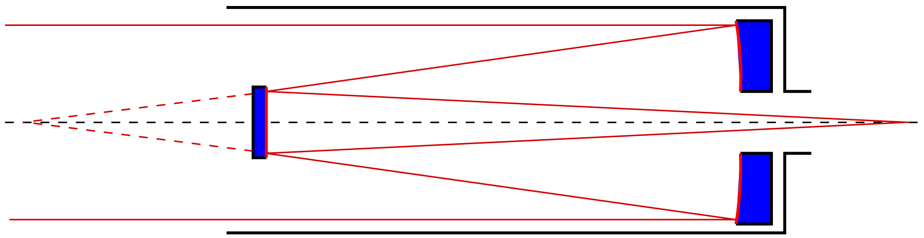
Imaging, I



Cassegrain telescope, after Wikipedia

Reminder: Optical telescopes are usually reflectors:

primary mirror → secondary mirror → detector

Main characteristics of a telescope:

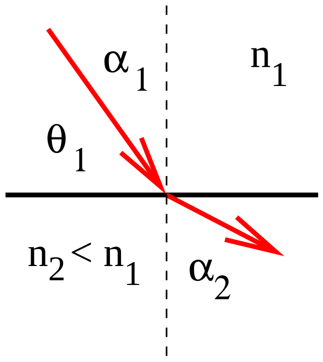
- collecting area (i.e., open area of telescope, $\sim \pi d^2/4$, where d: telescope diameter)
- for small telescopes: angular resolution,

$$\theta = 1.22\,\frac{\lambda}{d} \tag{3.9}$$

but do not forget the seeing!

Imaging, II

Optical telescopes are based on principle that reflection “just works” with metallic surfaces. For X-rays, things are more complicated...



Snell’s law of refraction:

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{n_2}{n_1} = n \tag{3.10}$$

where n index of refraction, and $\alpha_{1,2}$ angle wrt. surface normal. If $n \gg 1$: Total internal reflection

Total reflection occurs for $\alpha_2 = 90^\circ$, i.e. for

$$\sin\alpha_{1,\mathrm{c}} = n \quad\Longleftrightarrow\quad \cos\theta_\mathrm{c} = n \tag{3.11}$$

with the critical angle $\theta_\mathrm{c} = \pi/2 - \alpha_{1,\mathrm{c}}$.
Clearly, total reflection is only possible for $n < 1$

Light in glass at glass/air interface: $n = 1/1.6 \Longrightarrow \theta_\mathrm{c} \sim 50^\circ \Longrightarrow$ principle behind optical fibers.

Imaging, III

In general, the index of refraction is given by the Maxwell relation,

$$n = \sqrt{\epsilon\mu} \tag{3.12}$$

where ϵ is the dielectricity constant and where $\mu \sim 1$ is the permeability of the material.

For free electrons (e.g., in a metal), Jackson (1975, eq. 7.59) shows that

$$\epsilon = 1 - \left(\frac{\omega_\mathrm{p}}{\omega}\right)^2 \quad \text{with} \quad \omega_\mathrm{p}^2 = \frac{4\pi n Z e^2}{m_\mathrm{e}} \tag{3.13}$$

where ω_p is called the plasma frequency and where n is the number density of atoms and Z is the nuclear charge.
(i.e., nZ is the number density of electrons)

With $\omega = 2\pi\nu = 2\pi c/\lambda$, Eq. (3.13) becomes

$$\epsilon = 1 - \frac{nZe^2}{\pi m_\mathrm{e} c^2}\lambda^2 = 1 - \frac{nZr_e}{\pi}\lambda^2 \tag{3.14}$$

$r_\mathrm{e} = e^2/m_\mathrm{e}c^2 \sim 2.8 \times 10^{-13}$ cm is the classical electron radius.

Imaging, IV

$$n = \sqrt{1 - \frac{nZr_\mathrm{e}}{\pi}\lambda^2} \sim 1 - \frac{nZr_\mathrm{e}}{2\pi}\lambda^2 = 1 - \frac{\rho}{(Z/A)m_\mathrm{u}}\frac{r_\mathrm{e}}{2\pi}\lambda^2 =: 1 - \delta \tag{3.15}$$

Z: atomic number, A: atomic weight ($Z/A \sim 0.5$), ρ: density, $m_\mathrm{u} = 1\,\mathrm{amu} = 1.66 \times 10^{-24}\,\mathrm{g}$

Critical angle for X-ray reflection:

$$\cos\theta_\mathrm{c} = 1 - \delta \tag{3.16}$$

Since $\delta \ll 1$, Taylor ($\cos x \sim 1 - x^2/2$):

$$\theta_\mathrm{c} = \sqrt{2\delta} = 5.6' \left(\frac{\rho}{1\,\mathrm{g\,cm^{-3}}}\right)^{1/2} \frac{\lambda}{1\,\mathrm{nm}} \tag{3.17}$$

So for $\lambda \sim 1$ nm: $\theta_\mathrm{c} \sim 1^\circ$.

