mass function (derivation: see homework):

$$f_M := \frac{PK_1^3}{2\pi G} = \frac{M_2^3 \sin^3 i}{(M_1+M_2)^2} \qquad (12.2)$$

where the observables: K_1 & P

and the unknowns: M_1: mass of "primary star", M_2: mass of (unseen) "secondary star" & i: inclination

$\Longrightarrow f_M$ is lower limit for M_2, since for $M_1 = 0$: $M_2 = f_M/\sin^3 i \geq f_M$

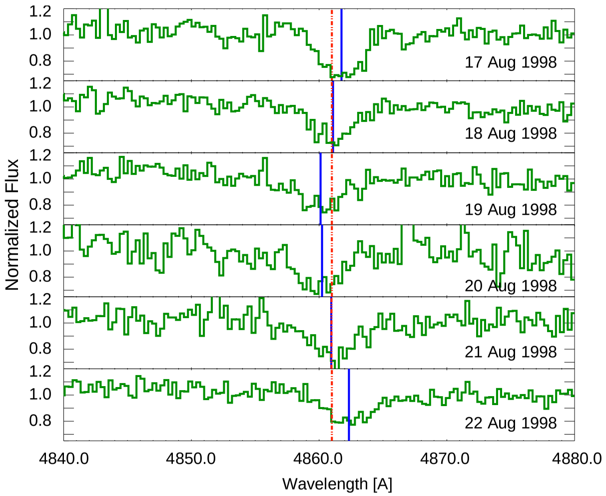
In some cases, knowing $f(M)$ is enough to determine M_2:

Often, the mass of the primary, M_1, can be derived from aquantitative spectral analysis. In this case two unknowns remain: M_2 and $\sin i$.

As $|\sin i| \leq 1$: improved lower limit for M_2.

Determine lower mass limits for all types of invisible faint companions: Exoplanets, brown dwarfs, faint normal stars, neutron star and black hole binaries.

Mass function



HDE 226868/Cyg X-1; Pottschmidt (2001)

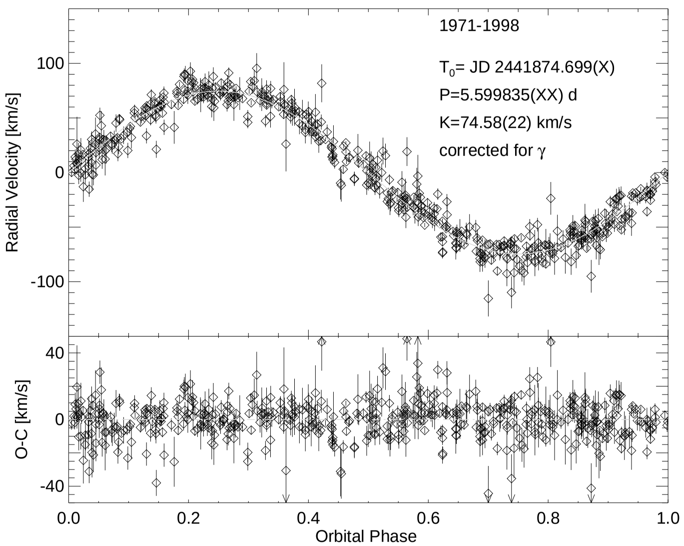
Motion of star visible through Doppler shift in stellar spectrum:

$$\frac{\Delta\lambda}{\lambda} = \frac{v_\mathrm{r}}{c} = \frac{v}{c}\sin i \cos \omega t$$

For virtually all stars, classical Doppler effect is enough; once $v \gtrsim 0.1c$, however, use relativistic Doppler effect,

$$\nu_\mathrm{obs} = \nu_\mathrm{em}\sqrt{\frac{1+v/c}{1-v/c}}$$

Mass function

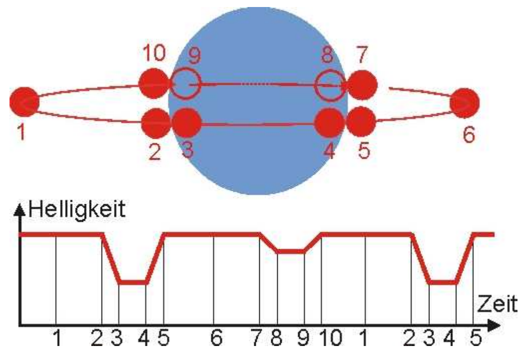


Best fit radial velocity curve of HDE 226868/Cyg X-1 using data spanning more than 30 years.

Pottschmidt et al. (2001)

Stellar Diameters

Eclipsing Binaries



Determination of diameters d_A and d_B from eclipse timing:

Duration of eclipse:

$$d_A + d_B = v(t_5 - t_2) \quad (12.3)$$

Duration of eclipse egress:

$$d_A - d_B = v(t_4 - t_3) \quad (12.4)$$

therefore:

