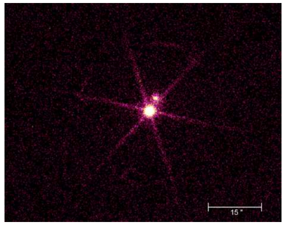
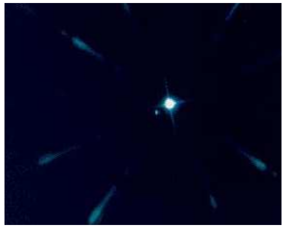
End-Stages of Stellar Evolution

White Dwarfs



Sirius A+B: *Chandra*
(X-rays; *WD is bright*)

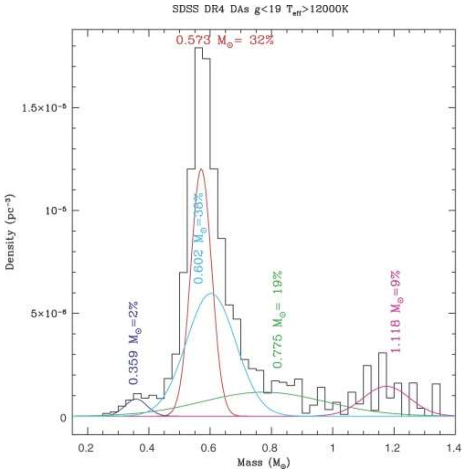


McDonald Observatory
(optical; *WD is faint*)

White Dwarfs: Sirius B

- Companion to the brightest star Sirius
- cannot be seen with the naked eye.
- Analyzing the motion of Sirius from 1833 to 1844, Friedrich Wilhelm Bessel (1844) concluded that Sirius must have an unseen companion.
- Sirius B was not actually observed until 1862 January 31 by Alvan Graham Clark.
- Star B's peculiar high temperature, small size, and great density were not established until 1925 by Walter Adams.

White Dwarfs

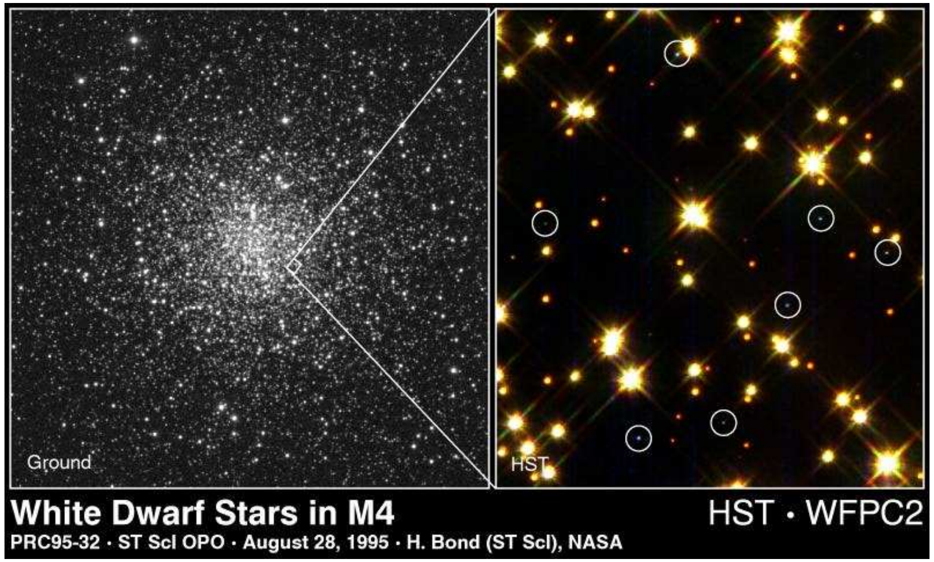


mass distribution of 1733 white dwarfs

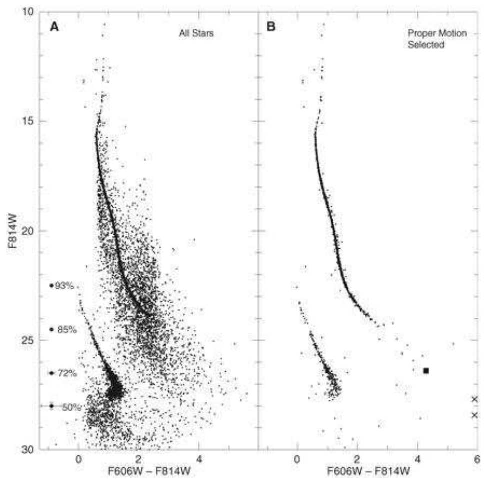
(Kepler et al. 2007, MNRAS 375, 1315)