Imaging, V

Typical parameters for selected elements

	Z	ρ g cm^{-3}	nZ e^{-} Å^{-3}
C	6	2.26	0.680
Si	14	2.33	0.699
Ag	47	10.50	2.755
W	74	19.30	4.678
Au	79	19.32	4.666

After Als-Nielsen & McMorrow (Tab. 3.1)

To increase θ_c: need material with high ρ

$\Longrightarrow$ gold (*XMM-Newton*) or iridium (*Chandra*).

For more information on mirrors etc., see, e.g., Aschenbach, 1985, Rep Prog Phys 48, 579, or Als-Nielsen & McMorrow, 2004, Elements of Modern X-ray Physics, Wiley

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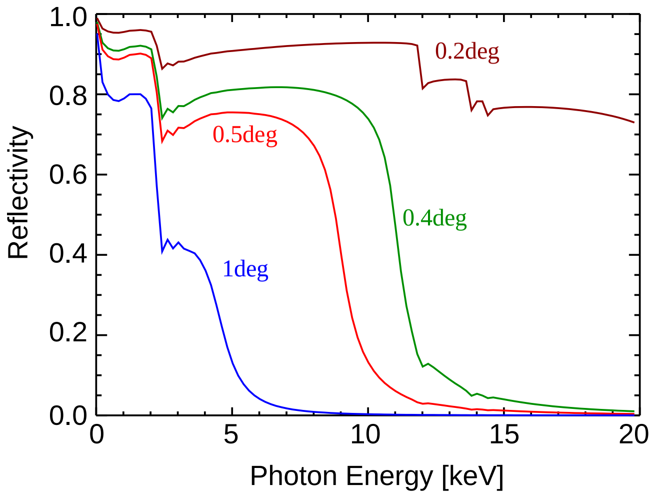
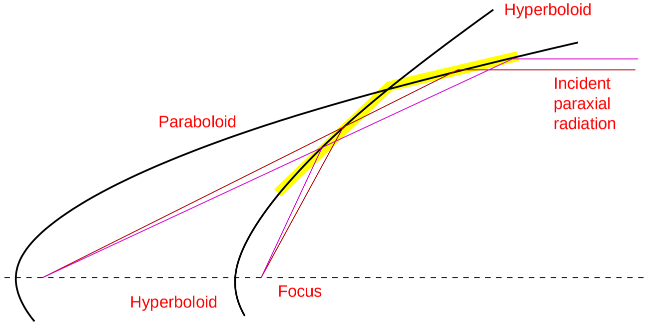
Imaging, VI



Reflectivity for Gold

X-rays: Total reflection only works in the soft X-rays and only under grazing incidence $\Longrightarrow$ grazing incidence optics.

Wolter Telescopes, I



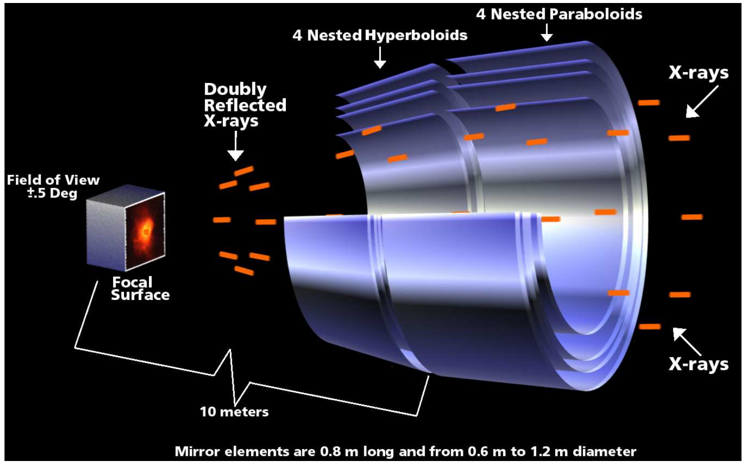
after ESA

To obtain manageable focal lengths ($\sim$ 10 m), do imaging with telescope using two reflections on a parabolic and a hyperboloidal mirror (“Wolter type I”)

(Wolter, 1952, for X-ray microscopes, Giacconi, 1961, for UV- and X-rays).