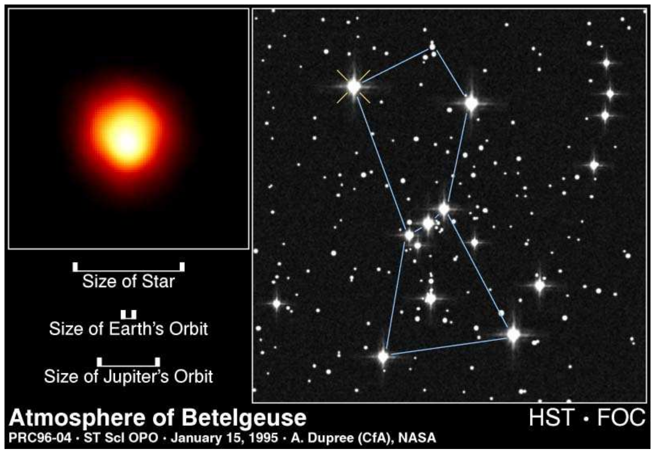
$$d_A = v(t_5 - t_2 + t_4 - t_3) \quad (12.5)$$

$$d_B = v(t_5 - t_2 + t_4 + t_3) \quad (12.6)$$

Note: requires extremely accurate photometry

Resulting radii are independent of distance

Stellar Angular Diameters

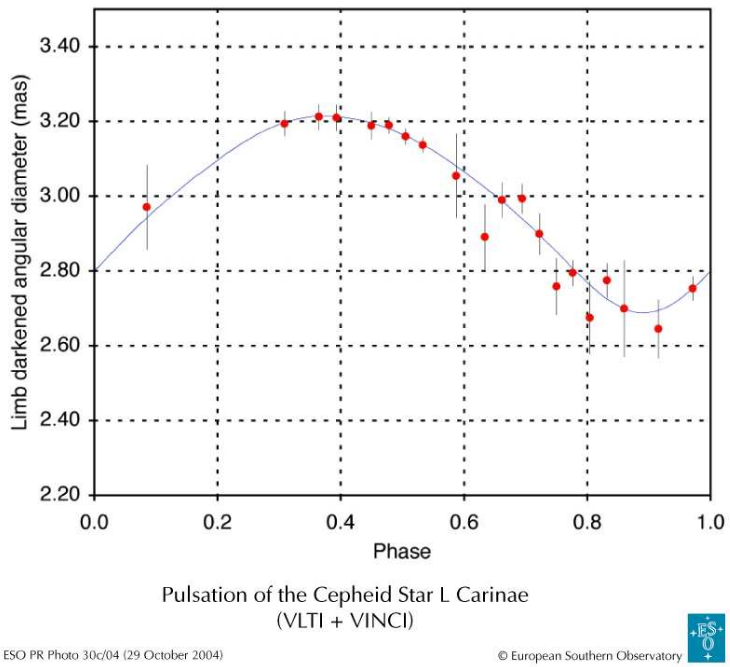


direct imaging:

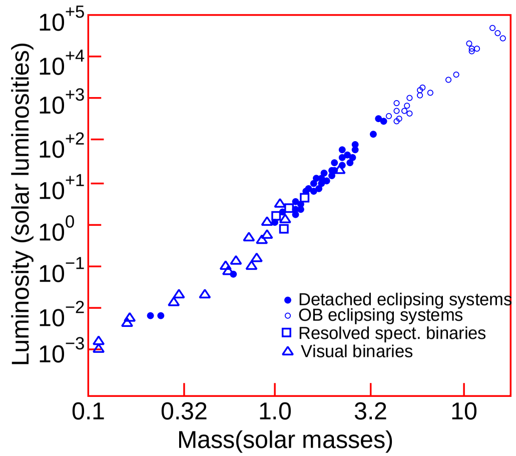
- requires very high resolution
- Betelgeuse: angular diameter: 54 mas
- distance needs to be known to determine the linear diameter

The Four Auxiliary Telescopes at Paranal

ESO PR Photo 51c/06 (22 December 2006) © ESO



Application: Mass-Luminosity Relation



Masses and Radii of binary stars:

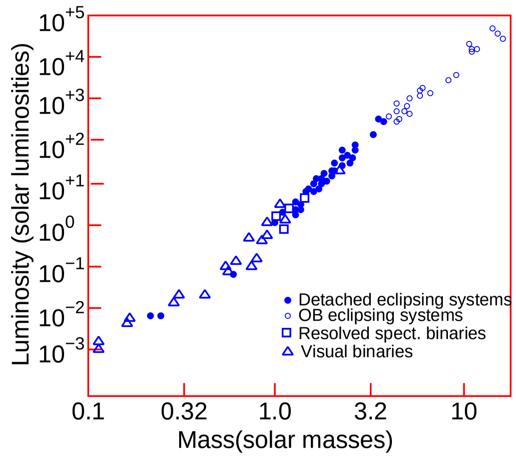
Radii R *and* temperatures T_{eff} give the luminosity:

$$L = 4\pi R^2 T_{\text{eff}}^4 \qquad (12.7)$$

Mass measured from binaries

$\Longrightarrow$ determine mass-luminosity relationship

Mass-Lumosity relation



Empirical result:

$$\frac{L}{L_\odot} = \begin{cases} 0.23\left(\frac{M}{M_\odot}\right)^{2.3} & (M < 0.43\,M_\odot) \\ \left(\frac{M}{M_\odot}\right)^{4.0} & (M \geq 0.43\,M_\odot) \end{cases}$$

$\Longrightarrow$ more massive stars have extremely larger luminosities!

(factor 2 in $M \longrightarrow$ factor 8 in L).

sometimes, one also sees $L \propto M^{3.3}\ldots$

Direct consequence:

More massive stars live much shorter lives