White Dwarfs

1. End stages of evolution of stars born with $M \lesssim 8\,M_\odot$
2. typically $M \sim 0.6 M_\odot$
3. mainly consist of C and O
4. Radius $\sim$ Earth
5. typical density $\rho \sim 10^6\,\mathrm{g\,cm^{-3}}$



White Dwarfs



Color-magnitude diagram of the NGC 6397 White Dwarf sequence (Richer et al. 2006, Science 313, 936)

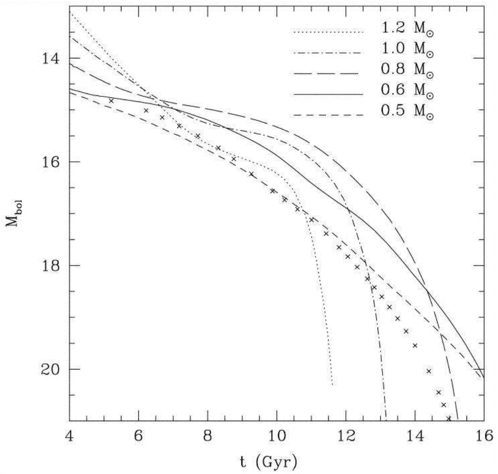
globular clusters are the oldest building blocks of the Galaxy

$\Longrightarrow$ many stars have already died

$\Longrightarrow$ GCs must host a large number of white dwarfs

---

White Dwarfs



white dwarf cooling tracks
Chabrieyer et al. 2005 (ApJ 542, 216)

- white dwarfs are stabilized by the pressure of the degenerate electron gas
- they can not shrink
- cooling of the ionic gas takes a very long time
- at low temperature: crystalization, crystal structure similar to diamond

White dwarfs are diamonds in the sky

White Dwarfs

For a degenerate gase, the equation of state: $(P = P(T, \rho))$ for is

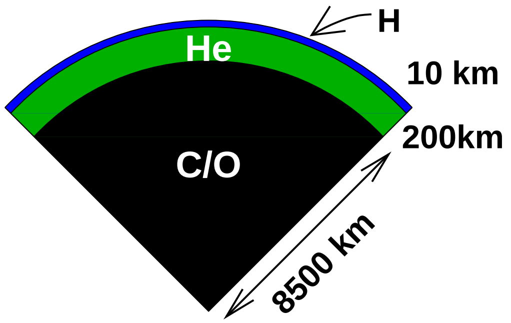
$$P \propto \begin{cases} \rho^{5/3} & \text{(non-relativistic)} \\ \rho^{4/3} & \text{(relativistic)} \end{cases} \tag{14.1}$$

independent of T!

WD structure can be determined from hydrostatic equilibrium alone:

Mass structure (mass conservation)	Pressure structure (hydrostatic equilibrium)
$\dfrac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho(r)$	$\dfrac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\dfrac{GM(r)}{r^2}$

White Dwarfs



White dwarfs come in two flavors:

DA: H present in spectrum (∼80% of all WD)

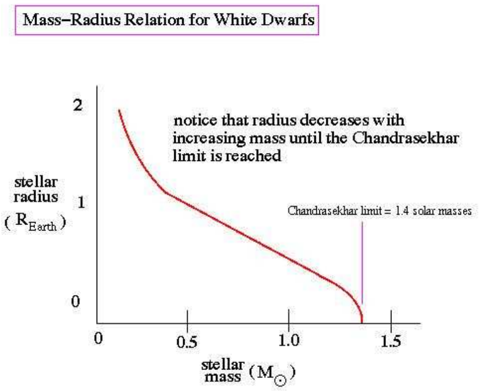
DB: He present in spectrum (∼ the rest)

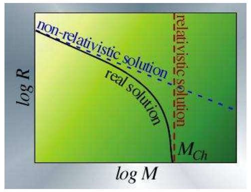
plus a few oddballs

Structure: gravitationally settled, so DB's really do not have any H since it would "swim on top"

