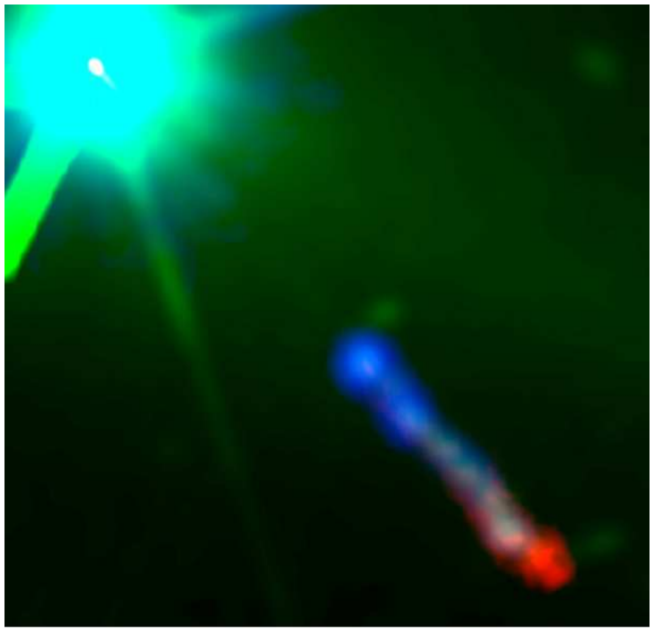
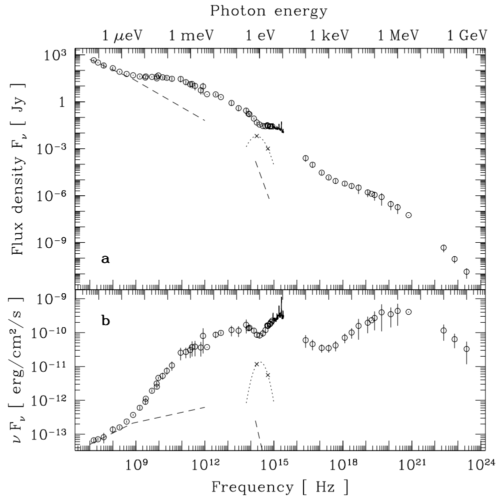
Radiative Processes



`http://chandra.harvard.edu/press/06_releases/press_062006.html` and Jester et al. (2006)
X-ray (blue; *Chandra*), optical (green; *HST*), IR (red; *Spitzer*) image of the quasar 3C273 (overexposed) and its jet.

Blazar Broadband Spectra



(Türler et al., 1999, Fig. 6)

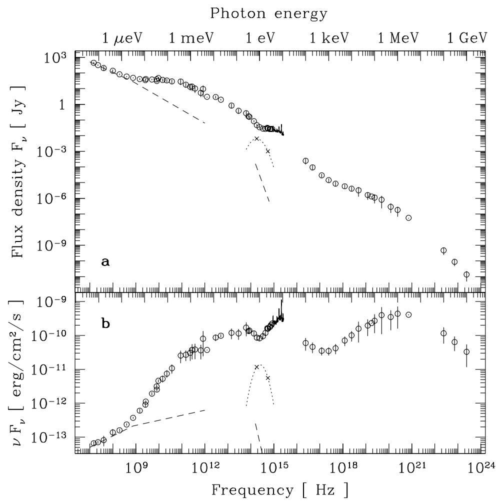
Broad-band spectrum of radio loud AGN 3C 273: Spectral Energy Distribution (SED) is flat $\Longrightarrow$ similar energy output at all wavebands!

Shown is a νf_ν-plot, where ν: frequency, f_ν: flux density at frequency ν (units of f_ν are erg s^{-1} cm^{-2} Hz^{-1}). Since

$$\int_{\nu_1}^{\nu_2} \nu f_\nu \, d\nu = \int_{\ln \nu_1}^{\ln \nu_2} f_\nu \, d \ln \nu \qquad (4.1)$$

plotting νf_ν in a log-log-plot gives a measure of the energy emitted per frequency decade.

Blazar Broadband Spectra



(Türler et al., 1999, Fig. 6)

Broad band power-law spectrum: "non-thermal radiation"

The broad-band emission of 3C 273 is mainly from its jet:

- radio – X-rays: synchrotron radiation from relativistic electrons.
- X-rays – gamma-rays: Synchrotron-self Compton radiation: Compton upscattered synchrotron photons

In addition thermal (=black body) emission from stars in the optical.

