## Question 1: *Synchrotron Self-Absorption*

For an electron gas, the energy density is $u = nkT/(\gamma_{\rm SH} - 1)$ where $\gamma_{\rm SH}$ is the ratio of specific heats, which is 4/3 for a relativistic gas and 5/3 for a non-relativistic gas.

a) Convince yourself that it makes sense to associate a temperature $T_{\rm e}$ with electrons of particle energy $\gamma m_{\rm e} c^2$ by setting

$$\gamma m_{\rm e} c^2 = 3kT_{\rm e} \tag{w1.1}$$

b) In the Rayleigh-Jeans-limit, the flux of a source with angular size $\Omega$ is

$$S_\nu = \frac{2kT}{\lambda^2}\Omega \tag{w1.2}$$

Taking the above into account, show that for optically thick synchrotron radiation

$$S_\nu \propto \nu^{5/2} B^{-1/2} \tag{w1.3}$$

Note: $\gamma \sim (\nu/\nu_L)^{1/2}$ and $\nu_L = eB/m_{\rm e}c$

From question a

$$T_{\rm e} = \gamma m_{\rm e} c^2/3k \propto \frac{\nu^{1/2}}{B^{1/2}} \tag{s1.1}$$

Therefore in the optically thick case

$$S_n u \propto \nu^2 \nu^{1/2} B^{-1/2} \propto \nu^{5/2} B^{-1/2} \tag{s1.2}$$