$\Longrightarrow$ layered, "onion-like" structure

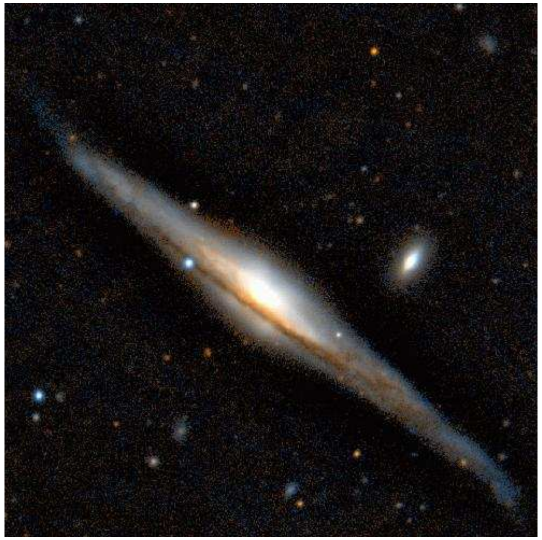
White Dwarfs





- Subrahmanyan Chandrasekhar, 1910–1995
- Nobel prize 1983
- Radius decreases with increasing mass: $R \propto M^{1/3}$
- Chandrasekhar limit: relativistic limit:

Mass must be less than 1.4 $M_\odot$



Type II SN2001cm in NGC5965 (2.56 m NOT, Håkon Dahle; NORDITA)

Evolution of more massive stars: fusion up to ^{56}Fe, then no energy gain $\Longrightarrow$ no pressure balance in centre $\Longrightarrow$ supernova explosion of type II.
energy release: 10^{46} W ($10^{20} L_\odot$; about 1% in light, rest in neutrinos)

Neutron Stars

Neutron stars form after the core collapse of massive stars.

See later for physics of supernovae.

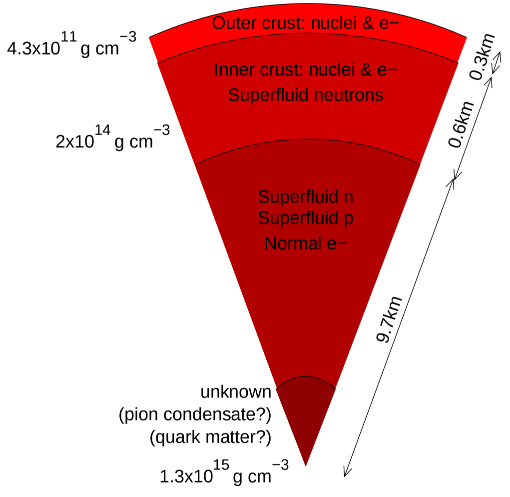
During the collapse the densities get so high that neutronization sets in:

$$\mathrm{p} + \mathrm{e}^- \longrightarrow \mathrm{n} + \nu_\mathrm{e}$$

General properties:

- Pressure mainly through degenerate neutrons (similar to degenerate electrons for WD!).
- Typical density: $\rho \sim 10^{14}\,\mathrm{g\,cm^{-3}}$ (nuclear densities)
- Typical radius: 10…15 km (Nuremberg!)
- surface gravity $\sim 10^{11} \times$ Earth
- Detailed structure not yet fully understood

Neutron Stars: Structure



Crust: perhaps crystallized?

Atoms become elongated along B-field line on surface

Internal structure unclear:

- Supraconducting matter
- Suprafluidity (i.e., fluid with no viscosity)
- central composition unknown

(after Shapiro & Teukolsky)

Neutron Stars: Rotation

During collapse, angular momentum is conserved (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

Neutron Stars: Rotation

During collapse, angular momentum is conserved (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

Angular momentum conservation ($J_{\text{before}} = J_{\text{NS}}$):

$$\frac{2}{5}M_{\text{before}}R_{\text{before}}^2\omega_{\text{before}} = \frac{2}{5}M_{\text{NS}}R_{\text{NS}}^2\omega_{\text{NS}}$$

or

$$\omega_{\text{NS}} = \left(\frac{M_{\text{before}}}{M_{\text{NS}}}\right)\left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2\omega_{\text{before}} \quad \text{or} \quad P_{\text{NS}} \sim \left(\frac{R_{\text{NS}}}{R_{\text{before}}}\right)^2 P_{\text{before}}$$

(where P: rotation period)