Synchrotron Radiation

Many sources studied today show broad-band spectra dominated by synchrotron radiation.

Synchrotron-Radiation (=Magnetobremsstrahlung): Radiation emitted by relativistic electrons in a magnetic field.

Outline for the following discussion of theory of synchrotron-radiation: Short and qualitative description. See Rybicki & Lightman (1979, Chapters 3, 6, and 7) and the lecture on "Radiation Processes" for details.

Structure:

1. Motion of electrons in magnetic fields,
2. Look at emission from a single electron,
3. Consider electron distribution and opacity effects to obtain the final spectrum.

Relativistic Motion

Moving electron in magnetic field ($\boldsymbol{E} = 0$): In Gaussian units, the Lorentz-Force is

$$\frac{d\boldsymbol{p}}{dt} = \frac{e}{c}\boldsymbol{v} \times \boldsymbol{B} \quad \text{where} \quad \boldsymbol{p} = \frac{m_e \boldsymbol{v}}{\sqrt{1-\beta^2}} = \gamma m_e \boldsymbol{v} \tag{4.2}$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \text{and where} \quad \beta = \frac{v}{c} \tag{4.3}$$

Therefore the acceleration is

$$\frac{d\boldsymbol{v}}{dt} = \frac{e}{c\gamma m_e}\boldsymbol{v} \times \boldsymbol{B} \tag{4.4}$$

Since $\boldsymbol{v} \times \boldsymbol{B}$ is always perpendicular to $\boldsymbol{v}$ and $\boldsymbol{B}$, the component of $\boldsymbol{v}$ along the B-field does not change. This constant perpendicular force results to a helical motion around the B-field line with the frequency

$$\omega_B = \frac{eB}{\gamma m_e c} = \frac{\omega_L}{\gamma} \tag{4.5}$$

where the Larmor frequency (also Cyclotron frequency, gyrofrequency)

$$\omega_L = 2\pi\nu_L = \frac{eB}{m_e c} \tag{4.6}$$

Numerical values

Numerically, the Larmor frequency is

$$\nu_L = 2.8 B_{1\,\mathrm{G}}\,\mathrm{MHz} \tag{4.7}$$

The radius of the orbit (Larmor radius) is

$$R_L = \frac{\gamma v_\perp}{\omega_L} \sim 2\,\mathrm{AU} \cdot \frac{E}{1\,\mathrm{GeV}} \cdot \left(\frac{B}{10^{-6}\,\mathrm{G}}\right)^{-1} \tag{4.8}$$

$$\sim 300\,\mathrm{km} \cdot \frac{E}{1\,\mathrm{GeV}} \cdot \left(\frac{B}{1\,\mathrm{G}}\right)^{-1} \tag{4.9}$$

i.e., small on cosmical scales

Units and orders of magnitude:

- $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$,
- the typical B-field in the interstellar medium is $\sim 10^{-6}\,\mathrm{G}$,
- close to the centers of AGN $B \sim 1\,\mathrm{G}$.

Radiated Energy

Motion around B-field lines: acceleration.

But accelerated charges emit radiation (Larmor's formula):

$$P = \frac{dW}{dt} = \frac{q^2\dot{v}^2}{4\pi c^3}\int \sin^2\theta\, d\Omega = \frac{2q^2\dot{v}^2}{3c^3} \tag{4.10}$$

Assumption of isotropic velocity distribution, relativistic electrons ($\beta \longrightarrow 1$), and a messy derivation (see Rybicki & Lightman) yields for the average emitted power of an electron in a B-field

$$\langle P_{em}\rangle = \frac{4}{3}\beta^2\gamma^2 c\sigma_T U_B \tag{4.11}$$

with $U_B = B^2/8\pi$, the magnetic field energy density, and $\sigma_T = 8\pi e^2/(3m_e^2c^4)$, the Thomson cross section.