But: small collecting area ($A \sim \pi r^2 l/f$ where f: focal length)

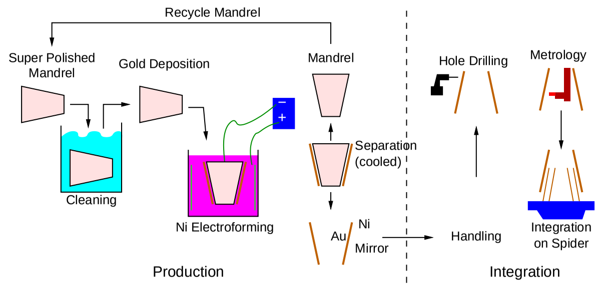
Wolter Telescopes, II



NASA/CXC (numbers are for *Chandra*)

Solution to small collecting area of individual mirror: nested mirrors

Mirror manufacture, I



(after ESA)

Recipe for making an X-ray mirror:

1. Produce mirror negative (“Mandrels”): Al coated with Kanigen nickel (Ni+10% phosphorus), super-polished [0.4 nm roughness]).
2. Deposit 250 nm Au onto Mandrel
3. Deposit 1 mm Ni onto mandrel (“electro-forming”, 10 μm/h)
4. Cool Mandrel with liquid N. Au sticks to Nickel
5. Verify mirror on optical bench.

Total production time of one mirror: 12 d, for XMM: 3×58 mirrors.

Mirror manufacture, II

Gold plastered mandrel for one of the XMM mirrors before electroforming the Ni shell onto the gold.

ESA picture 96.05.006-070

Mirror manufacture, III

. . . insertion of Mandrel into electroforming bath

ESA picture 96.12.002-016

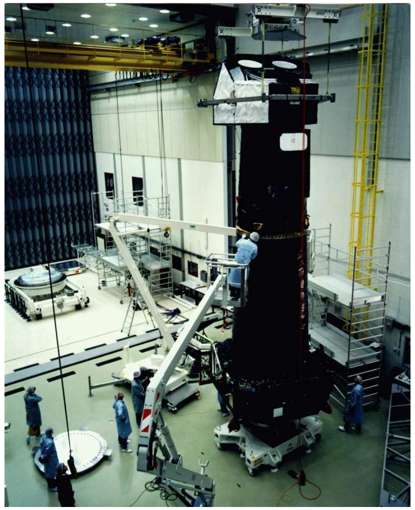
Mirror manufacture, IV

. . . and the mirror is done

ESA picture 96.12.002-093

XMM-Newton: launched 10 Dec. 1999 from Kourou on an Ariane 5

Practical Example: XMM-Newton, II



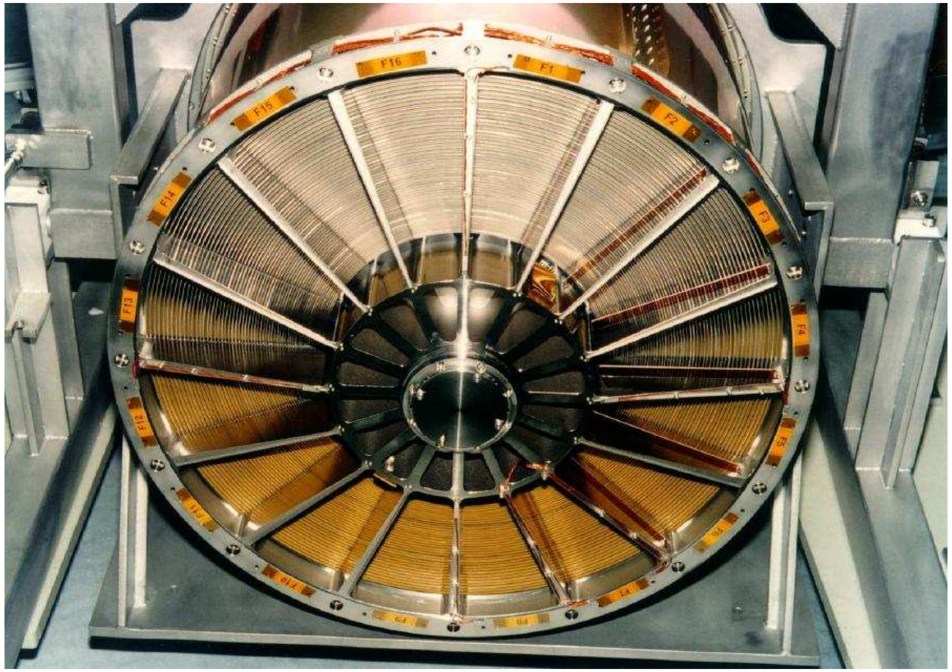
D. de Chambure, XMM-Newton Project, ESA/ESTEC