Neutron Stars: Rotation

During collapse, angular momentum is conserved (Explosion: symmetric)

Total angular momentum of homogeneous sphere:

$$J = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

Angular momentum conservation ($J_{\text{before}} = J_{\text{NS}}$):

$$\frac{2}{5}M_{\text{before}}R_{\text{before}}^2\omega_{\text{before}} = \frac{2}{5}M_{\text{NS}}R_{\text{NS}}^2\omega_{\text{NS}}$$

or

$$\omega_{\text{NS}} = \left(\frac{M_{\text{before}}}{M_{\text{NS}}}\right)\left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2\omega_{\text{before}} \quad \text{or} \quad P_{\text{NS}} \sim \left(\frac{R_{\text{NS}}}{R_{\text{before}}}\right)^2 P_{\text{before}}$$

(where P: rotation period)

Example: $R_{\text{before}} = 700000\,\text{km}$ (sun), $R_{\text{NS}} = 15\,\text{km}$, $P_{\text{Sun}} = 27\,\text{d} \Longrightarrow P_{\text{NS}} = 0.001\,\text{s}$

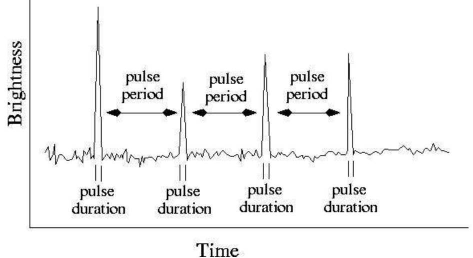
Neutron Stars are extremely fast rotators.

close to break-up speed!

Pulsars

Discovery: Bell & Hewish (1967):

Radio Pulsar

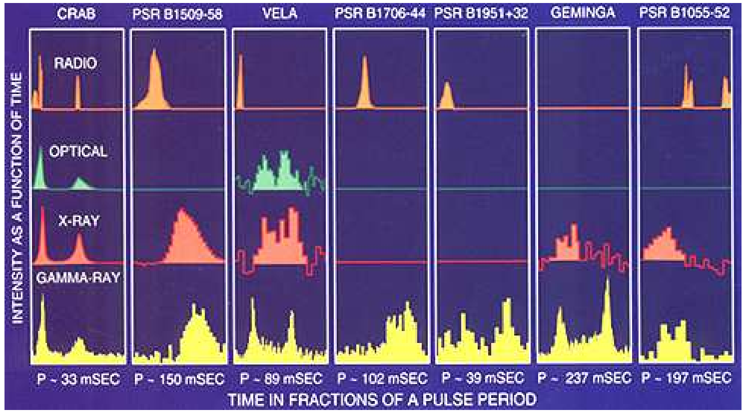


radio emission is pulsed,

very short periods: milliseconds to a few seconds

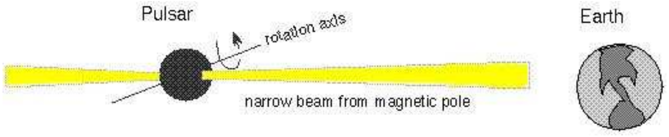
Pulsars

Pulsars at different wavelengths



Pulsations not only in the radio regime, but also at optical, X-ray, and γ-ray wavelength, but not in all cases.

Pulsars



If the narrow synchrotron beam passes over the Earth, we see the neutron star flash on and off like a lighthouse beam does for ships at sea.

Pulses due to the lighthouse effect caused by rapid rotation.

Rotation period:

$$P = \frac{2\pi R}{v_{\text{rot}}} \tag{14.2}$$

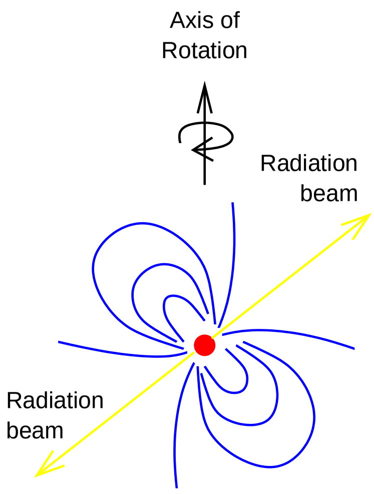
Rotation speed at the surface must be smaller the speed of light. $\Longrightarrow R < \frac{Pc}{2\pi}$

Shortest periods observed: $P \sim 1\,\text{ms}$

$\Longrightarrow R < 50\,\text{km}$

Pulsars are neutron stars!