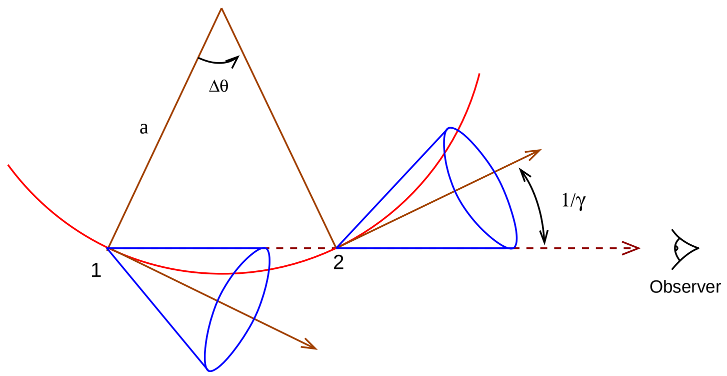
Note: Since $E = \gamma m_e c^2 \Longrightarrow P \propto E^2 U_B$.

Note: $P_{em} \propto \sigma_T \propto m_e^{-2} \Longrightarrow$ Synchrotron radiation from charged particles with larger mass (protons,...) is negligible.

Note: Life-time of particles of energy E is

$$t_{1/2} \sim \frac{E}{P} \propto \frac{1}{B^2 E} = 5\,\mathrm{s}\left(\frac{B}{1\,\mathrm{T}}\right)^{-2}\gamma^{-1} = 1.6\times10^7\,\mathrm{years}\left(\frac{B}{10^{-7}\,\mathrm{T}}\right)^{-2}\gamma^{-1} \tag{4.12}$$

Radiated Energy

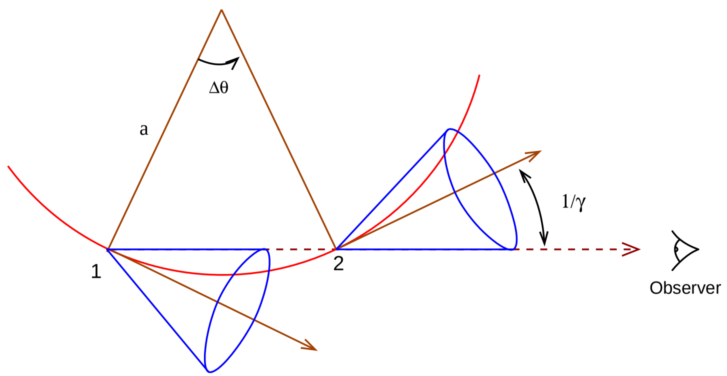


(after Fig. 6.2 of Rybicki & Lightman, 1979)

Relativistic electrons: radiation is forward beamed into cone with opening angle $\Delta\theta \sim 1/\gamma$. In the Electron frame of rest: beam passes observer during time

$$\Delta t = \frac{\Delta\theta}{\omega_B} = \frac{m_e c\gamma}{eB}\frac{2}{\gamma} = \frac{2}{\omega_L} \tag{4.13}$$

Radiated Energy



(after Fig. 6.2 of Rybicki & Lightman, 1979)

Observer frame: Doppler effect! (electron is closer to us at end of time interval)

$\Longrightarrow$ observed pulse duration:

$$\tau = \left(1 - \frac{v}{c}\right)\Delta t = (1-\beta)\Delta t \tag{4.14}$$

Radiated Energy

For $\gamma \gg 1$, i.e., $\beta = v/c \sim 1$

$$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} = (1+\beta)(1-\beta) \approx 2(1-\beta) \tag{4.15}$$

such that

$$\tau = (1-\beta)\Delta t = \frac{1}{2}\left(1 - \frac{v^2}{c^2}\right)\Delta t = \frac{1}{\gamma^2\omega_L} \tag{4.16}$$

Thus the characteristic frequency of the radiation is given by

$$\omega_c = \gamma^2\omega_L = \frac{eB}{m_e c}\left(\frac{E}{m_e c^2}\right)^2 \tag{4.17}$$

Short gyration pulses $\Longrightarrow$ broad spectrum (Heisenberg: $\Delta\omega\Delta t > 1$) with the highest frequency in the regime of $\nu_c = \omega_c/2\pi$.