Properties of *XMM-Newton*'s Mirrors:

Focal length	7500 mm
outer mirror radius	350 mm
inner mirror radius	153 mm
axial mirror length	600 mm
outer mirror thickness	1.07 mm
inner mirror thickness	0.47 mm
mirror substrate	Ni
coating	Au
number of mirrors per module	58

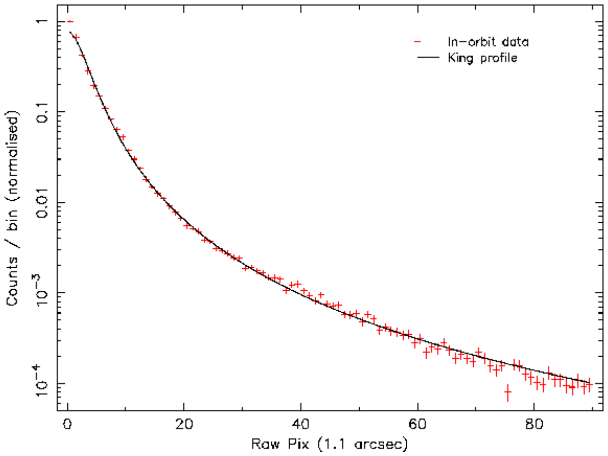
Gondoin et al., 1998 / XMM-SOC-CAL-TN-0002

D. Parker/ESA



Gondoin et al., 1998 / XMM-SOC-CAL-TN-0002

Practical Example: XMM-Newton, V



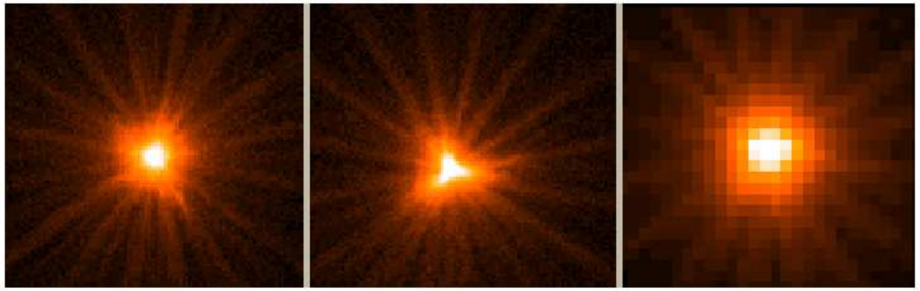
ESA/XMM-User's Handbook

The point spread function (PSF) of on-axis sources is rather good.

Characterization of mirror quality: Half Energy Width, i.e., circle within 50% of the detected energy are found; for XMM-Newton: 20″ at 1.5 keV, 40″ at 8 keV.

Ground calibration, e.g., at MPE's PANTER facility in Neuried.

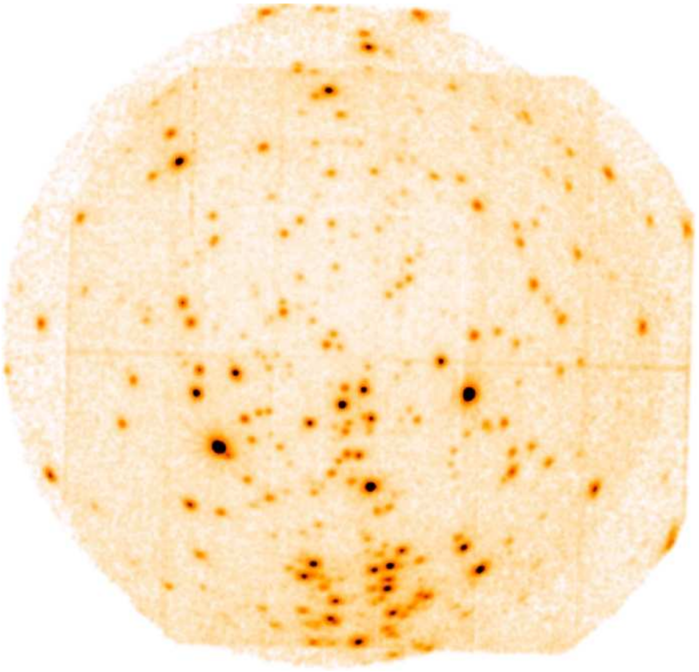
Practical Example: XMM-Newton, VI



110″ × 110″ images; ESA/XMM-User's Handbook

Image of the on-axis PSF for the three telescopes and detectors on XMM-Newton. Intensity profiles are scaled with a square-root to bring out details.

Note: EPIC-pn camera has larger pixels; see later.



The off-axis PSF is strongly asymmetric.