Pulsars



“Lighthouse model” for pulsars

Another conserved observable: magnetic flux: $\Phi = BR^2$

Magnetic field after SN:

$$B_{\text{NS}} = \left(\frac{R_{\text{before}}}{R_{\text{NS}}}\right)^2 B_{\text{before}}$$

$\Longrightarrow$ neutron stars have strong magnetic fields (typical: $B \sim 10^6 \dots 10^8\,\text{T}$)

Radio pulsars are fast rotating (isolated) neutron stars with strong magnetic fields.

The sounds of pulsars

- PSR 0329 – a normal pulsar ($P = 0.714519\,\text{s}$)
- PSR 0833 – the Vela pulsar, a faster, younger pulsar in the Vela supernova remnant ($P = 89\,\text{msec}$)
- Crab pulsar – the youngest pulsar ($P = 33\,\text{ms}$)
- B1937 – one of the fastest pulsars ($P = 0.00155780644887275\,\text{s}$)

See/hear `http://www.jb.man.ac.uk/~pulsar/Education/Sounds/sounds` for more examples.

Black Holes

Degenerate neutron gas: Chandrasekhar theory applies.

However, modified hydrostatic equation (GRT)

equation of state much more complicated than for white dwarfs

Neutron stars also have upper mass limit: Oppenheimer Volkoff limit.

Detailed mass limit unknown, causality considerations give $M \sim 3\,M_\odot$ (for “stiff equation of state” the sound speed becomes greater than speed of light at this mass)

Compact objects with mass above Oppenheimer Volkoff limit: Black Holes

More conservative astronomers: “Black Hole Candidates”.

Black Holes

In more modern usage (but still Newtonian!):

Total energy of a mass m:

$$E = E_{\text{pot}} + E_{\text{kin}} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

Mass m is unbound if $E > 0$, i.e., for

$$v \geq v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

Black Hole: Body of mass M and radius R for which $v_{\text{escape}} > c$, where c is the speed of light.

This is the case if

$$R \leq R_{\text{S}} = \frac{2GM}{c^2} \sim 3\,\text{km}\frac{M}{M_\odot}$$

the Schwarzschild Radius.

Einstein

Albert Einstein (1879–1955)

Special Relativity (1905):

- Speed of light has the same value in *all* frames of reference
- Observer with constant velocity measure the same physical laws

From these axioms follows:

$\Longrightarrow$ Space and time are relative (“4D-space-time”)

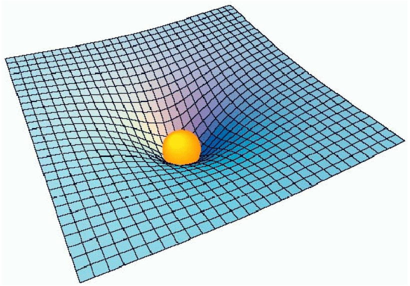
$\Longrightarrow E = mc^2$
(“Mass and Energy are equivalent”)

Einstein

Albert Einstein (1879–1955)

General relativity (1916):

- Mass curves space (“Metric”)

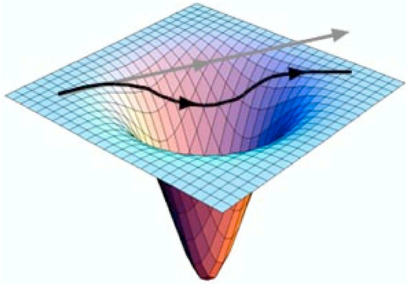


Einstein

Albert Einstein (1879–1955)

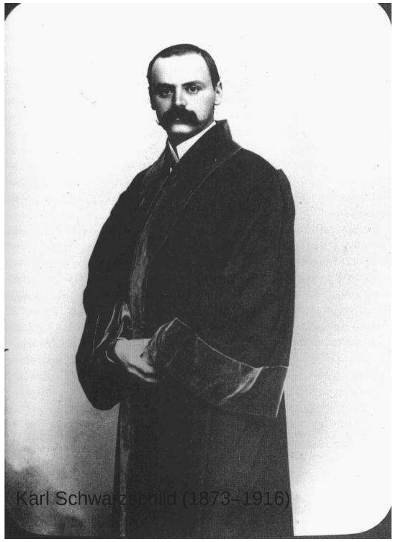
General relativity (1916):