Nonthermal Synchrotron Radiation

For an electron distribution, $n(\gamma)$, the emitted spectrum is found by properly weighting contributions of electrons with different energies:

$$P_\nu = \int_1^\infty P_\nu(\gamma)n(\gamma)d\gamma \tag{4.18}$$

Most important case: nonthermal synchrotron radiation, where electrons have a power-law distribution

$$n(\gamma)d\gamma = n_0\gamma^{-p}d\gamma \quad . \tag{4.19}$$

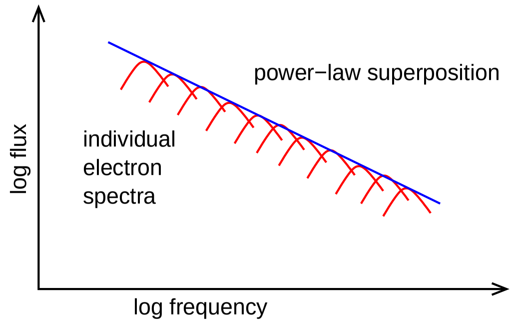
The spectral energy distribution P_ν of an electron with total energy $E = \gamma m_e c^2$ can be written as

$$P_\nu(\gamma) = \frac{4}{3}\beta^2\gamma^2 c\sigma_T U_B \phi_\nu(\gamma) \tag{4.20}$$

where the spectral shape is described by a function $\phi_\nu(\gamma)$ with

$$\int \phi_\nu(\gamma)\,d\gamma = 1 \quad . \tag{4.21}$$

Nonthermal Synchrotron Radiation



after (Shu, 1991, Fig. 18.4)

Assume that photons are only emitted at the characteristic frequency $\gamma^2\nu_L$ (Eq. 4.17).
this is a good approximation since the spectrum emitted by an electron has a strong peak at that frequency
Therefore

$$\phi_\nu(\gamma) \sim \delta(\nu - \gamma^2\nu_L) \qquad (4.22)$$

Therefore the emitted power at frequency ν (=spectrum) is

$$P_\nu = \int_1^\infty \langle P_\nu(\gamma)\rangle\, n(\gamma)\,\mathrm{d}\gamma \qquad (4.23)$$

Nonthermal Synchrotron Radiation

Therefore, for the electron power-law distribution Eq. 4.19

$$P_\nu = \int_1^\infty \frac{4}{3}\beta^2\gamma^2 c\sigma_T U_B \delta(\nu - \gamma^2\nu_L) n_0 \gamma^{-p}\,\mathrm{d}\gamma \qquad (4.24)$$

since $\gamma \gg 1$: $\beta \approx 1$

$$= A\int_1^\infty \gamma^{2-p}\delta(\nu - \gamma^2\nu_L)\,\mathrm{d}\gamma \qquad (4.25)$$

substituting $\nu' = \gamma^2\nu_L$, i.e., $\mathrm{d}\nu' = \nu_L 2\gamma\,\mathrm{d}\gamma$

$$= B\int_{\nu_L}^\infty \gamma^{1-p}\delta(\nu - \nu')\,\mathrm{d}\nu' \qquad (4.26)$$

since $\gamma = (\nu'/\nu_L)^{1/2}$, we find

$$P_\nu = \frac{2}{3} c\sigma_T n_0 \frac{U_B}{\nu_L}\left(\frac{\nu}{\nu_L}\right)^{-\frac{p-1}{2}} \qquad (4.27)$$

The spectrum of an electron power-law distribution is a power-law!

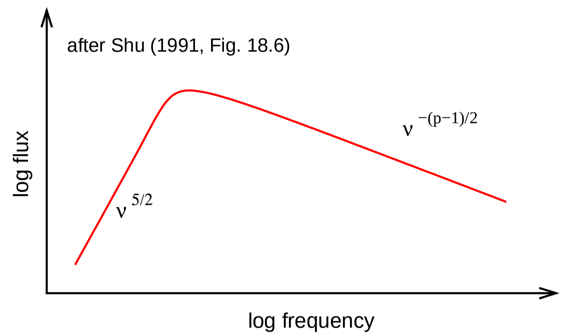
Summary

What we have done so far:

1. Motion of the electron
2. Radiation characteristic from relativistic motion
3. Doppler-effect
4. Integration over electron distribution

It is possible to do the same analytically without any approximations. This is too complicated to be done here. See the references for details.

Synchrotron Self-Absorption



after Shu (1991, Fig. 18.6)

At low ν: synchrotron emitting electrons can absorb synchrotron photons:
synchrotron self-absorption.

For a power law electron distribution $\propto E^{-p}$, total spectral shape is:

For low frequencies: $P_\nu \propto B^{-1/2}\nu^{5/2}$ (independent of p!)
For large frequencies: $P_\nu \propto \nu^{-(p-1)/2}$

One often uses the terms optically thick/thin to describe the absorbed/unabsorbed part of a synchrotron spectrum. The turnover describes the $\tau = 1$ surface, e.g., of a jet. In general: $\tau \propto R$ (R: size of the emitting region). More compact regions are optically thick, more extended regions are optically thin.

Comptonization

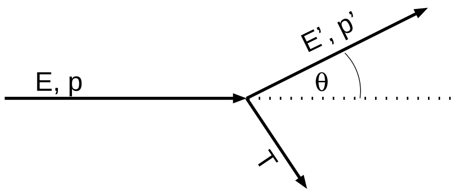
Comptonization: Upscattering of low-energy photons by inverse Compton collisions in a hot electron gas.

Astronomically important in

- galactic black hole candidates
- active galactic nuclei

Literature: Blumenthal & Gould (1970), Górecki & Wilczewski (1984), Hua & Titarchuk (1995), Pozdnyakov, Sobol & Sunyaev (1983), and Sunyaev & Titarchuk (1980).

Comptonization



Compton scattering: scattering of photons off electrons: $\Longrightarrow$ Scattering: photon changes direction

$\Longrightarrow$ Momentum change

$\Longrightarrow$ Energy change!

Dynamics of scattering gives energy/wavelength change:

$$E' = \frac{E}{1+\frac{E}{m_e c^2}(1-\cos\theta)} \sim E\left(1 - \frac{E}{m_e c^2}(1-\cos\theta)\right) \quad \Longleftrightarrow \quad \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) \qquad (4.28)$$

where $h/m_e c = 2.426 \times 10^{-10}$ cm (Compton wavelength).

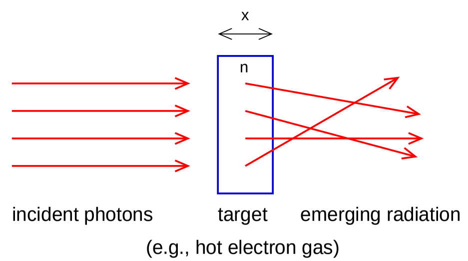
Averaging over θ, for $E \ll m_e c^2$:

$$\frac{\Delta E}{E} \approx -\frac{E}{m_e c^2} \qquad (4.29)$$

E.g., at 6.4 keV, $\Delta E \approx 0.2$ keV.

Thomson scattering: low energy limit of Compton scattering, initial and final wavelength are identical.

Cross section



Consider a medium of thickness x irradiated by N_{incident} photons.

In a medium that is sufficiently thin, such that only single scattering is important, the number of interactions (= number of photons scattered out of the beam), N_{tot}, is given by

$$N_{\text{tot}} = N_{\text{incident}} \cdot \frac{N_{\text{target}}}{A} \cdot \sigma \qquad (4.30)$$

where N_{target}: number of target electrons, A: area of the target, and σ is the total cross section.

It is often better to write this equation as

$$N_{\text{tot}} = N_{\text{incident}} \cdot \frac{n_{\text{target}} A x}{A} \cdot \sigma = N_{\text{incident}} n_{\text{target}} x \sigma \qquad (4.31)$$

where n_{target}: number density of target particles.

Analoguously the differential cross section, $d\sigma/d\Omega$ is defined through

$$N(\Omega) = N_{\text{incident}} n_{\text{target}} x \frac{d\sigma}{d\Omega} d\Omega \qquad (4.32)$$

($N(\Omega)$: number of interactions resulting in photons scattered into spatial direction Ω).

Optical Depth and Mean Free Path

According to Eq. (4.31), over a distance dx the number of photons in a photon beam *decreases* because of scattering:

$$dN_{\text{photons}} = -n\sigma N_{\text{photons}}\, dx \qquad (4.33)$$

Separation of variables gives the number of photons emerging from a medium of thickness ℓ:

$$\int_{N_0}^{N(\ell)} \frac{dN}{N} = -\int_0^{\ell} n\sigma\, dx \quad \Longrightarrow \quad N(\ell) = N_{\text{incident}}\, \mathrm{e}^{-n\sigma\ell} \qquad (4.34)$$

which defines the optical depth, τ, of the medium:

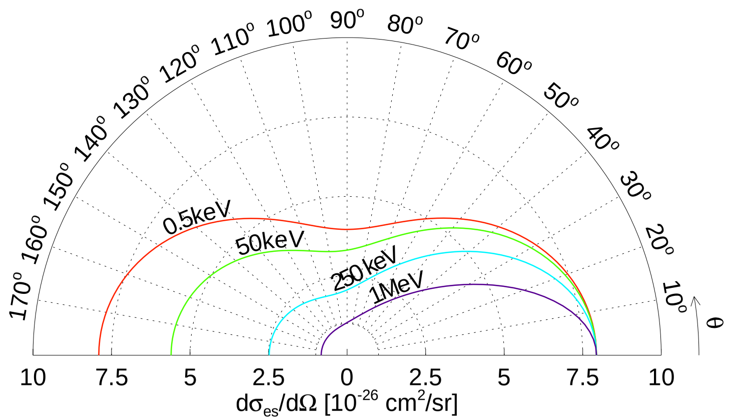
$$\tau = n\sigma\ell \qquad (4.35)$$

The mean free path of a photon, i.e., the average distance traveled between scattering events, is

$$\text{mfp} = \langle\ell\rangle = \frac{\int_0^\infty \ell \mathrm{e}^{-n\sigma\ell}\, d\ell}{\int_0^\infty \mathrm{e}^{-n\sigma\ell}\, d\ell} = \frac{1}{n\sigma} \quad \Longleftrightarrow \quad n\sigma\langle\ell\rangle = 1 \qquad (4.36)$$

The mean free path of a photon is the distance corresponding to an optical depth of unity, $\tau = 1$.

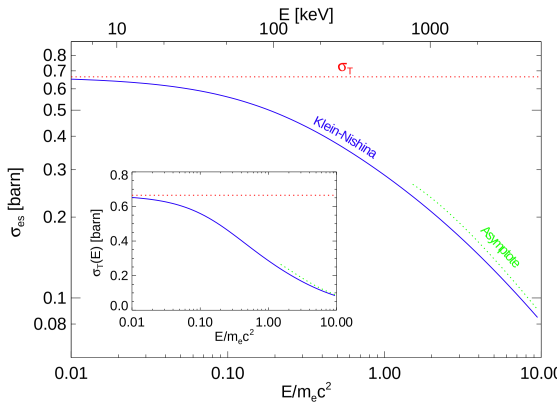
Klein-Nishina Formula



For unpolarized radiation, the cross-section for Compton scattering is given by the Klein-Nishina formula, which can only be derived in quantum-electrodynamics:

$$\frac{\mathrm{d}\sigma_{\mathrm{es}}}{\mathrm{d}\Omega} = \frac{3}{16\pi}\sigma_{\mathrm{T}}\left(\frac{E'}{E}\right)^2\left(\frac{E}{E'}+\frac{E'}{E}-\sin^2\theta\right) \tag{4.37}$$

Klein-Nishina Formula



Integrating Klein-Nishina over 4π sr gives the total cross-section:

$$\sigma_{\mathrm{es}} = \frac{3}{4}\sigma_{\mathrm{T}}\left[\frac{1+x}{x^3}\cdot\left\{\frac{2x(1+x)}{1+2x}-\ln(1+2x)\right\}+\frac{1}{2x}\ln(1+2x)-\frac{1+3x}{(1+2x)^2}\right] \tag{4.38}$$

where $x = E/m_{\mathrm{e}}c^2$.

For $x \gg 1$,

$$\sigma \sim \frac{3}{8}\sigma_{\mathrm{T}}\frac{1}{x}\left(\ln 2x + \frac{1}{2}\right) \tag{4.39}$$

Energy Exchange

For non-stationary electrons, energy after scattering can be derived by Lorentz transforming into electron's frame of rest and back:

1. Lab system $\Rightarrow$ electron's frame of rest:

$$E_{\mathrm{FoR}} = E_{\mathrm{Lab}}\gamma(1-\beta\cos\theta) \tag{4.40}$$

2. Scattering occurs, gives E'_{FoR}.
3. Electron's frame of rest $\Rightarrow$ Lab system:

$$E'_{\mathrm{Lab}} = E'_{\mathrm{FoR}}\gamma(1+\beta\cos\theta') \tag{4.41}$$

Therefore, if electron is relativistic:

$$E'_{\mathrm{Lab}} \sim \gamma^2 E_{\mathrm{Lab}} \tag{4.42}$$

since (on average) θ, θ' are $\mathcal{O}(\pi/2)$.

Thus: Energy transfer is *very* efficient.