- Mass curves space ("Metric")
- Light moves through curved space



---

post-Einstein



Karl Schwarzschild (1873–1916)

Directly after publication of GRT:

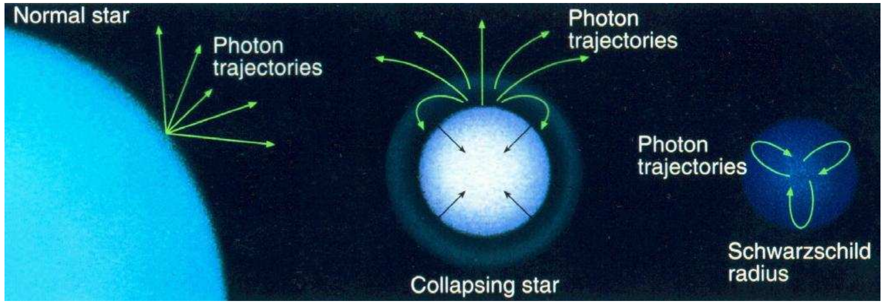
$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 \mathrm{d}t^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} \mathrm{d}r^2$$

(Schwarzschild Metric).

Describes "shape of space" in vicinity of mass M.

---

post-Einstein



$R > R_S$ $R \sim R_S$ $R < R_S$

Behavior of light is determined from location of emission, in dependence from the Schwarzschild Radius:

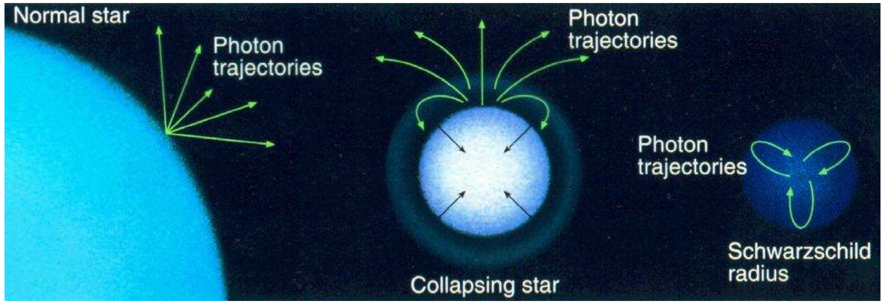
$$R_S = \frac{2GM}{c^2} \sim 3\,\mathrm{km}\,\frac{M}{M_\odot}$$

Same value as in Newtonian derivation!

J.N. Imamura

---

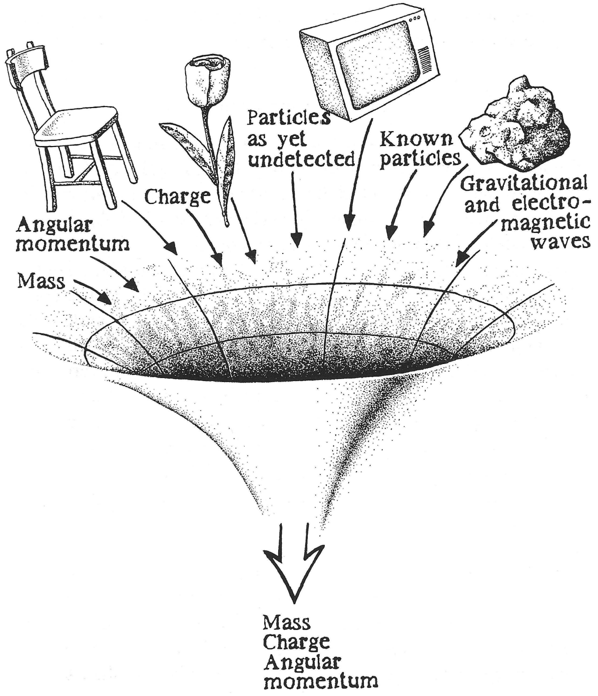
post-Einstein



$R > R_S$ $R \sim R_S$ $R < R_S$

Black hole in GRT: Bodies smaller than their Schwarzschild radius.

J.N. Imamura



Black holes are very simple physical objects, determined by

- Mass
- (Charge)
- Angular momentum

"It's black, and it looks like a hole. I'd say it's a black hole."