Thermal Electrons

Energy gain of photon field depends on electron distribution. In general, complicated calculation, however, for thermal electrons, energy gain can be obtained with a very elegant calculation:

In electron frame of rest,

$$\frac{\Delta E}{E} = -\frac{E}{m_{\mathrm{e}}c^2} \tag{4.29}$$

For electrons with a Maxwell distribution, a similar relation must hold to first order (Taylor!):

$$\frac{\Delta E}{E} = -\frac{E}{m_{\mathrm{e}}c^2} + \alpha\frac{kT_{\mathrm{e}}}{m_{\mathrm{e}}c^2} \tag{4.43}$$

where α is so far unknown. In thermodynamical equilibrium, photons and electrons interact only through scattering

$\Longrightarrow$ Photons have Bose-Einstein distribution,

$$N(E) = KE^2\exp\left(-\frac{E}{kT_{\mathrm{e}}}\right) \quad \text{with} \quad \langle E\rangle = 3kT_{\mathrm{e}} \quad \text{and} \quad \left\langle E^2\right\rangle = 12(kT_{\mathrm{e}})^2 \tag{4.44}$$

In equilibrium, $\Delta E = 0 \Longrightarrow$

$$\langle\Delta E\rangle = 0 = \frac{\alpha kT_{\mathrm{e}}}{m_{\mathrm{e}}c^2}\langle E\rangle - \frac{\langle E^2\rangle}{m_{\mathrm{e}}c^2} = (\alpha-4)\frac{3(kT_{\mathrm{e}})^2}{m_{\mathrm{e}}c^2} \tag{4.45}$$

such that $\alpha = 4$.

Compton y

We have seen that

$$\frac{\Delta E}{E} \approx \frac{4kT_e - E}{m_e c^2} =: A \qquad (4.46)$$

where A is the Compton amplification factor. Thus:

$E \lesssim 4kT_e \Longrightarrow$ Photons gain energy, gas cools down.
$E \gtrsim 4kT_e \Longrightarrow$ Photons loose energy, gas heats up.

A generalization of the Compton amplification factor for relativistic energies is

$$A = 1 + 4\Theta \frac{K_3(1/\Theta)}{K_2(1/\Theta)} \approx 4\Theta + 16\Theta^2 \qquad (4.47)$$

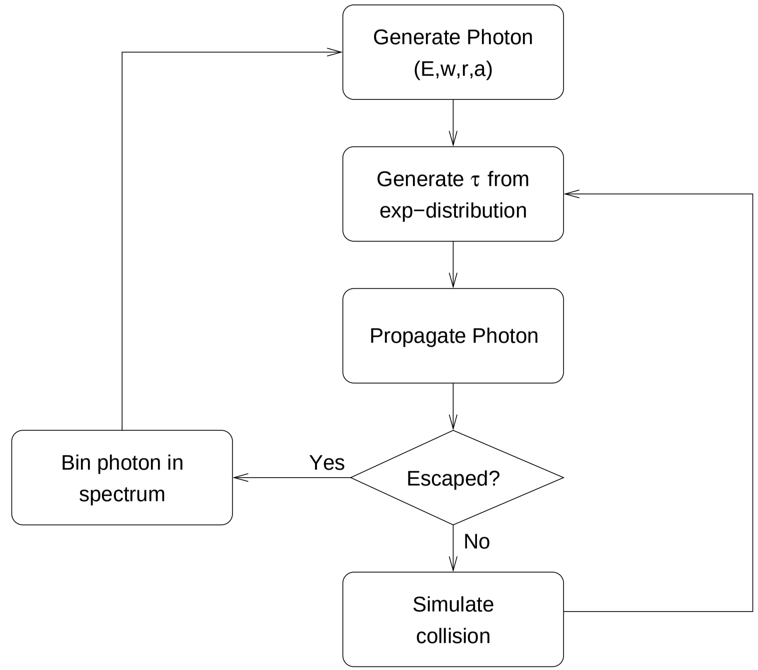
where $K_i(x)$: modified Bessel functions of 2nd kind (Zdziarski 1985).

Total relative energy change by traversal of hot ($E \ll kT_e$) medium with optical depth $\tau_e = n_e \sigma_T l$:

$$(\text{rel. energy change}) = \frac{\text{rel. energy change}}{\text{scattering}} \times (\text{\# scatterings}) \qquad (4.48)$$

$$y = \frac{4kT_e}{m_e c^2} \max(\tau_e, \tau_e^2) \qquad (4.49)$$

"Compton y-Parameter"



Approximate spectral shape

Photon spectra can be found by analytically solving the Kompaneets equation, a very complicated partial differential equation. See Sunyaev & Titarchuk (1980) for examples. Solution is only possible for special cases and simple geometries.

For the most common case, unsaturated Comptonization, one obtains

$$I(x) \propto \begin{cases} x^3 \exp(-x) & \text{for } x \gg 1 \\ x^{3-\Gamma} & \text{for } x \ll 1 \end{cases} \qquad (4.50)$$

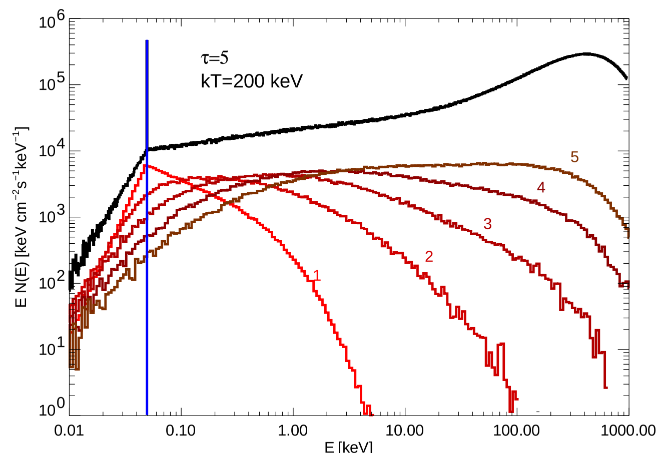
where

$$\Gamma = \frac{3}{2} \mp \sqrt{\frac{9}{4} + \frac{4}{y}} \qquad (4.51)$$

where $-$-root for $y \gg 1$, $+$-root for $y \ll 1$, and average for $y \sim 1$. Typical sources have $y \sim 1$, i.e., power law with photon index ~ 1.5

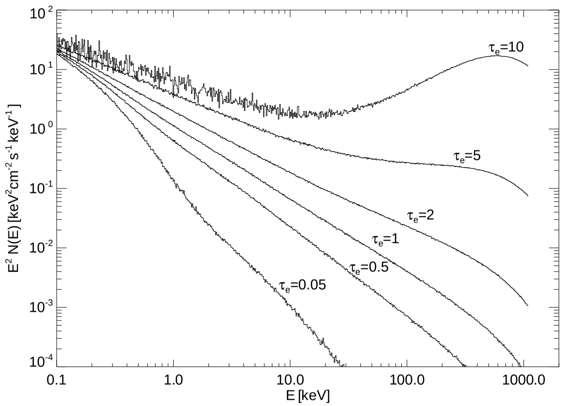
General solution: Possible via the Monte Carlo method (e.g., Pozdnyakov, Sobol & Sunyaev, 1983)

Results: Spectrum



Monte Carlo simulation shows: Spectrum is $\Longrightarrow$ Power law with exponential cut-off (here: with additional "Wien hump", see next slide)

Results: Spectrum



Sphere with $kT_e = 0.7 m_e c^2$ ($\sim$ 360 keV), seed photons come from center of sphere.

- $y \ll 1$: pure power-law.
- $y < 1$: power-law with exponential cut-off
- $y \gg 1$: "Saturated Comptonization".

Saturated Comptonization has never been observed because cloud would cool down quickly.

Compton Catastrophe

One can show that the net power gain of the photon field in Comptonization off electrons with gamma-factor γ is

$$P_{\text{compt}} = \frac{4}{3}\sigma_T c\gamma^2\beta^2 U_{\text{rad}} \tag{4.52}$$

where U_{rad} is the radiation density of the incident photon field.

Power emitted by synchrotron radiation in a B-field of energy density U_B was

$$P_{\text{synch}} = \frac{4}{3}\sigma_T c\gamma^2\beta^2 U_B \tag{4.11}$$

Magnetized plasma: synchrotron photons are inverse Compton scattered by the electrons:

$$\frac{P_{\text{compt}}}{P_{\text{synch}}} = \frac{U_{\text{rad}}}{U_B} \tag{4.53}$$

QED: synchrotron radiation is inverse Compton scattering off virtual photons of the B-field.

$U_{\text{rad}} > U_B \Longrightarrow P_{\text{compt}} > P_{\text{synch}}$

$\Longrightarrow$ (synchrotron) photon field will undergo dramatic amplification

$\Longrightarrow$ very efficient cooling of electrons by inverse Compton losses (Compton catastrophe).

As a result, the brightness temperature of radio sources is limited to 10^{12} K.

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